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a-KOIDEALS OF SETS

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Abstract. *In this paper we study a-coideals of sets in constructive mathematics, which are introduced in the papers* [9] *and* [10].

1. Introduction

For all notions of sets and functions in constructive mathematics we used here, the reader is refered to the books [1], [5], [7], [11] and [12], and to the papers [8], [9] and [10]. Examples of a-coideals and their applications are obtained in the paper [10]. The present paper contains some new results concerning a-coideals of sets, as for example a notion of basis of an a-coideal, two propositions of the decomposition property of a-coideals, and some constructions of a-coideals on the product of sets.

2. Preliminaries

To define a set X we mast explain how to construct members of X using objects that have been and describe what it means for two elements of X to be equal, and must satisfy the following

$$(\forall x \in X)\ (x = x),$$
$$(\forall xy \in X)\ (x = y \Rightarrow y = x),$$
$$(\forall xyz \in X)\ (x = y \wedge y = z \Rightarrow x = z).$$

If S is a subset of X and $x \in X$, we write $\neg(x \in S)$ if and only if $x \in S$ is impossible. The second more important relation in constructive mathematics is the *diversity* relation, a strong notion of inequality. The basic properties of the diversity relation are:

$$(\forall x \in X)\ \neg(x \neq x),$$
$$(\forall xy \in X)\ (x \neq y \Rightarrow y \neq x).$$

Note that, by our implicit assumption of extensionality, a diversity relation satisfies the condition

$$(\forall xyz \in X)\ (x \neq y \wedge y = z \Rightarrow x \neq z).$$

A diversity that also satisfies the condition

$$(\forall xyz \in X)\ (x \neq z \Rightarrow x \neq y \vee y \neq z)$$

is called *apartness*. An apartness is *tight* if and only if

$$(\forall xy \in X)\ (\neg(x \neq y) \Rightarrow x = y).$$

Let z be an arbitrary element of a set $(X, =, \neq)$ and let $Y \equiv \{y \in X \ :\ y \neq z\}$. Then we have

$$(*) \qquad (\forall st \in X)(s \in Y \Rightarrow s \neq t \vee t \in Y).$$

Not every set has this property. A subset Y of a set $(X, =, \neq)$ is *strongly extensional* if and only if the condition $(*)$. A subset Y of X is *inhabited* if

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and only if $(\exists x \in X)(x \in Y)$. An empty set $\varnothing$ is a set cannot be inhabited. If Y and Z are subsets of a set $(X, =, \neq)$, we defined

$$Y =_2 Z \Leftrightarrow Y \subseteq Z \wedge Z \subseteq Y .$$

With this equality, the collection of all subsets of X is a set $\mathcal{P}(x)$ called a *power set* of X. Let x be an arbitrary element of a set X and let Y be a subset of X. We write $x \# Y$ if and only if $(\forall y \in Y)(x \neq y)$, and define a diversity relation on $\mathcal{P}(X)$ by

$$Y \neq_2 Z \Leftrightarrow (\exists y \in Y)(y \# Z) \vee (\exists z \in Z)(z \# Y).$$

Note that, if $(X, =_X, \neq_X)$ and $(Y, =_Y, \neq_Y)$ are sets with diversity relations, then the Cartesian product $X \times Y$ of sets X and Y has the cannonical equality and diversity relations given by

$$(x, y) =_d (u, v) \Leftrightarrow x =_X u \wedge y =_Y v ,$$
$$(x, y) \neq_d (u, v) \Leftrightarrow x \neq_X u \vee y \neq_Y v .$$

