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The Exponential Cubic B-Spline Collocation Method for the Kuramoto-Sivashinsky Equation

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Abstract. In this study the Kuramoto–Sivashinsky (KS) equation has been solved using the collocation method, based on the exponential cubic B-spline approximation together with the Crank Nicolson. KS equation is fully integrated into a linearized algebraic equations. The results of the proposed method are compared with both numerical and analytical results by studying two text problems. It is found that the simulating results are in good agreement with both exact and existing numerical solutions.

1. Introduction

The Kuramoto–Sivashinsky (KS) equation, which is frequently encountered in the study of continuous media, exhibits complex chaotic behavior and have the following form

$$u_t + uu_x + \alpha u_{xx} + \vartheta u_{xxxx} = 0,\ x \in [x_0, x_N],\ t \in (0, T] \tag{1}$$

with the boundary conditions

$$u(x_0, t) = g_0, u(x_N, t) = g_1 \tag{2}$$
$$u_x(x_0, t) = 0,\ u_x(x_N, t) = 0 \tag{3}$$
$$u_{xx}(x_0, t) = 0,\ u_{xx}(x_N, t) = 0 \tag{4}$$

and initial condition

$$u(x, 0) = u_0,$$

where ε and ϑ are arbitrary constants related to the growth of linear stability and surface tension, respectively. The equation includes terms of linear growth u_{xx}, high order dissipation u_{xxxx} and nonlinear advection uu_x. When ϑ is zero, the surface tension term is removed and the equation gets reduced to Burgers equation. It is used as model equation in a number of applications including concentration waves and plasma physics, flame propagation and reaction-diffusion combustion dynamics, free surface film-flows and two face flows in cylindrical or plain geometries. Due to its wide applications, it has attracted considerable attention to be found analytical and numerical solutions. Thus, the solutions of the KS equation have been obtained by

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using many methods including finite difference methods[9, 10], tanh-function method [8], discontinuous Galerkin method [25], Chebyshev spectral collocation method [1], pseudo-spectral method [9], radial basis function (RBFs) based mesh-free method [16], He's variational iteration method [14], meshless method [23].

An alternative exponential B-spline basis functions to the polynomial B-spline basis functions have been introduced by Mc Cartin and its properties are given in his studies[3–5]. The exponential B-spline functions are extensively used for the computer aided design, the curve and surface approximation. The exponential B-spline basis functions include free parameters which determines the shapes of the B-splines. This may give good approximation for data having sharp changes. Recently the exponential B-spline functions are used to set up numerical methods for the differential equations. Solutions of singularly perturbed problem are given using some variant of the exponential B-spline collocation methods in the studies [7, 15, 22]. The numerical solutions of the equal width, generalized Burgers–Fisher, Korteweg-de Vries (KdV), Convection-Diffusion and Generalized Long Wave Equations are obtained by using the exponential B-spline collocation method [12, 13, 18, 20, 21]. Spline functions are employed for establishing the algorithms to compute solutions of KS equation. The novel B-spline based Galerkin finite element approach is presented in the study[6]. The orthogonal cubic spline collocation method [2] is given to solve the KS equation. The quintic B-spline collocation scheme [19] is presented by R.C. Mittal, Geeta Arora[19]. The Septic B-spline collocation algorithm is set up to find numerical solutions of the KS-equation in the paper[17]. Since the KS equation include the fourth order derivatives, the KS equation should turn into the system of partial differential equation by using space splitted technique to be able to apply the exponential B-spline based collocation method. The exponential B-splines have got the second order continuity over the defined region. The first and second order continuity requirements of the exponential B-spline based approximation function are guarantied over the problem domain for the system of partial differential equation including the second order derivatives.

In this research, the (KS) equation have been solved numerically using collocation method based on Crank-Nicolson for the time integration and exponential cubic B-spline functions for the space integration. The performance of the method is shown by studying two text problems. Results and graphical solutions are given in the section of the numerical methods to make comparison with some earlier studies.

