Zbornik radova Filozofskog fakulteta u Nišu
Serija Matematika 6:2 (1992), 239–245

THE PSEUDOWEIGHT AND SPLITTABILITY OF A TOPOLOGICAL SPACE

Lj. Kočinac

(Received 30.10.1991.; Revised 20.06.1992.)

Abstract. *We prove that for every Lindelöf space X the pseudoweight of X is equal to the splittable pseudoweight of X. We also prove some other results involving the splittable pseudoweight. The divisibility degree of a topological space is defined and studied. Some cardinal inequalities involving the divisibility degree are proved. It is proved that every compact divisible space is metrizable.*

0. Introduction

Let $\mathcal{P}$ be a class of topological spaces. A topological space X is said to be **splittable over** $\mathcal{P}$ if for every $A \subset X$ there exist a space $Y \in \mathcal{P}$ and a continuous mapping $f : X \to Y$ such that $f(X) = Y$ and $f^{-1}f(A) = A$ (see [2] and also [5], [18]). When X is splittable over the familly of all subsets of the space $\mathbb{R}^\omega$ we say simply that X is **splittable.** If φ is a topological cardinal function we define the splittable version φ_s of φ by

$$\varphi_s(X) = \min\{\tau : X \text{ is splittable over the class of all spaces } Y \text{ with } \varphi(Y) \leq \tau\},$$

where X is a topological space. For some results involving different splittable versions of cardinal functions we refer to [4], [6], [7], [8], [14], [15], [16].

Here we prove that for every Lindelöf space X we have $pw_s(X) = pw(X)$, where $pw(X)$ denotes the pseudoweight of X. We also prove some other results involving the splittable pseudoweight. In the second part of the paper we define and study the divisibility degree of a space and prove some cardinal inequalities using this cardinal function. In particular, it is proved that a compact is metrizable if and only if it is divisible.

We use the usual topological terminology and notation following [10]; for definitions and results on cardinal functions we refer to [1], [12] and [13]. w, pw, L, wL, s, e, ψ , t denote the weight, pseudoweight, Lindelöf number, weak Lindelöf number, spread, extent, pseudocharacter and tightness, respectively. $cL(X)$ denotes the smallest cardinal τ such that for any closed $A \subset X$ and any family $\mathcal{U}$ of open subsets of X for which $A \subset \cup\mathcal{U}$ there is a subfamily $\mathcal{V}$ of $\mathcal{U}$ with $|\mathcal{V}| \leq \tau$ and $A \subset \cup\overline{\mathcal{V}}$ (see, for example, [7], [18]). The *cl*-**cardinality** of a space X, denoted by $clard(X)$, is

1991 Mathematics Subject Classification: 54A25, 54C10.

Supported by Grant 0401A of FNS through Math. Inst. SANU

the smallest cardinal τ such that every subset of X is a union of $\leq \tau$ many closed subsets of X (see [9]).

All spaces in this paper are T_1, all mappings are continuous and all cardinals are infinite.

We shall need the following known lemma:

LEMMA. *If S is a set of cardinality $\leq 2^\tau$, then there exists a point separating family γ of subsets of S having cardinality $\leq \tau$.* ■

[Recall that γ is point separating if for any $p, q \in S$, $p \neq q$, there is some $A \in \gamma$ such that $p \in A$, $q \notin A$.]

1. The pseudoweight and splittability

THEOREM 1.1. *For every T_1-space X we have $pw_s(X) \leq pw(X) \leq L(X)pw_s(X)$. In particular, for every Lindelöf space X, $pw_s(X) = pw(X)$.*

PROOF. Put $L(X)pw_s(X) = \tau$. Let A be a subset of X. Choose a space Y with $pw(Y) \leq \tau$ and a continuous mapping $f : X \to Y$ such that $Y = f(X)$ and $A = f^{-1}f(A)$. We have [1]: $|Y| \leq 2^{pw(Y)} \leq 2^\tau$ and thus $|f(A)| \leq 2^\tau$. Hence, $e(A) = e(\cup\{f^{-1}(y) : y \in f(A)\}) \leq 2^\tau \cdot \tau = 2^\tau$. As A was an arbitrary subset of X we have $\mathrm{he}(X) = s(X) \leq 2^\tau$. Since X is a T_1-space, by the well known theorem of Hajnal-Juhász [1], [12], [13] we obtain $|X| \leq 2^{s(X)\psi(X)} \leq 2^{2^\tau}$. According to Lemma there exists a point separating family $\mathcal{S}$ of subsets of X having cardinality $\leq 2^\tau$; let $\mathcal{S} = \{S_\alpha : \alpha \in 2^\tau\}$.

