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LOCALLY SCC (HCC) SPACES AND SOME ONE-POINT EXTENSIONS

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Abstract. *In this paper we further investigate the results given in [8,9 and 10]. In section 2 we consider locally HCC(SCC,CC) spaces. A topological space X is called locally HCC(SCC,CC) space if for every $x \in X$ there exists a neighbourhood U of the point x such that the closure of U in X is a HCC(SCC,CC) subspace of X. In section 3 we consider a one point-extension of a space X related to the property $\mathcal{P}$, where $\mathcal{P}$ is one of the properties: compactness, countablle compactness, strong countable compactness, hypercountable compactnes or Lindelöfness. If (X,τ) is a Hausdorff locally $\mathcal{P}$ space, then there exists a one-point extension $(\omega X, \omega)$ which is a Hausdorff space and has the property $\mathcal{P}$. In section 4 we consider some relations between HCC and SCC properties*

1. Introduction

The closure of a subset A of a space X is denoted by $cl_X(A)$. In this paper we assume that all spaces are Hausdorff ($T_2 - spaces$). For notions and definitions not given here see [4,6,9].

Definition 1.1. *A space X is HCC (hypercountably compact) if every σ-compact set in X has compact closure in X* (see [9]).

Definition 1.2. *A space X is SCC (strongly countably compact) if every countable subset in X has compact closure in X* (see [5]).

It is easy to see that every HCC space is a SCC space. The converse is not necessarily true (see [9] example 2.1).

Proposition 1.3. *Let X be a first countable SCC space. Then X is a HCC space* ([9]).

Definition 1.4. *A topological space X is called CC (countably compact) if every countable open cover of X has a finite subcover.*

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Proposition 1.5. *Each CC (SCC, HCC) subset of a first countable space X is closed in X* ([9]).

That the condition of first countability cannot be omitted from proposition 1.5 can be seen from the following example.

Example 1.6. Let M be the Cartesian product of the interval $I = [0, 1]$ with the usual topology and $[0, \omega_1]$, where $[0, \omega_1]$ is the space of ordinals less then or equal to the first uncountable ordinal with the order topology. Let $X = M - \{(\alpha, \omega_1) : \frac{1}{3} < \alpha < \frac{2}{3}\}$ be the subspace of M. The space X is not countably compact since the subset $A = \{(\frac{2}{3} + \frac{1}{n}, \omega_1) : \ n \in N\}$ of X has no limit point in X and X is not first countable. According to the Proposition 2.8 in [9] the subspace $Y = [0, 1] \times [0, \omega_1)$ of X is a HCC space and $cl_X(Y) = X$.

Definition 1.7. *Let $\mathcal{P}$ be a topological property. A space X is called a locally $\mathcal{P}$ space if for every $x \in X$ there exists a neighbourhood U of the point x such that subspace $cl_X(U)$ of X has the property $\mathcal{P}$.*

2. Locally SCC (HCC) spaces

Definition 2.1. *A topological space X is called locally HCC (SCC, CC) if for every $x \in X$ there exists a neighbourhood U of the point x such that $cl_X(U)$ is a HCC (SCC, CC) subspace of X.*

Since the property HCC (SCC, CC, compactness) is hereditary with respect to closed subsets and finite unions (see [9]), it is easy to see that every HCC (SCC, CC) space is locally HCC (SCC, CC) and every locally compact space is a locally HCC (SCC, CC) space.

Proposition 2.2. *Every open subspace of a regular HCC (SCC, CC) space is locally HCC (SCC, CC).*

Proof. Let X be a regular HCC (SCC, CC) space and Y be any open subspace of X. Since X is regular for, every $x \in Y$ there exists a neighbourhood U of the point x in the space X such that $cl_X(U) \subset Y$ and $cl_X(U)$ is a HCC (SCC, CC) space. Hence Y is a locally HCC (SCC, CC) space. □

Proposition 2.3.. *Every closed subspace of a locally HCC (SCC, CC) space is locally HCC (SCC, CC).*

