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A Decomposition of Some Types of Mixed Soft Continuity in Soft Topological Spaces

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Abstract. In this paper, we study the concept of soft sets which is introduced by Molodtsov [5] and the notion of soft continuity is introduced by Zorlutuna et al. [8]. We give the definition of (τ_1, τ_2) - semi open soft (resp. (τ_1, τ_2) - pre open soft, (τ_1, τ_2) - α - open soft, (τ_1, τ_2) - β - open soft) set via two soft topologies. We introduce mixed semi - soft (resp. mixed pre - soft, mixed α - soft, mixed β - soft) continuity between two soft topological spaces $(X, \tau_1, A), (X, \tau_2, A)$ and a soft topological space (Y, τ, B). Also we prove that a function is mixed α - soft continuous if and only if it is both mixed pre - soft continuous and mixed semi - soft continuous.

1. Introduction

Some set theories can be dealt with unclear concepts such as rough sets theory, fuzzy sets theory etc. Unfortunately, these theories are not sufficient to deal with some difficulties and encounter some problems. In 1999, Molodtsov [5] has introduced the soft set theory as a general mathematical tool for dealing with these problems. He has accomplished that very significant applications of soft set theory such as solving some complications in economics, social science, medical science, engineering etc. There has been some important studies on the applications of soft sets theory. Some authors have studied soft sets theory and investigated some basic properties of this theory.

In 2003, Maji, Biswas and Roy [4] introduced the equality of two soft sets, subset of a soft set, null soft set, absolute soft set etc. In 2009, Ali, Feng, Liu, Min and Shabir [1] investigated several operations using soft sets and introduced some new notions such as the restricted intersection, the restricted union etc. In 2011, Shabir and Naz [6] defined some notions such as soft topological space, soft interior, soft closure etc. Also, Hussain and Ahmad [2] researched some properties of soft topological space.

The concept of continuity is an important concept in general topology, fuzzy topology, generalized topology etc. as well as in all branches of mathematics. Recently, we have seen the introduction of some types of continuity. Also decompositions of these continuities are investigated. In these days, continuity of functions is defined in soft topological spaces. In 2012, Zorlutuna, Akdag, Min and Atmaca [8] introduced the image and inverse image of a soft set under a function.

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We consider two soft topologies τ_1 and τ_2 over X in the whole paper. The aim of this present paper is to introduce the notions of $(\tau_1\tau_2, \tau)$ - semi open (resp. $(\tau_1\tau_2, \tau)$ - pre open, $(\tau_1\tau_2, \tau)$ - α - open, $(\tau_1\tau_2, \tau)$ - β - open) soft sets and mixed semi - soft (resp. mixed pre - soft, mixed α - soft, mixed β - soft) continuity between two soft topological spaces $(X, \tau_1, A), (X, \tau_2, A)$ and a soft topological space (Y, τ, B). We prove that a function mixed α - continuous if and only if it is both mixed pre - soft continuous and mixed semi - soft continuous as a decomposition of mixed α - continuity. Furthermore, we show that $(\tau_1\tau_2, \tau)$ - semi open soft set and $(\tau_1\tau_2, \tau)$ - pre open soft set are independent of each other giving some examples. Finally, we investigate relationships among τ_1 - soft continuity, $(\tau_1\tau_2, \tau)$ - semi - soft continuity, $(\tau_1\tau_2, \tau)$ - pre - soft continuity, $(\tau_1\tau_2, \tau)$ - α - soft continuity and $(\tau_1\tau_2, \tau)$ - β - soft continuity and these relations are shown in DIAGRAM - II.

2. Preliminaries

In this section, we recall some known definitions and theorems.

Let X be an initial universal set, E be a non-empty set of parameters and $A, B \subseteq E$.

Soft Sets:

Definition 2.1. *[5] A pair (F, A), where F is a mapping from A to $P(X)$, is called a soft set over X. The family of all soft sets on X is denoted by $SS(X)_E$.*

Definition 2.2. *[4] Let (F, A) and (G, B) be two soft sets over a common universe X. Then (F, A) is said to be a soft subset of (G, B) if $A \subseteq B$ and $F(e) \subseteq G(e)$, for all $e \in A$. This relation is denoted by $(F, A)\widetilde{\subseteq}(G, B)$.*

(F, A) is said to be soft equal to (G, B) if $(F, A)\widetilde{\subseteq}(G, B)$ and $(G, B)\widetilde{\subseteq}(F, A)$. This relation is denoted by $(F, A) = (G, B)$.

