Filomat 29:10 (2015), 2199–2205
DOI 10.2298/FIL1510199S

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Intrinsic Shape - The Proximate Approach

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Abstract. In this paper we present the intrinsic approach to shape based on proximate sequences and nets of functions, and establish equivalence of different definitions. At the end are presented the results obtained recently, by use of the intrinsic approach to shape.

The notion of shape was introduced by K. Borsuk in 1968 as a new classifcation of spaces from the point of view of their most important global topological properties. Main references about shape are the books of Borsuk [5] and of Mardesic and Segal [13] The approaches in both books are using external elements for describing shape of a space: neighborhoods in some external space where the original space is embedded, or an inverse sequence (system) of ANRs or polyhedra.

On the other hand there exists an internal characterization of shape, an approach without external spaces. In this intrinsic approach to shape there are two main branches:

1) approach by proximate sequences and proximate nets
2) approach by multivalued functions.

About the second approach we refer to articles: [7], [10], [11], [20] and [21].

The subject of this paper is the intrinsic approach by proximate sequences and nets, and the results obtained by this approach. More detailed explanation of the papers connected with this approach is presented in Sections 2 and 3.

1. Intrinsic approach to shape by proximate sequences and nets

The idea of ε - continuity (continuity up to $\varepsilon > 0$) leads to continuity up to some covering $\mathcal{V}$ i.e., $\mathcal{V}$ - continuity, and the corresponding $\mathcal{V}$ - homotopy. Here, will be presented a short description of the intrinsic approach presented in [27] and [28]. Let X, Y be topological spaces. For collections $\mathcal{U}$ and $\mathcal{V}$ of subsets of X, $\mathcal{U} \prec \mathcal{V}$ means that $\mathcal{U}$ refines $\mathcal{V}$, i.e., each $U \in \mathcal{U}$ is contained in some $V \in \mathcal{V}$.

By a covering we understand a covering consisting of open sets.

Definition 1.1. *Suppose $\mathcal{V}$ is a covering of Y. A function $f : X \to Y$ is $\mathcal{V}$ - continuous at pont x, if there exists a neighborhood U_x of x, and $V \in \mathcal{V}$ such that $f(U_x) \subseteq V$.*

A function is $\mathcal{V}$ - continuous, if it is $\mathcal{V}$ - continuous at every point $x \in X$. In this case, the family of all U_x form a covering of X . By this, $f : X \to Y$ is $\mathcal{V}$ - continuous if there exists a covering $\mathcal{U}$ of X, such that for any $x \in X$,

2010 *Mathematics Subject Classification*. Primary 54C56; Secondary 55P55, 54H20
Keywords. Intrinsic shape, proximate sequence, proximate net, continuity over a covering, ε - continuity
Received: 01 September 2014; Accepted: 13 July 2015
Communicated by Miroslav Ćirić
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there exists a neighborhood $U \in \mathcal{U}$ of x, and $V \in \mathcal{V}$ such that $f(U) \subseteq V$. We denote shortly: there exists $\mathcal{U}$, such that $f(\mathcal{U}) \prec \mathcal{V}$.

If $f : X \to Y$ is $\mathcal{V}$ - continuous, then is $\mathcal{W}$ - continuous for any $\mathcal{W}$ such that $\mathcal{V} \prec \mathcal{W}$.
If $\mathcal{V}$ is a covering of Y, and $V \in \mathcal{V}$, the star of V, is the open set $st(V) = \{W \in \mathcal{V} | W \bigcap V \neq \emptyset\}$.
We form a new covering of Y, $st(\mathcal{V}) = \{ \mathrm{st}(V) | V \in \mathcal{V}\}$.

Definition 1.2. *The functions $f, g : X \to Y$ are $\mathcal{V}$ -homotopic, if there exists a function $F : I \times X \to Y$ such that:*

1) *$F : X \times I \to Y$ is $st(\mathcal{V})$ - continuous,*
2) *$F : X \times I \to Y$ is $\mathcal{V}$ - continuous at all points of $X \times \partial I$,*
3) *$F(0, x) = f(x)$, $F(1, x) = g(x)$.*

The relation of $\mathcal{V}$ -homotopy is denoted by $f \underset{\mathcal{V}}{\sim} g$. This is an equivalence relation.