If the relations $\neq_X$ and $\neq_Y$ are (tight) apartness, then the relation $\neq_d$ are (tight) apartness. A *relation* f between elements of a set $(X, =_X, \neq_X)$ and a set $(Y, =_Y, \neq_Y)$ is a subset of the Cartesian product $X \times Y$. For f we say:

f is *total* relation if and only if

$$(\forall x \in X)(\exists y \in Y)((x, y) \in f);$$

f is *onto* if and only if

$$(\forall y \in Y)(\exists x \in X)((x, y) \in f);$$

f is *injective* if and only if

$$(\forall x x' \in X)(\forall y y' \in Y)((x, y) \in f \wedge (x', y') \in f \wedge y =_Y y' \Rightarrow x =_X x');$$

f is an *embedding* if and only if

$$(\forall x x' \in X)(\forall y y' \in Y)((x, y) \in f \wedge (x', y') \in f \wedge x \neq_X x' \Rightarrow y \neq_Y y');$$

f is a *function* if and only if

$$(\forall x x' \in X)(\forall y y' \in Y)((x, y) \in f \wedge (x', y') \in f \wedge x =_X x' \Rightarrow y =_Y y');$$

f is *strongly extensional* if and only if

$$(\forall x x' \in X)(\forall y y' \in Y)((x, y) \in f \wedge (x', y') \in f \wedge y \neq_Y y' \Rightarrow x \neq_X x').$$

The set $D(f) \equiv \{x \in X \ : \ (\exists y \in Y)((x, y) \in f)\}$ is called the *domain* of f, and the set $R(f) \equiv \{y \in Y \ : \ (\exists x \in X)((x, y) \in f)\}$ is called the *range* of f.

3. Basic definitions

Let X be a set. A *family of elements* of X, indexed by a set I, is a function f from I to X. A *family of subsets* of X is a family of elements of $\mathcal{P}(X)$. A family K of subsets of a set X will be called an *a-coideal* ([9],[10]), if and only if $X \in K$, $Z_1 \subseteq Z_2 \wedge Z_1 \in K \Rightarrow Z_2 \in K$, $Z_1 \cup Z_2 \in K \Rightarrow Z_1 \in K \vee Z_2 \in K$. If X is a set with a diversity relation, then a-coideal K is *strong* if and only if $\varnothing \# K$. As in the classical case of filters (b-coideals (in constructive mathematics) [10]) ([3]), we can define the *basis* of an a-coideal: the family $B \subseteq \mathcal{P}(X)$ is called a *basis* of an a-coideal of a set X if and only if

$$\varnothing \# B,$$
$$(\forall S, V \in \mathcal{P}(X))(S \cup V \in B \Rightarrow S \in B \vee V \in B).$$

If B is a basis of an a-coideal of a set X, then the family $\{Z \subseteq X \ : \ (\exists S \in B)(S \subseteq Z)\}$ is the a-coideal of X induced by B.

Let K_1 and K_2 be two a-coideal of a set X. As in the classical case ([2]), we shall say that K_1 and K_2 have the *decomposition property* if and only if there are sets $A \in K_1$ and $B \in K_2$ such that $A \cup B =_2 X$ and $A \cap B =_2 \varnothing$; for sets A, B we say that they *realize* the decomposition property of K_1 and K_2.

4. Functions and a-coideals

In this section we shall characterize some relations between functions and a-coideals.

LEMMA 1. *Let K_X be an a-coideal of a set X and let us take that $Y \subseteq X$. Then the family $K_Y \equiv \{Z \cap Y (\neq_2 \varnothing) : Z \in K_X\}$ is an a-coideal of the set Y.*

PROOF.. Let us take that $S_1 \cup S_2 \in K_Y$, where S_1 and S_2 are subsets of Y, i.e. let $(\exists Z \in K_X)(S_1 \cup S_2 =_2 Z \cap Y)$¿ Let us take Z_1 and Z_2 in $\mathcal{P}(X)$ arbitrarily so that $S_1 \subseteq Z_1$, $S_2 \subseteq Z_2$, $S_1 =_2 Z_1 \cap Y$, $S_2 =_2 Z_2 \cap Y$ and $Z \subset Z_1 \cup Z_2$. Then $Z_1 \in K_X$ or $Z_2 \in K_X$. Therefore, $S_1 \in K_Y$ or $S_2 \in K_Y$.
Suppose that $S_1 \subset S_2$ and $S_1 \in K_Y$, i.e. suppose that $(\exists Z \in K_X)(S_1 =_2 Z \cap Y)$. As $S_2 \cup Z \in K_X$ and $S_2 =_2 (Z \cap Y) \cup S_2 =_2 (Z \cup S_2) \cap (Y \cup S_2) =_2 (Z \cup S_2) \cap Y$, we have $S_2 \in K_Y$. Therefore, the family K_Y is an a-coideal of the set Y. □