Proof. Let Y be a closed subspace of a locally HCC (SCC, CC) space X. For every $x \in Y$ there exists a neighbourhood U of the point x in the space X such that $cl_X(U)$ is a HCC (SCC, CC) space. The intersection $Y \cap U$ is a neighbourhood of the point x in the subspace Y, and the closure $cl_Y(Y \cap U) = cl_X(Y \cap U) \cap Y$ of this neighbourhood in Y, being a closed subset of the HCC (SCC, CC) space $cl_X(U)$, is HCC (SCC, CC). □

Lemma 2.4. *For every compact subspace A of a regular locally HCC (SCC, CC) space X and every open set $V \subset X$ that contains A, there exists an open set $U \subset X$ such that $A \subset U \subset cl_X(U) \subset V$ and $cl_X(U)$ is HCC (SCC, CC).*

PROOF. For every $x \in A$ we can take a neighbourhood $V(x)$ of the point x such that $cl_X(V(x)) \subset V$ and a neighbourhood $W(x)$ of x such that $cl_X(W(x))$ is a HCC (SCC, CC) subspace of X. The set $cl_X(U(x))$, where $U(x) = V(x) \cap W(x)$, is HCC (SCC, CC). Since A is compact there exists a finite set $\{x_1, x_2, ..., x_n\} \subset A$ such that $A \subset U = U(x_1) \cup U(x_2) \cup ... \cup U(x_n)$. The set $cl_X(U)$ is HCC (SCC, CC) and we clearly have $cl_X(U) \subset cl_X(V(x_1)) \cup ... \cup cl_X(V(x_n)) \subset V$. This completes the proof. □

Remark Properties HCC, SCC, CC in Propositions 2.2, 2.3 and Lemma 2.4 can be replaced by any topological property which is hereditary with respect to closed sets and finite unions. For example, paracompactnes or Lindelöf property.

In a similar way we can prove the following two statements:

Proposition 2.5. *Every open subspace of a regular locally HCC (SCC, CC) space is locally HCC (SCC, CC).*

Theorem 2.6. *If X is a regular locally HCC (SCC, CC) space, then every subspace of X that can be represented in the form $U \cap V$, where U is closed in X and V is open in X, is also locally HCC (SCC, CC).*

Proposition 2.7. *The sum $\bigoplus\{X_\alpha,\ \alpha \in A\}$, is regular and locally HCC (SCC, CC) if and only if all spaces X_α , $\alpha \in A$, are regular and locally HCC (SCC, CC).*

PROOF. If the sum $\bigoplus\{X_\alpha,\ \alpha \in A\}$, is regular and locally HCC (SCC, CC), then all X_α, $\alpha \in A$, are locally HCC (SCC, CC) by Propositions 2.2 awnd 2.3.

Conversely, if all X_α, $\alpha \in A$, are regular and locally HCC (SCC, CC), then for every $x \in X = \bigoplus\{X_\alpha,\ \alpha \in A\}$, there exist an $\alpha_o \in A$ such that $x \in X_{\alpha_0}$ and a neighbourhood U in X_{α_0} whose closure is HCC (SCC, CC). Clearly, the subset U is a neighbourhood of x in X and the closure of U in X, being identical to the closure of U in X_{α_0}, is HCC (SCC, CC).

Remark. The sum $\bigoplus\{X_\alpha,\ \alpha \in A\}$, is locally HCC (SCC) if all spaces X_α, $\alpha \in A$, are Hausdorff and locally HCC (SCC).

Lemma 2.8. *Let $f : X \to Y$ be a continuous and open mapping of a locally SCC space X onto a first-countable space Y. Then the space Y is locally HCC.*

PROOF. Let y be a point of Y; take an arbitrary point $x \in f^{-1}(y)$ and a neighbourhood U of x such that $cl_X(U)$ is a SCC subspace of X. Since the mapping f from X onto Y is continuous and open, the image $f(U)$ is a neighbourhood of y in Y and $f(cl_X(U))$ is a SCC subspace of Y. According to Propositions 1.3 and 1.5, $f(cl_X(U))$ is a closed HCC subspace of Y. Furthermore, the closure $cl_Y(f(U)) \subset f(cl_X(U))$. Hence Y is a locally HCC space. This completes the proof. □

That the condition of first countability cannot be omitted from Lemma 2.8 can be seen from the following example.

