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On Certain Double A-summability Methods

Ekrem Savas[a]

[a]*Istanbul Ticaret University, Department of Mathematics, Üsküdar-Istanbul/Turkey*

Abstract. The aim of this paper is to continue our investigations in line of our recent paper, Savas [24] and [26]. We introduce the notion of A^I- double statistical convergence which includes the new summability methods studied in [24] and [23] as special cases and make certain observations on this new and more general summability method.

1. Introduction

The idea of convergence of a real sequence has been extended to statistical convergence by Fast [6] and later also by Schoenberg [32] as follows: Let K be a subset of $\mathbb{N}$. Then asymptotic density of K is denoted by

$$\lim_{n\to\infty} \frac{1}{n} |\{k \le n : k \in K\}|$$

where the vertical bars denoted the cardinality of the enclosed set.

A sequence (x_k) of real numbers is said to be statistically convergent to L if for arbitrary $\epsilon > 0$ the set K$(\epsilon) = \{n \in N : |x_n - L| \ge \epsilon\}$ has natural density zero. Statistical convergence turned out to be one of the most active areas of research in summability theory after the works of Fridy [8] and Salat [27]. More works on statistically convergence can be find from [1], [19], [30] and [33].

The notion of statistical convergence was further extended to I-convergence [14] using the notion of ideals of $\mathbb{N}$. Many interesting investigations using the ideals can be found in ([3], [2], [13], [15],[29], [28], [36] and [35]). In particular in [24] and [23] ideals were used to introduce new concepts of double I-statistical convergence, double I-lacunary statistical convergence and double I_λ -statistical convergence.

Natural density was generalized by Freeman and Sember in [9] by replacing C_1 with a nonnegative regular summability matrix $A = (a_{n,k})$. Thus, if K is a subset of N then the A-density of K is given by $\delta_A(K) = \lim_n \sum_{k\in K} a_{n,k}$ if the limit exists.

On the other hand, the idea of A-statistical convergence was introduced by Kolk [12] using a non-negative regular matrix A (which subsequently included the ideas of statistical, lacunary statistical or $\lambda-$ statistical convergence as special cases). More recent work in this line can be found in ([5],[18], [26]) and [27] where many references can be found.

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Email address: ekremsavas@yahoo.com (Ekrem Savas)

In [20] the notion of convergence for double sequences was presented by A. Pringsheim. Also, in [10] and [21] the four dimensional matrix transformation $(Ax)_{m,n} = \sum_{k,l=1}^{\infty,\infty} a_{m,n,k,l}x_{k,l}$ was studied extensively by Hamilton and Robison. In their work and throughout this paper, the four dimensional matrices and double sequences have real-valued entries unless specified otherwise.

In this paper, by using the above two approaches we introduce the idea of A^I- double statistical convergence and make certain observations.

2. Preliminaries

Throughout the paper $\mathbb{N}$ will denote the set of all positive integers. A family $I \subset 2^Y$ of subsets of a nonempty set Y is said to be an ideal in Y if (i) $A, B \in I$ implies $A \cup B \in I$; (ii) $A \in I, B \subset A$ implies $B \in I$, while an admissible ideal I of Y further satisfies $\{x\} \in I$ for each $x \in Y$. If I is a proper ideal in Y (i.e. $Y \notin I, Y \neq \phi$) then the family of sets $F(I) = \{M \subset Y : \text{there exists } A \in I : M = Y \setminus A\}$ is a filter in Y. It is called the filter associated with the ideal I. Throughout I will stand for a proper non-trivial admissible ideal of $\mathbb{N}$.

A sequence $\{x_k\}_{k\in\mathbb{N}}$ of real numbers is said to be $I-$convergent to $x \in \mathbb{R}$ if for each $\varepsilon > 0$ the set $A(\varepsilon) = \{n \in \mathbb{N} : |x_n - x| \geq \varepsilon\} \in I$ [14].

Before continuing with this paper we present some definitions. By the convergence in a double sequence we mean the convergence on the Pringsheim sense that is, a double sequence $x = (x_{k,l})$ has **Pringsheim limit** L (denoted by P-lim $x = L$) provided that given $\epsilon > 0$ there exists $N \in \mathbf{N}$ such that $\left|x_{k,l} - L\right| < \epsilon$ whenever $k, l > N$ [20]. We shall describe such an x more briefly as "**P-convergent**".