LEMMA 2. *Let K_X be an a-coideal of a set X and let us take that $X \subseteq Y$. Then the family $K_Y \equiv \{S \subseteq Y : (\exists Z \in K_X)(Z \subseteq S)\}$ is a basisof an a-coideal of the set Y.*

PROOF. Suppose that $S_1 \cup S_2 \in K_Y$ $(S_1, S_2 \in 2^Y)$, i.e. suppose that $(\exists Z \in K_X)(Z \subseteq S_1 \cup S_2)$. As $K_X \ni Z =_2 Z \cap (S_1 \cup S_2) =_2 (Z \cap S_1) \cup (Z \cap S_2)$ and $Z \cap S_1 \in K_X$ or $Z \cap S_2 \in K_X$, we have $(\exists Z \cap S_1 \in K_X)(Z \cap S_1 \subseteq S_1)$ or $(\exists Z \cap S_2 \in K_X)(Z \cap S_2 \subseteq S_2)$. So, $S_1 \in K_Y$ or $S_2 \in K_Y$.
Let S be an arbitrary element of K_Y. Then there is Z in K_X such that $\varnothing \neq_2 Z \subseteq S$. Therefore, the family K_Y is a basis of an a-coideal of the set Y. □

THEOREM 3. *Let $f : (X, =_X, \neq_X) \to (Y, =_Y, \neq_Y)$ be a strongly extensional function of sets with apertnesses and let K_X be an a-coideal of the set X such that $(\forall Z \in K_X)(Z \cap D(f) \neq_2 \varnothing)$. Then the a-coideal K_X of X induces an a-coideal K_Y of Y.*

PROOF. (i). Let K_X be an a-coideal of a set X and let $f : X \to Y$ be a strongly extensional function of sets. Then by Lemma 1, the family $\{Z \cap D(f) (\neq_2 \varnothing) : Z \in K_X\}$ is an a-coideal of the set $D(f)$.
(ii). Suppose that the function f is total and let K_X be an a-coideal of the set X. Then, if $Z \in K_X$, then $f(Z) \neq_2 \varnothing$. Let us take that $S_1, S_2 \in \mathcal{P}(X)$ such that $S_1 \subseteq S_2$ and $S_1 \in K = \{f(Z) : Z \in K_X\}$. Then $S_1 =_2 f(Z_1)$ for some $Z_1 \in K_X$. Let us take $Z_2 =_2 f^{-1}(S_2)$. Then $Z_1 \subseteq Z_2$ and $(Z_1 \subseteq Z_2 \wedge Z_1 \in K_X \Rightarrow Z_2 \in K_X)$. Further, there exists $Z_2 \in K_X$ such that $S_2 =_2 f(Z_2)$. So, $S_2 \in K$. Let S_1, S_2 be arbitrary elements of $\mathcal{P}(R(f))$ and $(\exists Z \in K_X)(S_1 \cup S_2 =_2 f(Z))$. Let us take that $Z_1 =_2 f^{-1}(S_1)$ and $Z_2 =_2 f^{-1}(S_2)$. Then $Z_1 \cup Z_2 =_2 f^{-1}(S_1) \cup f^{-1}(S_2) =_2 f^{-1}(S_1 \cup S_2) =_2 f^{-1}f(Z) \supseteq Z \in K_X$ and $Z_1 \in K_X$ or $Z_2 \in K_X$. Therefore, there exists $Z_1 \in K_X$ such that $S_1 =_2 f(Z_1)$ or there exists $Z_2 \in K_X$ such that $S_2 =_2 f(Z_2)$. Thus, the family K is an a-coideal of the set $R(f)$.
(iii). The a-coideal K of $R(f)$ can be extended, by Lemma 2, to an a-coideal of the set Y. □

