Zbornik radova Filozofskog fakulteta u Nišu
Serija Matematika 6:2 (1992), 215–238

$(p2,l)$-SYMMETRY THREE-DIMENSIONAL SPACE GROUPS ${G_3}^{l,p2}$ I. SYMMORPHIC GROUPS, II. HEMISYMMORPHIC GROUPS, III. ASYMMORPHIC GROUPS

SLAVIK V. JABLAN

(Received 09.10.1991; Revised 15.01.1992)

ABSTRACT. *By use of antisymmetric characteristic method, $(p2,l)$-symmetry three dimensional space groups ${G_3}^{l,p2}$ $(p=3,4,6)$, are derived.*

Crystallographic $(p2)$-symmetry three-dimensional space groups ${G_3}^{p2}$ $(p=3,4,6)$ are derived by Yu.S.Karpova (Yu.S.Chubarova) [1,2,3,4]. From 73 symmorphic space groups G_3 are derived 1025 junior ${G_3}^{p2}(96{G_3}^{32}+438{G_3}^{42}+491{G_3}^{62})$, from 54 hemisymmorphic G_3 are derived 945 junior ${G_3}^{p2}$ $(75{G_3}^{32}+444{G_3}^{42}+426G^{62})$, and from 103 asymmorphic G_3 are derived 1650 junior ${G_3}^{p2}(138{G_3}^{32}+785G^{42}+727{G_3}^{62})$; this means, the category ${G_3}^{p2}(p=3,4,6)$ consists of 3620 junior groups $(309{G_3}^{32}+1667{G_3}^{42}+1644{G_3}^{62})$ [3,4]. By the use of the generalized antisymmetric characteristic method (AC -method) [5,6], we will correct some of the results mentioned, and derive all crystallographic $(p2,l)$-symmetry three-dimensional space groups ${G_3}^{l,p2}(p=3,4,6)$.

1. Some general remarks on $(p2)$- and $(p2,l)$-symmetry

The $(p2)$-symmetry is a particular case of the general P -symmetry with $P=D_p$, where D_p is the irregular dihedral permutation group, generated by the permutations $e_1=(1\cdots p)$ and $e_2=(2\ p)\cdots([\frac{p+1}{2}]\ p-[\frac{p+1}{2}]+2)$.

$$ {e_1}^p={e_2}^2=(e_1e_2)^2=E. $$

For $p=4q+2$ $(q\in N)$, the group D_p is reducible, so the relationship

$$ D_{4q+2}\cong\{{e_1}^2,e_2\}\times\{{e_1}^{2q+1}\}=D_{2q+1}\times C_2 $$

holds, where by $\{{e_1}^2,e_2\}$ and $\{{e_1}^{2q+1}\}$ are denoted, respectively, the groups D_{2q+1} and C_2 generated by ${e_1}^2,e_2$ and ${e_1}^{2q+1}$ [4,7].

By introducing l antiidentity transformations $e_3,\cdots,e_{l+2}$ [8,9] $(l\in N)$ commuting between themeselves and with e_1,e_2 , we have $(p2,l)$-symmetry, with the group $P=D_p\times{C_2}^l$. At $p=4q+2$, because of the reducibility of the group D_{4q+2} , we have the relationship $D_{2q+1}\times{C_2}^l=D_{4q+2}\times{C_2}^{l-1}$, pointing out the equality of the $((2q+1)2,l)$- and $((4q+2)2,l-1)$-symmetry.

In this work only junior groups of complete $(p2,l)$-symmetry will be considered. Every junior $(p2)$-symmetry group $G^{(p2)}$ is derived from certain generating

1991 Mathematics Subject Classification: Primary 20H15.

Supported by Grant 0401A of FNS through Math. Inst. SANU

symmetry group G as well as every junior $(p2,l)$-symmetry group $G^{(p2,l)}$ is derived from certain junior $(p2)$-symmetry group [4,7,8].

THEOREM 1. a) *a $(p2,l)$-symmetry group $G^{(p2,l)}$ is the junior $(p2,l)$-symmetry group if all relations given in the presentation of its generating symmetry group G remain satisfied after replacing the generators of the group G by the corresponding $(p2,l)$-symmetry group generators;*

b) *a junior $(p2,l)$-symmetry group is called the M^m -type $(p2,l)$- symmetry group, if it is a M^m -type group regarded as a l - multiple antisymmetry group;*

c) *a junior M^m -type $(p2,l)$-symmetry group $G^{(p2,l)}$ is a group of the complete $(p2,l)$-symmetry, if every e_i -transformation $(i=1,\cdots,l+2)$ can be obtained in the group $G^{(p2,l)}$ as an independent $(p2,l)$-symmetry transformation.*

If only the condition c) it is not satisfied, such a group $G^{(p2,l)}$ is the uncomplete junior $(p2,l)$-symmetry group of the M^m -type.

Because of the afore mentioned relation between $((2q+1)2,l)$- and $((4q+2)2, l-1)$-symmetry groups, for the derivation of all crystallographic $(p2,l)$-symmetry groups $(p=3,4,6)$, (32)- and (42)-symmetry groups will be sufficient. The derivation will be realised by the use of generalized AC :

DEFINITION 1. Let all the products of $(p2)$-symmetry generators of a group $G^{(p2)}$, within which every generator participates once at the most, be formed, and then subsets of transformations equivalent with regard to $(p2)$-symmetry, be separated. The resulting system is called the antisymmetric characteristic of the group $G^{(p2)}$ and denoted by $AC(G^{(p2)})$[5, 6].

In every transition from the antisymmetric characteristic $AC(G)$ of a generating group G to the antisymmetric characteristic $AC(G^{(p2)})$ of a $(p2)$-symmetry group $G^{(p2)}$ derived from it, at $p=2q+1$ every e_1 -transformation has no influence on the existential conditions for the junior $(p2,l)$-symmetry groups of the M^m -type (Theorem 1.) and does not produce any change of the $AC(G)$; its possible change can be produced only by e_2 - transformation. Hence, we can conclude that the derivation of $((2q+1)2,l)$-symmetry groups from a $((2q+1)2)$-symmetry group coincides with the derivation of $(l+1)$-multiple antisymmetry M^m - type groups $(2 \le m \le l+1)$ from its corresponding e_2 -antisymmetry group. Having in mind the equality of $((2q+1)2,l)$- and $((4q+2)2,l-1)$-symmetry, the only non-trivial problem is the derivation of $(p2,l)$-symmetry groups at $p=0(\mathrm{mod}\ 4)$ [5,6].

The problem of differing between complete and uncomplete $(p2,l)$-symmetry junior M^m -type groups can be solved by the use of the homomorphism of the subgroup $C_p=\{e_1\}$ of the group D_p to the group C_2 at $p=0(\mathrm{mod}\ 2)$:

$$e_1{}^{2k-1} \to e_1, \quad e_1{}^{2k} \to E \quad (1 \le k \le (p+1)/2) \quad [6].$$

2. Symmorphic $(p2,l)$-symmetry three-dimensional space groups $G_3{}^{l,p2}$ $(p=3,4,6)$

The application of the theoretical assumptions given above will be illustrated by example of complete $(p2,l)$-symmetry junior three-dimensional space groups of the M^m -type $(p=3,4,6)$ derived in the family with the common generating symmetry group $G=7s(P2/m),\{a,b,c\}(2:m)$ with the $AC:\{m,cm\}\{2,2a,2b,2ab\}$ belonging to the AC -equivalency class VII [10, Tab.1]. At $p=3$ we have two junior (32)-symmetry groups [1]:

1) $\{a,b,c^{(3}\}(2:m^{2)})$,

2) $\{a^{(3},b,c\}(2^{2)}:m)$.

Because of the e_2 -transformation $m^{2)}$, the AC of the first group is of the form $\{e_2, e_2\}\{E, E, E, E\}$ and of the type $(2)(5)^1$, and the AC of the second is of the form $\{E, E\}\{e_2, e_2, e_2, e_2\}$ and of the same type $(3)(5)^1$. Hence, for the both of them $N_1 = 7, N_2 = 64, N_3 = 700, N_4 = 6720$ [10]. So, we have the following complete (32,1)- symmetry groups:

$\{\underline{a}, b, c^{(3}\}(2 : m^2)$, $\{a, b, \underline{c}^{(3}\}(2 : m^2)$, $\{a, b, c^{(3}\}(\underline{2} : m^2)$,
$\{\underline{a}, b, \underline{c}^{(3}\}(2 : m^2)$, $\{\underline{a}, b, c^{(3}\}(2 : \underline{m}^2)$, $\{a, b, \underline{c}^{(3}\}(\underline{2} : m^2)$,
$\{a, b, c^{(3}\}(\underline{2} : \underline{m}^2)$, $\{\underline{a}^{(3}, b, c\}(2^{2)} : m)$, $\{a^{(3}, b, \underline{c}\}(2^{2)} : m)$,
$\{a^{(3}, b, c\}(2^{2)} : \underline{m})$, $\{\underline{a}^{(3}, b, \underline{c}\}(2^{2)} : m)$, $\{\underline{a}^{(3}, b, c\}(2^{2)} : \underline{m})$,
$\{a^{(3}, b, \underline{c}\}(\underline{2}^{2)} : m)$, $\{a^{(3}, b, c\}(\underline{2}^{2)} : \underline{m})$.

Because of the equality between (32,1)- symmetry and (62)- symmetry, to the 14 complete (32,1)-symmetry groups obtained correspond the 14 (62)-symmetry junior groups:

$\{a^{(2}, b, c^{(3}\}(2 : m^{2)})$, $\{a, b, c^{(6}\}(2 : m^{2)})$, $\{a, b, c^{(3}\}(2^{(2} : m^{2)})$,
$\{a^{(2}, b, c^{(6}\}(2 : m^{2)})$, $\{a^{(2}, b, c^{(3}\}(2 : m^{22)})$, $\{a, b, c^{(6}\}(2^{(2} : m^{2)})$,
$\{a, b, c^{(3}\}(2^{(2} : m^{22)})$, $\{a^{(6}, b, c\}(2^{2)} : m)$, $\{a^{(3}, b, c^{(2}\}(2^{2)} : m)$,
$\{a^{(3}, b, c\}(2^{2)} : m^{(2)})$, $\{a^{(6}, b, c^{(2}\}(2^{2)} : m)$, $\{a^{(6}, b, c\}(2^{2)} : m^{(2)})$,
$\{a^{(3}, b, c^{(2}\}(2^{22)} : m)$, $\{a^{(3}, b, c\}(2^{22)} : m^{(2)})$.

According to the equality of $(32, l)$- and $(62, l-1)$-symmetry, we may conclude that from these 14 (62)-symmetry groups, $N_1 = 128$ complete (62,1)-symmetry junior groups of the M^1 -type, $N_2 = 1400$ complete (62,2)-symmetry junior groups of the M^2 -type and $N_3 = 13420$ complete (62,3)-symmetry junior groups of the M^3 -type, will be derived.

At $p = 0(\mathrm{mod}2)$, the form and, consequently, the type of $AC(G^{(p2)})$ is obtained by the use of the homomorphism mentioned in Chapter 1. By treating in this way 6 (42)-symmetry groups, we have the following results: the first three of them, $\{a, b, c^{(4}\}(2 : m^{2)})$, $\{a, b, c^{(4}\}(2^{(2} : m^{2)})$ and $\{a^{(2}, b, c^{(4}\}(2 : m^{2)})$ possess AC of the form $\{e_2, e_1e_2\}\{E, E, E, E\}$ and of the type $(4)(5)^2$, and for each of them $N_1 = 8, N_2 = 64, N_3 = 448$. For example, from the first group $\{a, b, c^{(4}\}(2 : m^{2)})$, the 8 complete (42,1)- symmetry junior groups of the M^1 -type:

$\{\underline{a}, b, c^{(4}\}(2 : m^{2)})$, $\{a, b, c^{(4}\}(\underline{2} : m^{2)})$, $\{\underline{a}, b, \underline{c}^{(4}\}(2 : m^{2)})$,
$\{\underline{a}, b, c^{(4}\}(2 : \underline{m}^{2)})$, $\{a, b, \underline{c}^{(4}\}(\underline{2} : m^{2)})$, $\{a, b, c^{(4}\}(\underline{2} : \underline{m}^{2)})$,
$\{\underline{a}, b, \underline{c}^{(4}\}(2 : \underline{m}^{2)})$, $\{a, b, \underline{c}^{(4}\}(\underline{2} : \underline{m}^{2)})$,

will be obtained, ect. For the three other (42)-symmetry groups, $\{a^{(4}, b, c\}(2^2 : m)$, $\{a^{(4}, b, c\}(2^2 : m^{(2})$ and $\{a^{(4}, b, c^{(2}\}(2^2 : m)$, their AC are of the form $\{E, E\}$ $\{e_2, e_2, e_1e_2, e_1e_2\}$ and of the type $(4)(5)^2$, so for each of them, $N_1 = 11, N_2 = 132, N_3 = 1344$ [10]. For example, from the first group $\{a^{(4}, b, c\}(2^2 : m)$, the 11 complete (42,1)-symmetry junior groups of the M^1 -type:

$\{a^{(4}, \underline{b}, c\}(2^2 : m)$, $\{a^{(4}, b, \underline{c}\}(2^2 : m)$, $\{a^{(4}, b, c\}(2^2 : \underline{m})$,
$\{\underline{a}^{(4}, b, \underline{c}\}(2^2 : m)$, $\{\underline{a}^{(4}, b, c\}(2^2 : \underline{m})$, $\{a^{(4}, \underline{b}, \underline{c}\}(2^2 : m)$,
$\{a^{(4}, \underline{b}, c\}(2^2 : \underline{m})$, $\{a^{(4}, b, \underline{c}\}(\underline{2}^2 : m)$, $\{a^{(4}, b, c\}(\underline{2}^2 : \underline{m})$,
$\{\underline{a}^{(4}, b, \underline{c}\}(\underline{2}^2 : m)$, $\{\underline{a}^{(4}, b, c\}(\underline{2}^2 : \underline{m})$,

will be obtained, etc.

3. Partial catalogue of symmorphic $(p2,l)$-symmetry three-dimensional space groups $G_3{}^{l,p2}$ $(p=3,4,6)$

In the same manner, the partial catalogue of all complete $(p2,l)$-symmetry junior symmorphic three-dimensional space groups of the M^m -type $G_3{}^{l,p2}(p=3,4,6)$, is realized. According to the work [10], this partial catalogue gives the possibility for their complete cataloguation.