Definition 2.1. *Let $A = (a_{m,n,k,l})$ denote a four dimensional summability method that maps the complex double sequences x into the double sequence Ax where the mn-th term to Ax is as follows:*

$$(Ax)_{m,n} = \sum_{k,l=1,1}^{\infty,\infty} a_{m,n,k,l}x_{k,l}.$$

Such transformation is said to be non-negative if $a_{m,n,k,l}$ is nonnegative for all m, n, k and l.
In 1926 Robison presented a four dimensional analog of the definition of regularity for double sequences in which he added an additional assumption of boundedness. This assumption was made because a double sequence which is P-convergent is not necessarily bounded. In addition, to this definition we also presented a Silverman-Toeplitz type characterization of the regularity of four dimensional matrices.This characterization is called the Robison-Hamilton characterization. A double sequence x is bounded if and only if there exists a positive number M such that $\left|x_{k,l}\right| < M$ for all k and l.

Definition 2.2. *The four dimensional matrix A is said to be **RH-conservative** if it maps every bounded P-convergent sequence into a P-convergent sequence.*

Theorem 2.1. *(Hamilton [10], Robison [21]) The four dimensional matrix A is RH-conservative if and only if*

RH_1: $P\text{-}\lim_{m,n} a_{m,n,k,l} = c_{k.l}$ *for each k and l;*
RH_2: $P\text{-}\lim_{m,n} \sum_{k,l=1,1}^{\infty,\infty} a_{m,n,k,l} = a$;
RH_3: $P\text{-}\lim_{m,n} \sum_{k=1}^{\infty} \left|a_{m,n,k,l} - c_{k,l}\right| = 0$ *for each l;*
RH_4: $P\text{-}\lim_{m,n} \sum_{l=1}^{\infty} \left|a_{m,n,k,l} - c_{k,l}\right| = 0$ *for each k;*
RH_5: $\sum_{k,l=1,1}^{\infty,\infty} \left|a_{m,n,k,l}\right| < A$ *for all* (m, n); *and*
RH_6: *there exist finite positive integers A and B such that*
$\sum_{k,l>B} \left|a_{m,n,k,l}\right| < A$.

When these conditions are satisfied, we have

$$P-\lim_{m,n} Y_{m,n} = \mu(a-\sum_{k,l} c_{k,l}) + \sum_{k,l} c_{k,l}x_{k,l}$$

where $\mu = P-\lim_{k,l} x_{k,l}$, the double series $\sum_{k,l=1,1}^{\infty,\infty} c_{k,l}(x_{k,l}-\mu)$ is always absolutely P-convergent.

Definition 2.3. *The four dimensional matrix A is said to be* ***RH-regular*** *if it maps every bounded P-convergent sequence into a P-convergent sequence with the same P-limit.*

Theorem 2.2. *(Hamilton [10], Robison [21]) The four dimensional matrix A is RH-regular if and only if*

RH_1: $P\text{-}\lim_{m,n} a_{m,n,k,l} = 0$ *for each k and l;*
RH_2: $P\text{-}\lim_{m,n} \sum_{k,l=1,1}^{\infty,\infty} a_{m,n,k,l} = 1$;
RH_3: $P\text{-}\lim_{m,n} \sum_{k=1}^{\infty} \left|a_{m,n,k,l}\right| = 0$ *for each l;*
RH_4: $P\text{-}\lim_{m,n} \sum_{l=1}^{\infty} \left|a_{m,n,k,l}\right| = 0$ *for each k;*
RH_5: $\sum_{k,l=1,1}^{\infty,\infty} \left|a_{m,n,k,l}\right|$ *is P-convergent; and*
RH_6: *there exist finite positive integers A and B such that* $\sum_{k,l>B} \left|a_{m,n,k,l}\right| < A$.

Let $K \subset N \times N$ be a two-dimensional set to positive integers and let $K(m,n)$ be the numbers of (i,j) in K such that $i \leq n$ and $j \leq M$. The two-dimensional analogues of natural density can be defined as follows: The lower asymptotic density of a set $K \subset N \times N$ is define as

$$\delta^2(K) = \liminf_{m,n} \frac{K(m,n)}{mn}.$$

In case the double sequence $\frac{K(m,n)}{mn}$ has a limit in the Pringsheim sense then we say that K has a double natural density as

$$P-\lim_{m,n} \frac{K(m,n)}{mn} = \delta^2(K).$$

Let $K \subset N \times N$ be a two-dimensional set of positive integers, then the A-density of K is given by

$$\delta_A^2(K) = P-\lim_{m,n} \sum_{(k,l)\in K} a_{m,n,k,l}$$

provided that the limit exists. The notion of double asymptotic density for double sequence was presented by Mursaleen and Edely [18] and Tripathy [34] independently as follows:
A real double sequence $x = (x_{k,l})$ is said to be P-statistically convergent to L provided that for each $\varepsilon > 0$

$$P-\lim_{mn} \frac{1}{mn}\{(k,l) : k < m \text{ and } k < n, |x_{k,l}-L| \geq \varepsilon\} = 0.$$