2. Exponential Cubic B-Spline Collocation Method

In this section we will carry on the space and temporal discretization of the time dependent one dimensional KS equation. We will obtain fully-discrete KS-equation in the from of the recursive algebraic equation. Knots are equally distributed over the problem domain $[a, b]$ as

$$\pi : a = x_0 < x_1 < \ldots < x_N = b$$

with mesh spacing $h = (x_N - x_0)/N$. The exponential B-splines, $B_i(x)$,with knots at the points of π together with fictitious knots $x_{-3}, x_{-2}, x_{-1}, x_{N+1}, x_{N+2}, x_{N+3}$ outside the problem domain$[a, b]$ can be defined as

$$B_i(x) = \begin{cases} b_2\left((x_{i-2} - x) - \dfrac{1}{p}\left(\sinh(p\,(x_{i-2} - x))\right)\right) & [x_{i-2}, x_{i-1}]\,, \\ a_1 + b_1\,(x_i - x) + c_1 \exp\left(p\,(x_i - x)\right) + d_1 \exp\left(-p\,(x_i - x)\right) & [x_{i-1}, x_i]\,, \\ a_1 + b_1(x - x_i) + c_1 \exp\left(p\,(x - x_i)\right) + d_1 \exp\left(-p\,(x - x_i)\right) & [x_i, x_{i+1}]\,, \\ b_2\left((x - x_{i+2}) - \dfrac{1}{p}(\sinh\left(p\,(x - x_{i+2})\right))\right) & [x_{i+1}, x_{i+2}]\,, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

where

$$a_1 = \frac{phc}{phc - s},\ b_1 = \frac{p}{2}[\frac{c(c-1)+s^2}{(phc-s)(1-c)}],\ b_2 = \frac{p}{2(phc-s)},$$

$$c_1 = \frac{1}{4}[\frac{\exp(-ph)(1-c)+s(\exp(-ph)-1)}{(phc-s)(1-c)}]$$

$$d_1 = \frac{1}{4}[\frac{\exp(ph)(c-1)+s(\exp(ph)-1)}{(phc-s)(1-c)}]$$

and $c = \cosh(ph)$, $s = \sinh(ph)$, p is a free parameter. When $p = 1$, graph of the exponential cubic B-splines over the interval [0.1] is depicted in Fig. 1.

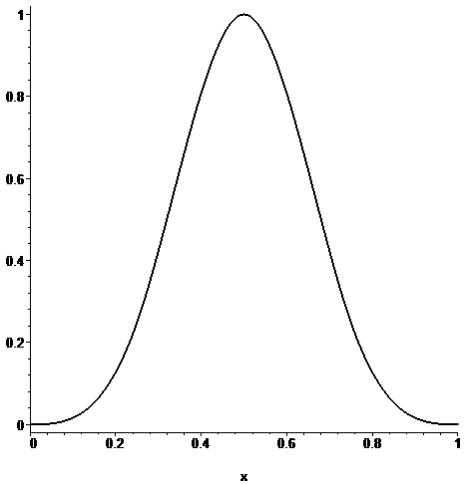


Figure 1: Solutions of KS equation

$\{B_{-1}(x), B_0(x), \cdots, B_{N+1}(x)\}$ forms a basis for the functions defined over the interval $[a, b]$. Each basis function $B_i(x)$ is twice continuously differentiable. The values of $B_i(x), B_i'(x)$ and $B_i''(x)$ at the knots x_i can be computed from Eq.(5) and are documented in Table 1.