2s $(P\bar{1})\,\{a,b,c\}(\tilde{2})$, $AC:\{\tilde{2},\tilde{2}a,\tilde{2}b,\tilde{2}c,\tilde{2}ab,\tilde{2}ac,\tilde{2}bc,\tilde{2}abc\}$, II

1) $\{a^{(3},b,c\}(\tilde{2}^{2)})$, $(9)^1$, $N_1=1$ $N_2=1$ $N_3=1$

2) $\{a^{(4},b,c\}(\tilde{2}^{2)})$, $(25)^2$, $N_1=1$ $N_2=1$

3s $(P2)\,\{a,b,c\}(2)$, $AC:\{c\}\{2,2a,2b,2ab\}$, III

1) $\{a^{(3},b,c\}(2^{2)})$, $(2)(5)^1$, $N_1=4$ $N_2=16$ $N_3=56$

2) $\{a^{(4},b,c\}(2^{2)})$, $(2)(9)^2$, $N_1=6$ $N_2=24$

3) $\{a^{(4},b,c^{(2}\}(2^{2)})$, $(2)(9)^2$, $N_1=6$ $N_2=24$

4s $(B2)\,\{a,b,(a+c)/2\}(2)$, $AC:\{2,2b\}\{2(a+c)/2,2b(a+c)/2\}$, IV

1) $\{a,b^{(3},(a+c)/2\}(2^{2)})$, $(3)(3)^1$, $N_1=3$ $N_2=6$

2) $\{a^{(2},b,(a+c)/2^{2)}\}(2^{22)})$, $(3)(3)^2$, $N_1=1$

3) $\{a^{(2},b,(a+c)/2^{22)}\}(2^{2)})$, $(3)(3)^2$, $N_1=1$

4) $\{a,b^{(4},(a+c)/2\}(2^{2)})$, $(4)(4)^2$, $N_1=4$

5) $\{a^{(2},b,(a+c)/2^{(4}\}(2^{2)})$, $(3)(3)^2$, $N_1=1$

6) $\{a^{(2},b,(a+c)/2^{(4}\}(2^{22)})$, $(3)(3)^2$, $N_1=1$

5s $(Pm)\,\{a,b,c\}(m)$, $AC:\{a,b,ab\}\{m,mc\}$, V

1) $\{a,b,c^{(3}\}(m^{2)})$, $(4)(3)^1$, $N_1=4$ $N_2=22$ $N_3=112$

2) $\{a,b,c^{(4}\}(m^{2)})$, $(4)(4)^2$, $N_1=4$ $N_2=16$

3) $\{a^{(2},b,c^{(4}\}(m^{2)})$, $(4)(4)^2$, $N_1=4$ $N_2=16$

6s $(Bm)\,\{a,b,(a+c)/2\}(m)$, $AC:\{m\}\{(a+c)/2,b(a+c)/2\}$, VI

1) $\{a,b,(a+c)/2^{(3}\}(m^{2)})$, $(2)(3)^1$, $N_1=4$ $N_2=12$

2) $\{a^{(2},b,(a+c)/2^{2)}\}(m^{22)})$, $(2)(3)^2$, $N_1=2$

3) $\{a^{(2},b,(a+c)/2^{22)}\}(m^{2)})$, $(2)(3)^2$, $N_1=2$

4) $\{a,b,(a+c)/2^{(4}\}(m^{2)})$, $(2)(3)^2$, $N_1=2$

5) $\{a,b,(a+c)/2^{(4}\}(m^{22)})$, $(2)(3)^2$, $N_1=2$

6) $\{a,b^{(2},(a+c)/2^{(4}\}(m^{2)})$, $(2)(3)^2$, $N_1=2$

7) $\{a,b^{(2},(a+c)/2^{(4}\}(m^{22)})$, $(2)(3)^2$, $N_1=2$

7s $(P2/m)\,\{a,b,c\}(2:m)$, $AC:\{m,cm\}\{2,2a,2b,2ab\}$, VII

1) $\{a,b,c^{(3}\}(2:m^{2)})$, $(3)(5)^1$, $N_1=7$ $N_2=64$ $N_3=700$

2) $\{a^{(3},b,c\}(2^{2)}:m)$, $(3)(5)^1$, $N_1=7$ $N_2=64$ $N_3=700$

3) $\{a,b,c^{(4}\}(2:m^{2)})$, $(4)(5)^2$, $N_1=8$ $N_2=64$ $N_3=448$

4) $\{a,b,c^{(4}\}(2^{(2}:m^{2)})$, $(4)(5)^2$, $N_1=8$ $N_2=64$ $N_3=448$

5) $\{a^{(2},b,c^{(4}\}(2:m^{2)})$, $(4)(5)^2$, $N_1=8$ $N_2=64$ $N_3=448$

6) $\{a^{(4},b,c\}(2^{2)}:m)$, $(3)(9)^2$, $N_1=11$ $N_2=132$ $N_3=1344$

7) $\{a^{(4},b,c\}(2^{2)}:m^{(2)})$, $(3)(9)^2$, $N_1=11$ $N_2=132$ $N_3=1344$

8) $\{a^{(4},b,c^{(2}\}(2^{2)}:m)$, $(3)(9)^2$, $N_1=11$ $N_2=132$ $N_3=1344$

8s $(B2/m)\,\{a,b,(a+c)/2\}(2:m)$, $AC:\{m\}\{2,2b\}\{(a+c)/2,b(a+c)/2\}$, VIII

1) $\{a,b,(a+c)/2^{(3}\}(2:m^{2)})$, $(2)(3)(3)^1$, $N_1=8$ $N_2=60$ $N_3=336$

2) $\{a,b^{(3},(a+c)/2\}(2^{2)}:m)$, $(2)(3)(3)^1$, $N_1=8$ $N_2=60$ $N_3=336$

3) $\{a^{(2},b,(a+c)/2^{2)}\}(2^{22)}:m^{22)})$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$

4) $\{a^{(2},b,(a+c)/2^{22)}\}(2^{2)}:m^{2)})$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$

5) $\{a,b,(a+c)/2^{(4}\}(2:m^{2)})$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$
6) $\{a,b,(a+c)/2^{(4}\}(2:m^{22)})$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$
7) $\{a,b,(a+c)/2^{(4}\}(2^{(2}:m^{2)})$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$
8) $\{a,b,(a+c)/2^{(4}\}(2^{(2}:m^{22)})$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$
9) $\{a^{(2},b,(a+c)/2^{(4}\}(2^{2)}:m)$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$
10) $\{a^{(2},b,(a+c)/2^{(4}\}(2^{22)}:m)$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$
11) $\{a^{(2},b,(a+c)/2^{(4}\}(2^{2)}:m^{(2})$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$
12) $\{a^{(2},b,(a+c)/2^{(4}\}(2^{22)}:m^{(2})$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$
13) $\{a,b^{(4},(a+c)/2\}(2^{2)}:m)$, $(2)(4)(4)^2$, $N_1=12$ $N_2=96$
14) $\{a,b^{(4},(a+c)/2\}(2^{2)}:m^{(2})$, $(2)(4)(4)^2$, $N_1=12$ $N_2=96$
15) $\{a,b^{(2},(a+c)/2^{(4}\}(2:m^{2)})$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$
16) $\{a,b^{(2},(a+c)/2^{(4}\}(2:m^{22)})$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$

9s $(P222)\,\{a,b,c\}(2:2')$, $AC:\{\{c\}\{2,2a,2b,2ab\},\{b\}\{2',2'a,2'c,2'ac\},\{a\}\{22',22'b,22'c,22'bc\}\}\{\{2,2',22'\},\{2a,2'a,22'\},\{2',2b,22'b\},\{2'a,2ab,22'b\},\{2,2'c,22'c\},\{2a,2'ac,22'c\},\{2b,2'c,22'bc\},\{2ab,2'ac,22'bc\}\}$, IX

1) $\{a,b,c^{(3}\}(2:2^{2)})$, $(4)(5,(5,5))^1$, $N_1=8$ $N_2=96$ $N_3=1516$ $N_4=20160$
2) $\{a,b,c^{(4}\}(2:2^{2)})$, $(6)(5,(9,9))^2$, $N_1=8$ $N_2=108$ $N_3=1344$
3) $\{a,b,c^{(4}\}(2^{(2}:2^{2)})$, $(6)(5,(9,9))^2$, $N_1=8$ $N_2=108$ $N_3=1344$
4) $\{a^{(2},b,c^{(4}\}(2:2^{2)})$, $(6)(5,(9,9))^2$, $N_1=8$ $N_2=108$ $N_3=1344$
5) $\{a^{(2},b^{(2},c^{(4}\}(2:2^{2)})$,$(6)(5,(9,9))^2$, $N_1=8$ $N_2=108$ $N_3=1344$

10s $(C222)\,\{a,(a+b)/2,c\}(2:2')$, $AC:\{(a+b)/2\}\{(2',22'),(2'c,22'c\}$, VIII

1) $\{a,(a+b)/2^{(3},c\}(2^{2)}:2^{2)})$, $(2)(4,4)^1$, $N_1=14$ $N_2=168$ $N_3=1344$
2) $\{a,(a+b)/2,c^{(3}\}(2:2^{2)})$, $(2)(3,3)^1$, $N_1=8$ $N_2=60$ $N_3=336$
3) $\{a^{(2},(a+b)/2^{2)},c\}(2:2^{22)})$, $(2)(3,3)^2$, $N_1=6$ $N_2=24$
4) $\{a^{(2},(a+b)/2^{2)},c^{(2}\}(2:2^{22)})$, $(2)(3,3)^2$, $N_1=6$ $N_2=24$
5) $\{a^{(2},(a+b)/2^{22)},c\}(2:2^{22)})$, $(2)(3,3)^2$, $N_1=6$ $N_2=24$
6) $\{a^{(2},(a+b)/2^{2)},c^{(2}\}(2:2^{22)})$, $(2)(3,3)^2$, $N_1=6$ $N_2=24$
7) $\{a,(a+b)/2^{(4},c\}(2^{2)}:2^{2)})$, $(2)(4,4)^2$, $N_1=12$ $N_2=96$
8) $\{a,(a+b)/2^{(4},c\}(2^{22)}:2^{22)})$, $(2)(4,4)^2$, $N_1=12$ $N_2=96$
9) $\{a,(a+b)/2^{(4},c\}(2^{2)}:2^{2')})$, $(2)(4,4)^2$, $N_1=12$ $N_2=96$
10) $\{a,(a+b)/2^{(4},c\}(2^{22)}:2^{22')})$, $(2)(4,4)^2$, $N_1=12$ $N_2=96$
11) $\{a,(a+b)/2^{(4},c^{(2}\}(2^{2)}:2^{2)})$, $(2)(4,4)^2$, $N_1=12$ $N_2=96$
12) $\{a,(a+b)/2^{(4},c^{(2}\}(2^{22)}:2^{22)})$, $(2)(4,4)^2$, $N_1=12$ $N_2=96$
13) $\{a,(a+b)/2,c^{(4}\}(2:2^{2)})$, $(2)(3,3)^2$, $N_1=6N_2=24$
14) $\{a,(a+b)/2,c^{(4}\}(2^{(2}:2^{2)})$, $(2)(3,3)^2$, $N_1=6$ $N_2=24$
15) $\{a,(a+b)/2^{(2},c^{(4}\}(2:2^{2)})$, $(2)(3,3)^2$ $N_1=6$ $N_2=24$
16) $\{a,(a+b)/2^{(2},c^{(4}\}(2^{(2}:2^{2)})$, $(2)(3,3)^2$, $N_1=6$ $N_2=24$

11s $(I222)\,\{a,b,(a+b+c)/2\}(2:2')$, $AC:\{(a+b+c)/2\}\{2,2',22'\}$, X

1) $\{a,b,(a+b+c)/2^{(3}\}(2:2^{2)})$, $(2)(6)^1$, $N_1=4$ $N_2=12$
2) $\{a,b,(a+b+c)/2^{(4}\}(2:2^{2)})$, $(2)(6)^2$, $N_1=2$
3) $\{a,b,(a+b+c)/2^{(4}\}(2:2^{22)})$, $(2)(6)^2$, $N_1=2$
4) $\{a,b,(a+b+c)/2^{(4}\}(2^{(2}:2^{2)})$, $(2)(6)^2$, $N_1=2$
5) $\{a,b,(a+b+c)/2^{(4}\}(2^{(2}:2^{22)})$, $(2)(6)^2$, $N_1=2$
6) $\{a^{(2},b^{(2},(a+b+c)/2^{2)}\}(2:2^{22)})$, $(2)(6)^2$, $N_1=2$
7) $\{a,b,(a+b+c)/2^{22)}\}(2:2^{2)})$, $(2)(6)^2$, $N_1=2$

12s $(F222)\,\{a,(a+b)/2,(a+c)/2\}(2:2')$, $AC:\{\{2,2(a+b)/2\},\{2',2'(a+c)/2\},\{22',$

$22'(a+b)/2(a+c)/2\}\}$, XI

1) $\{a,(a+b)/2,(a+c)/2^{(3}\}(2:2^{2)})$, $(3,(3,3))^1, N_1=5$ $N_2=42$ $N_3=336$
2) $\{a,(a+b)/2,(a+c)/2^{(4}\}(2:2^{2)})$, $(3,(4,4))^2, N_1=6$ $N_2=48$
3) $\{a,(a+b)/2,(a+c)/2^{(4}\}(2:2^{22)})$, $(3,(4,4))^2, N_1=6$ $N_2=48$
4) $\{a,(a+b)/2,(a+c)/2^{(4}\}(2^{(2}:2^{2)})$, $(3,(4,4))^2, N_1=6$ $N_2=48$
5) $\{a,(a+b)/2,(a+c)/2^{(4}\}(2^{(2}:2^{22)})$, $(3,(4,4))^2, N_1=6$ $N_2=48$
6) $\{a,(a+b)/2^{(2},(a+c)/2^{(4}\}(2:2^{2)})$, $(3,(4,4))^2, N_1=6$ $N_2=48$
7) $\{a,(a+b)/2^{(2},(a+c)/2^{(4}\}(2:2^{22)})$, $(3,(4,4))^2, N_1=6$ $N_2=48$

13s $(Pmm2)$, $\{a,b,c\}(2m)$, $AC:\{c\}\{\{m,am\},\{2m,2bm\}\}$, XII

1) $\{a^{(3},b,c\}(2^{2)}m^{2)})$, $(2)(3,3)^1, N_1=16$ $N_2=300$ $N_3=5712$ $N_4=80640$
2) $\{a^{(4},b,c\}(2^{2)}m^{2)})$, $(2)(3,4)^2, N_1=20$ $N_2=384$ $N_3=5376$
3) $\{a^{(4},b,c\}(2^{2)}m^{2')})$, $(2)(3,4)^2, N_1=20$ $N_2=384$ $N_3=5376$
4) $\{a^{(4},b,c^{2)}\}(2^{2)}m^{2)})$, $(2)(3,4)^2, N_1=20$ $N_2=384$ $N_3=5376$
5) $\{a^{(4},b,c^{2)}\}(2^{2)}m^{2')})$, $(2)(3,4)^2, N_1=20$ $N_2=384$ $N_3=5376$
6) $\{a^{(4},b^{2)},c\}(2^{2)}m^{2)})$, $(2)(3,4)^2, N_1=20$ $N_2=384$ $N_3=5376$
7) $\{a^{(4},b^{2)},c^{2)}\}(2^{2)}m^{2)})$, $(2)(3,4)^2, N_1=20$ $N_2=384$ $N_3=5376$