Definition 2.3. *[1] The complement of a soft set (F,A) is defined as*

$$(F, A)^c = (F^c, A),$$

where $F^c(e) = (F(e))^c = X - F(e)$ for all $e \in A$.

Definition 2.4. *[6] The difference of two soft sets (F,A) and (G,A) is defined as*

$$(F, A) - (G, A) = (F - G, A),$$

where $(F - G)(e) = F(e) - G(e)$ for all $e \in A$.

Definition 2.5. *[6] Let (F, A) be a soft set over X and $x \in X$. x is said to be in the soft set (F, A) and is denoted by $x \in (F, A)$ if $x \in F(e)$ for all $e \in A$.*

Definition 2.6. *[4] Let (F, A) be a soft set over X. Then*

1. *(F, A) is said to be a null soft set if $F(e) = \emptyset$, for all $e \in A$. This is denoted by $\widetilde{\emptyset}$.*
2. *(F, A) is said to be an absolute soft set if $F(e) = X$, for all $e \in A$. This is denoted by $\widetilde{X}$.*

Soft Topology:

Definition 2.7. *[6] Let τ be the collection of soft sets over X. Then τ is said to be a soft topology on X if*

1. *$\widetilde{\emptyset}, \widetilde{X} \in \tau$,*
2. *the intersection of any two soft sets in τ belongs to τ,*
3. *the union of any number of soft sets in τ belongs to τ.*

The triple (X, τ, E) is called a soft topological space over X. The members of τ are said to be $\tau-$ soft open sets or soft open sets in X. A soft set over X is said to be closed soft in X if its complement belongs to τ. The set of all open soft sets over X denoted by $OS(X, \tau, E)$ or $OS(X)$ and the set of all closed soft sets denoted by $CS(X, \tau, E)$ or $CS(X)$.

Definition 2.8. *[6] Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. The soft closure of (F, E), denoted by $cl(F, E)$ is the intersection of all closed soft super sets of (F, E).*

Definition 2.9. *[8] Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. The soft interior of (F, E), denoted by $int(F, E)$ is the union of all open soft subsets of (F, E).*

Theorem 2.10. *[2] Let (X, τ, E) be a soft topological space over X, (F, E) and (G, E) are two soft sets over X. Then*

1. $cl(\widetilde{\emptyset}) = \widetilde{\emptyset}$ *and* $cl(\widetilde{X}) = \widetilde{X}$.
2. $(F, E)\widetilde{\subseteq}cl(F, E)$.
3. (F, E) *is a closed soft set if and only if* $(F, E) = cl(F, E)$.
4. $cl(cl(F, E)) = cl(F, E)$.
5. $(F, E)\widetilde{\subseteq}(G, E)$ *implies* $cl(F, E)\widetilde{\subseteq}cl(G, E)$.
6. $cl((F, E)\widetilde{\cup}(G, E)) = cl(F, E)\widetilde{\cup}cl(G, E)$.
7. $cl((F, E)\widetilde{\cap}(G, E))\widetilde{\subseteq}cl(F, E)\widetilde{\cap}cl(G, E)$.

Theorem 2.11. *[2] Let (X, τ, E) be a soft topological space over X and (F, E) and (G, E) are two soft sets over X. Then*

1. $int\widetilde{\emptyset} = \widetilde{\emptyset}$ *and* $int\widetilde{X} = \widetilde{X}$.
2. $int(F, E)\widetilde{\subseteq}(F, E)$.
3. $int(int(F, E)) = int(F, E)$.
4. (F, E) *is a soft open set if and only if* $int(F, E) = (F, E)$.
5. $(F, E)\widetilde{\subseteq}(G, E)$ *implies* $int(F, E)\widetilde{\subseteq}int(G, E)$.
6. $int(F, E)\widetilde{\cap}int(G, E) = int((F, E)\widetilde{\cap}(G, E))$.
7. $int(F, E)\widetilde{\cup}int(G, E)\widetilde{\subseteq}int((F, E)\widetilde{\cup}(G, E))$.

