



Dynamics of Stochastic Holling II Predator-Prey under Markovian-Switching with Jumps

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Abstract. In this paper, a stochastic Holling II predator-prey model under Markovian switching with jumps is investigated. The aim is to find out how the Markovian switching and the jump noise affect the dynamics of this model. Firstly, we study the properties of the solutions, for example, the existence and uniqueness of the global positive solution, the uniform boundedness of the p th moment and the pathwise estimation. Secondly, sufficient criteria for extinction and strong persistence in the mean are established. Results show that jump noise can essentially change the nature of the system, i.e., it can make strongly persistent species extinct and extinct species persistent. We also observe that both the overall extinction and strong persistence in the mean have close relationships with the stationary probability distribution of the Markov chain. Finally, numerical examples are introduced to illustrate the results.

1. Introduction

In the ecological sciences, the dynamical behavior of interacting species is one of the main problems which are extensively studied. The interspecific interactions among species such as competition, cooperation and predation are the main research targets. So the dynamic relationship between predators and their prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance [1]. Since the relationship between predator and prey are usually complicated and diverse, Holling [2] gave three different kinds of functional response of the predator to the prey density. The classical deterministic predator-prey system with Holling II functional

2010 *Mathematics Subject Classification.* Primary 60H10; Secondary 34F05, 92D25

Keywords. Holling II predator-prey model, Markov chain, Jumps, Persistence and extinction

Received: 04 March 2014; Accepted: 03 April 2014

Communicated by Miljana Jovanović

Research supported by the Natural Science Foundation of P.R. China (Nos.1140113611501148) Natural Science Foundation of Shandong Province, China (Nos. ZR2014AL008), the Fundamental Research Funds for the Central Universities of China (No. 15CX02080A) Natural Scientific Research Innovation Foundation in Harbin Institute of Technology (HIT.NSRIF.2015103), Scientific Research Foundation of Harbin Institute of Technology at Weihai (HIT(WH)201420).

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response has the following form:

$$\begin{aligned}\frac{dx(t)}{dt} &= x(t) \left(r_1 - a_{11}x(t) - \frac{a_{12}y(t)}{1+x(t)} \right), \\ \frac{dy(t)}{dt} &= y(t) \left(-r_2 + \frac{a_{21}x(t)}{1+x(t)} - a_{22}y(t) \right),\end{aligned}\tag{1}$$

where $x(t)$ and $y(t)$ denote the prey and predator population size, respectively, at time t ; r_1 is the intrinsic growth rate of the prey; r_2 denotes the death rate of predator; a_{11} measures the strength of competition among individuals of prey species; a_{12} is the capturing rate; a_{21} is the conversion rate; a_{22} is the strength of competition among individuals of species predator. In recent years, qualitative analysis of the predator-prey model (1) with Holling II functional response and its extension have been studied by several authors, for example, see [3], [4], [5], [6], [7], [8], [9], [10] and [11].

On the other hand, population dynamics are inevitably affected by various environmental noise. So stochastic population dynamics have attracted a great attention of many mathematicians and biologists in recent years. One type of noise we know well is the white noise. There are many good papers on the study of population models perturbed by white noise (see [12]-[18] and references cited therein). And stochastic predator-prey models are also been well studied, for example, [19], [20] and [21]. They mainly focus on the qualitative analysis, such as persistence and extinction, permanence, global stability and so on. If the intrinsic growth rate of the prey population and the death rate of the predator population are affected by environmental white noise, then the stochastic predator-prey with Holling II functional response takes the following form:

$$\begin{aligned}dx(t) &= x(t) \left(r_1 - a_{11}x(t) - \frac{a_{12}y(t)}{1+x(t)} \right) dt + \sigma_1 x(t) dB_1(t), \\ dy(t) &= y(t) \left(-r_2 + \frac{a_{21}x(t)}{1+x(t)} - a_{22}y(t) \right) dt + \sigma_2 y(t) dB_2(t),\end{aligned}\tag{2}$$

where $B_i(t)$ is mutually independent Brownian motion, σ_i is a positive constant representing the intensity of the white noise, $i = 1, 2$.

However, some sudden environment shocks, for example, earthquakes, hurricanes, epidemics, may occur sometimes, and the solutions of population models are no longer continuous at this time. So stochastic models (2) perturbed only by white noise can not fit the reality well. Bao et al. [22], [23] suggested that stochastic models driven by Poisson random measure can describe these phenomena sometimes. The stochastic predator-prey with Holling II functional response driven by Poisson random measure can be described as

$$\begin{aligned}dx(t) &= x(t) \left(r_1 - a_{11}x(t) - \frac{a_{12}y(t)}{1+x(t)} \right) dt + \sigma_1 x(t) dB_1(t) + \int_{\mathbf{Y}} c_1(u) x(t^-) N(dt, du), \\ dy(t) &= y(t) \left(-r_2 + \frac{a_{21}x(t)}{1+x(t)} - a_{22}y(t) \right) dt + \sigma_2 y(t) dB_2(t) + \int_{\mathbf{Y}} c_2(u) y(t^-) N(dt, du),\end{aligned}\tag{3}$$

here $x(t^-)$ is the left limit of $x(t)$, N is a Poisson counting measure with characteristic measure λ on a measurable subset \mathbf{Y} of $(0, \infty)$ with $\lambda(\mathbf{Y}) < \infty$, and $\tilde{N}(dt, du) := N(dt, du) - \lambda(du)dt$.

Sometimes the population systems may be affected by color noise. Color noise can be described as a random switching among different regimes of environment. The switching is memoryless and the waiting time for the next switch follows an exponential distribution [24]. Hence the random switching can be modeled by a finite-state Markov chain. Then model (3) under Markov switching becomes the following model:

$$\begin{aligned}dx(t) &= x(t) \left(r_1(\xi(t)) - a_{11}(\xi(t))x(t) - \frac{a_{12}(\xi(t))y(t)}{1+x(t)} \right) dt + \sigma_1(\xi(t))x(t) dB_1(t) + \int_{\mathbf{Y}} c_1(\xi(t), u) x(t^-) N(dt, du), \\ dy(t) &= y(t) \left(-r_2(\xi(t)) + \frac{a_{21}(\xi(t))x(t)}{1+x(t)} - a_{22}(\xi(t))y(t) \right) dt + \sigma_2(\xi(t))y(t) dB_2(t) + \int_{\mathbf{Y}} c_2(\xi(t), u) y(t^-) N(dt, du),\end{aligned}\tag{4}$$

where $\xi(t)$ is a continuous-time Markov chain with finite-state space $S = \{1, 2, \dots, n\}$. And the generator $\Gamma = (\gamma_{ij})_{n \times n}$ is given by

$$\mathbb{P}\{\xi(t + \Delta) = j | (\xi(t) = i)\} = \begin{cases} \gamma_{ij}\Delta + o(\delta), & \text{if } i \neq j, \\ 1 + \gamma_{ii}\Delta + o(\delta), & \text{if } i = j, \end{cases}$$

where $\Delta > 0$, $\gamma_{ij} \geq 0$ ($i \neq j$) is the transition rate from i to j while $\sum_{i=1}^n \gamma_{ij} = 0$. Eq.(4) in regime $j \in S$ obeys

$$\begin{aligned} dx(t) &= x(t) \left(r_1(j) - a_{11}(j)x(t) - \frac{a_{12}(j)y(t)}{1+x(t)} \right) dt + \sigma_1(j)x(t)dB_1(t) + \int_{\mathcal{Y}} c_1(j, u)x(t^-)N(dt, du), \\ dy(t) &= y(t) \left(-r_2(j) + \frac{a_{21}(j)x(t)}{1+x(t)} - a_{22}(j)y(t) \right) dt + \sigma_2(j)y(t)dB_2(t) + \int_{\mathcal{Y}} c_2(j, u)y(t^-)N(dt, du). \end{aligned}$$

Regime-switching jump diffusions have numerous applications in insurance, biology and medicine, economics and so on [25]. In recent years, many important research papers on the basic properties of regime-switching jump diffusions have been reported, for instance, G. Yin and F. Xi [25], Z. Yang and G. Yin [26]. But to the best of our knowledge, to this day, the survival analysis for the predator-prey population systems with regime-switching jump diffusions has not been reported. Motivated by these, in this paper, we mainly consider asymptotic behaviors of stochastic Holling II predator-prey model (4) under Markovian-switching with jumps.