14s $(Cmm2)$ $\{a,(a+b)/2,c\}(2m)$, $AC:\{(a+b)/2\}\{c\}\{m,2m\}$, XII

1) $\{a,(a+b)/2^{(3},c\}(2^{2)}m)$, $(2)(2)(4)^1$, $N_1=14$ $N_2=168$ $N_3=1344$
2) $\{a^{(2},(a+b)/2^{2)},c\}(2m^{22)})$, $(2)(2)(3)^2$, $N_1=8$ $N_2=48$
3) $\{a^{(2},(a+b)/2^{2)},c^{2)}\}(2m^{22)})$, $(2)(2)(3)^2$, $N_1=8$ $N_2=48$
4) $\{a^{(2},(a+b)/2^{22)},c\}(2m^{2)})$, $(2)(2)(3)^2$, $N_1=8$ $N_2=48$
5) $\{a^{(2},(a+b)/2^{22)},c^{2)}\}(2m^{2)})$, $(2)(2)(3)^2$, $N_1=8$ $N_2=48$
6) $\{a,(a+b)/2^{(4},c\}(2^{2)}m)$, $(2)(2)(4)^2$, $N_1=12$ $N_2=96$
7) $\{a,(a+b)/2^{(4},c\}(2^{22)}m)$, $(2)(2)(4)^2$, $N_1=12$ $N_2=96$
8) $\{a,(a+b)/2^{(4},c\}(2^{2)}m^{(2)})$, $(2)(2)(4)^2$, $N_1=12$ $N_2=96$
9) $\{a,(a+b)/2^{(4},c\}(2^{22)}m^{(2)})$, $(2)(2)(4)^2$, $N_1=12$ $N_2=96$
10) $\{a,(a+b)/2^{(4},c^{2)}\}(2^{2)}m)$, $(2)(2)(4)^2$, $N_1=12$ $N_2=96$
11) $\{a,(a+b)/2^{(4},c^{2)}\}(2^{22)}m)$, $(2)(2)(4)^2$, $N_1=12$ $N_2=96$
12) $\{a,(a+b)/2^{(4},c^{2)}\}(2^{2)}m^{(2)})$, $(2)(2)(4)^2$, $N_1=12$ $N_2=96$
13) $\{a,(a+b)/2^{(4},c^{2)}\}(2^{22)}m^{(2)})$, $(2)(2)(4)^2$, $N_1=12$ $N_2=96$

15s $(Bmm2)$ $\{a,b,(a+c)/2\}(2m)$, $AC:\{(a+c)/2\}\{m\}\{2m,2mb\}$, X

1) $\{a,b^{(3},(a+c)/2\}(2^{2)}m)$, $(2)(2)(3)^1$, $N_1=10$ $N_2=96$ $N_3=672$
2) $\{a^{(3},b,(a+c)/2^{(-3}\}(2^{2)}m^{2)})$, $(2)(2)(3)^1$, $N_1=10$ $N_2=96$ $N_3=672$
3) $\{a^{(2},b,(a+c)/2^{2)}\}(2^{22)}m^{22)})$, $(2)(2)(3)^2$, $N_1=8$ $N_2=48$
4) $\{a^{(2},b,(a+c)/2^{2)}\}(2^{22)}m^{22')})$, $(2)(2)(3)^2$, $N_1=8$ $N_2=48$
5) $\{a^{(2},b^{2)},(a+c)/2^{2)}\}(2^{22)}m^{22)})$, $(2)(2)(3)^2$, $N_1=8$ $N_2=48$
6) $\{a^{(2},b,(a+c)/2^{22)}\}(2^{2)}m^{2)})$, $(2)(2)(3)^2$, $N_1=8$ $N_2=48$
7) $\{a^{(2},b,(a+c)/2^{22)}\}(2^{2)}m^{2')})$, $(2)(2)(3)^2$, $N_1=8$ $N_2=48$
8) $\{a^{(2},b^{2)},(a+c)/2^{22)}\}(2^{2)}m^{2)})$, $(2)(2)(3)^2$, $N_1=8$ $N_2=48$
9) $\{a^{(2},b,(a+c)/2^{(4}\}(2^{2)}m^{2)})$, $(2)(2)(3)^2$, $N_1=8$ $N_2=48$
10) $\{a^{(2},b,(a+c)/2^{(4}\}(2^{2)}m^{2')})$, $(2)(2)(3)^2$, $N_1=8$ $N_2=48$
11) $\{a^{(2},b,(a+c)/2^{(4}\}(2^{22)}m^{22)})$, $(2)(2)(3)^2$, $N_1=8$ $N_2=48$
13) $\{a^{(2},b^{2)},(a+c)/2^{(4}\}(2^{2)}m^{2)})$, $(2)(2)(3)^2$, $N_1=8$ $N_2=48$
14) $\{a^{(2},b^{2)},(a+c)/2^{(4}\}(2^{22)}m^{22)})$, $(2)(2)(3)^2$, $N_1=8$ $N_2=48$
15) $\{a,b^{(4},(a+c)/2\}(2^{2)}m)$, $(2)(2)(4)^2$, $N_1=12$ $N_2=96$
16) $\{a,b^{(4},(a+c)/2\}(2^{2)}m^{(2)})$, $(2)(2)(4)^2$, $N_1=12$ $N_2=96$

17) $\{a, b^{(4}, (a+c)/2^{(2}\}(2^{2)}m)$, $(2)(2)(4)^2$, $N_1 = 12$ $N_2 = 96$
18) $\{a, b^{(4}, (a+c)/2^{(2}\}(2^{2)}m^{(2})$, $(2)(2)(4)^2$, $N_1 = 12$ $N_2 = 96$

16s $(Imm2)\{a, b, (a+b+c)/2\}(2m)$, $AC : \{(a+b+c)/2\}\{m, 2m\}$, VI
1) $\{a, b^{(3}, (a+b+c)/2^{(-3}\}(2^{2)}m)$, $(2)(4)^1$, $N_1 = 6$ $N_2 = 24$
2) $\{a^{(2}, b^{2)}, (a+b+c)/2^{2)}\}(2m^{22)})$, $(2)(3)^2$, $N_1 = 2$
3) $\{a^{(2}, b^{2)}, (a+b+c)/2^{22)}\}(2m^{2)})$, $(2)(3)^2$, $N_1 = 2$
4) $\{a, b^{2)}, (a+b+c)/2^{(4}\}(2^{2)}m)$, $(2)(4)^2$, $N_1 = 4$
5) $\{a, b^{2)}, (a+b+c)/2^{(4}\}(2^{22)}m)$, $(2)(4)^2$, $N_1 = 4$
6) $\{a, b^{2)}, (a+b+c)/2^{(4}\}(2^{2)}m^{(2})$, $(2)(4)^2$, $N_1 = 4$
7) $\{a, b^{2)}, (a+b+c)/2^{(4}\}(2^{22)}m^{(2})$, $(2)(4)^2$, $N_1 = 4$
8) $\{a^{(2}, b^{2)}, (a+b+c)/2^{(4}\}(2m^{2)})$, $(2)(3)^2$, $N_1 = 2$
9) $\{a^{(2}, b^{2)}, (a+b+c)/2^{(4}\}(2m^{22)})$, $(2)(3)^2$, $N_1 = 2$

17s $(Fmm2)\{a, (a+b)/2, (a+c)/2\}(2m)$, $AC : \{(a+c)/2, (a+c)/2(a+b)/2\}\{m, 2m\}$ $\{(a+c)/2m, (a+c)/2(a+b)/22m\}$, XIV
1) $\{a, (a+b)/2^{(3}, (a+c)/2\}(2^{2)}m)$, $(3)(4)(4)^1$, $N_1 = 14$ $N_2 = 168$ $N_3 = 1344$
2) $\{a, (a+b)/2^{(4}, (a+c)/2\}(2^{2)}m)$, $(4)(4)(4)^2$, $N_1 = 12$ $N_2 = 96$
3) $\{a, (a+b)/2^{(4}, (a+c)/2\}(2^{22)}m)$, $(4)(4)(4)^2$, $N_1 = 12$ $N_2 = 96$
4) $\{a, (a+b)/2^{(4}, (a+c)/2\}(2^{2)}m^{(2})$, $(4)(4)(4)^2$, $N_1 = 12$ $N_2 = 96$
5) $\{a, (a+b)/2^{(4}, (a+c)/2\}(2^{22)}m^{(2})$, $(4)(4)(4)^2$, $N_1 = 12$ $N_2 = 96$
6) $\{a, (a+b)/2^{(4}, (a+c)/2^{(2}\}(2^{2)}m)$, $(4)(4)(4)^2$, $N_1 = 12$ $N_2 = 96$
7) $\{a, (a+b)/2^{(4}, (a+c)/2^{(2}\}(2^{22)}m)$, $(4)(4)(4)^2$, $N_1 = 12$ $N_2 = 96$
8) $\{a, (a+b)/2^{(4}, (a+c)/2^{(2}\}(2^{2)}m^{(2})$, $(4)(4)(4)^2$, $N_1 = 12$ $N_2 = 96$
9) $\{a, (a+b)/2^{(4}, (a+c)/2\}(2^{22)}m^{(2})$, $(4)(4)(4)^2$, $N_1 = 12$ $N_2 = 96$

18s $(Pmmm)\{a, b, c\}(2m : 2')$, $AC : \{\{m, ma\}, \{2m, 2mb\}, \{22'm, 22'mc\}\}$, XV
1) $\{a, b, c^{(3}\}(2m : 2^{2)})$, $((3,(3,3))^1, N_1 = 16, N_2 = 450, N_3 = 17220, N_4 = 685440,$ $N_5 = 19998720$
2) $\{a, b, c^{(4}\}(2m : 2^{2)})$, $((3,3),4)^2, N_1 = 20, N_2 = 624, N_3 = 22848, N_4 = 645120$
3) $\{a, b, c^{(4}\}(2^{(2}m : 2^{2)})$, $((3,3),4)^2, N_1 = 20, N_2 = 624, N_3 = 22848, N_4 = 645120$
4) $\{a, b, c^{(4}\}(2m^{(2} : 2^{2)})$, $((3,3),4)^2, N_1 = 20, N_2 = 624, N_3 = 22848, N_4 = 645120$
5) $\{a^{(2}, b, c^{(4}\}(2m : 2^{2)})$, $((3,3),4)^2, N_1 = 20, N_2 = 624, N_3 = 22848, N_4 = 645120$
6) $\{a^{(2}, b, c^{(4}\}(2^{(2}m : 2^{2)})$,$((3,3),4)^2, N_1 = 20, N_2 = 624, N_3 = 22848, N_4 = 645120$
7) $\{a^{(2}, b^{2)}, c^{(4}\}(2m : 2^{2)})$,$((3,3),4)^2, N_1 = 20, N_2 = 624, N_3 = 22848, N_4 = 645120$

19s $(Cmmm)\{a, (a+b)/2, c\}(2m : 2')$, $AC : \{(a+b)/2\}\{m, 2m\}\{22'm, 22'mc\}$, XVI
1) $\{a, (a+b)/2^{(3}, c\}(2^{2)}m : 2^{2)})$, $(2)(3)(4)^1, N_1 = 22, N_2 = 504, N_3 = 10752,$ $N_4 = 161280$
2) $\{a, (a+b)/2, c^{(3}\}(2m : 2^{2)})$, $(2)(3)(3)^1, N_1 = 16, N_2 = 300, N_3 = 5712,$ $N_4 = 80640$
3) $\{a^{(2}, (a+b)/2^{2)}, c\}(2m^{22)} : 2^{22)})$, $(2)(3)(3)^2, N_1 = 14, N_2 = 216, N_3 = 2688$
4) $\{a^{(2}, (a+b)/2^{2)}, c\}(2^{(2}m^{22)} : 2^{22)})$, $(2)(3)(3)^2, N_1 = 14, N_2 = 216, N_3 = 2688$
5) $\{a^{(2}, (a+b)/2^{2)}, c^{2)}\}(2m^{22)} : 2^{22)})$, $(2)(3)(3)^2, N_1 = 14, N_2 = 216, N_3 = 2688$
6) $\{a^{(2}, (a+b)/2^{22)}, c\}(2m^{2)} : 2^{2)})$, $(2)(3)(3)^2, N_1 = 14, N_2 = 216, N_3 = 2688$
7) $\{a^{(2}, (a+b)/2^{22)}, c\}(2^{(2}m^{2)} : 2^{2)})$, $(2)(3)(3)^2, N_1 = 14, N_2 = 216, N_3 = 2688$
8) $\{a^{(2}, (a+b)/2^{22)}, c^{2)}\}(2m^{2)} : 2^{2)})$, $(2)(3)(3)^2, N_1 = 14, N_2 = 216, N_3 = 2688$
9) $\{a, (a+b)/2, c^{(4}\}(2m : 2^{2)})$, $(2)(3)(4)^2, N_1 = 20, N_2 = 384, N_3 = 5376$
10) $\{a, (a+b)/2^{(2}, c^{(4}\}(2m : 2^{2)})$, $(2)(3)(4)^2, N_1 = 20, N_2 = 384, N_3 = 5376$
11) $\{a, (a+b)/2, c^{(4}\}(2^{(2}m : 2^{2)})$, $(2)(3)(4)^2, N_1 = 20, N_2 = 384, N_3 = 5376$
12) $\{a, (a+b)/2, c^{(4}\}(2m^{(2} : 2^{2)})$, $(2)(3)(4)^2, N_1 = 20, N_2 = 384, N_3 = 5376$