For every $\alpha \in 2^\tau$ fix a mapping $f_\alpha : X \to Y_\alpha$ from X onto a space Y_α with $pw(Y_\alpha) \leq \tau$ such that $f_\alpha^{-1}f_\alpha(S_\alpha) = S_\alpha$. Let $\mathcal{B}_\alpha$ be a pseudobase for Y_α having cardinality $\leq \tau$. We are going to prove that the following holds:

(*) for every $x \in S_\alpha$ and every $y \notin S_\alpha$ there is $V_x \in \mathcal{B}_\alpha$ such that $x \in f_\alpha^{-1}(V_x)$ and $y \notin f_\alpha^{-1}(V_x)$.

Indeed, $f_\alpha(y) \notin S_\alpha$ so that $f_\alpha(x) \neq f_\alpha(y)$. Therefore, there exists a member V_x in $\mathcal{B}_\alpha$ such that $f_\alpha(x) \in V_x$ and $f_\alpha(y) \notin V_x$. This V_x satisfies (*).

Put now $\mathcal{B} = \cup\{f^{-1}(\mathcal{B}_\alpha) : \alpha \in 2^\tau\}$, $\mathcal{C} = \{X\backslash B : B \in \mathcal{B}\}$. By (*) $\mathcal{C}$ is a point separating collection of closed subsets of X and $|\mathcal{C}| \leq 2^\tau$. Therefore, $\mathcal{U} = \{X\backslash C : C \in \mathcal{C}\}$ is a pseudobase for X such that $|\mathcal{U}| = |\mathcal{C}| \leq 2^\tau$ which means that $pw(X) \leq 2^\tau$. X is a T_1-space so that we have (see [1], [12], [13]): $|X| \leq pw(X)^{L(X)\psi(X)} \leq (2^\tau)^{\tau\cdot\tau} = 2^\tau$ (note that $\psi(X) \leq \tau$ because X is splittable over a class of spaces Y with $\psi(Y) \leq pw(Y) \leq \tau$). Applying once again Lemma one can find a point separating family $\{T_\alpha : \alpha \in \tau\}$ of subsets of X of cardinality $\leq \tau$. Repeating the proof of the previous part of the theorem we get a pseudobase $\mathcal{U}^*$ for X of cardinality $\leq \tau$. So, $pw(X) \leq \tau = L(X)pw_s(X)$.

If X is a Lindelöf space, then $pw(X) \leq pw_s(X)$. On the other hand, the inequality $pw_s(X) \leq pw(X)$ is always true and we have $pw_s(X) = pw(X)$. ■

Every compact (= compact Hausdorff space) with a countable pseudobase has a countable base [1], [12], so that by the previous theorem we get the following result.

COROLLARY 1.2. *If a compact X is splittable over the class of all spaces of countable pseudoweight, then X is metrizable.* ■

From the proof of Theorem 1.1 we obtain:

COROLLARY 1.3. *For every T_1-space X , $|X| \leq 2^{L(X)pw_s(X)}$.* ■

We shall now prove some other relations between pw and pw_s.

THEOREM 1.4. (i) *For every T_1-space X, $pw(X) \leq 2^{e(X)pw_s(X)}$;*

(ii) *For every T_2-space X, $pw(X) \leq 2^{cL(X)pw_s(X)}$;*

(iii) *For every normal space X, $pw(X) \leq 2^{wL(X)pw_s(X)}$.*

PROOF. (i) Let $e(X)pw_s(X) = \tau$. Choose a space Y with $pw(Y) \leq \tau$ and a mapping $f : X \to Y$ such that $f^{-1}f(A) = A$. From $|f(A)| \leq |Y| \leq 2^{pw(Y)} \leq 2^\tau$ it follows that A is the union of $\leq 2^\tau$ closed subsets of $X : A = \cup\{f^{-1}(y) : y \in f(A)\}$. For every $y \in f(A)$, $e(f^{-1}(y)) \leq e(X) \leq \tau$, so that $e(A) \leq 2^\tau \cdot \tau = 2^\tau$. This means that $he(X) \leq 2^\tau$. But $he(x) = s(X)$ [12]. On the other hand, $\psi(X) \leq \tau$. Since X is a T_1-space we have $|X| \leq 2^{s(X)\psi(X)} \leq 2^{2^\tau}$. It remains now to work as in the first part of the proof of Theorem 1.1 which will give $pw(X) \leq 2^\tau$.