In this case we write $St_2\text{-}\lim_{k,l} x_{k,l} = L$ and denote the set of all statistical convergent double sequences by St_2. It is clear that a convergent double sequence is also St_2-convergent but the converse is not true, in general. Also St_2-convergent double sequence need not be bounded.
Throughout e will denote a sequence all of whose elements are 1. Also as usual,

$$l''_\infty = \left\{ x = (x_{k,l}) : \|x\| = \sup_{k,l} \left|x_{k,l}\right| < \infty \right\}.$$

3. Main Results

Now we introduce the main concept of this paper, namely the notion of A_2^I-statistical convergence.

Definition 3.1. *Let $A = (a_{m,n,k,l})$ be a non-negative RH-regular four dimensional matrix . A sequence $(x_{k,l})$ is said to be A^I- double statistically convergent to L if for any $\epsilon > 0$ and $\delta > 0$,*

$$\left\{ m, n \in \mathbb{N} \times \mathbb{N} : \sum_{k,l \in K_2(x-Le,\varepsilon)} a_{m,n,k,l} \geq \delta \right\} \in I$$

where $K_2(x - Le, \epsilon) = \left\{ k, l \in \mathbb{N} \times \mathbb{N} : \left| x_{k,l} - L \right| \geq \epsilon \right\}$. In this case we write $x_{k,l} \xrightarrow{A_2^I - st} L$. We denote the class of all A_2^I-statistically convergent sequences by $S_A^2(I)$.

(1) If we take $A = (C, 1, 1)$,i.e., the double Cesàro matrix then A_2^I-statistical convergence becomes I-double statistical convergence [23].

(3) Let us consider the following notations and definitions. The double sequence $\theta_{r,s} = \{(k_r, l_s)\}$ is called double lacunary if there exist two increasing sequences of integers such that

$$k_0 = 0, h_r = k_r - k_{r-1} \to \infty \text{ as } r \to \infty,$$

$$l_0 = 0, h_s = l_s - l_{s-1} \to \infty \text{ as } s \to \infty,$$

and let $\bar{h}_{r,s} = h_r h_s$, $\theta_{r,s}$ is determine by $I_{r,s} = \{(i, j) : k_{r-1} < i \leq k_r \ \& \ l_{s-1} < j \leq l_s\}$. If we take

$$a_{r,s,k,l} = \begin{cases} \frac{1}{\bar{h}_{r,s}}, & \text{if } (k, l) \in I_{r,s}; \\ 0 & \text{otherwise.} \end{cases}$$

then A_2^I-statistical convergence coincides with I- double lacunary statistical convergence [23].

(4) As a final illustration let

$$a_{i,j,k,l} = \begin{cases} \frac{1}{\bar{\lambda}_{i,j}}, & \text{if } k \in I_i = [i - \lambda_i + 1, i] \text{ and } l \in L_j = [j - \lambda_j + 1, j] \\ 0, & \text{otherwise} \end{cases}$$

where we shall denote $\bar{\lambda}_{i,j}$ by $\lambda_i \mu_j$. Let $\lambda = (\lambda_i)$ and $\mu = (\mu_j)$ be two non-decreasing sequences of positive real numbers such that each tending to ∞ and $\lambda_{i+1} \leq \lambda_i + 1, \lambda_1 = 0$ and $\mu_{j+1} \leq \mu_j + 1, \mu_1 = 0$. Then A_2^I-statistical convergence coincides with $I_\lambda-$ double statistical convergence [24].

Non-trivial examples of such sequences can be seen from ([24], [23]).

Also note that for $I = I_{fin}, A_2^I$-statistical convergence becomes A- double statistical convergence [25].

We now prove the following result which establishes the topological character of the space $S_A^2(I)$.