13) $\{a,(a+b)/2^{(2},c^{(4}\}(2^{(2}m:2^{2)})$, $(2)(3)(4)^2, N_1=20, N_2=384, N_3=5376$
14) $\{a,(a+b)/2^{(2},c^{(4}\}(2m^{(2}:2^{2)})$, $(2)(3)(4)^2, N_1=20, N_2=384, N_3=5376$
15) $\{a,(a+b)/2^{(4},c\}(2^{2)}m:2^{2)})$, $(2)(3)(4)^2, N_1=20, N_2=384, N_3=5376$
16) $\{a,(a+b)/2^{(4},c^{2)}\}(2^{2)}m:2^{2)})$, $(2)(3)(4)^2, N_1=20, N_2=384, N_3=5376$
17) $\{a,(a+b)/2^{(4},c\}(2^{22)}m:2^{2)})$, $(2)(3)(4)^2, N_1=20, N_2=384, N_3=5376$
19) $\{a,(a+b)/2^{(4},c\}(2^{2)}m:2^{22)})$, $(2)(3)(4)^2, N_1=20, N_2=384, N_3=5376$
20) $\{a,(a+b)/2^{(4},c^{2)}\}(2^{22)}m:2^{2)})$, $(2)(3)(4)^2, N_1=20, N_2=384, N_3=5376$
21) $\{a,(a+b)/2^{(4},c^{2)}\}(2^{2)}m^{(2}:2^{2)})$, $(2)(3)(4)^2, N_1=20, N_2=384, N_3=5376$
22) $\{a,(a+b)/2^{(4},c\}(2^{22)}m^{(2}:2^{2)})$, $(2)(3)(4)^2, N_1=20, N_2=384, N_3=5376$
23) $\{a,(a+b)/2^{(4},c\}(2^{22)}m:2^{22)})$, $(2)(3)(4)^2, N_1=20, N_2=384, N_3=5376$
24) $\{a,(a+b)/2^{(4},c\}(2^{2)}m^{(2}:2^{22)})$, $(2)(3)(4)^2, N_1=20, N_2=384, N_3=5376$
25) $\{a,(a+b)/2^{(4},c^{2)}\}(2^{22)}m^{(2}:2^{2)})$, $(2)(3)(4)^2, N_1=20, N_2=384, N_3=5376$
26) $\{a,(a+b)/2^{(4},c\}(2^{22)}m^{(2}:2^{22)})$, $(2)(3)(4)^2, N_1=20, N_2=384, N_3=5376$

20s $(Immm)\{a,b,(a+b+c)/2\}(2m:2')$, $AC:\{(a+b+c)/2\}\{m,2m,22'm\}$, XVII
1) $\{a,b,(a+b+c)/2^{(3}\}(2m:2^{2)})$, $(2)(6)^1$, $N_1=10$ $N_2=96$ $N_3=672$
2) $\{a,b,(a+b+c)/2^{(4}\}(2m:2^{2)})$, $(2)(6)^2$, $N_1=8$ $N_2=48$
3) $\{a,b,(a+b+c)/2^{(4}\}(2m:2^{22)})$, $(2)(6)^2$, $N_1=8$ $N_2=48$
4) $\{a,b,(a+b+c)/2^{(4}\}(2m^{(2}:2^{2)})$, $(2)(6)^2$, $N_1=8$ $N_2=48$
5) $\{a,b,(a+b+c)/2^{(4}\}(2m^{(2}:2^{22)})$, $(2)(6)^2$, $N_1=8$ $N_2=48$
6) $\{a,b,(a+b+c)/2^{(4}\}(2^{(2}m:2^{2)})$, $(2)(6)^2$, $N_1=8$ $N_2=48$
7) $\{a,b,(a+b+c)/2^{(4}\}(2^{(2}m:2^{22)})$, $(2)(6)^2$, $N_1=8$ $N_2=48$
8) $\{a^{(2},b^{2)},(a+b+c)/2^{(4}\}(2m^{2)}:2)$, $(2)(4)^2$, $N_1=4$ $N_2=16$
9) $\{a^{(2},b^{2)},(a+b+c)/2^{(4}\}(2m^{22)}:2)$, $(2)(4)^2$, $N_1=4$ $N_2=16$
10) $\{a^{(2},b^{2)},(a+b+c)/2^{2)}\}(2m^{22)}:2^{22)})$, $(2)(6)^2$, $N_1=8$ $N_2=48$
11) $\{a^{(2},b^{2)},(a+b+c)/2^{2)}\}(2^{(2}m^{22)}:2^{22)})$, $(2)(6)^2$, $N_1=8$ $N_2=48$
12) $\{a^{(2},b^{2)},(a+b+c)/2^{22)}\}(2m^{2)}:2^{2)})$, $(2)(6)^2$, $N_1=8$ $N_2=48$
13) $\{a^{(2},b^{2)},(a+b+c)/2^{22)}\}(2^{(2}m^{2)}:2)$, $(2)(6)^2$, $N_1=8$ $N_2=48$

21s $(Fmmm)\{a,(a+b)/2,(a+c)/2\}(2m:2')$, $AC:\{((a+b)/2,m),((a+c)/2,2m),((a+b)/2(a+c)/2,22'm)\}$, XVI
1) $\{a,(a+b)/2,(a+c)/2^{(3}\}(2m:2^{2)})$, $(4,(4,4))^1$, $N_1=18, N_2=432, N_3=10080, N_4=161280$
2) $\{a,(a+b)/2,(a+c)/2^{(4}\}(2m:2^{2)})$, $(4,4,4)^2$, $N_1=28, N_2=672, N_3=10752$
3) $\{a,(a+b)/2,(a+c)/2^{(4}\}(2m:2^{22)})$, $(4,4,4)^2$, $N_1=28, N_2=672, N_3=10752$
4) $\{a,(a+b)/2,(a+c)/2^{(4}\}(2m^{(2}:2^{2)})$, $(4,4,4)^2$, $N_1=28, N_2=672, N_3=10752$
5) $\{a,(a+b)/2,(a+c)/2^{(4}\}(2m^{(2}:2^{22)})$, $(4,4,4)^2$, $N_1=28, N_2=672, N_3=10752$
6) $\{a,(a+b)/2,(a+c)/2^{(4}\}(2^{(2}m:2^{2)})$, $(4,4,4)^2$, $N_1=28, N_2=672, N_3=10752$
7) $\{a,(a+b)/2,(a+c)/2^{(4}\}(2^{(2}m:2^{22)})$, $(4,4,4)^2$, $N_1=28, N_2=672, N_3=10752$
8) $\{a,(a+b)/2^{(2},(a+c)/2^{(4}\}(2m:2^{2)})$, $(4,4,4)^2$, $N_1=28, N_2=672, N_3=10752$
9) $\{a,(a+b)/2^{(2},(a+c)/2^{(4}\}(2m:2^{22)})$, $(4,4,4)^2$, $N_1=28, N_2=672, N_3=10752$
10) $\{a,(a+b)/2^{(2},(a+c)/2^{(4}\}(2m^{(2}:2^{2)})$, $(4,4,4)^2$, $N_1=28, N_2=672, N_3=10752$
11) $\{a,(a+b)/2^{(2},(a+c)/2^{(4}\}(2m^{(2}:2^{22)})$, $(4,4,4)^2$, $N_1=28, N_2=672, N_3=10752$
12) $\{a,(a+b)/2^{(2},(a+c)/2^{(4}\}(2^{(2}m:2^{2)})$, $(4,4,4)^2$, $N_1=28, N_2=672, N_3=10752$
13) $\{a,(a+b)/2^{(2},(a+c)/2^{(4}\}(2^{(2}m:2^{22)})$, $(4,4,4)^2$, $N_1=28, N_2=672, N_3=10752$

22s $(P4)\{a,b,c\}(4)$, $AC:\{c\}\{4,4a\}$, VI
1) $\{a^{2)},b^{2')},c\}(4^{(4)})$, $(2)(4)^2$, $N_1=4$
2) $\{a^{2)},b^{2')},c^{2)}\}(4^{(4)})$, $(2)(4)^2$, $N_1=4$

3) $\{a^{22)}, b^{22')}, c\}(4^{(4})$, $(2)(4)^2$, $N_1 = 4$
4) $\{a^{22)}, b^{22')}, c^{2)}\}(4^{(4})$, $(2)(4)^2$, $N_1 = 4$

24s $(P4mm)\{a,b,c\}(4m)$, $AC:\{c\}\{m\}\{4,4a\}$, XIII
1) $\{a,b,c\}(4^{(4}m^{2)})$, $(2)(2)(3)^2$, $N_1 = 8$ $N_2 = 48$
2) $\{a,b,c\}(4^{(4}m^{22)})$, $(2)(2)(3)^2$, $N_1 = 8$ $N_2 = 48$
3) $\{a,b,c^{2)}\}(4^{(4}m^{2)})$, $(2)(2)(3)^2$, $N_1 = 8$ $N_2 = 48$
4) $\{a,b,c^{2)}\}(4^{(4}m^{22)})$, $(2)(2)(3)^2$, $N_1 = 8$ $N_2 = 48$
5) $\{a^{(2},b^{2)},c\}(4^{(4}m^{2)})$, $(2)(2)(3)^2$, $N_1 = 8$ $N_2 = 48$
6) $\{a^{(2},b^{2)},c\}(4^{(4}m^{22)})$, $(2)(2)(3)^2$, $N_1 = 8$ $N_2 = 48$
7) $\{a^{(2},b^{2)},c^{2)}\}(4^{(4}m^{2)})$, $(2)(2)(3)^2$, $N_1 = 8$ $N_2 = 48$
8) $\{a^{(2},b^{2)},c^{2)}\}(4^{(4}m^{22)})$, $(2)(2)(3)^2$, $N_1 = 8$ $N_2 = 48$
9) $\{a^{2)},b^{2')},c\}(4^{(-4}m^{2)})$, $(2)(2)(4)^2$, $N_1 = 12$ $N_2 = 96$
10) $\{a^{2)},b^{2')},c\}(4^{(-4}m^{2')})$, $(2)(2)(4)^2$, $N_1 = 12$ $N_2 = 96$
12) $\{a^{2)},b^{2')},c^{2)}\}(4^{(-4}m^{2')})$, $(2)(2)(4)^2$, $N_1 = 12$ $N_2 = 96$
13) $\{a^{22)},b^{22')},c\}(4^{(4}m^{22)})$, $(2)(2)(4)^2$, $N_1 = 12$ $N_2 = 96$
14) $\{a^{22)},b^{22')},c\}(4^{(4}m^{22')})$, $(2)(2)(4)^2$, $N_1 = 12$ $N_2 = 96$
15) $\{a^{22)},b^{22')},c^{2)}\}(4^{(4}m^{22)})$, $(2)(2)(4)^2$, $N_1 = 12$ $N_2 = 96$
16) $\{a^{22)},b^{22')},c^{2)}\}(4^{(4}m^{22')})$, $(2)(2)(4)^2$, $N_1 = 12$ $N_2 = 96$

25s $(I4mm)\{a,b,(a+b+c)/2\}(4m)$, $AC:\{(a+b+c)/2\}\{4\}\{m\}$, XX
1) $\{a,b,(a+b+c)/2^{(2}\}(4^{(4}m^{2)})$, $N_1 = 4$
2) $\{a,b,(a+b+c)/2^{(2}\}(4^{(4}m^{22)})$, $N_1 = 4$
3) $\{a,b,(a+b+c)/2\}(4^{(4}m^{2)})$, $N_1 = 4$
4) $\{a,b,(a+b+c)/2\}(4^{(4}m^{22)})$, $N_1 = 4$
5) $\{a^{(2},b^{2)},(a+b+c)/2^{2)}\}(4^{22)}m^{22)})$, $N_1 = 4$
6) $\{a^{(2},b^{2)},(a+b+c)/2^{2)}\}(4^{22)}m^{22')})$, $N_1 = 4$
7) $\{a^{(2},b^{2)},(a+b+c)/2^{2)}\}(4^{(4}m^{22)})$, $N_1 = 4$
8) $\{a^{(2},b^{2)},(a+b+c)/2^{22)}\}(4^{2)}m^{2)})$, $N_1 = 4$
9) $\{a^{(2},b^{2)},(a+b+c)/2^{22)}\}(4^{2)}m^{2')})$, $N_1 = 4$
10) $\{a^{(2},b^{2)},(a+b+c)/2^{22)}\}(4^{(4}m^{2)})$, $N_1 = 4$
11) $\{a^{(2},b^{2)},(a+b+c)/2^{(4}\}(4^{2)}m^{2)})$, $N_1 = 4$
12) $\{a^{(2},b^{2)},(a+b+c)/2^{(4}\}(4^{22)}m^{22)})$, $N_1 = 4$
13) $\{a^{(2},b^{2)},(a+b+c)/2^{(4}\}(4^{2)}m^{2')})$, $N_1 = 4$
14) $\{a^{(2},b^{2)},(a+b+c)/2^{(4}\}(4^{22)}m^{22')})$, $N_1 = 4$

26s $(P\tilde{4})\{a,b,c\}(\tilde{4})$, $AC:\{\tilde{4},\tilde{4}b\}\{\tilde{4}c,\tilde{4}bc\}$, IV
1) $\{a,b,c^{(3}\}(\tilde{4}^{2)})$, $(3)(3)^1$, $N_1 = 3$ $N_2 = 6$
2) $\{a^{2')},b^{2)},c\}(4^{(4})$, $(4)(4)^2$, $N_1 = 4$
3) $\{a^{2')},b^{2)},c^{2)}\}(\tilde{4}^{(4})$, $(4)(4)^2$, $N_1 = 4$
4) $\{a^{22)},b^{22')},c\}(\tilde{4}^{(4})$, $(4)(4)^2$, $N_1 = 4$
5) $\{a^{22)},b^{22')},c^{2)}\}(\tilde{4}^{(4})$,$(4)(4)^2$, $N_1 = 4$
6) $\{a,b,c^{(4}\}(\tilde{4}^{2)})$, $(3)(3)2$, $N_1 = 1$
7) $\{a^{(2},b^{2)},c^{(4}\}(\tilde{4}^{2)})$, $(3)(3)^2$, $N_1 = 1$

27s $(I\tilde{4})\{a,b,(a+b+c)/2\}(\tilde{4})$, $AC:\{\tilde{4},\tilde{4}(a+b+c)/2\}$, XXI
1) $\{a,b,(a+b+c)/2^{(3}\}(4^{2)})$,$(3)^1$, $N_1 = 1$

28s $(P4/m)\{a,b,c\}(4:m)$, $AC:\{4,4a\}\{m,cm\}$, XXII
1) $\{a,b,c^{(3}\}(4:m^{2)})$, $(3)(3)^1$, $N_1 = 7$ $N_2 = 54$ $N_3 = 336$

2) $\{a^{2')}, b^{2)}, c\}(4^{(4} : m)$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
3) $\{a^{2')}, b^{2)}, c\}(4^{(4} : m^{(2})$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
4) $\{a^{2')}, b^{2)}, c^{2)}\}(4^{(4} : m)$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
5) $\{a^{22)}, b^{22')}, c\}(4^{(4} : m)$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
6) $\{a^{22)}, b^{22')}, c\}(4^{(4} : m^{(2})$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
7) $\{a^{22)}, b^{22')}, c^{2)}\}(4^{(4} : m)$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
8) $\{a, b, c^{(4}\}(4 : m^{2)})$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
9) $\{a, b, c^{(4}\}(4^{(2} : m^{2)})$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
10) $\{a^{(2}, b^{2)}, c^{(4}\}(4 : m^{2)})$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$