Remark 1.1. *Usually, the condition 2) of the previous definition is formulated as: 2′) there exists an neighbourhood N of $\partial I = \{0, 1\}$ such that $F_{|X \times N}$ is $\mathcal{V}$ - continuous.*

The formulations are actually the same.

The definition of homotopy with conditions 1) and 2), is crucial and neccessary for usual concatenation of homotopies used to prove transitivity of the homotopy (see Theorem 2.2 [27]).

A **proximate net** $(f_{\mathcal{V}}) : X \to Y$, is a net of functions $f_{\mathcal{V}} : X \to Y$, $f_{\mathcal{V}}$, is a $\mathcal{V}$ - continuous function, indexed by all coverings, such that if $\mathcal{V} \succ \mathcal{W}$ then $f_{\mathcal{V}}$ and $f_{\mathcal{W}}$ are $\mathcal{V}$ - homotopic.

Two proximate nets $(f_{\mathcal{V}})$, $\left(f'_{\mathcal{V}}\right) : X \to Y$ are homotopic if $f_{\mathcal{V}}$ and $f'_{\mathcal{V}}$ are $\mathcal{V}$ - homotopic, for all $\mathcal{V} \in \mathrm{cov}\, Y$ which we denote by $(f_{\mathcal{V}}) \sim \left(f'_{\mathcal{V}}\right)$. This is an equivalence relation.

If $(f_{\mathcal{V}}) : X \to Y$ and $(g_{\mathcal{W}}) : Y \to Z$ are proximate nets, then for a covering $\mathcal{W} \in \mathrm{cov}Z$, there exists a covering $\mathcal{V} \in \mathrm{cov}Y$ such that $g_{\mathcal{W}}(\mathcal{V}) \prec \mathcal{W}$. Then the **composition** of these two proximate nets is a proximate net $(h_{\mathcal{W}})$ defined by $(h_{\mathcal{W}}) = (g_{\mathcal{W}} f_{\mathcal{V}}) : X \to Z$.

Paracompact spaces and homotopy classes of proximate nets form the category whose isomorphisms induce classification which coincide with the standard shape classification, i.e., isomorphic spaces in this category have the same shape.

Considering only compact metric spaces, the approach can be simplified.Actually in a compact metric space there exists a sequence of finite coverings, $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots$ with the property that for any covering $\mathcal{V}$, there exists n, such that $\mathcal{V} \succ \mathcal{V}_n$. We call such a sequence - cofinal sequence of finite coverings.

This fact allows working with proximate sequences instead with proximate nets.

Definition 1.3. ***Definition 1.3*** *The sequence (f_n) of functions $f_n : X \to Y$ is a proximate sequence from X to Y, if there exists a cofinal sequence of finite coverings of Y, $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots$, and for all indices $m \geq n$, f_n and f_m are $\mathcal{V}_n$ - homotopic. In this case we say that (f_n) is a proximate sequence over $(\mathcal{V}_n)$.*

If (f_n) and (f'_n) are proximate sequences from X to Y, then there exists a cofinal sequence of finite coverings $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots$ such that (f_n) and (f'_n) are proximate sequences over $(\mathcal{V}_n)$.

Two proximate sequences (f_n) and $(f'_n) : X \to Y$ are homotopic if for some cofinal sequence of finite coverings $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots$, (f_n) and (f'_n) are proximate sequences over $(\mathcal{V}_n)$, and for all integers f_n and f'_n are $(\mathcal{V}_n)$ - homotopic.

If $(f_n) : X \to Y$ is a proximate sequence over $(\mathcal{V}_n)$ and $(g_k) : Y \to Z$ is a proximate sequence over $(\mathcal{W}_k)$, for a covering $\mathcal{W}_k$ of Z, there exists a covering $\mathcal{V}_{n_k}$ of Y such that $g_k(\mathcal{V}_{n_k}) \prec \mathcal{W}_k$. Then, the composition is the proximate sequence $(h_k) = (g_k f_{n_k}) : X \to Z$.