(ii) Let $cL(X)pw_s(X) = \tau$. As in the proof of (i) we get $hcL(X) \leq 2^\tau$. We shall check that the inequality $s(X) \leq hcL(X)$ holds (for every Hausdorff space X). Let A be a discrete subset of X. Then for every $x \in A$ choose a neighbourhood U_x of x such that $\overline{U}_x \cap A = \{x\}$. The family $\{U_x : x \in A\}$ is an open cover of A and since $hcL(X) \leq 2^\tau$ there is a subfamily $\{U_{x_\alpha} : x_\alpha \in A, \alpha \in 2^\tau\}$ of $\{U_x : x \in A\}$ such that $A \subset \cup\{\overline{U}_{x_\alpha} : \alpha \in 2^\tau\}$. For every $\alpha \in 2^\tau$ we have $A \cap \overline{U}_x = \{x_\alpha\}$ and so $|A| \leq 2^\tau$, i.e. $s(X) \leq 2^\tau$. Since X is a T_1-space and $\psi(X) \leq \tau$ we again have $|X| \leq 2^{2^\tau}$. Now the previous proof should be repeated.

(iii) The proof is almost the same as in (i) and (ii) if we take into account 2.35 in [13] and $hwL(X) = s(X)$ [12]. ■

2. Another aproach to splittability: divisibility

Let X be a topological space and A a subset of X. Following [3] we say that a family $\mathcal{S}$ of closed subsets of X is a **separator for** A if for each $x \in A$ and each $y \in X \backslash A$ there exists $S \in \mathcal{S}$ such that $x \in S$ and $y \notin S$. In [3], Arhangel'skii defined a space X to be **divisible** if for every $A \subset X$ there is a countable separator for A, and to be **strictly divisible** if for every $A \subset X$ there is a countable separator for A consisting of (closed) G_δ-sets. He also proved that every Lindelöf strictly divisible space has a G_δ-diagonal.

Clearly, every splittable space is (strictly) divisible. The following result shows that splittability and divisibility are closely connected.

THEOREM 2.1. *A perfectly normal space X is divisible if and only if it is splittable.*

PROOF. Let X be divisible and let A be a subset of X. Take a countable separator $\{F_i : i \in \omega\}$ for A. As X is perfectly normal every F_i is a zero-set: $F_i = f_i^{-1}(0)$, where $f_i : X \to \mathbb{R}$ is a continuous mapping. Denote by f the diagonal product $\Delta\{f_i : i \in \omega\} : X \to \mathbb{R}^\omega$. From the definition of a separator it is easily seen that $A = f^{-1}f(A)$, i.e. X is splittable. ■

From this theorem we conclude that not all perfectly normal spaces are divisible. In fact, we have the following

EXAMPLE 2.2. There exist metric spaces which are not divisible. Indeed, Proposition 6.2 in [5] gives an example of a metric locally compact space which is not splittable (see also Proposition 6.5 in the same paper). ■

EXAMPLE 2.3. (1) Every perfectly normal metacompact scattered space is divisible (see [5;Prop. 5.6]).
(2) Every scattered metrizable space is divisible ([5;Cor. 5.7]).
(3) A metric space of cardinality $\leq 2^{\omega}$ is divisible ([5;Cor. 2.21]).
(4) A left metric space is divisible ([5;Cor. 5.5]). ■

For a space X and a subset A of X we define

$$dvs(A, X) = \min\{\, \tau : \text{ there is a separator } \mathcal{S} \text{ for } A \text{ having cardinality} \leq \tau \,\}$$

and

$$dvs(X) = \sup\{\, dvs(A) : A \subset X \,\}.$$

The cardinal number $dvs(X)$ we shall call the **divisibility degree** of X.

From a remark due to Arhangel'skii [3] we actually have this simple, but useful result.

PROPOSITION 2.4. *For every T_1-space X we have*
(1) $dvs(X) \leq pw(X)$;
(2) $clard(X) \leq 2^{dvs(X)}$. ■

In fact, we have the following result.