COROLLARY 3.1. *Let $f_y : X \ni x \mapsto (x, y) \in X \times Y$ $(y \in Y)$ be a function and let K_X be an a-coideal of the set X. Then the family $K_y \equiv \{E \subseteq X \times Y : (\exists Z \in K_X)(Z \times \{y\} \subseteq E)\}$ is an a-coideal of the set $X \times Y$.*

COROLLARY 3.2. *Let $p_1 : X \times Y \ni (x,y) \mapsto x \in X$ be the first canonical projection and let K_{XY} be an a-coideal of the set $X \times Y$. Then the family $\{p_1(E) \subseteq X \ : \ E \in K_{XY}\}$ is an a-coideal of the set X.*

COROLLARY 3.3. *Let K_1 and K_2 be two a-coideals of a set X and L_1 $(i = 1,2)$ be a-coideal of a set Y induced by K_i $(i = 1,2)$ respectively, and by a function $f : X \to Y$. If S_1 and S_1 realize the decomposition property of L_1 and L_2, then the sets $f^{-1}(S_1 \cap R(f))$ and $f^{-1}(S_2 \cap R(f))$ realize the decomposition property of the a-coideals K_1 and K_2.*

COROLLARY 3.4. *Let K_1 and K_2 be two a-coideals of a set X and let L_1 and L_2 be a-coideals of a set Y induced by K_1 and K_2 respectively and by an injective function $f : X \to Y$. If A and B realize the decomposition property of K_1 and K_2, then there exists sets $S_1 \supseteq f(A)$ and $S_2 \supseteq f(B)$ which realize the decomposition property of L_1 and L_2.*

THEOREM 4. *Let $f : X \to Y$ be a strongly extensional function of sets and let K_Y be an a-coideal of the set Y such that $R(f) \cap S \neq_2 \varnothing$ for each $S \in K_Y$. Then the a-coideal K_Y of Y induces an a-coideal K_X of the set X.*

PROOF. 1. Let K_Y be an a-coideal of Y. Then the family $\{S \cap R(f) \ : \ S \in K_Y\}$ by Lemma 1, is an a-coideal of the set $R(f)$.

2. Suppose that the function f is onto and let K_Y be an a-coideal of the set Y. Then the family $K \equiv \{Z \cap D(f) \ : \ f(Z) \in K_Y\}$ is an a-coideal of the set $D(f)$. We have

(i). If $Z \in K$, i.e. if $(\exists S \in K_Y)(S \subseteq f(Z))$, then from $\varnothing \neq_2 S \subseteq f(Z)$ follows $Z \neq_2 \varnothing$.

(ii). Let Z_1 and Z_2 be arbitrary elements of $\mathcal{P}(D(f))$ such that $Z_1 \subseteq Z_2$ and $Z_1 \in K$. Then there exists $S_1 \in K_Y$ such that $S_1 \subseteq f(Z_1)$. As $S_1 \subseteq f(Z_1) \subseteq f(Z_2)$, so $f(Z_2) \in K_Y$. Therefore, $Z_2 \in K$.

(iii). Suppose that Z_1, Z_2 are arbitrary elements of $\mathcal{P}(D(f))$ such that $Z_1 \cup Z_2 \in K$, i.e. such that $f(Z_1 \cup Z_2) \in K_Y$. Thus $f(Z_1) \cup f(Z_2) \in K_Y$ and $f(Z_1) \in K_Y$ or $f(Z_2) \in K_Y$. Therefore $Z_1 \in K$ or $Z_2 \in K$.

3. The a-coideal K of the set $D(f)$ can be extended, by Lemma 2, to an a-coideal of the set X. □

COROLLARY 4.1. *Let $f : X \to Y$ be an injective function of sets X and Y with apertnesses and let K_Y be an a-coideal of Y such that $R(f) \cap S \neq_2 \varnothing$ for each $S \in K_Y$. Then the family $\{Z \subseteq X \ : \ (\exists S \in K_Y)(f^{-1}(S) \subseteq Z)\}$ is an a-coideal of the set X.*