Example 2.9. Let X be the space of real numbers with the usual topology. Let I be the set of integers, and $\mathcal{D}$ the decomposition whose members are I and all sets $\{x\}$ for x in $X \setminus I$. Then the projection of X onto the quotient space is closed and continuous, but the quotient space is neither locally countably compact nor satisfies the first axiom of countability [6,p.165].

Proposition 2.10. *Let $\{X_i : i \in \{1,2,...,n\}\}$ be a family of non-empty locally HCC (SCC) spaces. Then the product space $X = \sqcap\{X_i : i \in \{1,2,...,n\}\}$ is a locally HCC (SCC) space.*

PROOF. Let x be a point in $X = \sqcap\{X_i : i \in \{1,2,...,n\}\}$. Take $x = (x_1, x_2, ..., x_n) \in X_1 \times X_2 \times ... \times X_n$. Since X_i is locally HCC (SCC), for $i \in \{1,2,...,n\}$ there exists a neighbourhood V_i of the point x_i in X_i such that $cl_{X_i}(V_i)$ is a HCC (SCC) subspace of X_i. The set $V = V_1 \times V_2 \times ... \times V_n$ is a neighbourhood of x in $X_1 \times X_2 \times ... \times X_n$. By Proposition 2.8 in [9] the set $U = \sqcap\{cl_{X_i}(V_i) : i \in \{1,2,...,n\}\}$ is a closed HCC (SCC) subspace of X. Clearly X is a locally HCC (SCC) space. □

Proposition 2.11. *Let $\{X_a : a \in A\}$ be a family of non-empty first-countable spaces and let $X = \sqcap\{X_a : a \in A\}$ be the product of spaces X_a, $a \in A$. If X is a locally HCC (SCC) space, then every X_a, $a \in A$, is a locally HCC space.*

PROOF. Since the projections $p_a : X \longrightarrow X_a$ from X onto X_a are continuous and open mappings, this is a direct consequence of Proposition 2.8. □

It can be shown that every continuous image of a SCC (HCC) space is a SCC space. Example 2.4 in [9] shows that the continuous image of a HCC space need not be a HCC space.

Proposition 2.12. *Let X be the product of spaces X_a, $a \in A$. If X is a locally SCC (HCC) space, then there exists a finite set $A_0 \subset A$ such that every X_a, $a \in A \setminus A_0$ is a SCC space.*

PROOF. Suppose that $X = \sqcap\{X_a : a \in A\}$ is a non-empty locally SCC (HCC) space. The point $\{x_a\} \in X$ has a neighbourhood U such that $cl_X(U)$ is a SCC (HCC) subspace of X. Clearly, there exists a member $V = \sqcap\{V_a : a \in A\}$ of the canonical base for $\sqcap\{X_a : a \in A\}$ such that $\{x_a\} \in V \subset U$ and $V_a = X_a$ for $a \in A \setminus A_0$, where $A_0 \subset A$ and $card(A_0) \prec \aleph_0$. For each $a \in A \setminus A_0$, we have that $X_a = p_a(V)$, where p_a is the projection from $cl_X(V)$ onto X_a. Therefore, for every $a \in A \setminus A_0$, X_a is a SCC space. □

Proposition 2.13. *Let $f : X \longrightarrow Y$ be a perfect mapping of a space X onto a SCC(HCC) space Y. Then X is a SCC (HCC) space.*

PROOF. *Case* SCC: Let $f : X \longrightarrow Y$ be a perfect mapping onto a Hausdorff SCC space Y and let $A = \{a_n \in X : n \in N\}$ be any countable subset of X. Then $card(f(A)) \preceq \aleph_0$ and $cl_Y(f(A))$ is a compact (closed) subspace of Y. The set $K = f^{-1}(cl_Y(f(A)))$ is closed and compact in X. Furthermore, $A \subset K$. Thus the set $cl_X(A)$ is compact in X. Hence X is a SCC space.

Case HCC is proved in a similar way.