Compared with the literature, contributions and novelties of the current work are as follows:

- (1) We introduce three different environmental noise to the predator-prey model with Holling II functional response.
- (2) We give the sufficient criteria for the extinction and persistence of the predator and prey species.
- (3) We reveal that how the Markovian-switching affects the population dynamics.
- (4) We analyze the impacts of jump noise on the population dynamics.

The remaining part of this paper is organized as follows. In section 2, we show that the Holling II predator-prey model under Markovian switching with jumps has a unique global positive solution and consider the asymptotic pathwise behavior of the solution. We establish the sufficient conditions for the extinction, strong persistence in the mean of the predator and prey species in section 3. Results show the effect of color noise on the system is mainly determined by the stationary distribution of its Markov chain, and the jump noise can change the survival situations of the population. Finally we introduce some examples to illustrate our main results in section 4.

2. Properties of the Solution

Throughout this paper, $\mathbb{R}_+ := (0, \infty)$. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions. Assume further that Markov chain $\xi(t)$ is irreducible and has a unique stationary distribution $\pi = \{\pi_1, \pi_2, \dots, \pi_n\}$ which can be determined by equation

$$\pi\Gamma = 0 \tag{5}$$

subject to

$$\sum_{i=1}^n \pi_i = 1, \quad \text{and } \pi_i > 0, \quad \forall i \in S.$$

We assume that Markov chain $\xi(\cdot)$, Brownian motion $B(\cdot)$ and Poisson counting measure $N(\cdot)$ are mutually independent.

Throughout the rest of this paper, we always assume the following assumption holds:

(A). For each $i \in S$, $r_1(i)$, $r_2(i)$, $a_{11}(i)$, $a_{12}(i)$, $a_{21}(i)$, $a_{22}(i)$, $\sigma_1(i)$ and $\sigma_2(i)$ are positive constants, and there exist $c_{j*}(i) > -1$ and $c_j^*(i) > |c_{j*}(i)|$ such that $c_{j*}(i) \leq c_j(i, u) \leq c_j^*(i)$ for any $u \in \mathcal{Y}$, $j = 1, 2$.

For convenience and simplicity, let

$$\begin{aligned}\beta_1(\xi(t)) &= r_1(\xi(t)) - \frac{\sigma_1^2(\xi(t))}{2} + \int_Y \ln(1 + c_1(\xi(t), u))\lambda(du), \\ \beta_2(\xi(t)) &= -r_2(\xi(t)) - \frac{\sigma_2^2(\xi(t))}{2} + \int_Y \ln(1 + c_2(\xi(t), u))\lambda(du);\end{aligned}$$

and for any constant vector $f(i)_{i \in S}$, define

$$\check{f} = \min_{i \in S} f(i), \quad \check{f} = \max_{i \in S} f(i).$$

Theorem 2.1. For any initial value $(x(0), y(0)) \in \mathbb{R}_+^2$ and $\xi(0) \in S$, there is a unique global positive solution $(x(t), y(t))$ to Eq.(4) for all $t \geq 0$ almost surely.

Proof. Consider the following system

$$\begin{aligned}du(t) &= \left[\beta_1(\xi(t)) - a_{11}(\xi(t)) \exp\{u(t)\} - \frac{a_{12}(\xi(t)) \exp\{v(t)\}}{1 + \exp\{u(t)\}} \right] dt \\ &\quad + \sigma_1(\xi(t)) dB_1(t) + \int_Y \ln(1 + c_1(\xi(t), u)) \tilde{N}(dt, du), \\ dv(t) &= \left[\beta_2(\xi(t)) + \frac{a_{21}(\xi(t)) \exp\{u(t)\}}{1 + \exp\{u(t)\}} - a_{22}(\xi(t)) \exp\{v(t)\} \right] dt \\ &\quad + \sigma_2(\xi(t)) dB_2(t) + \int_Y \ln(1 + c_2(\xi(t), u)) \tilde{N}(dt, du),\end{aligned}\tag{6}$$

with the initial value $(u(0), v(0)) = (\ln x(0), \ln y(0))$. The coefficients of (6) is locally Lipschitz continuous, so system (6) has a unique local solution $(u(t), v(t))$ on $t \in [0, \tau)$, where τ is the explosion time [27], [28]. Hence from Itô's formula it follows that $(x(t), y(t)) = (\exp\{u(t)\}, \exp\{v(t)\})$ is the unique local solution to system (4) with initial value $(x(0), y(0))$ on $t \in [0, \tau)$. The proof of its global solution is almost identical to of equations with regime switching driven by Brownian motion, and we refer the reader to the account of [29] and [30]. \square

Based on this fundamental theorem, we can study the asymptotic behavior of the solution.

Theorem 2.2. For any initial value $(x(0), y(0)) \in \mathbb{R}_+^2$ and $\xi(0) \in S$, for any $p > 0$, the solution satisfies

$$\limsup_{t \rightarrow \infty} \mathbb{E}(x^p(t) + y^p(t)) \leq K,$$

where here, and in the sequel, we denote by K a generic positive constant.

Proof. Applying Itô's formula, we obtain

$$\mathbb{E}(e^t(x^p(t) + y^p(t))) = x^p(0) + y^p(0) + \mathbb{E} \int_0^t e^s F(s) ds,$$

where

$$\begin{aligned}F(t) &= -a_{11}(\xi(t)) p x^{p+1} + \left(1 + p r_1(\xi(t)) + \frac{p(p-1)}{2} \sigma_1^2(\xi(t)) + \int_Y ((1 + c_1(\xi(t), u))^p - 1) \lambda(du) \right) x^p - \frac{a_{12}(\xi(t)) p x^p y}{1+x} \\ &\quad - a_{22}(\xi(t)) p y^{p+1} + \left(1 - p r_2(\xi(t)) + \frac{p(p-1)}{2} \sigma_2^2(\xi(t)) + \int_Y ((1 + c_2(\xi(t), u))^p - 1) \lambda(du) \right) y^p + \frac{a_{21}(\xi(t)) p x y^p}{1+x} \\ &\leq -\hat{a}_{11} p x^{p+1} + \left(1 + p \check{r}_1 + \frac{p(p+1)}{2} \check{\sigma}_1^2 + \int_Y ((1 + \check{c}_1)^p - 1) \lambda(du) \right) x^p \\ &\quad - \hat{a}_{22} p y^{p+1} + \left(1 - p \hat{r}_2 + \frac{p(p+1)}{2} \check{\sigma}_2^2 + \int_Y ((1 + \check{c}_2)^p - 1) \lambda(du) + \check{a}_{21} p \right) y^p \\ &\leq K.\end{aligned}$$

Hence

$$\mathbb{E}(e^t(x^p(t) + y^p(t))) \leq x^p(0) + y^p(0) + Ke^t.$$

So

$$\limsup_{t \rightarrow \infty} \mathbb{E}(e^t(x^p(t) + y^p(t))) \leq K.$$

We complete the proof. \square

Now we give some lemmas which are used in the analysis in what follows.