Definition 2.12. *[8] Let $SS(X)_A$ and $SS(Y)_B$ be two families of soft sets, $u : X \to Y$ and $p : A \to B$ be mappings. Then the mapping $f_{pu} : SS(X)_A \to SS(Y)_B$ is defined as:*

1. *Let $(F, A) \in SS(X)_A$. The image of (F, A) under f_{pu}, written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is a soft set in $SS(Y)_B$ such that*

$$f_{pu}(F)(y) = \begin{cases} \bigcup_{x \in p^{-1}(y) \cap A} u(F(x)) & , p^{-1}(y) \cap A \neq \emptyset \\ \emptyset & , p^{-1}(y) \cap A = \emptyset \end{cases}$$

for all $y \in B$.

2. *Let $(G, B) \in SS(Y)_B$. The inverse image of (G, B) under f_{pu}, written as $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), p^{-1}(B))$, is a soft set in $SS(X)_A$ such that*

$$f_{pu}^{-1}(G)(x) = \begin{cases} u^{-1}(G(p(x))) & , p(x) \in B \\ \emptyset & , p(x) \notin B \end{cases}$$

for all $x \in A$.

Definition 2.13. *[8] Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then*

1. *The function f_{pu} is called soft continuous (briefly, soft - cts or τ - soft cts) if $f_{pu}^{-1}(G, B) \in \tau$ for all $(G, B) \in \tau^*$.*
2. *The function f_{pu} is called open soft if $f_{pu}(G, A) \in \tau^*$ for all $(G, A) \in \tau$.*

3. Some Mixed Soft Operations

In this section we give the definitions of some mixed types of soft operations and investigate some relations between each other and soft open sets.

Definition 3.1. *[7] Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. Then $(F,E) \in SS(X)_E$ is said to be*

1. (τ_1, τ_2) *- semi open soft if* $(F,E)\widetilde{\subseteq} cl_2(int_1(F,E))$;
2. (τ_1, τ_2) *- pre open soft if* $(F,E)\widetilde{\subseteq} int_1(cl_2(F,E))$;
3. (τ_1, τ_2) *-* α *- open soft if* $(F,E)\widetilde{\subseteq} int_1(cl_2(int_1(F,E)))$;
4. (τ_1, τ_2) *-* β *- open soft if* $(F,E)\widetilde{\subseteq} cl_2(int_1(cl_2(F,E)))$.

The complement of (τ_1, τ_2) - semi open (resp. (τ_1, τ_2) - pre open, (τ_1, τ_2) - α - open, (τ_1, τ_2) - β - open) soft set is called (τ_1, τ_2) - semi closed (resp. (τ_1, τ_2) - pre closed, (τ_1, τ_2) - α - closed, (τ_1, τ_2) - β - closed) soft.

Let $\tau = \tau_1 = \tau_2$ in Definition 3.1. Then we obtain the following corollary.

Corollary 3.2. *[3] Let X be an initial universe and E be a set of parameters. Let $\tau = \tau_1 = \tau_2$ be a soft topology on X. Then $(F,E) \in SS(X)_E$ is said to be*

1. *semi - open soft set if* $(F,E)\widetilde{\subseteq} cl(int(F,E))$;
2. *pre - open soft set if* $(F,E)\widetilde{\subseteq} int(cl(F,E))$;
3. α *- open soft set if* $(F,E)\widetilde{\subseteq} int(cl(int(F,E)))$;
4. β *- open soft set if* $(F,E)\widetilde{\subseteq} cl(int(cl(F,E)))$.

Now we give some relationships between τ_1 - soft open sets and defined soft sets in Definition 3.1.

Theorem 3.3. *Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. Then the following statements hold:*

1. *every τ_1 - soft open set is (τ_1, τ_2) - semi open soft.*
2. *every τ_1 - soft open set is (τ_1, τ_2) - pre open soft.*
3. *every τ_1 - soft open set is (τ_1, τ_2) - α - open soft.*
4. *every τ_1 - soft open set is (τ_1, τ_2) - β - open soft.*

Similarly, every τ_1 - soft closed set is (τ_1, τ_2) - semi closed (resp. (τ_1, τ_2) - pre closed, (τ_1, τ_2) - α - closed, (τ_1, τ_2) - β - closed) soft set.