Compact metric spaces and homotopy classes of proximate sequences form the shape category, i.e., isomorphic spaces in this category have the same shape. This notion of shape for compact metric spaces is the standard one; it is proven in [16].

2. Relation to other approaches

The first intrinsic approach to shape of compact metric spaces was made in 1974 by Felt ,[9]. In the paper [9], by $[(X, Y)]_{\mathcal{V}}$ is denoted the set of $\mathcal{V}$ - homotopy classes of $\mathcal{V}$ – continuous functions $f : X \to Y$. The homotopy class of f is denoted by $[f]_{\mathcal{V}}$. If $\mathcal{V} > \mathcal{V}'$ and $f \underset{\mathcal{V}'}{\sim} g$ then $f \underset{\mathcal{V}}{\sim} g$. So, the map $p_{\mathcal{V}\mathcal{V}'} : [X, Y]_{\mathcal{V}'} \to [X, Y]_{\mathcal{V}}$ defined by $p_{\mathcal{V}\mathcal{V}'}([f]_{\mathcal{V}'}) = [f]_{\mathcal{V}}$ is well defined. It is formed the inverse system in the category of sets and functions $([X, Y]_{\mathcal{V}}, p_{\mathcal{V}\mathcal{V}'}, \mathcal{V}$ finite covering). By $\varprojlim_{\mathcal{V}} [X, Y]_{\mathcal{V}}$ is denoted the inverse limit of this inverse system. In the paper [9] a bijection is established between $\varprojlim_{\mathcal{V}} [X, Y]_{\mathcal{V}}$ and the set of all (standard) shape morphisms from X to Y.

However, in this paper, it is not defined the composition of morphisms, and it is not formed the corresponding shape category. That his notion of shape morphism coincides with the one in the current paper, is established by the Theorem 1.3, proved in [27]. In fact it is shown that, there is a bijection between the set $\varprojlim_{\mathcal{V}} [X, Y]_{\mathcal{V}}$ and the set of homotopy classes $[(f_n)]$ of proximate sequences $(f_n) : X \to Y$.

In the paper [19], J.M.R. Sanjurjo gave an intrinsic description of the shape category of compact metric spaces. His approach is based on the notion of ε -continuity, similar notion to the notion of $\mathcal{V}$ - continuity. A function $f : X \to Y$ is ε - continuous, if for every $x \in X$ there is a neighborhood of x whose image lies in the ε - neighbourhood of $f(x)$. The functions $f, g : X \to Y$ are ε - homotopic, if there exists a ε - continuous function $F : X \times I \to Y$ such that for every $x \in X$, $F(x, 0) = f(x)$ and $F(x, 1) = g(x)$. The relation of ε - homotopy is an equivalence relation on the set of ε - continuous functions.

A proximate net, from X to Y, is a sequence of (not necessarily continuous) functions $f_n : X \to Y$ such that for every $\varepsilon > 0$ there is an index n_0 such that f_n is ε - homotopic to f_{n+1} for every $n \geq n_0$. We denote proximate nets with $(f_n) : X \to Y$, or just with (f_n) .

Two proximate nets (f_n) and (f'_n) are homotopic if for every $\varepsilon > 0$, f_n is ε - homotopic to f'_n for almost every n (i.e., there exists n_0 such that f_n is ε - homotopic to f'_n for all $n \geq n_0$). The symbol $[(f_n)]$ denotes a homotopy class.

The definition of composition is more involved.

At first is defined a null sequence $\varepsilon_1 \geq \varepsilon_2 \geq \cdots \geq \varepsilon_n \geq \ldots$ of positive numbers, as a sequence of positive numbers such that $\varepsilon_n \to 0$ when $n \to 0$. Let $[(f_n)] : X \to Y$ and $[(g_n)] : Y \to Z$ be classes of proximate nets, and $(f_n) : X \to Y$ and $(g_n) : Y \to Z$ be their representatives. A null sequence of positive numbers $\varepsilon_1 \geq \varepsilon_2 \geq \cdots \geq \varepsilon_n \geq \ldots$ can be chosen such that g_n is $\frac{\varepsilon_{n_0}}{2}$ homotopic to g_{n_0} for every $n \geq n_0$, and a null sequence $\delta_1 \geq \delta_2 \geq \cdots \geq \delta_n \geq \ldots$ can be chosen such that $d(g_n(y), g_n(y')) < \varepsilon_n$ for every δ_n - close points y and y' in Y, and sequence of indices $k_1 < k_2 < \cdots < k_n < \ldots$ is chosen such that f_k is $\frac{\delta_n}{2}$ homotopic to f_{k_n} for every $k \geq k_n$. Then, the composition of homotopy classes is defined by $[(g_n)][(f_n)] = [(g_n f_{k_n})]$.