Lemma 2.3. For any initial value $(x(0), y(0)) \in \mathbb{R}_+^2$ and $\xi(0) \in S$, the solution to system (4) has the property

$$\limsup_{t \rightarrow \infty} \frac{\ln x(t)}{t} \leq 0 \text{ a.s.}, \quad (7)$$

$$\limsup_{t \rightarrow \infty} \frac{\ln y(t)}{t} \leq 0 \text{ a.s.} \quad (8)$$

Proof. The proof is similar to the proof of Lemma 3.2 in [31], so we omit it here. \square

Lemma 2.4. If for any $i \in S$,

$$\beta_2(i) = -r_2(i) - \frac{\sigma_2^2(i)}{2} + \int_Y \ln(1 + c_2(i, u))\lambda(du) \geq 0, \quad (9)$$

then for any initial value $(x(0), y(0)) \in \mathbb{R}_+^2$ and $\xi(0) \in S$,

$$\liminf_{t \rightarrow \infty} \frac{\ln y(t)}{t} \geq 0 \text{ a.s.} \quad (10)$$

Proof. Consider the auxiliary equation

$$\begin{cases} dY(t) = Y(t)(-r_2(\xi(t)) - a_{22}(\xi(s))Y(t))dt + \sigma_2(\xi(t))Y(t)dB_2(t) + \int_Y c_2(\xi(t), u)Y(t^{-1})N(dt, du) \\ Y(0) = y(0) \end{cases} \quad (11)$$

From the stochastic comparison theorem it follows $y(t) \geq Y(t)$. Using the proof of Lemma 3.3 in [31], the solution of Eq. (11) has the property

$$\liminf_{t \rightarrow \infty} \frac{\ln Y(t)}{t} \geq 0,$$

hence the desired assertion (10) holds. \square

3. Persistence and Extinction

In the previous section, we have discussed some properties of the solution to the system (4). Now in this section we will analyze the survival of system (4). To proceed, we need some appropriate definitions.

Definition 3.1. [32]

- (1) $x(t)$ is said to be extinct if $\lim_{t \rightarrow \infty} x(t) = 0$ a.s..
- (2) $x(t)$ is said to be strongly persistent in the mean if $\liminf_{t \rightarrow \infty} t^{-1} \int_0^t x(s)ds > 0$ a.s..

Now we are in the position to give our main results of this section.

Theorem 3.2.

- (i) If $\sum_{i=1}^n \pi_i \beta_1(i) < 0$ and $\sum_{i=1}^n \pi_i \beta_2(i) < 0$, then the prey and predator are both extinct;
- (ii) If $\sum_{i=1}^n \pi_i \beta_1(i) < 0$ and $\sum_{i=1}^n \pi_i \beta_2(i) > 0$, then the prey population is extinct while the predator population is strongly persistent in mean;
- (iii) If $\sum_{i=1}^n \pi_i \beta_1(i) > 0$ and $\sum_{i=1}^n \pi_i (\beta_2(i) + a_{21}(i)) < 0$, then the predator population is extinct and the prey population is strongly persistent in mean;
- (iv) If for any $i \in S$, $\beta_2(i) > 0$ and $\sum_{i=1}^n \pi_i \beta_1(i) - \frac{\hat{a}_{12}}{\hat{a}_{22}} \sum_{i=1}^n \pi_i (\beta_2(i) + a_{21}(i)) > 0$, then both the predator population and the prey population are strongly persistent in the mean.

Proof. (i). Theorem 1 shows that the solution $(x(t), y(t))$ with positive initial value will remain in \mathbb{R}_+^2 . By the generalized Itô's formula, we get

$$d \ln x(t) = \left[\beta_1(\xi(t)) - a_{11}(\xi(t))x(t) - \frac{a_{12}(\xi(t))y(t)}{1+x(t)} \right] dt + \sigma_1(\xi(t))dB_1(t) + \int_{\mathbf{Y}} \ln(1 + c_1(\xi(t), u))\tilde{N}(dt, du).$$

$$d \ln y(t) = \left[\beta_2(\xi(t)) + \frac{a_{21}(\xi(t))x(t)}{1+x(t)} - a_{22}(\xi(t))y(t) \right] dt + \sigma_2(\xi(t))dB_2(t) + \int_{\mathbf{Y}} \ln(1 + c_2(\xi(t), u))\tilde{N}(dt, du).$$

Integrating from 0 to t yields

$$\ln x(t) = \ln x(0) + \int_0^t \beta_1(\xi(s))ds - \int_0^t a_{11}(\xi(s))x(s)ds - \int_0^t \frac{a_{12}(\xi(s))y(s)}{1+x(s)}ds + M_{11}(t) + M_{12}(t), \quad (12)$$

$$\ln y(t) = \ln y(0) + \int_0^t \beta_2(\xi(s))ds - \int_0^t a_{22}(\xi(s))y(s)ds + \int_0^t \frac{a_{21}(\xi(s))x(s)}{1+x(s)}ds + M_{21}(t) + M_{22}(t), \quad (13)$$

where $M_{j1}(t) = \int_0^t \sigma_j(\xi(s))dB_1(s)$ and $M_{j2}(t) = \int_0^t \int_{\mathbf{Y}} \ln(1 + c_j(\xi(s), u))\tilde{N}(ds, du)$, $j = 1, 2$ are all martingale terms. Then by Proposition 2.4 in [33],

$$\langle M_{j1} \rangle(t) = \int_0^t \sigma_j^2(\xi(s))ds \leq \check{\sigma}_j^2 t, \quad j = 1, 2,$$

$$\langle M_{j2} \rangle(t) = \int_0^t \int_{\mathbf{Y}} [\ln(1 + c_j(\xi(s), u))]^2 \lambda(du)ds \leq t[\ln(1 + \check{c}_j^*)]^2 \lambda(\mathbf{Y}), \quad j = 1, 2,$$

where $\langle M \rangle(t) := \langle M, M \rangle$ is Meyer's angle bracket process.