Proof.
1. Let (F,E) be τ_1 - soft open set. Then $int_1(F,E) = (F,E)$. Since $(F,E)\widetilde{\subseteq} cl_2(F,E)$ and $int_1(F,E) = (F,E)$, we have $(F,E)\widetilde{\subseteq} cl_2(int_1(F,E))$. Hence (F,E) is a (τ_1, τ_2) - semi open soft.
2. Let (F,E) be τ_1 - soft open set. Then $int_1(F,E) = (F,E)$. Since $(F,E)\widetilde{\subseteq} cl_2(F,E)$ and $int_1(F,E) = (F,E)$, we have $(F,E)\widetilde{\subseteq} int_1(cl_2(F,E))$. Hence (F,E) is a (τ_1, τ_2) - pre open soft.
3. Let (F,E) be τ_1 - soft open set. Then $int_1(F,E) = (F,E)$. Since $(F,E)\widetilde{\subseteq} cl_2(F,E)$ and $int_1(F,E) = (F,E)$, we have $(F,E)\widetilde{\subseteq} int_1(cl_2(F,E)) = int_1(cl_2(int_1(F,E)))$. Hence (F,E) is a (τ_1, τ_2) - α - open soft.
4. Let (F,E) be τ_1 - soft open set. Then $int_1(F,E) = (F,E)$. Since $(F,E)\widetilde{\subseteq} cl_2(F,E)$ and $int_1(F,E) = (F,E)$, we have $(F,E)\widetilde{\subseteq} cl_2(int_1(F,E))\widetilde{\subseteq} cl_2(int_1(cl_2(F,E)))$. Hence (F,E) is a (τ_1, τ_2) - β - open soft.

□

The converse of Theorem 3.3 is not always true as shown in the following examples.

Example 3.4.
1. *Let $X = \{a,b,c\}$, $E = \{e\}$, $\tau_1 = \{\widetilde{X}, \widetilde{\emptyset}, (F_1,E), (F_2,E), (F_3,E)\}$ and $\tau_2 = \{\widetilde{X}, \widetilde{\emptyset}\}$ where (F_1,E), (F_2,E), (F_3,E) are soft sets over X defined as follows:*
   $(F_1,E) = \{(e,\{a\})\}$, $(F_2,E) = \{(e,\{b\})\}$, $(F_3,E) = \{(e,\{a,b\})\}$. Then the soft set $(G,E) = \{(e,\{a,c\})\}$ is a (τ_1, τ_2) - semi open soft set, but it is not τ_1 - soft open.

2. *Let* $X = \{a, b, c\}$, $E = \{e\}$, $\tau_1 = \{\widetilde{X}, \widetilde{\emptyset}, (F_1, E), (F_2, E), (F_3, E)\}$ *and* $\tau_2 = \{\widetilde{X}, \widetilde{\emptyset}, (G, E)\}$ *where* $(F_1, E), (F_2, E), (F_3, E), (G, E)$ *are soft sets over X defined as follows:*
   $(F_1, E) = \{(e, \{a\})\}$, $(F_2, E) = \{(e, \{c\})\}$, $(F_3, E) = \{(e, \{a, c\})\}$, $(G, E) = \{(e, \{b, c\})\}$.
   Then the soft set $(H, E) = \{(e, \{b\})\}$ *is a* (τ_1, τ_2) *- pre open soft set, but it is not* τ_1 *- soft open.*
3. *Let* $X = \{a, b, c\}$, $E = \{e_1, e_2\}$, $\tau_1 = \{\widetilde{X}, \widetilde{\emptyset}, (F, E)\}$ *and* $\tau_2 = \{\widetilde{X}, \widetilde{\emptyset}\}$ *where* (F, E) *is soft set over X defined as follows:*
   $(F, E) = \{(e_1, \{a\}), (e_2, \{b\})\}$.
   Then the soft set $(G, E) = \{(e_1, \{a, c\}), (e_2, \{a, b\})\}$ *is a* (τ_1, τ_2) *-* α *- open soft set, but it is not* τ_1 *- soft open.*
4. *Let* $X = \{a, b, c\}$, $E = \{e_1, e_2\}$, $\tau_1 = \{\widetilde{X}, \widetilde{\emptyset}, (F, E)\}$ *and* $\tau_2 = \{\widetilde{X}, \widetilde{\emptyset}\}$ *where* (F, E) *is soft set over X defined as follows:*
   $(F, E) = \{(e_1, \{b\}), (e_2, \{c\})\}$.
   Then the soft set $(G, E) = \{(e_1, \{a, c\}), (e_2, X)\}$ *is a* (τ_1, τ_2) *-* β *- open soft set, but it is not* τ_1 *- soft open.*