Theorem 3.1. *$S_A^2(I) \cap l_\infty''$ is a closed subset of l_∞'' endowed with the superior norm.*

Proof. Suppose that $(x^{mn}) \subset S_A^2(I) \cap l_\infty''$ is a convergent sequence and it converges to $x \in l_\infty''$. We have to show that $x \in S_A^2(I) \cap l_\infty''$. Let $x^{mn} \xrightarrow{A_2^I - st} L_{mn}$ for all $(m, n) \in \mathbb{N} \times \mathbb{N}$. Take a sequence (ε_{mn}) where $\varepsilon_{mn} = \frac{1}{2^{m+1,n+1}}, \forall (m, n) \in \mathbb{N} \times \mathbb{N}$. We can find a positive integer N_{mn} such that $\|x - x^{mn}\|_\infty < \frac{\varepsilon_{mn}}{4}$, $\forall mn \geq N_{mn}$. Choose $0 < \delta < \frac{1}{3}$.

Now

$$A = \{(m,n) \in \mathbb{N}\times\mathbb{N} : \sum_{k,l\in M_1} a_{mnkl} < \delta\} \in F(I)$$

where

$$M_1 = \{(k,l) \in \mathbb{N}\times\mathbb{N} : |x_{k,l}^{mn} - L_{mn}| \geq \frac{\varepsilon_{mn}}{4}\}$$

and

$$B = \{(m,n) \in \mathbb{N}\times\mathbb{N} : \sum_{k,l\in M_2} a_{mnkl} < \delta\} \in F(I)$$

where $M_2 = \{(k,l) \in \mathbb{N} : |x_{k,l}^{m+1,n+1} - L_{m+1,n+1}| \geq \frac{\varepsilon_{mn}}{4}\}$.

Since $A \cap B \in F(I)$ and I is admissible, $A \cap B$ must be infinite. So we can choose $(m,n) \in A \cap B$ such that $|\sum_{k,l} a_{mnkl} - 1| < \frac{\delta}{2}$. But $\sum_{(k,l)\in M_1\cup M_2} a_{mnkl} \leq 2\delta < 1 - \frac{\delta}{2}$, while $\sum_{k,l} a_{mnkl} > 1 - \frac{\delta}{2}$.

Hence there must exist a $(k,l) \in \mathbb{N}\times\mathbb{N} \setminus (M_1 \cup M_2)$ and for which we have both $|x_{k,l}^{mn} - L_{mn}| < \frac{\varepsilon_{nm}}{4}$ and $|x_{k,l}^{m+1,n+1} - L_{m+1,n+1}| < \frac{\varepsilon_{mn}}{4}$. Then it follows that

$$\begin{aligned} |L_{mn} - L_{m+1,n+1}| &\leq |L_{mn} - x_{k,l}^{mn}| + |x_{k,l}^{mn} - x_{k,l}^{m+1,n+1}| + |x_{k,l}^{m+1,n+1} - L_{m+1,n+1}| \\ &\leq |L_{mn} - x_{k,l}^{mn}| + |x_{k,l}^{m+1,n+1} - L_{m+1,n+1}| + \|x - x^{mn}\|_\infty + \|x - x^{m+1,n+1}\|_\infty \\ &\leq \frac{\varepsilon_{mn}}{4} + \frac{\varepsilon_{mn}}{4} + \frac{\varepsilon_{mn}}{4} + \frac{\varepsilon_{mn}}{4} \\ &= \varepsilon_{mn}. \end{aligned}$$

This implies that (L_{mn}) is a Cauchy sequence in $\mathbb{R}$ and let $L_{mn} \to L \in \mathbb{R}$ as $m,n \to \infty$, Pringsheim sense. We shall prove that $x \xrightarrow{A_2^I - st} L$. Choose $\varepsilon > 0$ and $(m,n) \in \mathbb{N}\times\mathbb{N}$ such that $\varepsilon_{mn} < \frac{\varepsilon}{4}$, $\|x - x^{mn}\|_\infty < \frac{\varepsilon}{4}$, $|L_{mn} - L| < \frac{\varepsilon}{4}$. Now since

$$\sum_{k,l\in\{(k,l)\in\mathbb{N}\times\mathbb{N}:\ |x_{k,l}-L|\geq\varepsilon\}} a_{mnkl} \leq \sum_{k,l\in\{k,l:\ |x_{k,l}-x_{k,l}^{mn}|+|x_{k,l}^{mn}-L_{mn}|+|L_{mn}-L|\geq\varepsilon\}} a_{mnkl},$$

so it follows that

$$\left\{(m,n) \in \mathbb{N}\times\mathbb{N} : \sum_{k,l\in\{(k,l)\in\mathbb{N}\times\mathbb{N}:\ |x_{k,l}-L|\geq\varepsilon\}} a_{mnkl} \geq \delta\right\}$$
$$\subset \left\{(m,n) \in \mathbb{N}\times\mathbb{N} : \sum_{k,l\in\{(k,l)\in\mathbb{N}\times\mathbb{N}:\ |x_{k,l}^{mn}-L_{mn}|\geq\frac{\varepsilon}{2}\}} a_{mnkl} \geq \delta\right\} \in I$$

for any given $\delta > 0$. Since the set on the right hand side belongs to I, this shows that $x \xrightarrow{A_2^I - st} L$. This completes the proof of the result. □

Remark 1: We can say that the set of all bounded A_2^I-statistically convergent sequences of real numbers forms a closed linear subspace of l_∞''. Also it is obvious that $S_A^2(I) \cap l_\infty''$ is complete.