5. Two constructions of a-coideals of $X \times Y$

Let $(X, =_X, \neq_X)$ and $(Y, =_Y, \neq_Y)$ be sets with diversity relations. For any $E \subseteq X \times Y$, $x \in X$, $y \in Y$ let us take that ([2])

$$E_x^2 \equiv \{y \in Y \ : \ (x,y) \in E\}, \quad E_y^1 \equiv \{x \in X \ : \ (x,y) \in E\} \, .$$

THEOREM 5. *Let K_1 be an a-coideal of a set X and let K_2 be an a-coideal of a set Y. Then the family*

$$K_{12} \equiv \{E \subseteq X \times Y \ : \ \{x \in X \ : \ E_x^2 \in K_2\} \in K_1\}$$

is an a-coideal of the Cartesian product $X \times Y$ of sets X and Y.

PROOF. (i). Let $E \in K_{12}$. If $\{x \in X \ : \ E_x^2 \in K_2\} \in K_1$, then $\{x \in X \ : \ E_x^2 \in K_2\} \neq_2 \varnothing$, i.e. then there exists $x \in X$ such that $E_x^2 \in K_2$. So, $E_x^2 \neq_2 \varnothing$. Therefore $(\exists y \in Y)(\exists x \in X)((x,y) \in E)$, i.e. $E \neq \varnothing$.

(ii). As $(X \times Y)_x =_2 Y$ and $Y \in K_2$, we have that $\{x \in X : (X \times Y)_x =_2 Y \in K_2\} =_2 X$ and $X \in K_1$. Therefore, $X \times Y \in K_{12}$.

(iii). Let E_1, E_2 be arbitrary elements of $\mathcal{P}(X \times Y)$ cuch that $E_1 \subseteq E_2$ and $E_1 \in K_{12}$. Then $E^2_{1x} \subseteq E^2_{2x}$ and $\{x \in X : E^2_{1x} \in K_2\} \in K_1$. Thus $E_2 \in K_{12}$ because $\{x \in X : E^2_{1x} \in K_2\} \subseteq \{x \in X : E^2_{2x} \in K_2\}$.

(iv). Suppose that E_1, E_2 are elements of $\mathcal{P}(X \times Y)$ such that $E_1 \cup E_2 \in K_{12}$, i.e. such that $\{x \in X : (E_1 \cup E_2)^2_x \in K_2\} \in K_1$. Then $\{x \in X : E^2_{1x} \cup E^2_{2x} \in K_2\} \in K_1$ because $(E_1 \cup E_2)^2_x =_2 E^2_{1x} \cup E^2_{2x}$. Therefore for $\{x \in X : E^2_{1x} \in K_2\} \cup \{x \in X : E^2_{2x} \in K_2\} \in K_1$ follows $\{x \in X : E^2_{1x} \in K_2\} \in K_1$ or $\{x \in X : E^2_{2x} \in K_2\} \in K_1$, i.e. $E_1 \in K_{12}$ or $E_2 \in K_{12}$ because

$\{x \in X : E^2_{1x} \in K_2\} \cup \{x \in X : E^2_{2x} \in K_2\} =_2 \{x \in X : E^2_{1x} \cup E^2_{2x} \in K_2\}$. □

NOTE. Let K_1 be an a-coideal of a set X and let K_2 be an a-coideal of a set Y. Then the family

$$K_{21} \equiv \{E \subseteq X \times Y : \{y \in Y : E^1_y \in K_1\} \in K_2\}$$

is an a-coideal of the set $X \times Y$. When is $K_{12} = K_{21}$?

In the paper [10] we defined notions of an a-ideal and a b-ideal of sets:

a) A family $J_a \subseteq 2^X$ is called an *a-ideal* of X if and only if

$\varnothing \in J_a, \quad Y_1 \subseteq Y_2 \wedge Y_2 \in J_a \Rightarrow Y_1 \in J_a, \quad Y_1 \in J_a \wedge Y_2 \in J_a \Rightarrow Y_1 \cup Y_2 \in J_a$.

b) A family J_b of subsets of X is a *b-ideal* of X if and only if

$\varnothing \in J_b, \quad Y_1 \subseteq Y_2 \wedge Y_2 \in J_b \Rightarrow Y_1 \in J_b, \quad Y_1 \cap Y_2 \in J_b \Rightarrow Y_1 \in J_b \vee Y_2 \in J_b$.