29s $(I4/m)\,\{a, b, (a+b+c)/2\}(4 : m)$, $AC : \{(a+b+c)/2\}\{4\}\{m\}$, XX
1) $\{a, b, (a+b+c)/2^{(3}\}(4 : m^{2)})$, $N_1 = 6$ $N_2 = 24$
2) $\{a^{(2}, b^{2)}, (a+b+c)/2^{2)}\}(4^{22)}m)$, $N_1 = 4$
3) $\{a^{(2}, b^{2)}, (a+b+c)/2^{2)}\}(4^{22)}m^{(2})$, $N_1 = 4$
4) $\{a^{(2}, b^{2)}, (a+b+c)/2^{2)}\}(4^{(4}m)$, $N_1 = 4$
5) $\{a^{(2}, b^{2)}, (a+b+c)/2^{2)}\}(4^{(4}m^{(2})$, $N_1 = 4$
6) $\{a^{(2}, b^{2)}, (a+b+c)/2^{22)}\}(4^{2)}m)$, $N_1 = 4$
7) $\{a^{(2}, b^{2)}, (a+b+c)/2^{22)}\}(4^{2)}m^{(2})$, $N_1 = 4$
8) $\{a^{(2}, b^{2)}, (a+b+c)/2^{22)}\}(4^{(4}m)$, $N_1 = 4$
9) $\{a^{(2}, b^{2)}, (a+b+c)/2^{22)}\}(4^{(4}m^{(2})$, $N_1 = 4$
10) $\{a, b, (a+b+c)/2^{(4}\}(4m^{2)})$, $N_1 = 4$
11) $\{a, b, (a+b+c)/2^{(4}\}(4m^{22)})$, $N_1 = 4$
12) $\{a, b, (a+b+c)/2^{(4}\}(4^{(2}m^{2)})$, $N_1 = 4$
13) $\{a, b, (a+b+c)/2^{(4}\}(4^{(2}m^{22)})$, $N_1 = 4$
14) $\{a^{(2}, b^{2)}, (a+b+c)/2^{(4}\}(4^{2)}m^{2)})$, $N_1 = 4$
15) $\{a^{(2}, b^{2)}, (a+b+c)/2^{(4}\}(4^{22)}m^{22)})$, $N_1 = 4$

30s $(P422)\,\{a, b, c\}(4 : 2)$, $AC : \{4, 4a\}\{2, 2c\}$, XXII
1) $\{a, b, c^{(3}\}(4 : 2^{2)})$, $(3)(3)^1$, $N_1 = 7$ $N_2 = 54$ $N_3 = 336$
2) $\{a, b, c\}(4^{(4} : 2^{2)})$, $(3)(3)^2$, $N_1 = 5$ $N_2 = 24$
3) $\{a, b, c\}(4^{(4} : 2^{22)})$, $(3)(3)^2$, $N_1 = 5$ $N_2 = 24$
4) $\{a, b, c^{2)}\}(4^{(4} : 2^{2)})$, $(3)(3)^2$, $N_1 = 5$ $N_2 = 24$
5) $\{a, b, c^{2)}\}(4^{(4} : 2^{22)})$, $(3)(3)^2$, $N_1 = 5$ $N_2 = 24$
6) $\{a^{(2}, b^{2)}, c\}(4^{(4} : 2^{2)})$, $(3)(3)^2$, $N_1 = 5$ $N_2 = 24$
7) $\{a^{(2}, b^{2)}, c\}(4^{(4} : 2^{22)})$, $(3)(3)^2$, $N_1 = 5$ $N_2 = 24$
8) $\{a^{(2}, b^{2)}, c^{2)}\}(4^{(4} : 2^{2)})$, $(3)(3)^2$, $N_1 = 5$ $N_2 = 24$
9) $\{a^{(2}, b^{2)}, c^{2)}\}(4^{(4} : 2^{22)})$, $(3)(3)^2$, $N_1 = 5$ $N_2 = 24$
10) $\{a^{2')}, b^{2)}, c\}(4^{(4} : 2^{2)})$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
11) $\{a^{2')}, b^{2)}, c\}(4^{(4} : 2^{2')})$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
12) $\{a^{2')}, b^{2)}, c^{2)}\}(4^{(4} : 2^{2)})$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
13) $\{a^{22)}, b^{22')}, c\}(4^{(4} : 2^{22)})$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
14) $\{a^{22)}, b^{22')}, c^{2)}\}(4^{(4} : 2^{22)})$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
15) $\{a^{22)}, b^{22')}, c\}(4^{(4} : 2^{22')})$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
16) $\{a, b, c^{(4}\}(4 : 2^{2)})$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
17) $\{a, b, c^{(4}\}(4^{(2} : 2^{2)})$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
18) $\{a, b, c^{(4}\}(4^{(4} : 2^{2)})$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
19) $\{a, b, c^{(4}\}(4^{(-4} : 2^{2)})$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$
20) $\{a^{(2}, b^{2)}, c^{(4}\}(4 : 2^{2)})$, $(3)(4)^2$, $N_1 = 8$ $N_2 = 48$

21) $\{a^{(2},b^{2)},c^{(4}\}(4^{(4}:2^{2)})$, $(3)(4)^2$, $N_1=8$ $N_2=48$

31s $(I422)\,\{a,b,(a+b+c)/2\}(4:2)$, $AC:\{(a+b+c)/2\}\{4\}\{2\}$, XX

1) $\{a,b,(a+b+c)/2^{(3}\}(4:2^{2)})$, $N_1=6$ $N_2=24$
2) $\{a,b,(a+b+c)/2\}(4^{(4}:2^{2)})$, $N_1=4$
3) $\{a,b,(a+b+c)/2\}(4^{(4}:2^{22)})$, $N_1=4$
4) $\{a,b,(a+b+c)/2^{(2}\}(4^{(4}:2^{2)})$, $N_1=4$
5) $\{a,b,(a+b+c)/2^{(2}\}(4^{(4}:2^{22)})$, $N_1=4$
6) $\{a^{(2},b^{2)},(a+b+c)/2^{2)}\}(4^{22)}:2^{22)})$, $N_1=4$
7) $\{a^{(2},b^{2)},(a+b+c)/2^{2)}\}(4^{22')}:2^{22)})$, $N_1=4$
8) $\{a^{(2},b^{2)},(a+b+c)/2^{2)}\}(4^{(4}:2^{22)})$, $N_1=4$
9) $\{a^{(2},b^{2)},(a+b+c)/2^{22)}\}(4^{2)}:2^{2)})$, $N_1=4$
10) $\{a^{(2},b^{2)},(a+b+c)/2^{22)}\}(4^{2')}:2^{2)})$, $N_1=4$
11) $\{a^{(2},b^{2)},(a+b+c)/2^{22)}\}(4^{(4}:2^{2)})$, $N_1=4$
12) $\{a,b,(a+b+c)/2^{(4}\}(4:2^{2)})$, $N_1=4$
13) $\{a,b,(a+b+c)/2^{(4}\}(4:2^{22)})$, $N_1=4$
14) $\{a,b,(a+b+c)/2^{(4}\}(4^{(2}:2^{2)})$, $N_1=4$
15) $\{a,b,(a+b+c)/2^{(4}\}(4^{(2}:2^{22)})$, $N_1=4$
16) $\{a,b,(a+b+c)/2^{(4}\}(4^{(4}:2^{2)})$, $N_1=4$
17) $\{a,b,(a+b+c)/2^{(4}\}(4^{(4}:2^{22)})$, $N_1=4$
18) $\{a,b,(a+b+c)/2^{(4}\}(4^{(-4}:2^{2)})$, $N_1=4$
19) $\{a,b,(a+b+c)/2^{(4}\}(4^{(-4}:2^{22)})$, $N_1=4$
20) $\{a^{(2},b^{2)},(a+b+c)/2^{(4}\}(4^{2)}:2)$, $N_1=4$
21) $\{a^{(2},b^{2)},(a+b+c)/2^{(4}\}(4^{22)}:2)$, $N_1=4$

32s $(P\bar{4}2m)\,\{a,b,c\}(\tilde{4}:2)$, $AC:\{2\}\{\tilde{4},\tilde{4}b\}\{\tilde{4},\tilde{4}c\}$, VIII

1) $\{a,b,c^{(3}\}(\tilde{4}^{2)}:2^{2)})$, $(2)(3)(3)^1$, $N_1=8$ $N_2=60$ $N_3=336$
2) $\{a,b,c\}(\tilde{4}^{(4}:2^{2)})$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$
3) $\{a,b,c\}(\tilde{4}^{(4}:2^{22)})$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$
4) $\{a,b,c^{2)}\}(\tilde{4}^{(4}:2^{2)})$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$
5) $\{a,b,c^{2)}\}(\tilde{4}^{(4}:2^{22)})$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$
7) $\{a^{(2},b^{2)},c\}(\tilde{4}^{(4}:2^{22)})$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$
8) $\{a^{(2},b^{2)},c^{2)}\}(\tilde{4}^{(4}:2^{2)})$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$
9) $\{a^{(2},b^{2)},c^{2)}\}(\tilde{4}^{(4}:2^{22)})$, $(2)(3)(3)^2$, $N_1=6$ $N_2=24$
10) $\{a^{2')},b^{2)},c\}(\tilde{4}^{(4}:2^{2)})$, $(2)(3)(4)^2$, $N_1=12$ $N_2=96$
11) $\{a^{2')},b^{2)},c\}(\tilde{4}^{(4}:2^{2')})$, $(2)(3)(4)^2$, $N_1=12$ $N_2=96$
12) $\{a^{2')},b^{2)},c^{2)}\}(\tilde{4}^{(4}:2^{2)})$, $(2)(3)(4)^2$, $N_1=12$ $N_2=96$
13) $\{a^{22)},b^{22')},c\}(\tilde{4}^{(4}:2^{22)})$, $(2)(3)(4)^2$, $N_1=12$ $N_2=96$
14) $\{a^{22)},b^{22')},c\}(\tilde{4}^{(4}:2^{22')})$, $(2)(3)(4)^2$, $N_1=12$ $N_2=96$
15) $\{a^{22)},b^{22')},c^{2)}\}(\tilde{4}^{(4}:2^{22)})$, $(2)(3)(4)^2$, $N_1=12$ $N_2=96$
16) $\{a,b,c^{(4}\}(\tilde{4}^{2)}:2^{2)})$, $(2)(3)(4)^2$, $N_1=12$ $N_2=96$
17) $\{a,b,c^{(4}\}(\tilde{4}^{2)}:2^{2')})$, $(2)(3)(4)^2$, $N_1=12$ $N_2=96$
18) $\{a^{(2},b^{2)},c^{(4}\}(\tilde{4}^{2)}:2^{2)})$, $(2)(3)(4)^2$, $N_1=12$ $N_2=96$

33s $(P\bar{4}m2)\,\{a,b,c\}(\tilde{4}m)$, $AC:\{m,am\}\{\tilde{4},\tilde{4}c\}$, XXII

1) $\{a,b,c^{(3}\}(\tilde{4}^{2)}m)$, $(3)(3)^1$, $N_1=7$ $N_2=54$ $N_3=336$
2) $\{a,b,c\}(\tilde{4}^{(4}m^{2)})$, $(3)(3)^2$, $N_1=5$ $N_2=24$
3) $\{a,b,c\}(\tilde{4}^{(4}m^{22)})$, $(3)(3)^2$, $N_1=5$ $N_2=24$
4) $\{a,b,c^{2)}\}(\tilde{4}^{(4}m^{2)})$, $(3)(3)^2$, $N_1=5$ $N_2=24$

5) $\{a,b,c^{2)}\}(\tilde{4}^{(4}m^{22)})$, $(3)(3)^2$, $N_1=5$ $N_2=24$
6) $\{a^{(2},b^{2)},c\}(\tilde{4}^{(4}m^{2)})$, $(3)(3)^2$, $N_1=5$ $N_2=24$
7) $\{a^{(2},b^{2)},c\}(\tilde{4}^{(4}m^{22)})$, $(3)(3)^2$, $N_1=5$ $N_2=24$
8) $\{a^{(2},b^{2)},c^{2)}\}(\tilde{4}^{(4}m^{2)})$, $(3)(3)^2$, $N_1=5$ $N_2=24$
9) $\{a^{(2},b^{2)},c^{2)}\}(\tilde{4}^{(4}m^{22)})$, $(3)(3)^2$, $N_1=5$ $N_2=24$
10) $\{a^{2')},b^{2)},c\}(\tilde{4}^{(4}m^{2)})$, $(3)(3)^2$, $N_1=5$ $N_2=24$
11) $\{a^{2')},b^{2)},c\}(\tilde{4}^{(4}m^{2')})$, $(3)(3)^2$, $N_1=5$ $N_2=24$
12) $\{a^{2')},b^{2)},c^{2)}\}(\tilde{4}^{(4}m^{2)})$, $(3)(3)^2$, $N_1=5$ $N_2=24$
13) $\{a^{2')},b^{2)},c^{2)}\}(\tilde{4}^{(4}m^{2')})$, $(3)(3)^2$, $N_1=5$ $N_2=24$
14) $\{a^{22)},b^{22')},c\}(\tilde{4}^{(4}m^{22)})$, $(3)(3)^2$, $N_1=5$ $N_2=24$
15) $\{a^{22)},b^{22')},c\}(\tilde{4}^{(4}m^{22')})$, $(3)(3)^2$, $N_1=5$ $N_2=24$
16) $\{a^{22)},b^{22')},c^{2)}\}(\tilde{4}^{(4}m^{22)})$, $(3)(3)^2$, $N_1=5$ $N_2=24$
17) $\{a^{22)},b^{22')},c^{2)}\}(\tilde{4}^{(4}m^{22')})$, $(3)(3)^2$, $N_1=5$ $N_2=24$
18) $\{a,b,c^{(4}\}(\tilde{4}^{2)}m)$, $(3)(4)^2$, $N_1=8$ $N_2=48$
19) $\{a,b,c^{(4}\}(\tilde{4}^{2)}m^{(2)})$, $(3)(4)^2$, $N_1=8$ $N_2=48$
20) $\{a^{(2},b^{2)},c^{(4}\}(\tilde{4}^{2)}m)$, $(3)(4)^2$, $N_1=8$ $N_2=48$