Table 1: Values of $B_i(x)$ and its first and second derivatives at the knot points

x	x_{i-2}	x_{i-1}	x_i	x_{i+1}	x_{i+2}
B_i	0	$\frac{s-ph}{2(phc-s)}$	1	$\frac{s-ph}{2(phc-s)}$	0
B_i'	0	$\frac{p(1-c)}{2(phc-s)}$	0	$\frac{p(c-1)}{2(phc-s)}$	0
B_i''	0	$\frac{p^2s}{2(phc-s)}$	$-\frac{p^2s}{phc-s}$	$\frac{p^2s}{2(phc-s)}$	0

Now suppose that an approximate solution U_N to the unknown U is given by

$$U_N(x,t) = \sum_{i=-1}^{N+1} \delta_i B_i(x) \tag{6}$$

where δ_i are time dependent parameters to be determined from the boundary and initial conditions, collocation form of the KS equation. Evaluation of Eq. (6), its first and second derivatives at knots x_i using the Table 1 yields the nodal values U_i in terms of parameters

$$\begin{aligned} U_i &= U(x_i,t) = \frac{s-ph}{2(phc-s)}\delta_{i-1} + \delta_i + \frac{s-ph}{2(phc-s)}\delta_{i+1}, \\ U_i' &= U'(x_i,t) = \frac{p(1-c)}{2(phc-s)}\delta_{i-1} + \frac{p(c-1)}{2(phc-s)}\delta_{i+1} \\ U_i'' &= U''(x_i,t) = \frac{p^2 s}{2(phc-s)}\delta_{i-1} - \frac{p^2 s}{phc-s}\delta_i + \frac{p^2 s}{2(phc-s)}\delta_{i+1}. \end{aligned} \tag{7}$$

To be able to apply the exponential B-spline based collocation method, KS equation is space-splitted as

$$\begin{aligned} U_t + UU_x + \alpha V + \vartheta V_{xx} &= 0 \\ V - U_{xx} &= 0. \end{aligned} \tag{8}$$

This system includes the second-order derivatives so that smooth approximation can constructed with the combination of the exponential B-splines. The time integration of the space-splitted KS equation is performed by the Crank-Nicolson method as

$$\begin{aligned} \frac{U^{n+1}-U^n}{\Delta t} + \frac{(UU_x)^{n+1}+(UU_x)^n}{2} + \alpha\frac{V^{n+1}+V^n}{2} + \vartheta\frac{V_{xx}^{n+1}+V_{xx}^n}{2} &= 0 \\ \frac{V^{n+1}+V^n}{2} - \frac{{U_{xx}}^{n+1}+{U_{xx}}^n}{2} &= 0 \end{aligned} \tag{9}$$

where $U^{n+1} = U(x,(n+1)\Delta t)$ represent the solution at the $(n+1)$th time level. Here $t^{n+1} = t^n + \Delta t$, Δt is the time step, superscripts denote n th time level, $t^n = n\Delta t$.

One linearize terms $(UU_x)^{n+1}$and $(UUx)^n$ in (9) as [24]

$$\begin{aligned} (UUx)^{n+1} &= U^{n+1}U_x^n + U^n U_x^{n+1} - U^n U_x^n \\ (UUx)^n &= U^n U_x^n \end{aligned} \tag{10}$$

to obtain the time-integrated linearized the KS Equation:

$$\begin{aligned} \frac{2}{\Delta t}U^{n+1} - \frac{2}{\Delta t}U^n + U^{n+1}U_x^n + U^n U_x^{n+1} + \alpha\left(V^{n+1}+V^n\right) + \vartheta(V_{xx}^{n+1}+V_{xx}^n) &= 0 \\ \frac{V^{n+1}+V^n}{2} - \frac{{U_{xx}}^{n+1}+{U_{xx}}^n}{2} &= 0 \end{aligned} \tag{11}$$

To proceed with space integration of the (11), an approximation of U^n and V^n in terms of the unknown element parameters and exponential B-splines separately can be written as

$$U_N(x,t) = \sum_{i=-1}^{N+1}\delta_i B_i(x),\ V_N(x,t) = \sum_{i=-1}^{N+1}\phi_i B_i(x). \tag{12}$$