The ideal J is *strong* if and only if $X \# J$.

THEOREM 6. *Let J^b_1 be a b-ideal of a set X and let J^a_2 be an a-ideal of a set Y. Then the family*

$$K^{12}_{ab} \equiv \{E \subseteq X \times Y : \{x \in X : E^2_x \in J^a_2\} \in J^b_1\}$$

is an a-coideal of the set $X \times Y$.

PROOF. (i). Let $E \in K^{12}_{ab}$, i.e. let $\{x \in X : E^2_x \in J^a_2\} \in J^b_1$. As $\{x \in X : E^x_2 \in J^a_2\} \in J^b_1 \# X$ we have $(\exists x \in X)(E^2_x \# J^a_2)$. It is meant $(\exists x \in X)(\forall P \in J^a_2)(E^2_x \neq_2 P)$. Thus, $(\exists x \in X)(E^2_x \neq_2 \varnothing)$. So, $(\exists x \in X)(\exists y \in Y)((x, y) \in E)$. Therefore $E \neq \varnothing$.

(ii). Let $E_1 \subseteq E_2$ and $E_1 \in K^{12}_{ab}$. Then $E^2_{1x} \subseteq E^2_{2x}$ and $\{x \in X : E^2_x \in J^a_2\} \in J^b_1$. It follows $\{x \in X : E^2_{2x} \in J^a_2\} \in J^b_1$ because $\{x \in X : E^2_{2x} \in J^a_2\} \subseteq \{x \in X : E^2_{1x} \in J^a_2\}$. Therefore $E_2 \in K^{12}_{ab}$.

(iii). As $(X \times Y)_x =_2 Y \# J^b_2$, we have $\{x \in X : Y \in J^a_2\} =_2 \varnothing \in J^b_1$. So $X \times Y \in K^{12}_{ab}$.

(iv). Suppose that E_1, E_2 are arbitrary elements of $\mathcal{P}(X \times Y)$ such that $E_1 \cup E_2 \in K^{12}_{ab}$. Then $\{x \in X : (E_1 \cup E_2)^2_x \in J^a_2\} \in J^b_1$. As $\{x \in X : E^2_{1x} \in J^a_2\} \cap \{x \in X : E^2_{2x} \in J^a_2\} \subseteq \{x \in X : E^2_{1x} \cup E^2_{2x} \in J^a_2\}$ we have $\{x \in X : E^2_{1x} \in J^x_2\} \cap \{x \in X : E^2_{2x} \in J^a_2\} \in J^b_1$. From here, we have $\{x \in X : E^2_{1x} \in J^a_2\} \in J^b_1$ or $\{x \in X : E^2_{2x} \in J^a_2\} \in J^b_1$. So $E_1 \in K^{12}_{ab}$ or $E_2 \in K^{12}_{ab}$. □

NOTE II. Let J^a_1 be an a-ideal of a set X and let J^b_2 be a b-ideal of a set Y. Then the family

$$K^{21}_{ab} \equiv \{E \subseteq X \times Y : \{y \in Y : E^1_y \in J^a_1\} \in J^b_2\}$$

is an a-coideal of the set $X \times Y$. Let J_1^{ab} and J_2^{ab} be two ab-ideals of X and Y respectively. When is $K^{12} = K^{21}$?

NOTE III. Let K be an a-coideal of a set X. Then we have

$$(**) \qquad (\forall Z, S \in \mathcal{P}(X))(Z \cup S \in K \Rightarrow Z \in K \vee S \in K).$$

We can generalize the condition $(**)$ to the following case: as in the classical case ([4],[6]), for the family R of subsets of a set X we say that R is *open in the relation to function*

$$f : \mathcal{P}(X) \times \mathcal{P}(X) \to \mathcal{P}(X)$$

if and only if

$$(\forall Z, S \in \mathcal{P}(X))(f(Z,S) \in R \Rightarrow Z \in R \vee S \in R).$$

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