34s $(I\bar{4}2m)\,\{a,b,(a+b+c)/2\}(\tilde{4}:2)$, $AC:\{(a+b+c)/2\}\{\tilde{4}\}\{2\}$, XX
1) $\{a,b,(a+b+c)/2^{(3}\}(\tilde{4}^{2)}:2^{2)})$, $N_1=6$ $N_2=24$
2) $\{a,b,(a+b+c)/2\}(\tilde{4}^{(4}:2^{2)})$, $N_1=4$
3) $\{a,b,(a+b+c)/2\}(\tilde{4}^{(4}:2^{22)})$, $N_1=4$
4) $\{a,b,(a+b+c)/2^{(2}\}(\tilde{4}^{(4}:2^{2)})$, $N_1=4$
5) $\{a,b,(a+b+c)/2^{(2}\}(\tilde{4}^{(4}:2^{22)})$, $N_1=4$
6) $\{a^{(2},b^{2)},(a+b+c)/2^{2)}\}(\tilde{4}^{22)}:2^{22)})$, $N_1=4$
7) $\{a^{(2},b^{2)},(a+b+c)/2^{2)}\}(\tilde{4}^{22')}:2^{22)})$, $N_1=4$
8) $\{a^{(2},b^{2)},(a+b+c)/2^{2)}\}(\tilde{4}^{(4}:2^{22)})$, $N_1=4$
9) $\{a^{(2},b^{2)},(a+b+c)/2^{22)}\}(\tilde{4}^{2)}:2^{2)})$, $N_1=4$
10) $\{a^{(2},b^{2)},(a+b+c)/2^{22)}\}(\tilde{4}^{2')}:2^{2)})$, $N_1=4$
11) $\{a^{(2},b^{2)},(a+b+c)/2^{22)}\}(\tilde{4}^{(4}:2^{2)})$, $N_1=4$
12) $\{a,b,(a+b+c)/2^{(4}\}(\tilde{4}^{2)}:2^{2)})$, $N_1=4$
13) $\{a,b,(a+b+c)/2^{(4}\}(\tilde{4}^{22)}:2^{22)})$, $N_1=4$
14) $\{a,b,(a+b+c)/2^{(4}\}(\tilde{4}^{2)}:2^{2')})$, $N_1=4$
15) $\{a,b,(a+b+c)/2^{(4}\}(\tilde{4}^{22)}:2^{22')})$, $N_1=4$

35s $(I\bar{4}m2)\,\{a,b,(a+b+c)/2\}(\tilde{4}m)$, $AC:\{m\}\{\tilde{4},\tilde{4}(a+b+c)/2\}$, VI
1) $\{a,b,(a+b+c)/2^{(3}\}(\tilde{4}^{2)}m)$, $(2)(3)^1$, $N_1=4$ $N_2=12$
2) $\{a,b,(a+b+c)/2\}(\tilde{4}^{(4}m^{2)})$, $(2)(3)^2$, $N_1=2$
3) $\{a,b,(a+b+c)/2\}(\tilde{4}^{(4}m^{22)})$, $(2)(3)^2$, $N_1=2$
4) $\{a,b,(a+b+c)/2^{(2}\}(\tilde{4}^{(4}m^{2)})$, $(2)(3)^2$, $N_1=2$
6) $\{a^{(2},b^{2)},(a+b+c)/2^{2)}\}(\tilde{4}^{22)}m^{22)})$, $(2)(4)^2$, $N_1=4$
7) $\{a^{(2},b^{2)},(a+b+c)/2^{2)}\}(\tilde{4}^{22')}m^{22)})$, $(2)(4)^2$, $N_1=4$
8) $\{a^{(2},b^{2)},(a+b+c)/2^{22)}\}(\tilde{4}^{2)}m^{2)})$, $(2)(4)^2$, $N_1=4$
9) $\{a^{(2},b^{2)},(a+b+c)/2^{22)}\}(\tilde{4}^{2')}m^{2)})$, $(2)(4)^2$, $N_1=4$
10) $\{a,b,(a+b+c)/2^{(4}\}(\tilde{4}^{2)}m)$, $(2)(4)^2$, $N_1=4$
11) $\{a,b,(a+b+c)/2^{(4}\}(\tilde{4}^{2)}m^{(2)})$, $(2)(4)^2$, $N_1=4$
12) $\{a^{(2},b^{2)},(a+b+c)/2^{(4}\}(\tilde{4}m^{2)})$, $(2)(4)^2$, $N_1=4$
13) $\{a^{(2},b^{2)},(a+b+c)/2^{(4}\}(\tilde{4}m^{22)})$, $(2)(4)^2$, $N_1=4$

36s $(P4/mmm)\{a,b,c\}(4m:2)$, $AC:\{m\}\{4,4a\}\{4a2,4a2c\}$, XVI

1)	$\{a,b,c^{(3}\}(4m:2^{2)})$,	$(2)(3)(3)^1$,	$N_1=16, N_2=300, N_3=5712, N_4=80640$
2)	$\{a,b,c\}(4^{(4}m^{2')}:2^{2)})$,	$(2)(3)(3)^2$,	$N_1=14, N_2=216, N_3=2688$
3)	$\{a,b,c\}(4^{(4}m^{22')}:2^{22)})$,	$(2)(3)(3)^2$,	$N_1=14, N_2=216, N_3=2688$
4)	$\{a,b,c\}(4^{(4}m^{2)}:2^{2)})$,	$(2)(3)(3)^2$,	$N_1=14, N_2=216, N_3=2688$
5)	$\{a,b,c\}(4^{(4}m^{22)}:2^{22)})$,	$(2)(3)(3)^2$,	$N_1=14, N_2=216, N_3=2688$
6)	$\{a,b,c^{2)}\}(4^{(4}m^{2')}:2^{2)})$,	$(2)(3)(3)^2$,	$N_1=14, N_2=216, N_3=2688$
7)	$\{a,b,c^{2)}\}(4^{(4}m^{22')}:2^{22)})$,	$(2)(3)(3)^2$,	$N_1=14, N_2=216, N_3=2688$
8)	$\{a^{(2},b^{2)},c\}(4^{(4}m^{2')}:2^{2)})$,	$(2)(3)(3)^2$,	$N_1=14, N_2=216, N_3=2688$
9)	$\{a^{(2},b^{2)},c\}(4^{(4}m^{22')}:2^{22)})$,	$(2)(3)(3)^2$,	$N_1=14, N_2=216, N_3=2688$
10)	$\{a^{(2},b^{2)},c\}(4^{(4}m^{2)}:2^{2)})$,	$(2)(3)(3)^2$,	$N_1=14, N_2=216, N_3=2688$
11)	$\{a^{(2},b^{2)},c\}(4^{(4}m^{22)}:2^{22)})$,	$(2)(3)(3)^2$,	$N_1=14, N_2=216, N_3=2688$
12)	$\{a^{(2},b^{2)},c^{2)}\}(4^{(4}m^{2')}:2^{2)})$,	$(2)(3)(3)^2$,	$N_1=14, N_2=216, N_3=2688$
13)	$\{a^{(2},b^{2)},c^{2)}\}(4^{(4}m^{22')}:2^{22)})$,	$(2)(3)(3)^2$,	$N_1=14, N_2=216, N_3=2688$
14)	$\{a^{2')},b^{2)},c\}(4^{22)}m:2)$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3=5376$
15)	$\{a^{2')},b^{2)},c\}(4^{22)}m:2^{(2)}$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3=5376$
16)	$\{a^{2')},b^{2)},c\}(4^{22)}m^{(2}:2)$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3=5376$
17)	$\{a^{2')},b^{2)},c\}(4^{22)}m^{(2}:2^{(2)}$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3=5376$
18)	$\{a^{2')},b^{2)},c^{2)}\}(4^{22)}m:2)$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3=5376$
19)	$\{a^{2')},b^{2)},c^{2)}\}(4^{22)}m^{(2}:2)$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3=5376$
20)	$\{a^{22)},b^{22')},c\}(4^{2)}m:2)$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3=5376$
21)	$\{a^{22)},b^{22')},c\}(4^{2)}m:2^{(2)}$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3=5376$
22)	$\{a^{22)},b^{22')},c\}(4^{2)}m^{(2}:2)$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3=5376$
23)	$\{a^{22)},b^{22')},c\}(4^{2)}m^{(2}:2^{(2)}$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3=5376$
24)	$\{a^{22)},b^{22')},c^{2)}\}(4^{2)}m:2)$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3=5376$
25)	$\{a^{22)},b^{22')},c^{2)}\}(4^{2)}m^{(2}:2)$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3=5376$
26)	$\{a,b,c^{(4}\}(4m:2^{2)})$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3=5376$
27)	$\{a,b,c^{(4}\}(4m^{(2}:2^{2)})$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3=5376$
28)	$\{a,b,c^{(4}\}(4^{(2}m:2^{2)})$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3=5376$
29)	$\{a,b,c^{(4}\}(4^{(2}m^{(2}:2^{2)})$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3=5376$
30)	$\{a^{(2},b^{2)},c^{(4}\}(4m:2^{2)})$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3=5376$
31)	$\{a^{(2},b^{2)},c^{(4}\}(4m^{(2}:2^{2)})$,	$(2)(3)(4)^2$,	$N_1=20, N_2=384, N_3 5376$

37s $(I4/mmm)\{a,b,(a+b+c)/2\}(4m:2)$, $AC:\{(a+b+c)/2\}\{4\}\{2\}\{m\}$, XXIII

1)	$\{a,b,(a+b+c)/2^{(3}\}(4m:2^{2)})$,	$N_1=14$	$N_2=168$ $N_3=1344$
2)	$\{a,b,(a+b+c)/2\}(4^{(4}m^{2')}:2^{2)})$,	$N_1=12$	$N_2=96$
3)	$\{a,b,(a+b+c)/2\}(4^{(4}m^{22')}:2^{22)})$,	$N_1=12$	$N_2=96$
4)	$\{a,b,(a+b+c)/2\}(4^{(4}m^{2)}:2^{2)})$,	$N_1=12$	$N_2=96$
5)	$\{a,b,(a+b+c)/2\}(4^{(4}m^{22)}:2^{22)})$,	$N_1=12$	$N_2=96$
6)	$\{a,b,(a+b+c)/2^{(2}\}(4^{(4}m^{2')}:2^{2)})$,	$N_1=12$	$N_2=96$
7)	$\{a,b,(a+b+c)/2^{(2}\}(4^{(4}m^{22')}:2^{22)})$,	$N_1=12$	$N_2=96$
8)	$\{a,b,(a+b+c)/2^{(2}\}(4^{(4}m^{2)}:2^{2)})$,	$N_1=12$	$N_2=96$
9)	$\{a,b,(a+b+c)/2^{(2}\}(4^{(4}m^{22)}:2^{22)})$,	$N_1=12$	$N_2=96$
10)	$\{a^{(2},b^{2)},(a+b+c)/2^{2)}\}(4^{22)}m^{22)}:2^{22)})$,	$N_1=12$	$N_2=96$
11)	$\{a^{(2},b^{2)},(a+b+c)/2^{2)}\}(4^{22')}m^{22)}:2^{22)})$,	$N_1=12$	$N_2=96$
12)	$\{a^{(2},b^{2)},(a+b+c)/2^{2)}\}(4^{22)}m^{22')}:2^{22)})$,	$N_1=12$	$N_2=96$

13) $\{a^{(2},b^{2}),(a+b+c)/2^{2})\}(4^{22')}m^{22')}:2^{22)}),\quad N_1=12\quad N_2=96$
14) $\{a^{(2},b^{2}),(a+b+c)/2^{2})\}(4^{(4}m^{22')}:2^{22)}),\quad N_1=12\quad N_2=96$
16) $\{a^{(2},b^{2}),(a+b+c)/2^{22)}\}(4^{2)}m^{2)}:2^{2)}),\quad N_1=12\quad N_2=96$
17) $\{a^{(2},b^{2}),(a+b+c)/2^{22)}\}(4^{2')}m^{2)}:2^{2)}),\quad N_1=12\quad N_2=96$
18) $\{a^{(2},b^{2}),(a+b+c)/2^{22)}\}(4^{2)}m^{2')}:2^{2)}),\quad N_1=12\quad N_2=96$
19) $\{a^{(2},b^{2}),(a+b+c)/2^{22)}\}(4^{2')}m^{2')}:2^{2)}),\quad N_1=12\quad N_2=96$
20) $\{a^{(2},b^{2}),(a+b+c)/2^{22)}\}(4^{(4}m^{2')}:2^{2)}),\quad N_1=12\quad N_2=96$
21) $\{a^{(2},b^{2}),(a+b+c)/2^{22)}\}(4^{(4}m^{2)}:2^{2)}),\quad N_1=12\quad N_2=96$
22) $\{a,b,(a+b+c)/2^{(4}\}(4m:2^{2)}),\quad N_1=12\quad N_2=96$
23) $\{a,b,(a+b+c)/2^{(4}\}(4m:2^{22)}),\quad N_1=12\quad N_2=96$
24) $\{a,b,(a+b+c)/2^{(4}\}(4m^{(2}:2^{2)}),\quad N_1=12\quad N_2=96$
25) $\{a,b,(a+b+c)/2^{(4}\}(4m^{(2}:2^{22)}),\quad N_1=12\quad N_2=96$
26) $\{a,b,(a+b+c)/2^{(4}\}(4^{(2}m:2^{2)}),\quad N_1=12\quad N_2=96$
27) $\{a,b,(a+b+c)/2^{(4}\}(4^{(2}m:2^{22)}),\quad N_1=12\quad N_2=96$
28) $\{a,b,(a+b+c)/2^{(4}\}(4^{(2}m^{(2}:2^{2)}),\quad N_1=12\quad N_2=96$
29) $\{a,b,(a+b+c)/2^{(4}\}(4^{(2}m^{(2}:2^{22)}),\quad N_1=12\quad N_2=96$
30) $\{a^{(2},b^{2}),(a+b+c)/2^{(4}\}(4^{2)}m^{2)}):2),\quad N_1=12\quad N_2=96$
31) $\{a^{(2},b^{2}),(a+b+c)/2^{(4}\}(4^{22)}m^{22)}):2),\quad N_1=12\quad N_2=96$
32) $\{a^{(2},b^{2}),(a+b+c)/2^{(4}\}(4^{2)}m^{2')}:2),\quad N_1=12\quad N_2=96$
33) $\{a^{(2},b^{2}),(a+b+c)/2^{(4}\}(4^{22)}m^{22')}:2),\quad N_1=12\quad N_2=96$

$40s$ $(P3m1)\,\{(a,b),c\}(3m),\quad AC:\{c\}\{m\},\ XIX$
1) $\{(a,b),c\}(3^{(3}m^{2)}),\quad N_1=2$
2) $\{(a^{(3},b^{(3}),c\}(3m^{2)}),\quad N_1=2$

$41s$ $(P31m)\,\{(a,b),c\}(m3),\quad AC:\{c\}\{m\},\ XIX$
1) $\{(a,b),c\}(m^{2)}3^{(3}),\quad N_1=2$