Substitute Eqs (12) into (11) and collocate the resulting the equation at the knots $x_i, i = 0,...,N$ yields a linear algebraic system of equations:

$$\begin{aligned} &\left[\left(\tfrac{2}{\Delta t}+K_2\right)\alpha_1 + K_1\beta_1\right]\delta_{m-1}^{n+1} + (\alpha\alpha_1+\vartheta\gamma_1)\phi_{m-1}^{n+1} + \left[\left(\tfrac{2}{\Delta t}+K_2\right)\alpha_2\right]\delta_m^{n+1} + (\alpha\alpha_2+\vartheta\gamma_2)\phi_{m0}^{n+1} \\ &+\left[\left(\tfrac{2}{\Delta t}+K_2\right)\alpha_1 - K_1\beta_1\right]\delta_{m+1}^{n+1} + (\alpha\alpha_1+\vartheta\gamma_1)\phi_{m+1}^{n+1} \\ &= \tfrac{2}{\Delta t}\alpha_1\delta_{m-1}^n - (\alpha\alpha_1+\vartheta\gamma_1)\phi_{m-1}^n + \tfrac{2}{\Delta t}\alpha_2\delta_m^n - (\alpha\alpha_2+\vartheta\gamma_2)\phi_m^n + \tfrac{2}{\Delta t}\alpha_1\delta_{m+1}^n - (\alpha\alpha_1+\upsilon\vartheta_1)\phi_{m+1}^n \\ &-\gamma_1\delta_{m-1}^{n+1} + \alpha_1\phi_{m-1}^{n+1} - \gamma_2\delta_m^{n+1} + \alpha_2\phi_m^{n+1} - \gamma_1\delta_{m+1}^{n+1} + \alpha_1\phi_{m+1}^{n+1} \\ &= \gamma_1\delta_{m-1}^n - \alpha_1\phi_{m-1}^n + \gamma_2\delta_m^n - \alpha_2\phi_m^n + \gamma_1\delta_{m+1}^n - \alpha_1\phi_{m+1}^n, \quad m = 0...N,\ n = 0,1..., \end{aligned} \tag{13}$$

where

$$K_1 = \alpha_1\delta_{i-1} + \alpha_2\delta_i + \alpha_1\delta_{i+1}$$
$$K_2 = \beta_1\delta_{i-1} - \beta_1\delta_{i+1}$$

$$\alpha_1 = \frac{s-ph}{2(phc-s)},\ \alpha_2 = 1,\ \beta_1 = \frac{p(1-c)}{2(phc-s)},$$
$$\gamma_1 = \frac{p^2 s}{2(phc-s)},\ \gamma_2 = -\frac{p^2 s}{phc-s}$$

The system (13) can be converted the following matrices system;

$$\mathbf{A}\mathbf{x}^{n+1} = \mathbf{B}\mathbf{x}^n \tag{14}$$

where

$$\mathbf{A} = \begin{bmatrix} \nu_{m1} & \nu_{m2} & \nu_{m3} & \nu_{m4} & \nu_{m5} & \nu_{m2} & & & & \\ -\gamma_1 & \alpha_1 & -\gamma_2 & \alpha_2 & -\gamma_1 & \alpha_1 & & & & \\ & & \nu_{m1} & \nu_{m2} & \nu_{m3} & \nu_{m4} & \nu_{m5} & \nu_{m2} & & \\ & & -\gamma_1 & \alpha_1 & -\gamma_2 & \alpha_2 & -\gamma_1 & \alpha_1 & & \\ & & & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & & & \nu_{m1} & \nu_{m2} & \nu_{m3} & \nu_{m4} & \nu_{m5} & \nu_{m2} \\ & & & & -\gamma_1 & \alpha_1 & -\gamma_2 & \alpha_2 & -\gamma_1 & \alpha_1 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \nu_{m6} & \nu_{m7} & \nu_{m8} & \nu_{m9} & \nu_{m6} & \nu_{m7} & & & & \\ \gamma_1 & -\alpha_1 & \gamma_2 & -\alpha_2 & \gamma_1 & -\alpha_1 & & & & \\ & & \nu_{m6} & \nu_{m7} & \nu_{m8} & \nu_{m9} & \nu_{m6} & \nu_{m7} & & \\ & & \gamma_1 & -\alpha_1 & \gamma_2 & -\alpha_2 & \gamma_1 & -\alpha_1 & & \\ & & & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & & & \nu_{m6} & \nu_{m7} & \nu_{m8} & \nu_{m9} & \nu_{m6} & \nu_{m7} \\ & & & & \gamma_1 & -\alpha_1 & \gamma_2 & -\alpha_2 & \gamma_1 & -\alpha_1 \end{bmatrix}$$