The following is an immediate consequence of Proposition 2.13. □

Proposition 2.14. *If $f : X \longrightarrow Y$ of X is a perfect mapping of a space X onto a locally SCC (HCC) space Y, then X is a locally SCC (HCC) space.*

3. Some one-point extensions of topological spaces

Definition 3.1. *A pair (Y,p), is called an extension of a space X if the mapping $p: X \longrightarrow Y$ is a homeomorphic embedding of X in Y such that $cl_Y(p(X)) = Y$.*

A pair (Y,p), is called a one-point extension of a space X if the mapping $p:$ $X \longrightarrow Y$ is a homeomorphic embedding of X in Y such that $cl_Y(p(X)) = Y$ and the remainder $Y \setminus p(X)$ is one-point.

Definition 3.2. *Let $\mathcal{P}$ be a topological property. A pair (Y,p), is called the one-point extension of a space X related to the property $\mathcal{P}$ if the space Y has the property $\mathcal{P}$ and $p: X \longrightarrow Y$ is a homeomorphic embeding of X in Y such that $cl_Y(p(X)) = Y$ and the remainder $Y \setminus p(X)$ is one-point.*

Proposition 3.3. *Let $\mathcal{P}$ be a topological property which is hereditary with respect to closed subsets and finite unions. If (X,τ) is a Hausdorff locally $\mathcal{P}$ space which does not satisfy the property $\mathcal{P}$, then there exists a one-point extension $(\omega X, \omega)$ of the space X and $(\omega X, \omega)$ is a Hausdorff space.*

PROOF. Let $\mathcal{P}(X) = \{A : A \subset X, cl_X(A) = A$ and the subspace A has the property $\mathcal{P}\}$ and $\omega X = X \cup \{p\}$ where $p \notin X$. Since X is a locally $\mathcal{P}$ space and the property $\mathcal{P}$ is hereditary with respect to closed sets and finite unions, the family $\{\{p\} \cup (X \setminus A) : A \in \mathcal{P}(X)\}$ is an open base of the point $p \in \omega X$. Let ω be the topology on ωX defined as follows:

$$\omega = \tau \cup \{\{p\} \cup (X \setminus A) : A \in \mathcal{P}(X)\}$$

.

It is obvious from the way the topology ω is constructed that the space $(\omega X, \omega)$ has the following properties:

(i) The subspace X is open in ωX and $cl_{\omega X}(X) = \omega X$, hence ωX is a one-point extension of the space X.

(ii) ωX is Hausdorff. □

Proposition 3.4. *Let $\mathcal{P}$ be one of the following properties: compactness, countable compactness or Lindelöfness. If X is a locally $\mathcal{P}$ space, then the one-point extension $(\omega X, \omega)$ has the property $\mathcal{P}$.*

PROOF. Let $(\omega X, \omega)$ be the topological space defined in Proposition 3.3. The space X is a locally $\mathcal{P}$ space and the property $\mathcal{P}$ is hereditary with respect to closed sets and finite unions. By Proposition 3.3, ωX is a Hausdorff space. Let $\mathcal{U}$ be an open cover of the space ωX. Then there exists a member $U \in \mathcal{U}$ such that $p \in U$. The set $(X \setminus U)$ is a closed subspace of X and satisfies the property $\mathcal{P}$. The family $\mathcal{U} \setminus \{U\}$is an open cover of the subspace $(X \setminus U)$. Since $(X \setminus U)$ has the property $\mathcal{P}$, there exists a subcover $\mathcal{V} \subset \mathcal{U} \setminus \{U\}$. If $\mathcal{P}$ is the Lindelöf property then card$(\mathcal{V}) = \aleph_0$. If $\mathcal{P}$ countable compactness, then card$(\mathcal{U}) = \aleph_0$ and card$(\mathcal{V}) \prec \aleph_0$.If $\mathcal{P}$ is compactness, then card$(\mathcal{V}) \prec \aleph_0$.The family $\mathcal{V} \cup \{U\}$ is a subcover of ωX and card$(\mathcal{V} \cup \{U\}) =$ card$(\mathcal{V})$. This completes the proof. □

In the following example we give a locally SCC space which is not SCC.