Theorem 3.5. *Let X be an initial universe and E be a set of parameters. Let* τ_1 *and* τ_2 *be two soft topologies on X. Then the following statements hold:*

1. *every* (τ_1, τ_2) *-* α *- open soft set is* (τ_1, τ_2) *- semi open soft.*
2. *every* (τ_1, τ_2) *- semi open soft set is* (τ_1, τ_2) *-* β *- open soft.*
3. *every* (τ_1, τ_2) *- pre open soft set is* (τ_1, τ_2) *-* β *- open soft.*
4. *every* (τ_1, τ_2) *-* α *- open soft set is* (τ_1, τ_2) *- pre open soft.*

Similarly, every (τ_1, τ_2) *-* α *- closed soft set is* (τ_1, τ_2) *- semi closed (resp.* (τ_1, τ_2) *- pre closed) soft and every* (τ_1, τ_2) *- semi closed (resp.* (τ_1, τ_2) *- pre closed) soft set is* (τ_1, τ_2) *-* β *- closed soft.*

Proof. 1. Let (F, E) be (τ_1, τ_2) - α - open soft set. Then $(F, E)\widetilde{\subseteq}int_1(cl_2(int_1(F, E)))\widetilde{\subseteq}cl_2(int_1(F, E))$. Therefore, we have $(F, E)\widetilde{\subseteq}cl_2(int_1(F, E))$. Hence (F, E) is a (τ_1, τ_2) - semi open soft.

2. Let (F, E) be (τ_1, τ_2) - semi open soft set. Then $(F, E)\widetilde{\subseteq}cl_2(int_1(F, E))\widetilde{\subseteq}cl_2(int_1(cl_2(F, E)))$. Therefore, we have $(F, E)\widetilde{\subseteq}cl_2(int_1(cl_2(F, E)))$. Hence (F, E) is a (τ_1, τ_2) - β - open soft.
3. Let (F, E) be (τ_1, τ_2) - pre open soft set. Then $(F, E)\widetilde{\subseteq}int_1(cl_2(F, E))\widetilde{\subseteq}cl_2(int_1(cl_2(F, E)))$. Therefore, we have $(F, E)\widetilde{\subseteq}cl_2(int_1(cl_2(F, E)))$. Hence (F, E) is a (τ_1, τ_2) - β - open soft.
4. Let (F, E) be (τ_1, τ_2) - α - open soft set. Then $(F, E)\widetilde{\subseteq}int_1(cl_2(int_1(F, E)))\widetilde{\subseteq}int_1(cl_2(F, E))$. Therefore, we have $(F, E)\widetilde{\subseteq}int_1(cl_2(F, E))$. Hence (F, E) is a (τ_1, τ_2) - pre open soft.

□

The converse of Theorem 3.5 is not always true as shown in the following examples.

Example 3.6. 1. *Let* $X = \{a, b, c\}$, $E = \{e\}$, $\tau_1 = \{\widetilde{X}, \widetilde{\emptyset}, (F_1, E), (F_2, E), (F_3, E)\}$ *and* $\tau_2 = \{\widetilde{X}, \widetilde{\emptyset}, (G, E)\}$ *where* $(F_1, E), (F_2, E), (F_3, E), (G, E)$ *are soft sets over X defined as follows:*
   $(F_1, E) = \{(e, \{a\})\}$, $(F_2, E) = \{(e, \{b\})\}$, $(F_3, E) = \{(e, \{a, b\})\}$, $(G, E) = \{(e, \{a\})\}$.
   Then the soft set $(H, E) = \{(e, \{b, c\})\}$ *is a* (τ_1, τ_2) *- semi open soft set, but it is not* (τ_1, τ_2) *-* α *- open soft.*