In [19] is proven that the composition of classes is well defined, and that compact metric spaces together with homotopy classes of proximate net form a category isomorphic to standard shape category.

In [15] is proven that to every proximate net in the sense of Sanjurjo we can associate a proximate sequence consisting of the same functions. If two proximate nets are homotopic then the associated proximate sequences are homotopic. In [15] is shown that this correspondence is bijective and that in this way is obtained a categorical isomorphism between shape categories of Sanjurjo [19] and Shekutkovski [27].

In [11], Kieboom, using actually the same approach as in this paper, presented an intrinsic definition of shape for paracompact spaces and showed that for paracompact spaces his notion of shape is equivalent to the notion of shape from [13].

In the paper [6] is presented intrinsic definition of shape for all topological spaces considering only normal coverings. Instead of indexing proximate nets $(f_{\mathcal{V}})$ by normal coverings $\mathcal{V}$, in the paper proximate nets (f_a) are indexed by finite sets of coverings $a = \{\mathcal{V}_0, \mathcal{V}_1, \ldots, \mathcal{V}_n\}$ having a maximal element (i.e., a covering that refines all other coverings of that finite set, and is not refined by any other covering of that finite set). These sets are ordered by inclusion. This ordering is cofinite i.e., each a has only finite number of predecessors. It is shown that obtained shape category is isomorphic to the shape category for topological spaces from [13].

Proper shape is a type of shape more appropriate for non compact locally compact spaces. In this theory functions $f : X \to Y$ are proper i.e., for any compact D in Y, there exists a compact C in X such that $f(X \setminus C) \subset Y \setminus D$ [1]. This definition is more general and coincides with the usual one for continuous functions.

In [1], Akaike and Sakai develop proper n - shape theory using an intrinsic approach. Following the same line, in [23], the authors describe proper shape theory by intrinsic approach, and proved that the obtained notion of proper shape coincides with the previous main notion of proper shape (in [3], [22] is shown the equivalence of several external approaches to proper shape).

The intrinsic definition of the category of proper shape for locally compact separable metric spaces follows the steps in the beginning of Section 1. Only open covers consisting of sets with compact closure are considered. This additional condition is connected with proper functions.

Definition 2.1. *The proper functions $f, g : X \to Y$ are $\mathcal{V}$ - properly homotopic, if there exists a proper function $F : I \times X \to Y$ such that:*

1) *$F : X \times I \to Y$ is st($\mathcal{V}$) - continuous,*
2) *$F : X \times I \to Y$ is $\mathcal{V}$ - continuous at all points of $X \times \partial I$,*
3) *$F(0, x) = f(x)$, $F(1, x) = g(x)$.*

A **proper proximate net** $(f_{\mathcal{V}}) : X \to Y$ is a net of functions $f_{\mathcal{V}} : X \to Y$, $f_{\mathcal{V}}$ is a proper $\mathcal{V}$-continuous function, indexed by all coverings, such that if $\mathcal{V} > \mathcal{W}$ then $f_{\mathcal{V}}$ and $f_{\mathcal{W}}$ are properly $\mathcal{V}$ - homotopic.

The proper homotopy classes $[(f_{\mathcal{V}})]_p$ are morphisms of the category of proper shape.

Remark 2.1. *In the original approach there was an additional requirement coverings to be star finite. We mention that in the case of locally compact spaces any covering has a locally finite refinement iff any covering has a star finite refinement. So, for paracompact locally compact spaces any covering has:*

1) *locally finite refinement*
2) *star finite refinement.*

(i.e, both locally finite coverings and star finite coverings are cofinite in the set of all coverings) [8]. By this, the additional requirement can be omitted.