By the strong law of large numbers for local martingales [34] again, we then obtain

$$\lim_{t \rightarrow \infty} \frac{M_{j1}(t)}{t} = 0 \quad a.s. \quad \text{and} \quad \lim_{t \rightarrow \infty} \frac{M_{j2}(t)}{t} = 0 \quad a.s., \quad j = 1, 2. \quad (14)$$

From (12), we get

$$\ln x(t) \leq \ln x(0) + \int_0^t \beta_1(\xi(s))ds + M_{11}(t) + M_{12}(t).$$

Dividing by t on both sides and combining (14), we deduce

$$\begin{aligned} \limsup_{t \rightarrow \infty} \frac{\ln x(t)}{t} &\leq \limsup_{t \rightarrow \infty} \frac{1}{t} \int_0^t \beta_1(\xi(s))ds \\ &= \sum_{i=1}^n \pi_i \beta_1(i) \quad a.s., \end{aligned}$$

where we use the ergodic property of the Markov chain. Therefore, if $\sum_{i=1}^n \pi_i \beta_1(i) < 0$, then $\lim_{t \rightarrow \infty} x(t) = 0$ a.s., that is to say, the prey population $x(t)$ is extinctive. Furthermore, it implies that

$$\lim_{t \rightarrow \infty} \frac{1}{t} \int_0^t x(s) ds = 0 \text{ a.s.} \quad (15)$$

Similarly, dividing by t on both sides of (13) results in

$$\frac{\ln y(t)}{t} \leq \frac{\ln y(0)}{t} + \frac{1}{t} \int_0^t \beta_2(\xi(s)) ds + \frac{\check{a}_{21}}{t} \int_0^t x(s) ds + \frac{M_{21}(t)}{t} + \frac{M_{22}(t)}{t}.$$

By the ergodic property of the Markov chain, combining (14) and (15), we obtain

$$\limsup_{t \rightarrow \infty} \frac{\ln y(t)}{t} \leq \limsup_{t \rightarrow \infty} \frac{1}{t} \int_0^t \beta_2(\xi(s)) ds = \sum_{i=1}^n \pi_i \beta_2(i).$$

So if $\sum_{i=1}^n \pi_i \beta_2(i) < 0$, then $\lim_{t \rightarrow \infty} y(t) = 0$ a.s. which implies that the predator population is extinct.

(ii). From (i) it follows that if $\sum_{i=1}^n \pi_i \beta_1(i) < 0$, the prey population is extinct. Now we prove that the predator population is strongly persistent in the mean under the condition $\sum_{i=1}^n \pi_i \beta_2(i) > 0$. Dividing by t on both sides of (13) we obtain after rearrangement

$$\frac{1}{t} \int_0^t a_{22}(\xi(s)) y(s) ds = -\frac{\ln y(t)}{t} + \frac{\ln y(0)}{t} + \frac{1}{t} \int_0^t \beta_2(\xi(s)) ds + \frac{1}{t} \int_0^t \frac{a_{21}(\xi(s)) x(s)}{1+x(s)} ds + \frac{M_{21}(t)}{t} + \frac{M_{22}(t)}{t}.$$

Taking the inferior limit, together with (8), (14) and (15), we find

$$\begin{aligned} \liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t a_{22}(\xi(s)) y(s) ds &\geq \liminf_{t \rightarrow \infty} \left(-\frac{\ln y(t)}{t}\right) + \liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t \beta_2(\xi(s)) ds \\ &= -\limsup_{t \rightarrow \infty} \frac{\ln y(t)}{t} + \sum_{i=1}^n \pi_i \beta_2(i) \\ &\geq \sum_{i=1}^n \pi_i \beta_2(i) \text{ a.s.} \end{aligned}$$

Hence

$$\liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t y(s) ds \geq \frac{\sum_{i=1}^n \pi_i \beta_2(i)}{\check{a}_{22}} \text{ a.s.,}$$

so if $\sum_{i=1}^n \pi_i \beta_2(i) > 0$, then the predator population is strongly persistent in mean.

(iii). From (13) we can also get that

$$\frac{\ln y(t)}{t} \leq \frac{\ln y(0)}{t} + \frac{1}{t} \int_0^t \beta_2(\xi(s)) ds + \frac{1}{t} \int_0^t a_{21}(\xi(s)) ds + \frac{M_{21}(t)}{t} + \frac{M_{22}(t)}{t}.$$

It is easy to derive that

$$\limsup_{t \rightarrow \infty} \frac{\ln y(t)}{t} \leq \sum_{i=1}^n \pi_i (\beta_2(i) + a_{21}(i)) \text{ a.s.}$$

Hence it implies that if $\sum_{i=1}^n \pi_i (\beta_2(i) + a_{21}(i)) < 0$, then $\lim_{t \rightarrow \infty} y(t) = 0$ a.s. and

$$\lim_{t \rightarrow \infty} \frac{1}{t} \int_0^t y(s) ds = 0 \text{ a.s.} \quad (16)$$

On the other hand, (12) can lead to

$$\begin{aligned} \frac{1}{t} \int_0^t a_{11}(\xi(s))x(s)ds &= -\frac{\ln x(t)}{t} + \frac{\ln x(0)}{t} + \frac{1}{t} \int_0^t \beta_1(\xi(s))ds - \frac{1}{t} \int_0^t \frac{a_{12}(\xi(s))y(s)}{1+x(s)}ds + \frac{M_{11}(t)}{t} + \frac{M_{12}(t)}{t} \\ &\geq -\frac{\ln x(t)}{t} + \frac{\ln x(0)}{t} + \frac{1}{t} \int_0^t \beta_1(\xi(s))ds - \frac{1}{t} \int_0^t a_{12}(\xi(s))y(s)ds + \frac{M_{11}(t)}{t} + \frac{M_{12}(t)}{t}. \end{aligned}$$

This combining with (7), (14) and (16) yields

$$\liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t a_{11}(\xi(s))x(s)ds \geq \liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t \beta_1(\xi(s))ds = \sum_{i=1}^n \pi_i \beta_1(i) \text{ a.s.}$$

So we obtain that if $\sum_{i=1}^n \pi_i \beta_1(i) > 0$, then the prey population is strongly persistent in the mean.

(iv). In (ii), we have proved that under the condition $\sum_{i=1}^n \pi_i \beta_2(i) > 0$ the predator is strongly persistent in the mean. So if for any $i \in S$, $\beta_2(i) > 0$ holds, the same desired conclusion follows. Now we are in the position to prove that the prey population is strongly persistent in mean.

Firstly, we prove that

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \int_0^t a_{22}(\xi(s))y(s)ds \leq \sum_{i=1}^n \pi_i (\beta_2(i) + a_{21}(i)). \quad (17)$$

To prove this we note that by (13) we have

$$\frac{1}{t} \int_0^t a_{22}(\xi(s))y(s)ds \leq -\frac{\ln y(t)}{t} + \frac{\ln y(0)}{t} + \frac{1}{t} \int_0^t \beta_2(\xi(s))ds + \frac{1}{t} \int_0^t a_{21}(\xi(s))ds + \frac{M_{21}(t)}{t} + \frac{M_{22}(t)}{t}.$$

Taking the superior limit on both sides, using (10) and (14) yields

$$\begin{aligned} \limsup_{t \rightarrow \infty} \frac{1}{t} \int_0^t a_{22}(\xi(s))y(s)ds &\leq \limsup_{t \rightarrow \infty} \left(-\frac{\ln y(t)}{t} \right) + \sum_{i=1}^n \pi_i (\beta_2(i) + a_{21}(i)) \\ &\leq \sum_{i=1}^n \pi_i (\beta_2(i) + a_{21}(i)) \text{ a.s.} \end{aligned}$$