and

$$\begin{array}{lll} \nu_{m1} = \left(\frac{2}{\Delta t} + K_2\right)\alpha_1 + K_1\beta_1 & \nu_{m4} = (\alpha\alpha_2 + \vartheta\gamma_2) & \nu_{m7} = -(\alpha\alpha_1 + \vartheta\gamma_1) \\ \nu_{m2} = (\alpha\alpha_1 + \vartheta\gamma_1) & \nu_{m5} = \left(\frac{2}{\Delta t} + K_2\right)\alpha_1 - K_1\beta_1 & \nu_{m8} = \frac{2}{\Delta t}\alpha_2 \\ \nu_{m3} = \left(\frac{2}{\Delta t} + K_2\right)\alpha_2 & \nu_{m6} = \frac{2}{\Delta t}\alpha_1 & \nu_{m9} = -(\alpha\alpha_2 + \vartheta\gamma_2) \end{array}$$

The system (14) consist of $2N+2$ linear equation in $2N+6$ unknown parameters

$$\mathbf{x}^{n+1} = (\delta_{-1}^{n+1}, \phi_{-1}^{n+1}, \delta_0^{n+1}, \phi_0^{n+1}, \ldots, \delta_{n+1}^{n+1}, \phi_{n+1}^{n+1},).$$

To obtain a unique solution, an additional four constraints are needed. These are obtained from the imposition of the Robin boundary conditions so that $U_{xx}(a,t) = 0$, $V(a,t) = 0$ and $U_{xx}(b,t) = 0$, $V(b,t) = 0$ gives the following equations:

$$\gamma_1\delta_{-1} + \gamma_2\delta_0 + \gamma_1\delta_1 = 0$$
$$\alpha_1\phi_{-1} + \alpha_2\phi_0 + \alpha_1\phi_1 = 0$$
$$\gamma_1\delta_{N-1} + \gamma_2\delta_N + \gamma_1\delta_{N+1} = 0$$
$$\alpha_1\phi_{N-1} + \alpha_2\phi_N + \alpha_1\phi_{N+1} = 0$$

Elimination of the parameters $\delta_{-1}, \phi_{-1}, \delta_{N+1}, \phi_{N+1}$, from the Eq.,(13) using the above equations gives a solvable system of $2N+2$ linear equations including $2N+2$ unknown parameters. After finding the

unknown parameters via the application of a variant of Thomas algorithm, approximate solutions at the knots can be obtained by placing successive three parameters in the Eq.(7).

Initial parameters $\delta_i^0,, \phi_i^0, i = -1, \ldots, N+1$ are needed to start the iteration procedure (14). Thus the following requirements help to determine initial parameters:

$$
\begin{aligned}
&(U_N)_{xx}(a,0) = 0 = \gamma_1\delta_{-1}^0 + \gamma_2\delta_0^0 + \gamma_1\delta_1^0,\\
&(U_N)_{xx}(x_i,0) = \gamma_1\delta_{i-1}^0 + \gamma_2\delta_i^0 + \gamma_1\delta_{i+1}^0 = U_{xx}(x_i,0), i = 1,...,N-1\\
&(U_N)_{xx}(b,0) = 0 = \gamma_1\delta_{N-1}^0 + \gamma_2\delta_N^0 + \gamma_1\delta_{N+1}^0,\\
&(V_N)(a,0) = 0 = \alpha_1\phi_{-1}^0 + \alpha_2\phi_0^0 + \alpha_1\phi_1^0\\
&(V_N)(x_i,0) = \alpha_1\phi_{i-1}^0 + \alpha_2\phi_i^0 + \alpha_1\phi_{i+1}^0 = V(x_i,0), i = 1,...,N-1\\
&(V_N)(a,0) = \alpha_1\phi_{N-1}^0 + \alpha_2\phi_N^0 + \alpha_1\phi_{N+1}^0
\end{aligned}
$$

3. Numerical Validation

To see versatility of the present method, three numerical examples are studied in this section. The accuracy of the schemes is measured in terms of the following global relative error

$$
\text{GRE} = \sqrt{\frac{\sum_{j=1}^{N}\left|(U_N)_j^n - (U)_j^n\right|}{\sum_{j=1}^{N}\left|(U)_j^n\right|}} \tag{15}
$$

is used where U_N denotes numerical solution and U denotes analytical solution.

Numerical solution of KS equation (1) is obtained for $\alpha = 1$ and $\vartheta = 1$ with the exact solution given by

$$
u(x,t) = b + \frac{15}{19}d\left[e\tanh\left(k\left(x - bt - x_0\right)\right) + f\tanh^3\left(k\left(x - bt - x_0\right)\right)\right]
$$

the initial condition is taken from the exact solution together with boundary conditions given by (2-4). This example is studied in [11, 19, 25]. The above solution models the shock wave propagation with the speed b and initial position x_0.

Case 1: We have considered domain as $[x_0, x_N] = [-30, 30]$ with time step $\Delta t = 0.01$ and number of partitions = 150. In order to compare the solutions with [19] we have taken $b = 5$, $k = \frac{1}{2}\sqrt{\frac{11}{19}}$, $x_0 = -12$, $d = \sqrt{\frac{11}{19}}$, $.e = -9$, $f = 11$. A comparison of the global relative errors can be made among the proposed method, the Lattice Boltzman method and quintic B-spline collocation method in Table 2. We have tried to find best free parameters experientially in a predetermined interval. But we have not seen the effect of the free parameters on increasing the accuracy. We see that the quintic collocation method produces a little less error than the suggested method. The numerical results are plotted at different time step for $\Delta t = 0.005$ and $N = 400$ in Fig. 2 and Fig. 3 shows projection of the solution on the x-t plane. Solution obtained by exponential cubic B-spline collocation method is very close to the exact solutions due to the global relative error obtained in Table 2

Table 2: Comparison of global relative error for Example a at different time t, $N = 150$

Time(t)	$p = 1$	$p = 0.0000002770$	[19]	[11]
1	8.74634×10^{-4}	3.32908×10^{-4}	3.81725×10^{-4}	6.7923×10^{-4}
2	1.30146×10^{-3}	5.56364×10^{-4}	5.51142×10^{-4}	1.1503×10^{-3}
3	1.73971×10^{-3}	8.74899×10^{-4}	7.03980×10^{-4}	1.5941×10^{-3}
4	2.23657×10^{-3}	1.25164×10^{-3}	8.63662×10^{-4}	2.0075×10^{-3}