3. Results using intrinsic shape

In the paper [28] using the intrinsic definition of shape is proven an analogue of well known Borsuk's theorem: Let X and Y be compact metric spaces. Then for any approximative map f from X towards Y, there exists an unique map $\hat{f} : C(X) \to C(Y)$ such that for any component C_0 of X, the restriction $\hat{f}$ to C_0 is an approximative map from C_0 to $\hat{f}(C_0)$. Moreover, if f and g define the same shape morphism, then $\hat{f} = \hat{g}$.

The non-compact analogue of Borsuk theorem is:

Theorem 3.1. *Suppose X and Y are locally compact metric spaces with compact spaces of quasicomponents QX and QY. If a shape morphism $f : X \to Y$ is presented by a proximate net $(f_{\mathcal{V}}) : X \to Y$, then there exists a unique mapping $(f_{\mathcal{V}})_{\#} : QX \to QY$, such that if $(f_{\mathcal{V}}), (g_{\mathcal{V}}) : X \to Y$ are homotopic proximate nets i.e., define the same shape morphism then $(f_{\mathcal{V}})_{\#} = (g_{\mathcal{V}})_{\#}$.*

Moreover, for any quasicomponent Q of X, the restriction of f to any open set W containing $(f_{\mathcal{V}})_{\#}(Q)$, presented by the restriction of proximate net $(f_{\mathcal{V}}) : Q \to W$ is also a shape morphism.

Mention that the conclusion in the last sentence of the theorem is obtained, since the intrinsic approach is used i.e., in the proof are used the neighborhoods of the original space, not external spaces.

In [27] is defined strong shape for compact metric spaces by intrinsic approach. First definition of strong shape for metric compacta was in the paper [18]. We refer to book [14], and for equivalence of different approaches for metric compacta to [12]. The definition of strong shape in [27], is based on the notion of strong proximate sequence.

The sequence of pairs $(f_n, f_{n,n+1})$ of functions $f_n : X \to Y$ and $f_{n,n+1} : X \times I \to Y$, is a *strong proximate sequence* from X to Y, if there exists a cofinal sequence of finite coverings,$\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots$ of Y, such that for each natural number n, $f_n : X \to Y$ is a $(\mathcal{V}_n)$ - continuous function and $f_{n,n+1} : X \times I \to Y$, is a homotopy connecting $(\mathcal{V}_n)$ - continuous functions $f_n : X \to Y$ and $f_{n+1} : X \to Y$.

We say that $(f_n, f_{n,n+1})$ is a strong proximate sequence over $(\mathcal{V}_n)$.

If $(f_n, f_{n,n+1})$ and $(f'_n, f'_{n,n+1})$ are strong proximate sequences from X to Y, then there exists a cofinal sequence of finite coverings $(\mathcal{V}_n)$ such that $(f_n, f_{n,n+1})$ and $(f'_n, f'_{n,n+1})$ are strong proximate sequences over $(\mathcal{V}_n)$.

Two strong proximate sequences $(f_n, f_{n,n+1})$ and $(f'_n, f'_{n,n+1}) : X \to Y$ are ***homotopic*** if there exists a strong proximate sequence $(F_n, F_{n,n+1}) : X \times I \to Y$ over $(\mathcal{V}_n)$ such that

1) $F_n : X \times I \to Y$ is a homotopy between $\mathcal{V}_n$ – continuous maps f_n and f'_n,
2) $F_{n,n+1} : X \times I \times I \to Y$ is a $\mathrm{st}^2(\mathcal{V}_n)$ - continuous function, at all poins from $X \times \partial I^2$ is $st(\mathcal{V}_n)$ – continuous,
3) $F_{n,n+1}(x, t, 0) = f_{n,n+1}(x, t)$, $F_{n,n+1}(x, t, 1) = f'_{n,n+1}(x, t)$.

Homotopy classes of strong proximate sequences are morphisms of strong shape category.

In compact metric space, the existence of cofinal sequence of coverings $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots$ of Y, allows to define strong shape theory using only homotopies of second order.