PROPOSITION 2.5. *For every T_1-space X we have*

$$dvs(X) \leq pw_s(X) \leq pw(X).$$

PROOF. We shall prove the first inequality because the second one is obvious. Let $pw_s(X) = \tau$. Take a subset A of X and a point $y \in X \setminus$ A. Choose a space Y with $pw(Y) \leq \tau$ and a mapping $f : X \to Y = f(X)$ such that $f^{-1}f(A) = A$. Let $\mathcal{B}$ be a pseudobase for Y witnessing $pw(Y) \leq \tau$ and let $\{V_\alpha : \alpha \in \tau\}$ be a subfamily of $\mathcal{B}$ for which $\cap\{V_\alpha : \alpha \in \tau\} = \{f(y)\}$. The sets $f^{-1}(Y \setminus V_\alpha)$, $\alpha \in \tau$, are closed. On the other hand, if x is any member in A, then $f(x) \notin V_\beta$ for some $\beta \in \tau$ because $f(y) \notin f(A)$. Then $x \in f^{-1}(Y \setminus V_\beta)$, $y \notin f^{-1}(Y \setminus V_\beta)$ which means that $\{f^{-1}(Y \setminus V_\alpha) : \alpha \in \tau\}$ is a separator for A, i.e. $dvs(X) \leq \tau$. ■

Let us note that the following two obvious propositions hold.

PROPOSITION 2.6. *For every T_1-space we have*

$$\psi(X) \leq dvs(X). \quad ■$$

PROPOSITION 2.7. *For every space X we have*

$$dvs(X) = dvs_s(X). \quad ■$$

We are going now to prove some cardinal inequalities involving the divisibility degree. We start with a theorem which is, according to Proposition 2.5, an improvement of Theorem 1.1 and Corollary 1.3 (see [17]). Let us point out that this theorem should be compared with Theorems 2 and 3 in [3].

THEOREM 2.8. *For every T_1-space X we have:*
(a) $|X| \leq 2^{dvs(X)L(X)}$;
(b) $pw(X) \leq dvs(X)L(X)$.

PROOF. (a) Let $dvs(X)L(X) = \tau$ and let A be a subset of X. Since $dvs(X) \leq \tau$ we have $clard(X) \leq 2^\tau$ so that the set A can be represented as the union $A = \cup\{A_\alpha : \alpha \in 2^\tau\}$ of $\leq 2^\tau$ closed subsets of X. For each $\alpha \in 2^\tau$ we have $e(A_\alpha) \leq L(A_\alpha) \leq L(X) \leq \tau$, so that $e(A) \leq L(A) \leq 2^\tau \cdot \tau = 2^\tau$. This means $he(X) = s(X) \leq 2^\tau$. Since X is a T_1-space one has $|X| \leq 2^{s(X)\psi(X)} \leq 2^{2^\tau}$. According to Lemma it follows the existence of a point separating family $\mathcal{F}$ of subsets of X of cardinality $\leq 2^\tau$. For every $F \in \mathcal{F}$ let $\mathcal{S}_F$ be a separator for F having cardinality $\leq \tau$. Then $\mathcal{S} = \cup\{\mathcal{S}_F : F \in \mathcal{F}\}$ is a point separating collection of closed subsets of X and its cardinality is $\leq 2^\tau$. Therefore, $\mathcal{B} = \{X \backslash S : S \in \mathcal{S}\}$ is a pseudobase for X such that $|\mathcal{B}| \leq 2^\tau$, so that $pw(X) \leq 2^\tau$. As X is a T_1-space we have $|X| \leq pw(X)^{L(X)\psi(X)} \leq (2^\tau)^{\tau\cdot\tau} = 2^\tau$.