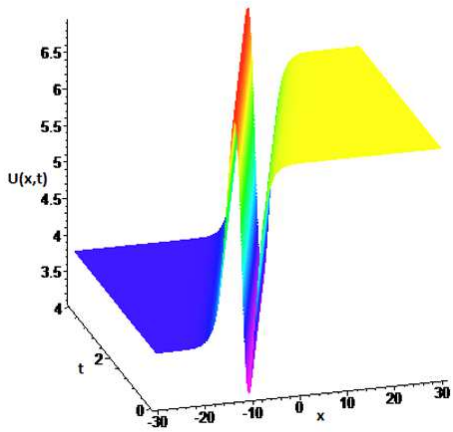


*Figure*2 : *Solutionsof KSequation*

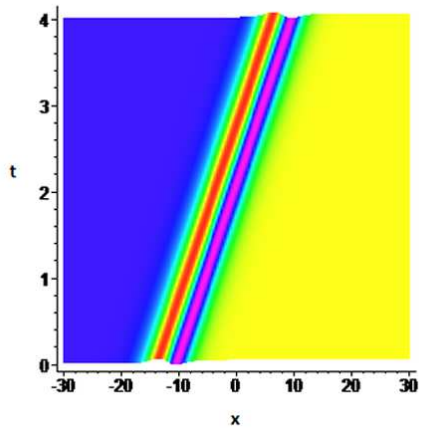


*Figure*3 : *Projectedsolutionsonx − tplane*

Case 2: We have run the algorithm with parameters $b = 5$, $k = \dfrac{1}{2\sqrt{19}}$, $d = \dfrac{1}{\sqrt{19}}$, $x_0 = -25, e = -3, f = 1$ over the domain $[x_0, x_N] = [-50, 50]$ with $\Delta t = 0.01$ and number of partitions as 200. In order to compare the solutions with [19] and [11], the global relative errors are recorded in Table 3.

Table 3 : Comparison of global relative errors at different time t, $N = 200$

Time(t)	$p = 1$	$p = 0.0000007606$	[19]	[11]
1	3.64671×10^{-5}	9.33798×10^{-6}	6.50927×10^{-6}	7.8808×10^{-6}
2	4.94999×10^{-5}	1.57173×10^{-5}	7.13154×10^{-6}	9.5324×10^{-6}
3	6.32739×10^{-5}	2.37302×10^{-5}	7.31029×10^{-6}	1.0891×10^{-5}
4	7.78406×10^{-5}	3.33367×10^{-5}	8.77659×10^{-6}	1.1793×10^{-5}

Fig. 4 shows the numerical solutions at different times for $\Delta t = 0.005, N = 400$. In Fig 5 view of the projected solutions is depicted onto the x-t plane.

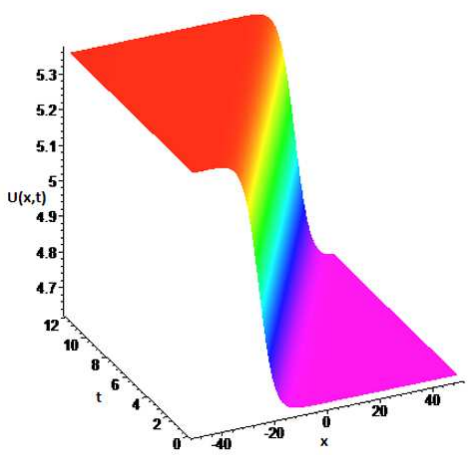


*Figure*4 : *Solutionsof KSequation*

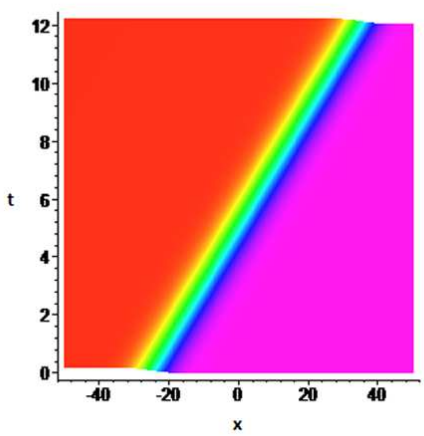


*Figure*5 : *Projectedsolutiononx − tplane*

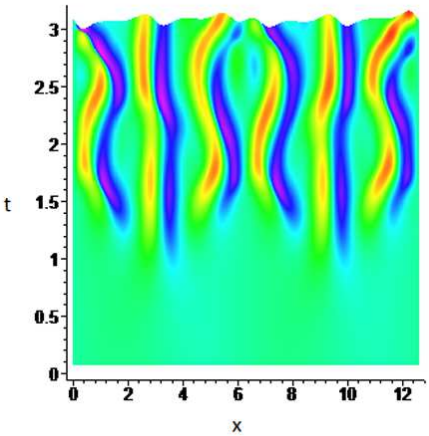
(b) This example represents chaotic behaviors with the initial condition,