In more general case of paracompact spaces, homotopies of all orders must be considered. In [18], is described the construction for (strongly) paracompact spaces. We form all finite sets of coverings of Y, $a = \{\mathcal{V}_0, \mathcal{V}_1, \ldots, \mathcal{V}_n\}$, having a maximal element (i.e., a covering that refines all other coverings of that finite set, and is not refined by any other covering of that finite set). The maximal element is denoted by $\max a$.

Let $\Delta^n \subset R^n$, $\Delta^n = \{(t_1, t_2) | 1 \geq t_1 \geq t_2 \geq \cdots \geq t_n \geq 0\}$ be the non standard n-simplex.

Definition 3.1. *A* ***coherent proximate net*** $\underline{f} : X \to Y$, *consists of functions* $\underline{f} = \{f_{\underline{a}} | \forall \underline{a} = (a_0, a_1, ..., a_n),\ a_n \subset ... \subset a_0\}$, *such that each* $f_{\underline{a}} : X \times \Delta^n \to Y$ *is* $st^n \max a_0$ *- continuous and is* $st^{n-1} \max a_0$ *- continuous on* $X \times \partial\Delta^n$, *and the following coherence condition is satisfied*

$$f_{\underline{a}}(t_1, t_2, ..., t_n, x) = \begin{cases} f_{a_1...a_n}(t_2, ..., t_n, x), & t_1 = 1 \\ f_{a_0...\hat{a}_i...a_n}(t_1, ..., \hat{t}_i, ..., t_n, x), & t_i = t_{i+1} \\ f_{a_0...a_{n-1}}(t_1, ..., t_{n-1}, x), & t_n = 0. \end{cases}$$

The coherent proximate net, will be shortly denoted by $\underline{f} = (f_{\underline{a}})$.

Definition 3.2. *Coherent proximate nets* $\underline{f}, \underline{g} : X \to Y$ *are* ***homotopic****, if there exists a coherent proximate net* $\underline{H} = (H_{\underline{a}})$ *, such that* $H_{\underline{a}} : X \times \Delta^n \times I \to Y$ *is* $st^{n+1} \max a_0$ *- continuous,* $st^n \max a_0$ – continuous *on* $X \times \partial(\Delta^n \times I)$ *and the following conditions are satisfied:*

$$H_{\underline{a}}(x, \underline{t}, 0) = f_{\underline{a}}(x, \underline{t}),$$
$$H_{\underline{a}}(x, \underline{t}, 1) = g_{\underline{a}}(x, \underline{t}).$$

Finite sets of coverings with a maximal element, are ordered by inclusion, and this ordering is cofinite i.e., each a has only finite number of predecessors. This fact, allows composition of coherent proximate nets to be defined, although not in an easy way. In this way category of strong shape is formed for (strongly) paracompact spaces [18]. In [2] is shown that strong shape category of metric compacta is a subcategory of the last category.

In [26] a proper shape is presented using an intrinsic definition and proximate sequences indexed only by finite coverings consisting of open sets with compact boundary. The construction follows the line from Section 2, and this notion of shape is denoted as proper shape over finite coverings.

One of the main results in proper shape is the following theorem: If X and Y have the same proper shape, then their end point compactifications FX and FY have the same shape [4]. In the paper is proved the following

Theorem 3.2. *X and Y are locally compact separable metric spaces with compact spaces of quasicomponents. If X and Y have the same proper shape over finite coverings then their end points compactifications FX and FY have the same shape.*

Based on recent techniques of intrinsic shape in [31], is defined proximate fundamental group, an invariant of pointed intrinsic shape of a space [33].

Finally, one of the main applications of shape is in dynamical systems. It seems that intrinsic approach to shape is the most natural way how shape theory should be applied to dynamical systems. In [29] using the intrinsic approach is generalized the theorem about shape of compact attractors in non-compact spaces: If A is global attractor for a given semi-dynamical system then the shape of the phase space coincides with the shape of the attractor.

In [32] is proven that components of chain recurrent set of a dynamical system defined on a compact manifold, possess a stronger connectivity known as joinability (or pointed 1 - movability in the sense of Borsuk).

In [34], using techniques of intrinsic shape is proved a theorem about the shape of members of Morse decomposition and shape of the chain recurrent set.

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