$42s$ $(R3m)\,\{a,b,c\}(m3),\quad AC:\{a\}\{m\},\ XIX$
1) $\{a,b,c\}(m^{2)}3^{(3}),\quad N_1=2$

$43s$ $(P\bar{6})\{(a,b),c\}(3:m),\quad AC:\{m,cm\},\ XXI$
1) $\{(a,b),c^{(3}\}(3:m^{2)}),(3)^1,\quad N_1=1$

$44s$ $(P321)\,\{(a,b),c\}(3:2),\quad AC:\{2,2c\},\ XXI$
1) $\{(a,b),c\}(3^{(3}:2^{2)}),\quad (3)^1,\quad N_1=1$
2) $\{(a,b),c^{(3}\}(3:2^{2)}),\quad (3)^1,\quad N_1=1$
3) $\{(a,b),c^{(3}\}(3^{(3}:2^{2)}),\quad (3)^1,\quad N_1=1$
4) $\{(a,b),c^{(3}\}(3^{(-3}:2^{2)}),\quad (3)^1,\quad N_1=1$

$45s$ $(P312)\,\{(a,b),c\}(2:3),\quad AC:\{2,2c\},\ XXI$
1) $\{(a,b),c\}(2^{2)}:3^{(3}),\quad (3)^1,\quad N_1=1$
2) $\{(a^{(3},b^{(3}),c^{(3}\}(2^{2)}:3),\quad (3)^1,\quad N_1=1$
3) $\{(a,b),c^{(3}\}(2^{2)}:3),\quad (3)^1,\quad N_1=1$
4) $\{(a,b),c^{(3}\}(2^{2)}:3^{(3}),\quad (3)^1,\quad N_1=1$
5) $\{(a,b),c^{(3}\}(2^{2)}:3^{(-3}),\quad (3)^1,\quad N_1=1$
6) $\{(a^{(3},b^{(3}),c^{(3}\}(2^{2)}:3),\quad (3)^1,\quad N_1=1$

$46s$ $(R32)\,\{a,b,c\}(2:3),\quad AC:\{2,2a\},\ XXI$
1) $\{a,b,c\}(2^{2)}:3^{(3}),\quad (3)^1,\quad N_1=1$
2) $\{a^{(3},b^{(3},c^{(3}\}(2^{2)}:3),\quad (3)^1,\quad N_1=1$
3) $\{a^{(3},b^{(3},c^{(3}\}(2^{2)}:3^{(3}),\quad (3)^1,\quad N_1=1$

4) $\{a^{(3},b^{(3},c^{(3}\}(2^{2)}:3^{(-3)}),\quad (3)^1,\quad N_1=1$

47s $(P\bar{6}2m)\,\{(a,b),c\}(3:m2),\quad AC:\{m\}\{2,2c\},\ VI$

1) $\{(a,b),c\}(3^{(3}:m2^{2)}),\quad (2)(3)^1,\quad N_1=4\quad N_2=12$

2) $\{(a,b),c^{(3}\}(3:m^{2)}2^{2)}),\quad (2)(3)^1,\quad N_1=4\quad N_2=12$

3) $\{(a,b),c^{(4}\}(3:m^{2)}2^{2)}),\quad (2)(4)^2,\quad N_1=4$

4) $\{(a,b),c^{(4}\}(3:m^{2')}2^{2)}),\quad (2)(4)^2,\quad N_1=4$

48s $(P\bar{6}m2)\,\{(a,b),c\}(2m:3),\quad AC:\{m\}\{2,2c\},\ VI$

1) $\{(a,b),c\}(2^{2)}m:3^{(3}),\quad (2)(3)^1,\quad N_1=4\quad N_2=12$

2) $\{(a,b),c^{(3}\}(2^{2)}m^{2)}:3),\quad (2)(3)^1,\quad N_1=4\quad N_2=12$

3) $\{(a^{(3},b^{(3}),c\}(2^{2)}m:3),\quad (2)(3)^1,\quad N_1=4\quad N_2=12$

4) $\{(a,b),c^{(4}\}(2^{2)}m^{2)}:3),\quad (2)(4)^2,\quad N_1=4$

5) $\{(a,b),c^{(4}\}(2^{2)}m^{2')}:3),\quad (2)(4)^2,\quad N_1=4$

49s $(P6)\,\{(a,b),c\}(6),\quad AC:\{6\}\{c\},\ XIX$

1) $\{(a^{(3},b^{(3}),c\}(6^{2)}),\quad N_1=2$

50s $(P6mm)\,\{(a,b),c\}(6m),\quad AC:\{6\}\{c\}\{m\},\ XX$

1) $\{(a,b),c^{(3}\}(6m^{2)}),\quad N_1=6\quad N_2=24$

2) $\{(a^{(3},b^{(3}),c\}(6^{2)}m),\quad N_1=6\quad N_2=24$

51s $(P\bar{3})\{(a,b),c\}(6),\quad AC:\{6,6c\},\ XXI$

1) $\{(a,b),c^{(3}\}(6^{2)}),\quad (3)^1,\quad N_1=1$

2) $\{(a^{(3},b^{(3}),c^{(3}\}(6^{2)}),\quad (3)^1,\quad N_1=1$

3) $\{(a^{(3},b^{(3}),c\}(6^{2)}),\quad (3)^1,\quad N_1=1$

52s $(R\bar{3})\,\{a,b,c\}(6),\quad AC:\{6,6a\},\ XXI$

1) $\{a^{(3},b^{(3},c^{(3}\}(6^{2)}),(3)^1,\quad N_1=1$

53s $(P6/m)\,\{(a,b),c\}(6:m),\quad AC:\{6\}\{m,cm\},\ VI$

1) $\{(a,b),c^{(3}\}(6:m^{2)}),\quad (2)(3)^1,\quad N_1=4\quad N_2=12$

2) $\{(a^{(3},b^{(3}),c\}(6^{2)}:m),\quad (2)(3)^1,\quad N_1=4\quad N_2=12$

3) $\{(a,b),c^{(4}\}(6:m^{2)}),\quad (2)(4)^2,\quad N_1=4$

4) $\{(a,b),c^{(4}\}(6(2:m^{2)}),\quad (2)(4)^2,\quad N_1=4$

54s $(P622)\,\{(a,b),c\}(6:2),\quad AC:\{6\}\{2,2c\},\ VI$

1) $\{(a,b),c\}(6^{(3}:2^{2)}),\quad (2)(3)^1,\quad N_1=4\quad N_2=12$

2) $\{(a^{(3},b^{(3}),c\}(6^{2)}:2),\quad (2)(3)^1,\quad N_1=4\quad N_2=12$

3) $\{(a,b),c^{(3}\}(6:2^{2)}),\quad (2)(3)^1,\quad N_1=4\quad N_2=12$

4) $\{(a,b),c^{(3}\}(6^{(-3}:2^{2)}),\quad (2)(3)^1,\quad N_1=4\quad N_2=12$

5) $\{(a,b),c^{(3}\}(6^{(-3}:2^{2)}),\quad (2)(3)^1,\quad N_1=4\quad N_2=12$

6) $\{(a,b),c^{(4}\}(6:2^{2)}),\quad (2)(4)^2,\quad N_1=4$

7) $\{(a,b),c^{(4}\}(6(2:2^{2)}),\quad (2)(4)^2,\quad N_1=4$

55s $(P\bar{3}m1)\,\{(a,b),c\}(3m:2),\quad AC:\{m\}\{2,2c\},\ VI$

1) $\{(a,b),c\}(3^{(3}m^{2)}:2^{2)}),\quad (2)(3)^1,\quad N_1=4\quad N_2=12$

2) $\{(a,b),c^{(3}\}(3m:2^{2)}),\quad (2)(3)^1,\quad N_1=4\quad N_2=12$

3) $\{(a^{(3},b^{(3}),c\}(3m^{2)}:2),\quad (2)(3)^1,\quad N_1=4\quad N_2=12$

4) $\{(a,b),c^{(4}\}(3m:2^{2)}),\quad (2)(4)^2,\quad N_1=4$

5) $\{(a,b),c^{(4}\}(3m^{(2}:2^{2)}),\quad (2)(4)^2,\quad N_1=4$

56s $(P\bar{3}1m)\,\{(a,b),c\}(2:m3),\quad AC:\{m\}\{2,2c\},\ VI$

1) $\{(a,b),c\}(2^{2)}:m^{2)}3^{(3}),\quad (2)(3)^1,\quad N_1=4\quad N_2=12$

2) $\{(a,b),c^{(3}\}(2^{2)}:m3),\quad (2)(3)^1,\quad N_1=4\quad N_2=12$

3) $\{(a^{(3},b^{(3}),c\}(2^{2)}:m3)$, $(2)(3)^1$, $N_1=4$ $N_2=12$
4) $\{(a^{(3},b^{(3}),c^{(3}\}(2^{2)}:m3)$, $(2)(3)^1$, $N_1=4$ $N_2=12$
5) $\{(a,b),c^{(4}\}(2^{2)}:m3)$, $(2)(4)^2$, $N_1=4$
6) $\{(a,b),c^{(4}\}(2^{2)}:m^{(2}3)$, $(2)(4)^2$, $N_1=4$

57s $(R\bar{3}1m)\{a,b,c\}(2:m3)$, $AC:\{m\}\{2,2c\}$, VI
1) $\{a,b,c\}(2^{2)}:m^{2)}3^{(3})$, $(2)(3)^1$, $N_1=4$ $N_2=12$
2) $\{a^{(3},b^{(3},c^{(3}\}(2^{2)}:m3)$, $(2)(3)^1$, $N_1=4$ $N_2=12$
3) $\{a^{(4},b^{(4},c^{(4}\}(2^{2)}:m3)$, $(2)(4)^2$, $N_1=4$
4) $\{a^{(4},b^{(4},c^{(4}\}(2^{2)}:m^{(2}3)$, $(2)(4)^2$, $N_1=4$

58s $(P6/mmm)\{(a,b),c\}(6:m2)$, $AC:\{6\}\{2\}\{m,cm\}$, XIII
1) $\{(a,b),c\}(6^{(-3}:m2^{2)})$, $(2)(2)(3)^1$, $N_1=10$ $N_2=96$ $N_3=672$
2) $\{(a^{(3},b^{(3}),c\}(6^{2)}:m2)$, $(2)(2)(3)^1$, $N_1=10$ $N_2=96$ $N_3=672$
3) $\{(a,b),c^{(3}\}(6:m^{2)}2^{2)})$, $(2)(2)(3)^1$, $N_1=10$ $N_2=96$ $N_3=672$
4) $\{(a,b),c^{(4}\}(6:m^{2)}2^{2)})$, $(2)(2)(4)^2$, $N_1=12$ $N_2=96$
5) $\{(a,b),c^{(4}\}(6:m^{2)}2^{2')})$, $(2)(2)(4)^2$, $N_1=12$ $N_2=96$
6) $\{(a,b),c^{(4}\}(6^{(2}:m^{2)}2^{2)})$, $(2)(2)(4)^2$, $N_1=12$ $N_2=96$
7) $\{(a,b),c^{(4}\}(6^{(2}:m^{2)}2^{2')})$, $(2)(2)(4)^2$, $N_1=12$ $N_2=96$

65s $(P\bar{4}3m)\{a,b,c\}(3/\tilde{4})$, $AC:\{\tilde{4}\}\{a\}$, XIX
1) $\{a,b,c\}(3^{(3}/4^{2)})$, $N_1=2$

66s $(I\bar{4}3m)\{a,b,(a+b+c)/2\}(3/\tilde{4})$, $AC:\{\tilde{4}\}\{(a+b+c)/2\}$, XIX
1) $\{a,b,(a+b+c)/2\}(3^{(3}/4^{2)})$, $N_1=2$

68s $(P432)\{a,b,c\}(3/4)$, $AC:\{4,4a\}$, XXI
1) $\{a,b,c\}(3^{(3}/4^{2)}),(3)^1$ $N_1=1$

69s $(I432)\{a,b,(a+b+c)/2\}(3/4)$, $AC:\{4\}\{(a+b+c)/2\}$, XIX
1) $\{a,b,(a+b+c)/2\}(3^{(3}/4^{2)})$, $N_1=2$

71s $(Pm3m)\{a,b,c\}(3/4m)$, $AC:\{m\}\{4,4a\}$, VI
1) $\{a,b,c\}(3^{(3}/4^{2)}m)$, $(2)(3)^1$ $N_1=4$

72s $(Im3m)\{a,b,(a+b+c)/2\}(3/4m)$, $AC:\{(a+b+c)/2\}\{4\}\{m\}$, XX
1) $\{a,b,(a+b+c)/2\}(3^{(3}/4^{2)}m)$, $N_1=6$ $N_2=24$
2) $\{a^{(2},b^{(2},(a+b+c)/2^{(4}\}(3/4^{2)}m^{2)})$, $N_1=4$
3) $\{a^{(2},b^{(2},(a+b+c)/2^{(4}\}(3/4^{2)}m^{2')})$, $N_1=4$
4) $\{a^{(2},b^{(2},(a+b+c)/2^{(4}\}(3/4^{22)}m^{22)})$, $N_1=4$
5) $\{a^{(2},b^{(2},(a+b+c)/2^{(4}\}(3/4^{22)}m^{22')})$, $N_1=4$

73s $(Fm3m)\{a,(a+b)/2,(a+c)/2\}(3/4m)$, $AC:\{4\}\{m\}$, XIX
1) $\{a,(a+b)/2,(a+c)/2\}(3^{(3}/4m)$, $N_1=2$

From the results given in the catalogue, we can first conclude that, because of the equality between (32,1)- and (62)-symmetry, there exist 496 (not 491 [1,3,4]) complete (32,1)-symmetry junior symmorphic three-dimensional space groups and, consequently, the same number of symmorphic (62)-symmetry groups. Such a disagreement with [1,3,4] implies the necessary corrections for the (62)-symmetry groups in the families with the generating groups 10s ($C222$) and 32s ($P\bar{4}2m$). For the first family, their correct number is 22 (not 18 [1]), and for the second it is 8 (not 7 [1]).

Complete numerical results are given in Table 1.

Table 1.