$$u(x,0) = \cos(\frac{x}{2})\sin(\frac{x}{2})$$

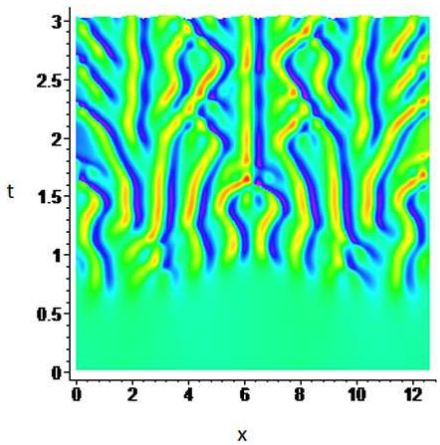
with the boundary condition

$$u_{xx}(0,t) = 0,\ u_{xx}(4\pi,t) = 0$$

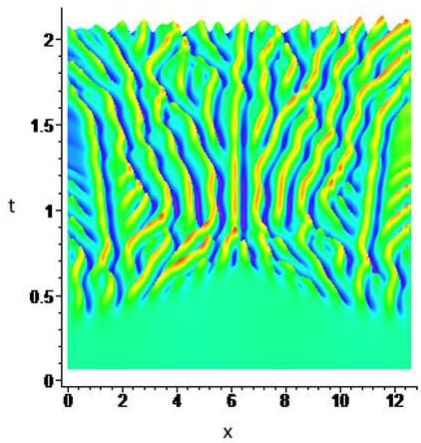
The computational domain $[x_0, x_N] = [0, 4\pi]$ is used with $N = 512$, $\Delta t = 0.001$, $\alpha = 1$. It is shown that KS-Equation is highly sensitive for choice of the parameter ϑ. In Figs. 6-9, we can observe the solution pattern exhibiting complete chaotic behaviors on the x-t plane, respectively. Figures illustrate that for the smaller value of ϑ, chaotic behavior starts to evolve earlier and seen more complex instabilities.



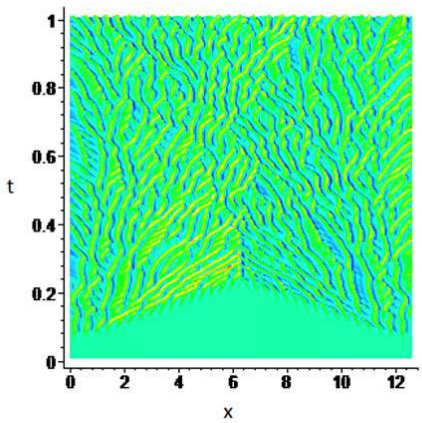
$Figure 6: \ Solution on x-t plane for \vartheta = 0.05$



$Figure 7: \ Solution on x-t plane for \vartheta = 0.02$



$Figure 8: \ Solution on x-t plane for \vartheta = 0.01$



$Figure 9: \ Solution on x-t plane for \vartheta = 0.002$

4. Conclusion

Numerical treatment of the KS equation is carried out in the paper. Exponential B-spline collocation algorithm gives the reliable solutions of the KS equations if compared with the existing results in literature. The free parameter of the exponential B-splines is determined to give the least error by scanning values with a small increment within a predetermined interval KS equation is reduced to linear system of algebraic equations that is solved with the Thomas algorithm. Numerical accuracy is similar to results of both quintic B-spline collocation and Bolzman methods and application of the method is simple. The complex chaotic behavior are modelled reliably by the presented method.

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