2. *Let* $X = \{a, b, c, d\}$, $E = \{e\}$, $\tau_1 = \{\widetilde{X}, \widetilde{\emptyset}, (F, E)\}$ *and* $\tau_2 = \{\widetilde{X}, \widetilde{\emptyset}\}$ *where* (F, E) *is soft set over X defined as follows:*
   $(F, E) = \{(e, \{a\})\}$.
   Then the soft set $(G, E) = \{(e, \{d\})\}$ *is a* (τ_1, τ_2) *-* β *- open soft set, but it is not* (τ_1, τ_2) *- semi open soft.*
3. *Let* $X = \{a, b, c, d\}$, $E = \{e\}$, $\tau_1 = \{\widetilde{X}, \widetilde{\emptyset}, (F_1, E), (F_2, E), (F_3, E)\}$ *and* $\tau_2 = \{\widetilde{X}, \widetilde{\emptyset}, (G_1, E), (G_2, E), (G_3, E), (G_4, E)\}$ *where* $(F_1, E), (F_2, E), (F_3, E), (G_1, E), (G_2, E), (G_3, E), (G_4, E)$ *are soft sets over X defined as follows:*
   $(F_1, E) = (G_1, E) = \{(e, \{a\})\}$, $(F_2, E) = (G_2, E) = \{(e, \{b\})\}$, $(F_3, E) = (G_3, E) = \{(e, \{a, b\})\}$, $(G_4, E) = \{(e, \{a, b, c\})\}$.
   Then the soft set $(H, E) = \{(e, \{a, d\})\}$ *is a* (τ_1, τ_2) *-* β *- open soft set, but it is not* (τ_1, τ_2) *- pre open soft.*
4. *Let* $X = \{a, b, c\}$, $E = \{e\}$, $\tau_1 = \{\widetilde{X}, \widetilde{\emptyset}, (F, E)\}$ *and* $\tau_2 = \{\widetilde{X}, \widetilde{\emptyset}, (G, E)\}$ *where* $(F, E), (G, E)$ *are soft sets over X defined as follows:*
   $(F, E) = \{(e, \{a\})\}$, $(G, E) = \{(e, \{b, c\})\}$.
   Then the soft set $(H, E) = \{(e, \{a, b\})\}$ *is a* (τ_1, τ_2) *- pre open soft set, but it is not* (τ_1, τ_2) *-* α *- open soft.*

Corollary 3.7. *We obtain the following diagram by combining Theorem 3.3* (3) *and Theorem 3.5.*

DIAGRAM - I

τ_1 - soft open $\longrightarrow$ (τ_1, τ_2) - α - open soft $\longrightarrow$ (τ_1, τ_2) - semi open soft
$\searrow$ $\swarrow$ $\swarrow$
(τ_1, τ_2) - pre open soft $\longrightarrow$ (τ_1, τ_2) - β - open soft

(τ_1, τ_2) - pre open soft set and (τ_1, τ_2) - semi open soft set are independent of each other as we have seen the following examples.

Example 3.8.
1. *Let* $X = \{a, b, c\}$, $E = \{e\}$ *and* τ_1, τ_2 *be soft topological spaces defined as Example 3.6* (1). *Then the soft set* $(H, E) = \{(e, \{b, c\})\}$ *is a* (τ_1, τ_2) *- semi open soft set, but it is not* (τ_1, τ_2) *- pre open soft set.*
2. *Let* $X = \{a, b, c\}$, $E = \{e\}$ *and* τ_1, τ_2 *be soft topological spaces defined as Example 3.6* (4). *Then the soft set* $(H, E) = \{(e, \{a, b\})\}$ *is a* (τ_1, τ_2) *- pre open soft set, but it is not* (τ_1, τ_2) *- semi open soft set.*

Theorem 3.9. *Let X be an initial universe and E be a set of parameters. Let* τ_1 *and* τ_2 *be two soft topologies on X.* (F, E) *is a* (τ_1, τ_2) *-* α *- open soft set if and only if* (F, E) *is both* (τ_1, τ_2) *- pre open soft and* (τ_1, τ_2) *- semi open soft set.*

Proof. Let (F, E) be a (τ_1, τ_2) - α - open soft set. Then $(F, E)\widetilde{\subseteq} int_1(cl_2(int_1(F, E)))$. Therefore $(F, E)\widetilde{\subseteq} int_1(cl_2(F, E))$ and $(F, E)\widetilde{\subseteq} cl_2(int_1(F, E))$. Hence (F, E) is both (τ_1, τ_2) - pre open soft and (τ_1, τ_2) - semi open soft.