Now we turn to the proof of the strong persistence of $x(t)$. Follow the proof of (iii) we find that

$$\begin{aligned} &\liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t a_{11}(\xi(s))x(s)ds \\ &\geq \liminf_{t \rightarrow \infty} \left\{ -\frac{\ln x(t)}{t} + \frac{\ln x(0)}{t} + \frac{1}{t} \int_0^t \beta_1(\xi(s))ds - \frac{1}{t} \int_0^t a_{12}(\xi(s))y(s)ds + \frac{M_{11}(t)}{t} + \frac{M_{12}(t)}{t} \right\} \\ &= -\limsup_{t \rightarrow \infty} \frac{\ln x(t)}{t} + \sum_{i=1}^n \pi_i \beta_1(i) - \limsup_{t \rightarrow \infty} \frac{1}{t} \int_0^t a_{12}(\xi(s))y(s)ds \\ &\geq \sum_{i=1}^n \pi_i \beta_1(i) - \frac{\check{a}_{12}}{\hat{a}_{22}} \sum_{i=1}^n \pi_i (\beta_2(i) + a_{21}(i)) \text{ a.s.,} \end{aligned}$$

where we used (7), (17) and the ergodic property of the Markov chain. Hence if $\sum_{i=1}^n \pi_i \beta_1(i) > \frac{\check{a}_{12}}{\hat{a}_{22}} \sum_{i=1}^n \pi_i (\beta_2(i) + a_{21}(i))$, then the prey population is strongly persistent in the mean.

This completes the proof. \square

Remark 3.3. For some $i \in S$, consider the subsystem

$$\begin{aligned} dx(t) &= x(t) \left(r_1(i) - a_{11}(i)x(t) - \frac{a_{12}(i)y(t)}{1+x(t)} \right) dt + \sigma_1(i)x(t)dB_1(t) + \int_Y c_1(i, u)x(t^-)N(dt, du), \\ dy(t) &= y(t) \left(-r_2(i) + \frac{a_{21}(i)x(t)}{1+x(t)} - a_{22}(i)y(t) \right) dt + \sigma_2(i)y(t)dB_2(t) + \int_Y c_2(i, u)y(t^-)N(dt, du). \end{aligned}$$

Similarly, we can derive that for this subsystem

- (i) If $\beta_1(i) < 0$ and $\beta_2(i) < 0$, then the prey and predator are both extinct;
- (ii) If $\beta_1(i) < 0$ and $\beta_2(i) > 0$, then the prey population is extinct while the predator population is strongly persistent in mean;
- (iii) If $\beta_1(i) > 0$ and $\beta_2(i) + a_{21}(i) < 0$, then the predator population is extinct and the prey population is strongly persistent in mean;
- (iv) If $\beta_2(i) > 0$ and $\beta_1(i) - \frac{a_{12}(i)}{a_{22}(i)}(\beta_2(i) + a_{21}(i)) > 0$, then both the predator population and the prey population are strongly persistent in the mean.

Remark 3.4. Now we analyze how the jump noise affects the population system (4). Suppose system (4) is not disturbed by the jump noise, that is to say, $c_j(\xi(t), u) \equiv 0$, $j = 1, 2$, we denote

$$\beta_1^0(i) = r_1(i) - \frac{\sigma_1^2(i)}{2}, \quad \beta_2^0(i) = -r_2(i) - \frac{\sigma_2^2(i)}{2}.$$

Then if $\sum_{i=1}^n \pi_i(\beta_2^0(i) + a_{21}(i)) < 0$ and $\sum_{i=1}^n \pi_i\beta_1^0(i) > 0$, the predator species is extinct and the prey species is strongly persistent in the mean; but if system is affected by jump noise, furthermore, the jump noise satisfy $\sum_{i=1}^n \pi_i\beta_1(i) < 0$ and $\sum_{i=1}^n \pi_i\beta_2(i) > 0$, the prey is extinct while the predator is strongly persistent in mean. So the jump noise can make the extinct species persistent and persistent species extinct. In general, if $c_j(\xi(t), u) > 0$, the jump noise is advantageous to the ecosystem; if $c_j(\xi(t), u) < 0$, it is disadvantageous.

4. Examples and Numerical Simulations

In this section, to support to our analytical results, we give out the numerical experiment.

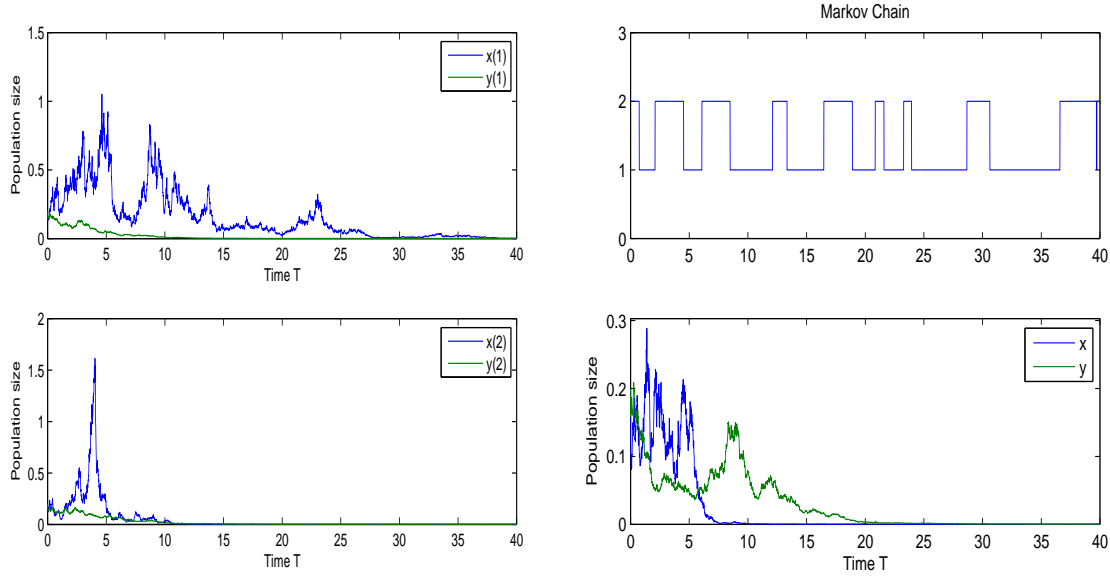


Figure 1: Numerical simulation of model (4) in both sub- Figure 2: Numerical simulation of the whole system (4) with systems with jump noise $c_1(1, u) = -0.2, c_1(2, u) = -0.25$, jump noise $c_1(1, u) = -0.2, c_1(2, u) = -0.25, c_2(1, u) = -0.1, c_2(2, u) = -0.1$. From this figure, we can see the prey species in both subsystems are extinctive.