Example3.5. Let $[0,\omega_1]([0,\omega_0])$ be the space of ordinals less than or equal to the first uncountable ordinal (first countable ordinal) with the order topology. The product space $[0,\omega_1] \times [0,\omega_0]$ is compact and normal (see Example 2.1 in [9]). Let $X_2 = [0,\omega_1] \times [0,\omega_0] \setminus \{(\omega_1, n) : n \in N\}$ be the subspace of X_1. Then X_2 is noncompact and normal in the subspace topology. Let $X_3 = X_2 \cup \{p\}(p \notin X_2)$ be the one point compactification of X_2. The space X_3 is compact and a T_1-space. The point (ω_1, ω_0) is an accumulation (limit) point of X_3. Let $X = X_3 \setminus \{(\omega_1, \omega_0)\}$ be the subspace of X_3. The space X is Hausdorff and SCC but not HCC. Furthermore, it is not locally compact. Every space $X_n = X \times \{n\}$, $n \in N$, is homeomorphic to X. Let $Y = \bigoplus\{X_n, n \in N\}$. By Proposition 2.7, the space Y is a locally SCC but it is not countably compact. Hence the space Y is locally SCC but not SCC.

Example 3.6.(A locally HCC space which is not HCC). Let $X = [0,\omega_1)$ be the space of ordinals less than the first uncountable ordinal with the order topology. The space X is HCC but it is not compact. Let $Y = exp(X)$ be the space of all non-empty closed subsets of X with finite topology. According to the results of M.S. Stanojević (see [11]) we have that Y is HCC but it is not locally compact. Every space $Y_n = Y \times \{n\}$, $n \in N$, is homeomorphic to Y. Let $W = \bigoplus\{Y_n : n \in N\}$. By Proposition 2.7, the space W is locally HCC but it is not countably compact. Hence the space W is locally HCC but not HCC.

Definition 3.7. *A topological space X is called an L-space if X is a T_2-space and the closure in X of each countable subset of X has the Lindelöf property* (see [8], Def. 2.1).

Definition 3.8. *A topological space X is called an L'-space if X is a T_2-space and the closure in X of each Lindelöf subspace of X has the Lindelöf property* (see [10], Def.5.2).

Recall the well-known characterization of compactness.

Proposition 3.9. *A topol ogical space is compact if and only if it is a countably compact space with the Lindelöf property* ([4]).

Proposition 3.10. *For every L-space X the following conditions are equivalent:*

1. *The space X is countably compact.*
2. *The space X is strongly countably compact (SCC)* (see [8]).

Proposition 3.11. *For every L'-space X the following conditions are equivalent:*

1. *The space X is countably compact.*
2. *The space X is strongly countably compact (SCC)*
3. *The space X is hypercountably compact (HCC)* (see [10]).

It is clear that every closed subspace of $L(L')$ space is an $L(L')$ subspace.

Proposition 3.12. *Let (X, τ) be an L-space which is locally SCC but not SCC. Then the one-point extension $(\omega X, \omega)$ of the space (X, τ) is a SCC space.*

Proof. Let A be any countable subset of $(\omega X, \omega)$ defined in Proposition 3.3. Then either $cl_{\omega X}(A) = cl_X(A) \cup \{p\}$ or $p \notin cl_{\omega X}(A)$.

Case I: If $p \notin cl_{\omega X}(A)$ then there exists an open neighbourhood $V = U \cup \{p\}$ of the point P such that $U \cap A = \emptyset$. Therefore the set $A \subset X \setminus U$. Since $X \setminus U$ is a closed SCC subspace of ωX, the closure $cl_{\omega X}(A) \subseteq X \setminus U$ and $cl_{\omega X}(A)$ is a compact subspace of ωX.

Case II: Let $cl_{\omega X}(A) = cl_X(A) \cup \{p\}$. We will now show that $cl_{\omega X}(A)$ is a compact subspace of ωX. Since X is an L-space, the closure $cl_{\omega X}(A) = cl_X(A) \cup \{p\}$ is a subspace of ωX with the Lindelöf property. Furthermore, $cl_{\omega X}(A)$ is a countably compact subspace of ωX(Proposition 3.4). By Proposition 3.9 the subspace $cl_{\omega X}(A)$ is compact. This completes the proof. □

Proposition 3.13. *Let (X,τ) be an L'-space which is locally HCC but not HCC. Then the one-point extension $(\omega X, \omega)$ of the space (X,τ) is a HCC space.*

PROOF. Let $(\omega X, \omega)$ be the topological space defined in Proposition 3.3. By Proposition 3.3, the space $(\omega X, \omega)$ is countably compact. Furthermore the space $(\omega X, \omega)$ is an L'-space, since X is an L'-space. By Proposition 3.11 the space $(\omega X, \omega)$ is HCC. This completes the proof. □

Question 3.14. Does there exist a Hausdorff locally SCC space (X,τ) which is not SCC such that the one-point extension $(\omega X, \omega)$ is not SCC?