We define another related summability method and establish its relation with A_2^I-statistical convergence.

Definition 3.2. *Let Let $A = (a_{m,n,k,l})$ be a non-negative RH-regular four dimensional matrix . Then we say that x is A_2^I-summable to L if the sequence $(A_{mn}(x))$ I-converges to L.*

For $I = I_d$, A_2^I-summability reduces to statistical double A-summability, [5].

Theorem 3.2. *If a sequence is bounded and A_2^I-statistically convergent to L then it is A_2^I-summable to L.*

Proof. Let $x = (x_{k,l})$ be bounded and A_2^I-statistically convergent to L and for $\varepsilon > 0$, let as before $K_2(\frac{\varepsilon}{2}) := \{(k,l) \in \mathbb{N}\times\mathbb{N} : |x_{k,l} - L| \geq \frac{\varepsilon}{2}\}$. Then

$$\begin{aligned} |A_{mn}(x) - L| &\leq |\sum_{(k,l)\notin K(\frac{\varepsilon}{2})} a_{mnkl}(x_{kl} - L)| + |\sum_{(k,l)\in K(\frac{\varepsilon}{2})} a_{mnkl}(x_{kl} - L)| \\ &\leq \frac{\varepsilon}{2}\sum_{k,l\notin K(\frac{\varepsilon}{2})} a_{mnkl} + \sup_{k,l}|(x_{kl} - L)|| \sum_{k,l\in K(\frac{\varepsilon}{2})} a_{mnkl}| \leq \frac{\varepsilon}{2} + B. \sum_{k,l\in K(\frac{\varepsilon}{2})} a_{mnkl}, \end{aligned}$$

where $B = sup_{k,l}|x_{k,l} - L|$. It now follows that

$$\{(m,n) \in \mathbb{N}\times\mathbb{N} : |A_{mn}(x) - L| \geq \varepsilon\} \subset \left\{(m,n) \in \mathbb{N}\times\mathbb{N} : \sum_{k\in K(\frac{\varepsilon}{2})} a_{mnkl} \geq \frac{\varepsilon}{2B}\right\}.$$

Since x is A_2^I- statistically convergent to L so the set on the right hand side belongs to I and this consequently implies that x is A_2^I- summable to L. □

The converse of the above result is not generally true.
Example 2.If $A = (a_{mnkl}) = (C,1,1)$, double Cesàro matrix and let

$$x_{kl} = (\begin{matrix} 1 & \text{if } k,l \text{ are odd} \\ 0 & \text{if } k,l \text{ are even.} \end{matrix}$$

Then $x = (x_{kl})$ is A_2-summable to $1/2$ and so is A_2^I-summable to $1/2$ for any admissible ideal I. But note that for any $L \in \mathbb{R}$ and for $0 < \epsilon < \frac{1}{2}$, $K_2(\epsilon) = ((k,l) \in \mathbb{N}\times\mathbb{N} : |x_{kl} - L| \geq \epsilon)$ contains either the set of all even integers or the set of all odd integers or both. Consequently $\sum_{k,l\in K_2(\epsilon)} a_{mnkl} = \infty$ for any $(k,l) \in \mathbb{N}\times\mathbb{N}$ and so for any $\delta > 0$,

$$\left\{(m,n) \in \mathbb{N}\times\mathbb{N} : \sum_{k,l\in K_2(\epsilon)} a_{mnkl} \geq \delta\right\} \notin I.$$

This shows that $x = (x_{kl})$ is not A_2^I-statistically convergent for any non-trivial ideal I.

We conclude this paper with the following theorem which shall give that continuity preserves the A_2^I-statistical convergence.

Theorem 3.3. *If for a sequence $x = (x_{kl})$, $x_{kl} \xrightarrow{A_2^I-st} L$ and g is a real valued function which is continuous then $g(x_{kl}) \xrightarrow{A_2^I-st} g(L)$.*

Proof. The proof can be established using standard techniques, so omitted. □

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