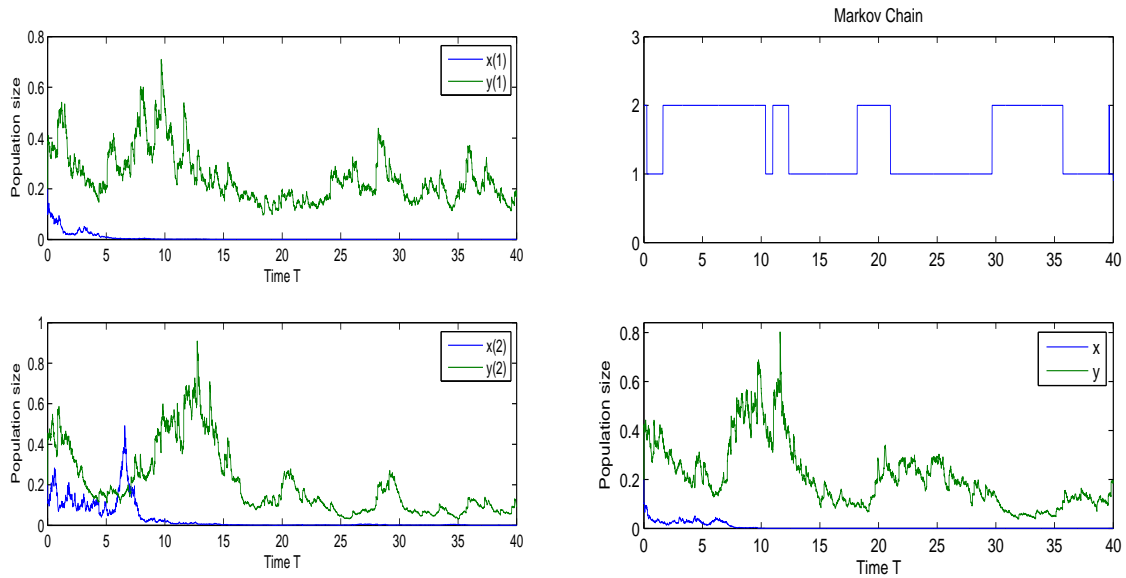


Figure 3: Numerical simulation of model (4) in both subsystems with jump noise $c_1(1, u) = -0.2, c_1(2, u) = -0.25, c_2(1, u) = 0.4, c_2(2, u) = 0.45$. From this figure, we can see the prey species are extinct while the predator species are strongly persistent in the mean in both subsystems.

Figure 4: Numerical simulation of the whole system (4) with jump noise $c_1(1, u) = -0.2, c_1(2, u) = -0.25, c_2(1, u) = 0.4, c_2(2, u) = 0.45$. From this figure, we can see that the prey species are extinct and predator are strongly persistent in the mean in the whole system.

Example 4.1 Consider system (4) with a right-continuous Markov chain taking values in $S = \{1, 2\}$. Let

the generator of the Markov chain is given by

$$\Gamma = \begin{pmatrix} -7 & 7 \\ 5 & -5 \end{pmatrix}.$$

By (5), we obtain the unique stationary distribution $\pi_1 = 5/12, \pi_2 = 7/12$.

Let $\lambda(Y) = 1$, the initial value $(x(0), y(0)) = (0.2, 0.2)$, $\xi(0) = 2$ and the coefficients be

$$\begin{aligned} r_1(1) &= 0.3, & a_{11}(1) &= 1, & a_{12}(1) &= 0.5, & \sigma_1(1) &= 0.7; \\ r_1(2) &= 0.4, & a_{11}(2) &= 1.1, & a_{12}(2) &= 0.5, & \sigma_1(2) &= 0.8; \\ r_2(1) &= 0.1, & a_{21}(1) &= 0.1, & a_{22}(1) &= 0.8, & \sigma_2(1) &= 0.3; \\ r_2(2) &= 0.12, & a_{21}(2) &= 0.1, & a_{22}(2) &= 0.7, & \sigma_2(2) &= 0.3. \end{aligned}$$

Case 1. If the jump noise $c_1(1, u) = -0.2, c_1(2, u) = -0.25, c_2(1, u) = -0.1, c_2(2, u) = -0.1$, then

$$\begin{aligned} \beta_1(1) &= r_1(1) - \frac{1}{2}\sigma_1^2(1) + \int_Y \ln(1 + c_1(1, u))\lambda(du) = -0.1681 < 0, \\ \beta_1(2) &= r_1(2) - \frac{1}{2}\sigma_1^2(2) + \int_Y \ln(1 + c_1(2, u))\lambda(du) = -0.2077 < 0, \\ \beta_2(1) &= -r_2(1) - \frac{1}{2}\sigma_2^2(1) + \int_Y \ln(1 + c_2(1, u))\lambda(du) = -0.2504 < 0, \\ \beta_2(2) &= -r_2(2) - \frac{1}{2}\sigma_2^2(2) + \int_Y \ln(1 + c_2(2, u))\lambda(du) = -0.2704 < 0. \end{aligned}$$

Hence from Remark 1 the species in both subsystems are extinctive, see Fig. 1. In the meanwhile, we note that

$$\sum_{i=1}^2 \pi_i \beta_1(i) = -0.1912 < 0, \quad \sum_{i=1}^2 \pi_i \beta_2(i) = -0.2621 < 0.$$

Therefore, by Theorem 3(i), both the prey and predator in the whole system (4) are extinct, Fig.2 confirms this.

Case 2. If the jump noise $c_1(1, u) = -0.2, c_1(2, u) = -0.25, c_2(1, u) = 0.4, c_2(2, u) = 0.45$, then the prey species $x(t)$ is extinct in subsystems and system (4) as in Case 1. But

$$\beta_2(1) = 0.1915 > 0, \quad \beta_2(2) = 0.2066 > 0, \quad \sum_{i=1}^2 \pi_i \beta_2(i) = 0.2033 > 0,$$

hence from Remark 1 and Theorem 3(ii) we obtain that the predator species in both subsystems and in the whole system (4) is strongly persistent in the mean. See Fig. 3 and 4.

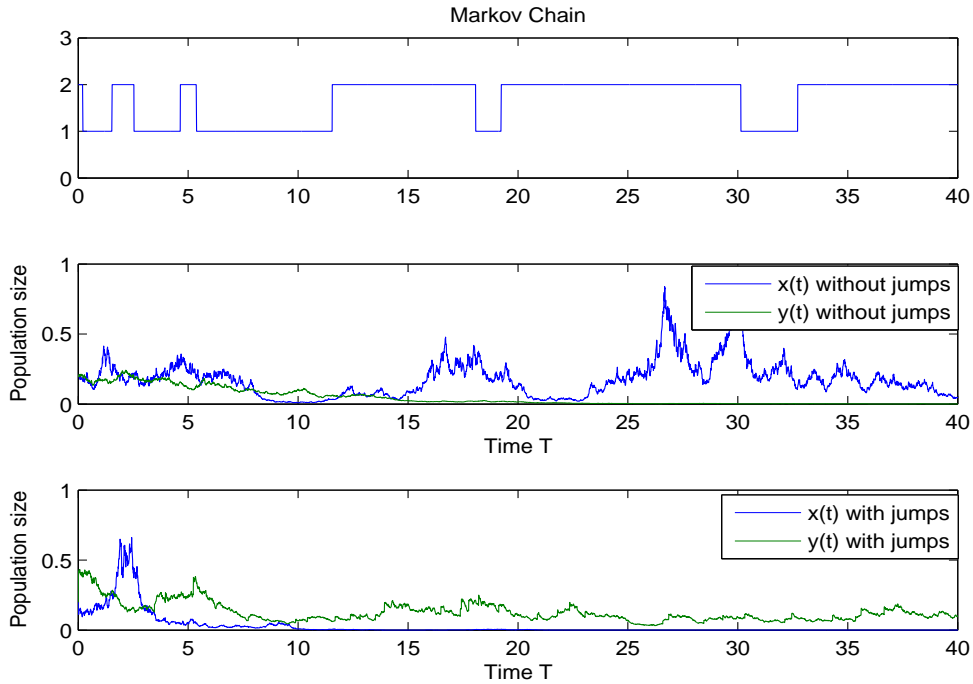


Figure 5: The first subfigure is the numerical simulation of Markov chain; the second subfigure is the numerical simulation of model (4) without jumps. The last subfigure is the numerical simulation of model (4) with jump noise $c_1(1, u) = -0.2, c_1(2, u) = -0.25, c_2(1, u) = 0.4, c_2(2, u) = 0.45$. Hence the jump noise can make persistent species extinct and extinct species strongly persistent.