Question 3.15. Does there exist a Hausdorff locally HCC space (X,τ) which is not HCC such that the one-point extension $(\omega X, \omega)$ is not HCC?

4. Some questions on relations between HCC and SCC properties

Let X be a topological T_2-space. Then:

(1) $exp(X)$ denotes the space of all non-empty closed subsets of X with the finite topology. The finite (Vietoris) topology on $exp(X)$ is generated by the collections of the form $\langle U_1, U_2, ..., U_n \rangle = \{F \in exp(X) : F \subset \cup\{U_i : i \in \{1,2,...,n\}\}$ and $F \cap U_i \neq \emptyset$, for $i \in \{1,2,...,n\}$ where $U_1, U_2, ..., U_n$ are open subsets of $X\}$.

(2) $\mathcal{K}(X)$ denotes the set of all nonempty compact subsets of X as a subspace of $exp(X)$.

(3) $\mathcal{F}_n(X) = \{F \subset X : F$ has at most n points$\} \subset \mathcal{K}(X)$.

Definition 4.1. *Let X be a topological space and let $exp(X)$ be the space of all none-mpty closed subsets of X with the finite topology. The space X is ECC if $exp(X)$ is a countably compact (CC) space.*

Proposition 4.2. *If X is SCC (strongly countably compact), then $exp(X)$ is pseudocompact. If in addition X is normal, then $exp(X)$ is SCC* ([5]).

Question 4.3. Is it true that every ECC space is SCC(HCC)?

Question 4.4. Is it true that every SCC(HCC) space is ECC?

Question 4.5. Is it true that every normal and SCC space is HCC?

It is known that there exists a Tychonoff SCC space X which is not HCC (see [9], Example 3.4.)

Question 4.6. Let X be a normal HCC space. Is it true that $exp(X)$ is a HCC space?

Proposition 4.7. *Every closed subspace of an ECC space is ECC.*

PROOF. Let Y be a closed subspace of an ECC space X. It is known that then the space $exp(Y)$ is a closed subspace of $exp(X)$. Since the property CC is hereditary with respect to closed sets, it is easy to see that the subspace $exp(Y)$ is a CC space. Hence Y is an ECC space. This completes the proof. □

Proposition 4.8. *If $f : X \longrightarrow Y$ is a continuous and closed mapping of a Tychonoff ECC space X onto a Tychonoff space Y, then the space Y is ECC.*

PROOF. The mapping $F : \exp(X) \longrightarrow \exp(Y)$ defined by $F(A) = f(A)$, for each $A \in exp(X)$ is a continuous surjection. Since $\exp(X)$ is a CC space and the CC property is an invariant of continuous mapping, the space $\exp(Y)$ is CC. Hence Y is ECC space. □

Proposition 4.9. *Let X be a Tychonoff ECC space. Then the product X^n is a CC space for every $n \in N$.*

PROOF. It is known that the mapping $p : X^n \longrightarrow \mathcal{F}_n(X) \subset \exp(X)$ defined by $p((x_1, x_2, \dots, x_n)) = \{x_1, x_2, \dots, x_n\}$ is a perfect mapping from X^n in $\mathcal{F}_n(X)$. Since the subspace $\mathcal{F}_n(X)$ is a closed subset of $\exp(X)$ then, by 4.7, $\mathcal{F}_n(X)$ ia a CC subspace of $\exp(X)$. Furthermore, the CC property is an inverse invariant of perfect mapping and X^n is a CC space for all $n \in N$. This completes the proof. □

Question 4.10. Let X be a Tychonoff ECC space. Is it true that then the product X^α is a CC space $(\alpha \succeq \aleph_0)$?

Question 4.11. Is it true that a hyperspace $\exp(X)$ is homeomorphic to $\mathcal{K}(X)$ if and only if X is a compact space?

This question was formulated by M. S. Stanojević and D. Milovančević in [11].

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