(b) Let $dvs(X)L(X) = \tau$. By (a) $|X| \leq 2^\tau$ so that, by Lemma, there is a point separating family $\mathcal{A}$ of subsets of X such that $|\mathcal{A}| \leq \tau$. For every $A \in \mathcal{A}$ we take a separator $\mathcal{S}_A$ for A of cardinality $\leq \tau$ and put $\mathcal{S} = \cup\{\mathcal{S}_A : A \in \mathcal{A}\}$. The collection $\mathcal{S}$ is point separating and has cardinality $\leq \tau$, so that $\mathcal{P} = \{X \backslash S : S \in \mathcal{S}\}$ is a pseudobase for X of cardinality $\leq \tau$, i.e. $pw(X) \leq \tau$. ■

REMARK 2.9. In fact, Theorem 2.8 is not a "proper" improvement of Theorem 1.1 and Corolary 1.3, because we are going to prove the equality

$$dvs(X)L(X) = L(X)pw_s(X).$$

We have already proved $dvs(X) \leq pw_s(X)$ so that $dvs(X)L(X) \leq L(X)pw_s(X)$. On the other hand, from $pw_s(X) \leq pw(X) \leq L(X)dvs(X)$ it follows $pw_s(X)L(X) \leq L(X)dvs(X)$. ■

For compact spaces we get the following nice result.

COROLLARY 2.10. ([17]) *Every divisible compact is metrizable.* ■

It is known that regular Lindelöf spaces are paracompact [10] and that every paracompact p-space with a G_δ-diagonal is metrizable. From the fact that every Lindelöf strictly divisible space has a G_δ-diagonal, we have

COROLLARY 2.11. *Every strictly divisible regular Lindelöf p-space* [1] *is metrizable.* ■

This corollary is related to the following result: every splittable paracompact p-space is metrizable [5] (see also [16]).

Since, obviously, a perfect space (= closed sets are G_δ) is divisible if and only if it is strictly divisible, we also have

COROLLARY 2.11′. *Every divisible perfect regular Lindelöf p-space is metrizable.* ■

REMARK 2.12. Corollary 2.10 can be also proved in the following way:

Let A be a subset of X. The set A is the union of $\leq 2^\omega$ closed and thus compact subsets of X. Because X is a T_2-space we can apply the following result from [11]

(see also [9]): if every subset of a T_2-space is a union of $\leq \lambda$ compact subsets of the space, then that space has cardinality $\leq \lambda$. So, $|X| \leq 2^{\omega}$. It is easy now to find out a countable pseudobase for X. ■

REMARK 2.13. Taking into account Proposition 2.6, we conclude that the part (a) of Theorem 2.8 is one type of the Arhangel'skii theorem: for every T_2-space X, $|X| \leq 2^{t(X)L(X)\psi(X)}$. In this connection, it should be remarked the following:

(1) for every splittable space X, $dvs(X) \leq \omega$; but there are splittable spaces having uncountable tightness.

(2) Corollary 2.10 shows that for any non-metrizable compact Y of countable tightness $t(Y) < dvs(Y)$ holds. ■

REMARK 2.14. In [5], it was proved: every pseudocompact splittable space is metrizable. After Corollaries 2.10 and 2.11 it is reasonable to ask whether a pseudocompact divisible space is metrizable. The answer is "No". The famous Mrówka's space $\Psi(\omega, \mathcal{A})$ [10; 3. 6. I] is a counterexample. Describe this space. A collection $\mathcal{A}$ of subsets of ω is called almost disjoint if for any two members $A, B \in \mathcal{A}$, the set $A \cap B$ is finite. There exists a maximal almost disjoint collection of infinite subsets of ω having cardinality 2^{ω} (see [12]). Take such a collection and topologize the set $\omega \cup \mathcal{A}$ as follows: the points of ω are isolated; basic neighbourhoods of a point $A \in \mathcal{A}$ are of the form $\{A\} \cup (A \setminus F)$ with F is a finite subset of ω. In this way one obtains the space $\Psi(\omega, \mathcal{A})$. It is known that $\Psi(\omega, \mathcal{A})$ is a Tychonoff, perfect, locally compact, pseudocompact (iff $\mathcal{A}$ is a maximal almost disjoint family), developable, non-normal (and so non-metrizable and non-splittable) space. Besides, $\mathcal{A}$ is a discrete subspace of $\Psi(\omega, \mathcal{A})$. But this space is divisible (even strictly divisible) because the family $\mathcal{D} = \{\omega \setminus \{n\} : n \in \omega\} \cup \{(\mathcal{A} \setminus f^{-1}(U_n)) \cup \{1, ..., n\} : n \in \omega\}$, where f is any one-to-one mapping from $\mathcal{A}$ onto a space Y of cardinality 2^{ω} with a countable base $\{U_n : n \in \omega\}$ (for instance, one can take $Y = \mathbb{R}$), is a closed countable divisor for every subset of $\Psi(\omega, \mathcal{A})$. ■

In connection with Theorem 1.4 and (b) in Theorem 2.8 we have the following result (which is, according to Proposition 2.5, an improvement of Theorem 1.4). We omit the proof of this theorem, because it is quite similar to the proofs in Theorem 1.4 and Theorem 2.8.