Case 3. We choose the jump noise $c_1(1, u) = c_1(2, u) = c_2(1, u) = c_2(2, u) = 0$, that is to say, the jump noise is not considered, then

$$\sum_{i=1}^2 \pi_i(\beta_2(i) + a_{21}(i)) = -0.057 < 0, \quad \sum_{i=1}^2 \pi_i\beta_1(i) = 0.070 > 0.$$

Thus by (iii) in Theorem 3, we can observe that the prey species in system (4) is strongly persistent in the mean while the predator is extinct.

If we choose $c_1(1, u) = -0.2, c_1(2, u) = -0.25, c_2(1, u) = 0.4, c_2(2, u) = 0.45$, as we said in Case 2, the prey species is extinct and the predator is persistent in the mean. Fig.5 confirms these. Hence we observe that the jump noise can make persistent species extinct and extinct species strongly persistent.

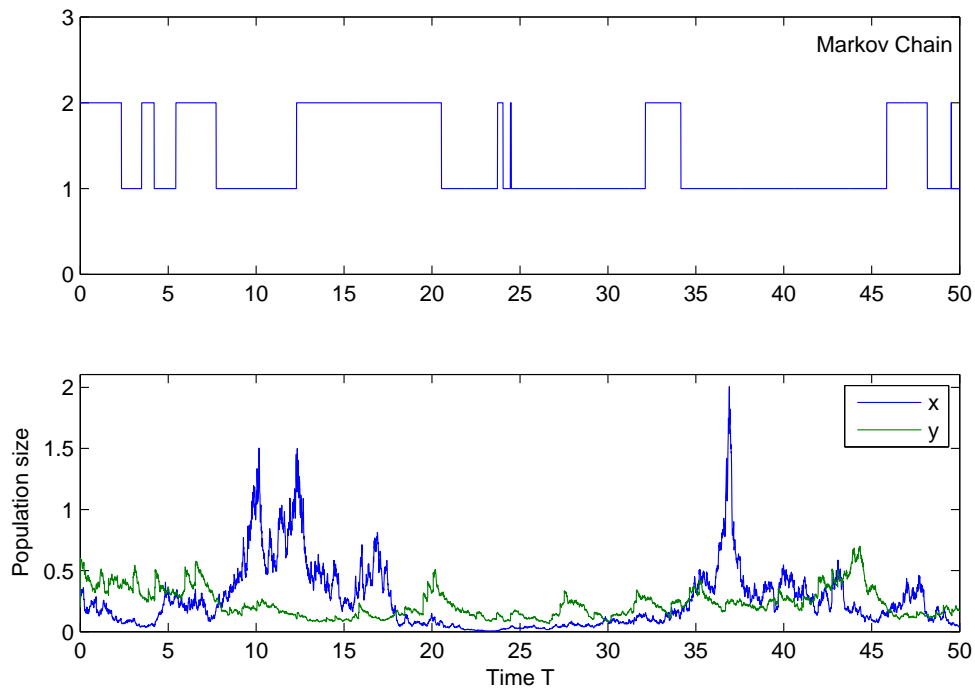


Figure 6: Numerical simulation of the whole system (4) with jump noise $c_1(1, u) = 0.3, c_1(2, u) = 0.2, c_2(1, u) = 0.4, c_2(2, u) = 0.45$. From this figure, we can see that both the prey and predator are strongly persistent in the mean in the whole system .

Case 4. We choose $c_1(1, u) = 0.3, c_1(2, u) = 0.2, c_2(1, u) = 0.4, c_2(2, u) = 0.45$, then

$$\beta_2(1) = 0.1915 > 0, \quad \beta_2(2) = 0.2066 > 0, \quad \sum_{i=1}^2 \pi_i \beta_2(i) = 0.2033 > 0,$$

moreover,

$$\sum_{i=1}^2 \pi_i \beta_1(i) - \frac{\check{a}_{12}}{\hat{a}_{22}} \sum_{i=1}^2 \pi_i (\beta_2(i) + a_{21}(i)) = 0.0708 > 0,$$

from (iv) in Theorem 3 it follows that both the predator $y(t)$ and the prey $x(t)$ are strongly persistent in the mean. See Fig. 6.

Example 4.2. In this example, we will analyze the impacts of Markovian switching on the population dynamics. Consider system (4) with Markov chain taking values in $S = \{1, 2\}$, then system (4) can be regarded as the result of the following two subsystems switching from one to another according to the law of Markov chain:

$$\begin{aligned} dx(t) &= x(t) \left(r_1(1) - a_{11}(1)x(t) - \frac{a_{12}(1)y(t)}{1+x(t)} \right) dt + \sigma_1(1)x(t)dB_1(t) + \int_Y c_1(1, u)x(t^-)N(dt, du), \\ dy(t) &= y(t) \left(-r_2(1) + \frac{a_{21}(1)x(t)}{1+x(t)} - a_{22}(1)y(t) \right) dt + \sigma_2(1)y(t)dB_2(t) + \int_Y c_2(1, u)y(t^-)N(dt, du), \end{aligned} \tag{18}$$

and

$$\begin{aligned} dx(t) &= x(t) \left(r_1(2) - a_{11}(2)x(t) - \frac{a_{12}(2)y(t)}{1+x(t)} \right) dt + \sigma_1(2)x(t)dB_1(t) + \int_Y c_1(2, u)x(t^-)N(dt, du), \\ dy(t) &= y(t) \left(-r_2(2) + \frac{a_{21}(2)x(t)}{1+x(t)} - a_{22}(2)y(t) \right) dt + \sigma_2(2)y(t)dB_2(t) + \int_Y c_2(2, u)y(t^-)N(dt, du). \end{aligned} \tag{19}$$

Let $\lambda(Y) = 1$, the initial value $(x(0), y(0)) = (0.2, 0.2)$, $\xi(0) = 2$. Assume that

$$\begin{aligned} r_1(1) &= 0.3, & a_{11}(1) &= 1, & a_{12}(1) &= 0.5, & \sigma_1(1) &= 0.7, & c_1(1, u) &= -0.2; \\ r_1(2) &= 0.4, & a_{11}(2) &= 1.1, & a_{12}(2) &= 0.5, & \sigma_1(2) &= 0.8, & c_1(2, u) &= -0.25; \\ r_2(1) &= 0.1, & a_{21}(1) &= 0.1, & a_{22}(1) &= 0.8, & \sigma_2(1) &= 0.3, & c_2(1, u) &= -0.1; \\ r_2(2) &= 0.12, & a_{21}(2) &= 0.1, & a_{22}(2) &= 0.7, & \sigma_2(2) &= 0.3, & c_2(2, u) &= 0.45. \end{aligned}$$

Now we analyze the effect of Markov switching.