Conversely, let (F, E) be both (τ_1, τ_2) - pre open soft and (τ_1, τ_2) - semi open soft. Then $(F, E)\widetilde{\subseteq} int_1(cl_2(F, E))$ and $(F, E)\widetilde{\subseteq} cl_2(int_1(F, E))$. Hence $(F, E)\widetilde{\subseteq} int_1(cl_2(cl_2(int_1(F, E)))) = int_1(cl_2(int_1(F, E)))$. Consequently (F, E) is (τ_1, τ_2) - α - open soft. □

4. Decomposition of Some Mixed Soft Continuities

In this section we introduce some mixed types of soft continuity and investigate some relations between them and soft continuity.

Definition 4.1. *Let* X, Y *be an initial universe,* $A, B \subseteq E$ *be two sets of parameters,* τ_1, τ_2 *be two soft topologies over X and* τ *be a soft topology over Y. Let* $u : X \to Y$ *and* $p : A \to B$ *be mappings. Let* $f_{pu} : SS(X)_A \to SS(Y)_B$ *be a function. Then* f_{pu} *is called*

1. *mixed semi - soft continuous (briefly,* $(\tau_1\tau_2, \tau)$ *- semi - soft cts) if* $f_{pu}^{-1}(G, B)$ *is* (τ_1, τ_2) *- semi open soft set for every* $(G, B) \in \tau$.
2. *mixed pre - soft continuous (briefly,* $(\tau_1\tau_2, \tau)$ *- pre - soft cts) if* $f_{pu}^{-1}(G, B)$ *is* (τ_1, τ_2) *- pre open soft set for every* $(G, B) \in \tau$.
3. *mixed* α *- soft continuous (briefly,* $(\tau_1\tau_2, \tau)$ *-* α *- soft cts) if* $f_{pu}^{-1}(G, B)$ *is* (τ_1, τ_2) *-* α *- open soft set for every* $(G, B) \in \tau$.
4. *mixed* β *- soft continuous (briefly,* $(\tau_1\tau_2, \tau)$ *-* β *- soft cts) if* $f_{pu}^{-1}(G, B)$ *is* (τ_1, τ_2) *-* β *- open soft set for every* $(G, B) \in \tau$.

Theorem 4.2. *Let* X, Y *be an initial universe,* $A, B \subseteq E$ *be two sets of parameters,* τ_1, τ_2 *be two soft topologies over X and* τ *be a soft topology over Y. Let* $u : X \to Y$ *and* $p : A \to B$ *be mappings. Let* $f_{pu} : SS(X)_A \to SS(Y)_B$ *be a function. Then*

1. *every* τ_1 *- soft continuous function is mixed semi - soft continuous function.*
2. *every* τ_1 *- soft continuous function is mixed pre - soft continuous function.*
3. *every* τ_1 *- soft continuous function is mixed* α *- soft continuous function.*
4. *every* τ_1 *- soft continuous function is mixed* β *- soft continuous function.*

Proof. Obvious from Theorem 3.3. □

Theorem 4.3. *Let X, Y be an initial universe, $A, B \subseteq E$ be two sets of parameters, τ_1, τ_2 be two soft topologies over X and τ be a soft topology over Y. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then*

1. *every mixed α - soft continuous function is mixed semi - soft continuous function.*
2. *every mixed semi - soft continuous function is mixed β - soft continuous function.*
3. *every mixed pre - soft continuous function is mixed β - soft continuous function.*
4. *every mixed α - soft continuous function is mixed pre - soft continuous function.*

Proof. Obvious from Theorem 3.5. □

Corollary 4.4. *We obtain the following diagram by combining Theorem 4.2* (3) *and Theorem 4.3.*

DIAGRAM - II

$$\begin{array}{ccccc} \tau_1 \text{ - soft cts} & \longrightarrow & (\tau_1\tau_2, \tau) \text{ - } \alpha \text{ - soft cts} & \longrightarrow & (\tau_1\tau_2, \tau) \text{ - semi - soft cts} \\ \searrow & & \swarrow & & \swarrow \\ & (\tau_1\tau_2, \tau) \text{ - pre - soft cts} & \longrightarrow & (\tau_1\tau_2, \tau) \text{ - } \beta \text{ - soft cts} & \end{array}$$

Theorem 4.5. *Let X, Y be an initial universe, $A, B \subseteq E$ be two sets of parameters, τ_1, τ_2 be two soft topologies over X and τ be a soft topology over Y. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then f_{pu} is a mixed α - soft continuous function if and only if it is both mixed pre - soft continuous function and mixed semi - soft continuous function.*

Proof. Obvious from Theorem 3.9. □

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