THEOREM 2.15. (α) *For every T_1-space X, $pw(X) \leq 2^{e(X)dvs(X)}$.*

(β) *For every T_2-space X, $pw(X) \leq 2^{cL(X)dvs(X)}$.*

(γ) *For every normal space X, $pw(X) \leq 2^{wL(X)dvs(X)}$.* ■

Acknowledgment. I would like to thank A. Arhangel'skii and A. Bella for some helpful comments on an earlier version of this paper.

REFERENCES

[1] А.В. АРХАНГЕЛЬСКИЙ, *Строение и классификация топологических пространств и кардинальные инварианты*, Успехи мат. наук 33(1978), 29–84.

[2] А.В. АРХАНГЕЛЬСКИЙ, *Некоторые новые направления в теории непрерывных отображений*, В: Непрерывные функции на топологических пространствах, ЛГУ, Рига, 1986, 5–35.

[3] A.V. ARHANGEL'SKII, *Some problems and lines of investigation in general topology,* Comment. Math. Univ. Carolinae 29(1988), 611-629.

[4] A.V. ARHANGEL'SKII, LJ. KOČINAC, *Concerning splittability and perfect mappings,* Publ. Inst. Math. (Beograd) 47(61)(1990), 127-131.

[5] А.В. АРХАНГЕЛЬСКИЙ, Д.Б. ШАХМАТОВ, *О поточечной аппроксимации произвольных функций счетными семействами непрерывных функций,* Труды Сем. И. Г. Петровского 13(1988), 206–207.

[6] A.V. ARHANGEL'SKII, B.E. SHAPIROVSKII, *On splittable spaces,* Abstr. Intern. Conf. on Topology, Varna, Sept. 24-29, 1990, pp. 9-10.

[7] A. BELLA, F. CAMMAROTO, LJ. KOČINAC, *Remarks on splittability of topological spaces, Q&A* in General Topology 9(1991), 88-99.

[8] A. BELLA, LJ. KOČINAC, *A short note on perfect splittability,* Zbornik rad. Fil. fak. (Niš), Ser. Mat. 4(1990), 101-103.

[9] JU. BREGMAN, B.ŠAPIROVSKI, A. ŠOSTAK, *On decomposition of a set into certain subsets and cl-cardinality of a topological space,* Učen. Zap. Latv. Univ., Ser. Matem. 1(1990), 73-85.

[10] R. ENGELKING, *General Topology,* PWN, Warszawa, 1977.

[11] J. GERLITS, A. HALNAL, Z. SZENTMIKLOSSY, *On the cardinality of certain Hausdorff spaces,* Math. Inst. of the Hung. Acad. Sci., Preprint $N^o 77$, 1989.

[12] R.E. HODEL, *Cardinal functions I,* In: K. Kunen & J.E. Vaughan (eds.), Handbook of Set-theoretic Topology, North Holland, Amsterdam, 1984, pp. 3-61.

[13] I. JUHÁSZ, *Cardinal functions in topology - ten years later,* Mathematical Centre Tracts 123, Amsterdam, 1980.

[14] LJ. KOČINAC, *Perfect $\mathcal{P}$-splittability of topological spaces,* Zbornik rad. Fil. fak. (Niš), Ser. Mat. 3(1989), 19-24.

[15] LJ. KOČINAC, *On (M, P)-splittability of topological spaces,* Supl. Rend. Circolo Mat. di Palermo, Ser. II 24(1990), 397-404.

[16] LJ. KOČINAC, *Metrizability and cardinal invariants using splittability,* Compt. Rend. Acad. Bulg. Sci. 43(1990), 9-12.

[17] LJ. KOČINAC, *Compact divisible spaces are metrizable,* Abstracts Amer. Math. Soc. (1992).

[18] LJ. KOČINAC, F. CAMMAROTO, A. BELLA, *Some results on splittability of topological spaces,* Atti Accademia Pelor. dei Pericolanti 68(1990), 41-60.

29. novembra 132, 37230 Aleksandrovac, Yugoslavia