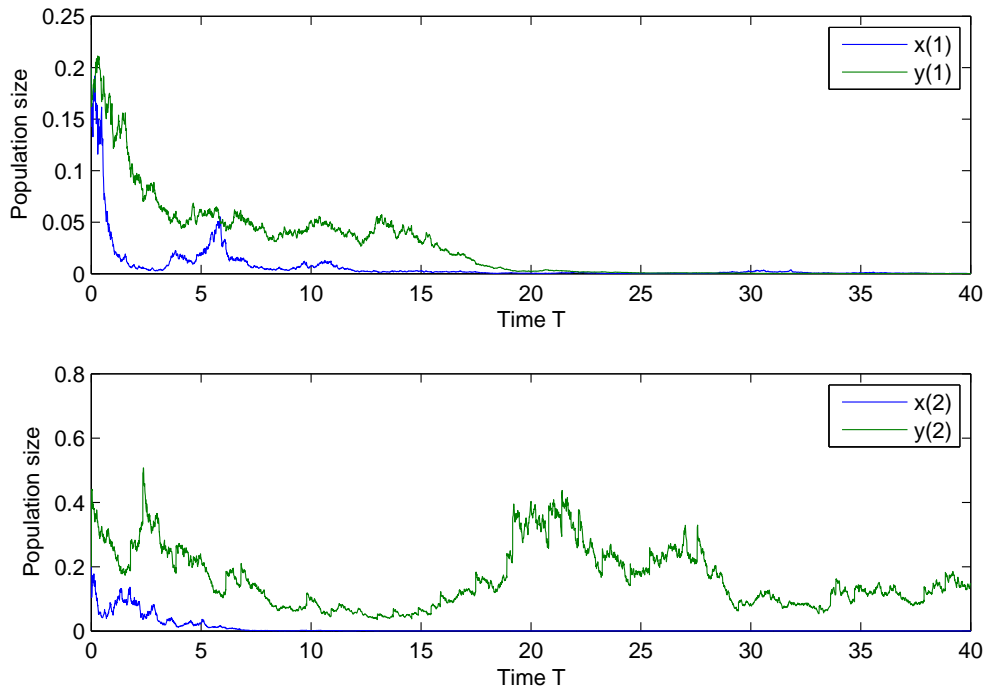


Figure 7: Numerical simulation of Example 2. The first subfigure is the numerical simulation of model (18). The second subfigure is the numerical simulation of model (19). From this figure, we observe that the prey species in both subsystems are extinct and the predator in subsystem (18) is extinct while the predator in subsystem (19) is strongly persistent in the mean.

Case 1. Let the generator of the Markov chain $\xi(t)$ be

$$\Gamma = \begin{pmatrix} -11 & 11 \\ 2 & -2 \end{pmatrix}.$$

Then the unique stationary distribution $\pi_1 = 2/13$, $\pi_2 = 11/13$. As we have obtained that the prey species

in both subsystems and in the whole system are extinct. Moreover, noting that

$$\beta_2(1) = -0.2504 < 0, \quad \beta_2(2) = 0.2066 > 0, \quad \sum_{i=1}^2 \pi_i \beta_2(i) = 0.1363 > 0,$$

combining Remark 1 and Theorem 3(ii), we observe that the predator in subsystem (18) is extinctive while the predator in subsystem (19) is strongly persistent in the mean. See Fig. 7. But as the result of Markovian switching, the whole behavior of predator in (4) is strongly persistent in the mean, see Fig.8.

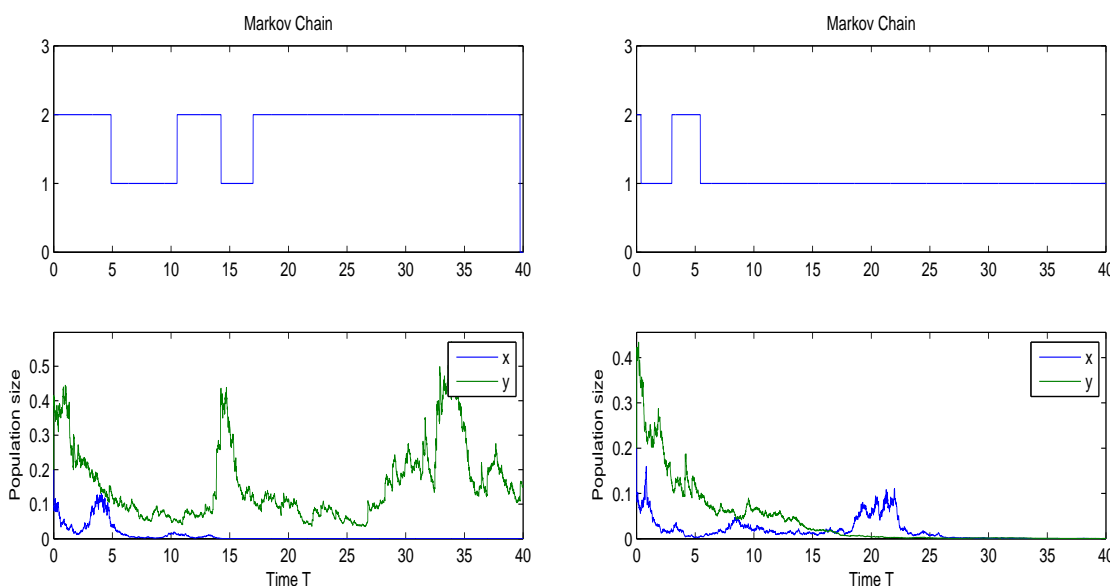


Figure 8: Numerical simulation of Example 2 with the generator of the Markov chain $\pi_1 = 2/13, \pi_2 = 11/13$. From this figure, we see the whole behavior of predator in (4) is strongly persistent in the mean.

Figure 9: Numerical simulation of Example 2 with the stationary probability of the Markov chain $\pi_1 = 11/13, \pi_2 = 2/13$. From this figure, we see the whole behavior of predator in (4) is extinctive.

Case 2. Let the generator of the Markov chain $\xi(t)$ be

$$\Gamma = \begin{pmatrix} -2 & 2 \\ 11 & -11 \end{pmatrix}.$$

It is easy to see that the unique stationary distribution $\pi_1 = 11/13, \pi_2 = 2/13$. The prey species in both subsystems and in the whole system are also extinctive. For the species predator, subsystem (18) is extinct while subsystem (19) is strongly persistent, and the whole behavior of predator in (4) is extinctive with the result of Markovian switching because $\sum_{i=1}^2 \pi_i \beta_2(i) = -0.1801 < 0$. Fig. 9 confirms this. These results also show that both the overall extinction and strong persistence in the mean of the predator have close relationships with the stationary probability distribution of the Markov chain.

5. Conclusions

In this paper we study the dynamical properties of a stochastic Holling II predator-prey model with Markovian switching driven by Poisson counting measure. We show that this model has a unique global positive solution and study the asymptotic behavior of the solution. Afterwards, we establish the sufficient

conditions for strong persistence in the mean and extinction of the solution. According to the movement of Markov chain, our model switches from one regime to another regime, and the overall behavior of species will be extinct or persistent in the mean depending on the stationary probability distribution of the Markov chain, though some individual equations in system (4) are extinct while some are persistent in the mean. Another interesting result is that under some conditions jump noise can make the extinct species persistent and the strongly persistent species extinct.

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