	(32)	(42)	(62)								
$2s$	1	1	1	$24s$		16		$48s$	3	2	12
$3s$	1	2	4	$25s$		14		$49s$	1		2
$4s$	1	5	3	$26s$	1	6	3	$50s$	2		12
$5s$	1	2	4	$27s$	1	3	1	$51s$	3	1	3
$6s$	1	6	4	$28s$	1	9	7	$52s$	1	1	1
$7s$	2	6	14	$29s$	1	14	6	$53s$	2	2	8
$8s$	2	14	16	$30s$	1	20	7	$54s$	5	2	20
$9s$	1	4	8	$31s$	1	20	6	$55s$	3	2	12
$10s$	2	14	22	$32s$	1	17	8	$56s$	4	2	16
$11s$	1	6	4	$33s$	1	19	7	$57s$	2	2	8
$12s$	1	6	5	$34s$	1	14	6	$58s$	3	4	30
$13s$	1	6	16	$35s$	1	12	4	$63s$		2	
$14s$	1	12	14	$36s$	1	30	16	$65s$	1		2
$15s$	2	16	20	$37s$	1	32	14	$66s$	1		2
$16s$	1	8	6	$40s$	2		4	$67s$	1		
$17s$	1	8	14	$41s$	1		2	$68s$	1		1
$18s$	1	6	16	$42s$	1		2	$69s$	1	2	2
$19s$	2	24	38	$43s$	1	1	1	$70s$	1		
$20s$	1	12	10	$44s$	4	1	4	$71s$	1		4
$21s$	1	12	18	$45s$	6	1	6	$72s$	1	4	6
$22s$		4		$46s$	4	1	4	$73s$	1		2
$23s$		6		$47s$	2	2	8				

	(32, 1)	(42, 1)	(62, 1)	(32, 2)	(42, 2)	(62, 2)
$2s$	1	1	1	1	1	1
$3s$	4	12	16	16	48	56
$4s$	3	8	6	6		
$5s$	4	8	22	22	32	112
$6s$	4	12	12	12		
$7s$	14	57	128	128	588	1400
$8s$	16	96	120	120	480	672
$9s$	8	32	96	96	432	1516
$10s$	22	120	228	228	768	1680
$11s$	4	12	12	12		
$12s$	5	36	42	42	288	336
$13s$	16	120	300	300	2304	5712
$14s$	14	128	168	168	960	1344
$15s$	20	144	192	192	960	2688
$16s$	6	24	24	24		
$17s$	14	96	168	168	768	1344
$18s$	16	120	450	450	3744	17220
$19s$	38	444	804	804	8208	16464
$20s$	10	88	96	96	512	672
$21s$	18	336	432	432	8064	10080
$22s$		16				
$24s$		160			1152	
$25s$		56				

26*s*	3	18	6	6		
27*s*	1					
28*s*	7	72	54	54	432	336
29*s*	6	56	24	24		
30*s*	7	136	54	54	768	336
31*s*	6	80	24	24		
32*s*	8	156	60	60	1056	336
33*s*	7	104	54	54	528	336
34*s*	6	56	24	24		
35*s*	4	40	12	12		
36*s*	16	528	300	300	9504	5712
37*s*	14	384	168	168	3072	1344
40*s*	4					
41*s*	2					
42*s*	2					
43*s*	1					
44*s*	4					
45*s*	6					
46*s*	4					
47*s*	8	8	24	24		
48*s*	12	8	36	36		
49*s*	2					
50*s*	12		48	48		
51*s*	3					
52*s*	1					
53*s*	8	8	24	24		
54*s*	20	8	60	60		
55*s*	12	8	36	36		
56*s*	16	8	48	48		
57*s*	8	8	24	24		
58*s*	30	48	288	288	384	2016
65*s*	2					
66*s*	2					
68*s*	1					
69*s*	2					
71*s*	4					
72*s*	6	16	24	24		
73*s*	2					

	(32, 3)	(42, 3)	(62, 3)	(32, 4)	(42, 4)	(62, 4)	(32, 5)
2*s*	1						
3*s*	56						
5*s*	112						
7*s*	1400	5376	13440	13440			
8*s*	672						
9*s*	1516	5376	20160	20160			
10*s*	1680						
12*s*	336						
13*s*	5712	32256	80640	80640			
14*s*	1344						

15s	2688						
17s	1344						
18s	17220	137088	685440	685440	3870720	19998720	19998720
19s	16464	112896	241920	241920			
20s	672						
21s	10080	129024	161280	161280			
28s	336						
30s	336						
32s	336						
33s	336						
36s	5712	129024	80640	80640			
37s	1344						
58s	2016						

For the complete $(p2,l)$-symmetry junior symmorphic three- dimensional space groups of the M^m-type the numbers ${N_m}^{p2}$ $(p=3,4,6)$ are the following:

${N_0}^{p2} = 96^{32} + 438^{42} + 496^{62} = 1030;$
${N_1}^{p2} = 496^{32} + 3876^{42} + 4709^{62} = 9081;$
${N_2}^{p2} = 4709^{32} + 45053^{42} + 71713^{62} = 121475;$
${N_3}^{p2} = 71713^{32} + 551040^{42} + 1283520^{62} = 1906273;$
${N_4}^{p2} = 1283520^{32} + 3870720^{42} + 19998720^{62} = 25152960;$
${N_5}^{p2} = 19998720^{32} = 19998720.$

4. Hemisymmorphic $(p2,l)$-symmetry three-dimensional space groups ${G_3}^{l,p2}$ $(p=3,4,6)$

Crystallographic $(p2)$-symmetry three-dimensional space groups ${G_3}^{p2}$ $(p=3,4,6)$ are derived by Yu.S.Karpova (Yu.S.Chubarova) [1,2,3,4]. From 54 hemisymmorphic G_3 are derived 945 junior ${G_3}^{p2}$ $(75{G_3}^{32} + 444{G_3}^{42} + 426G^{62})$. By the use of the generalized antisymmetric characteristic method (AC-method) [5,6], some of the results mentioned are corrected, and derived all crystallographic $(p2,l)$-symmetry three-dimensional space groups ${G_3}^{l,p2}$ $(p=3,4,6)$.

Using the theoretical background given in the §1, §2, all hemisymmorphic $(p2,l)$-symmetry three-dimensional space groups are derived, resulting in the complete catalogue of the corresponding groups.

From the results given in the catalogue, we can first conclude that, because of the equality between (32,1)- and (62)-symmetry, there exist 413 (not 426 [2,3,4]) complete (32,1)-symmetry junior hemisymmorphic three-dimensional space groups and, consequently, the same number of hemisymmorphic (62)-symmetry groups. Such a disagreement with [2,3,4] implies the necessary corrections for the (62)-symmetry groups in the families with the generating groups 1h (Pb), 2h (Bb), 3h ($P2/b$), 4h ($B2/b$), 21h (Cmma), 22h (Ccca) and 53h ($Pn3n$). For these families, the correct number of (62)-symmetry groups is, respectively, 3 (not 4 [2]), 1 (not 2 [2]), 12 (not 13 [2]), 8 (not 10 [2]), 36 (not 38 [2]), 22 (not 24 [2]) and 2 (not 6 [2]).

The complete numerical results are given in Table 2.

Table 2.

	(32)	(42)	(62)								
1h	1	2	3	19h	1	6	5	37h	1	20	6
2h	1	4	1	20h	2	22	24	38h	1	28	14
3h	2	9	12	21h	2	22	36	39h	2		
4h	2	10	8	22h	2	24	22	40h	1		
5h	1	3	7	23h	2	18	24	41h	1		
6h	2	10	20	24h	1	4	4	42h	2	2	4

$7h$	2	11	8	$25h$		8		$43h$	3	2	6
$8h$	1	4	3	$26h$		8		$44h$	2		4
$9h$	1	4	6	$27h$		6		$45h$	3	2	6
$10h$	1	8	6	$28h$		10		$46h$	4	2	8
$11h$	2	12	20	$29h$	1	11	4	$47h$	2	2	4
$12h$	2	16	12	$30h$	1	14	4	$48h$	3	4	18
$13h$	2	10	12	$31h$	1	12	4	$51h$	1		
$14h$	2	6	12	$32h$	1	8	4	$52h$	1		
$15h$	1	6	6	$33h$	1	8	3	$53h$	1		2
$16h$	1	2	2	$34h$	1	10	4	$54h$	1		2
$17h$	2	14	26	$35h$	1	22	10				
$18h$	2	12	17	$36h$	1	26	10				

	(32, 1)	(42, 1)	(62, 1)	(32, 2)	(42, 2)	(62, 2)
$1h$	3	8	6	6		
$2h$	1					
$3h$	12	58	72	72	264	336
$4h$	8	36	24	24		
$5h$	7	24	54	54	144	336
$6h$	20	96	192	192	672	1344
$7h$	8	28	24	24		
$8h$	3	4	6	6		
$9h$	6	16	24	24		
$10h$	6	24	24	24		
$11h$	20	112	192	192	768	1344
$12h$	12	64	48	48		
$13h$	12	40	48	48		
$14h$	12	24	48	48		
$15h$	6	20	24	24		
$16h$	2					
$17h$	26	184	456	456	3168	8568
$18h$	17	90	150	150	528	1008
$19h$	5	18	24	24	36	84
$20h$	24	208	264	264	1440	2016
$21h$	36	416	768	768	8160	16128
$22h$	22	240	252	252	1920	2016
$23h$	24	176	264	264	1248	2016
$24h$	4	8	12	12		
$25h$		24				
$26h$		32				
$28h$		40				
$29h$	4	28	12	12		
$30h$	4	40	12	12		
$31h$	4	32	12	12		
$32h$	4	20	12	12		
$33h$	3	8	6	6		
$34h$	4	32	12	12		
$35h$	10	208	96	96	1440	672
$36h$	10	232	96	96	1536	672
$37h$	6	80	24	24		
$38h$	14	336	168	168	2688	1344
$42h$	4					
$43h$	6					
$44h$	4					
$45h$	6					
$46h$	8					
$47h$	4					
$48h$	18	16	72	72		
$53h$	2					
$54h$	2					

	(32, 3)	(42, 3)	(62, 3)	(32, 4)
$3h$	336			
$5h$	336			
$6h$	1344			
$11h$	1344			
$17h$	8568	43008	120960	120960
$18h$	1008			
$19h$	84			
$20h$	2016			
$21h$	16128	118272	241920	241920
$22h$	2016			
$23h$	2016			
$35h$	672			
$36h$	672			
$38h$	1344			

For the complete $(p2,l)$-symmetry junior hemisymmorphic three-dimensional space groups of the M^m-type the numbers $N_m{}^{p2}$ $(p = 3,4,6)$ are the following:

$N_0{}^{p2} = 75^{32} + 444^{42} + 413^{62} = 932;$
$N_1{}^{p2} = 413^{32} + 3022^{42} + 3498^{62} = 6933;$
$N_2{}^{p2} = 3498^{32} + 24012^{42} + 37884^{62} = 65394;$
$N_3{}^{p2} = 37884^{32} + 161280^{42} + 362880^{62} = 562044;$
$N_4{}^{p2} = 362880^{32} = 362880.$

5. Asymmorphic $(p2,l)$-symmetry three–dimensional space groups $G_3{}^{l,p2}$ $(p = 3,4,6)$

Crystallographic $(p2)$-symmetry three-dimensional space groups $G_3{}^{p2}$ $(p = 3,4,6)$ are derived by Yu.S.Karpova (Yu.S.Chubarova) [1,2,3,4]. From 103 asymmorphic G_3 are derived 1650 junior $G_3{}^{p2}$ $(138G_3{}^{32} + 785G_3{}^{42} + 727G^{62})$. By the use of the generalized antisymmetric characteristic method (AC-method) [5,6] some of the results mentioned are corrected, and derived all crystallographic $(p2,l)$-symmetry three-dimensional space groups $G_3{}^{l,p2}$ $(p = 3,4,6)$.

Using the theoretical background given in the §1, §2, all asymmorphic $(p2,l)$-symmetry three-dimensional space groups are derived and listed in the corresponding catalogue.

From the results given in the catalogue, we can first conclude that, because of the equality between (32,1)- and (62)-symmetry, there exist 725 (not 727 [2,3,4]) complete (32,1)-symmetry junior asymmorphic three-dimensional space groups and consequently, the same number of asymmorphic (62)-symmetry groups.

The complete numerical results are given in Table 3.

Table 3.

	(32)	(42)	(62)									
$1a$	1	1	1	$34a$		4		$67a$	1	20	6	
$2a$	2	6	8	$35a$		4		$70a$	1	1	1	
$3a$	2	7	6	$36a$		14		$71a$	1	1	1	
$4a$	2	9	11	$37a$		12		$72a$	2	1	2	
$5a$	2	10	10	$38a$		8		$73a$	2	1	2	
$6a$	1	4	4	$39a$		6		$74a$	1			
$7a$	2	6	9	$40a$	1	4	6	$75a$	1			
$8a$	1	2	1	$41a$	1	14	4	$76a$	1			
$9a$	2	6	14	$42a$	1	8	3	$77a$	1	16		
$10a$	2	10	8	$43a$	1	12	4	$78a$	1	56		
$11a$	2	6	8	$44a$	1	8	4	$79a$	2	8	4	
$12a$	2	6	4	$45a$	1	8	4	$80a$	2	32	4	
$13a$	2	10	12	$46a$	1	10	6	$81a$	3	2	6	

14*a*	3	24	48	47*a*	1	14	7	82*a*	2	2	4
15*a*	3	22	30	48*a*	1	4	2	83*a*	2	2	4
16*a*	3	16	30	49*a*	1	4	2	84*a*	2	2	8
17*a*	3	18	18	50*a*	1	8	4	85*a*	2	2	8
18*a*	3	28	42	51*a*	1	4	2	86*a*	5	2	10
19*a*	3	22	42	52*a*	1	10	4	87*a*	3	4	18
20*a*	2	10	22	53*a*	1	6	2	88*a*	3	4	18
21*a*	1	4	10	54*a*	1	16	10	93*a*	1		
22*a*	2	9	17	55*a*	1	22	10	94*a*	1		
23*a*	3	18	30	56*a*	1	14	6	95*a*	1		
24*a*	2	15	17	57*a*	1	16	6	96*a*	1		2
25*a*	2	12	10	58*a*	1	12	6	97*a*	1		
26*a*	3	14	18	59*a*	1	12	6	98*a*	1		1
27*a*	2	10	10	60*a*	1	30	10	99*a*	1		2
28*a*	3	14	18	61*a*	1	28	10	100*a*	1		2
29*a*	1	4	6	62*a*	1	20	8	101*a*	1		2
30*a*		2		63*a*	1	20	6	102*a*	1		2
31*a*		2		64*a*	1	20	6	103*a*	1		4
32*a*		4		65*a*	1	20	6				
33*a*		4		66*a*	1	14	6				

	(32, 1)	(42, 1)	(62, 1)	(32, 2)	(42, 2)	(62, 2)
1*a*	1	1	1	1		
2*a*	8	20	32	32	64	112
3*a*	6	10	12	12		
4*a*	11	54	81	81	288	504
5*a*	10	36	36	36		
6*a*	4	12	12	12		
7*a*	9	18	30	30		
8*a*	1					
9*a*	14	48	108	108	288	672
10*a*	8	24	24	24		
11*a*	8	16	24	24		
12*a*	4					
13*a*	12	40	48	48		
14*a*	48	408	900	900	7200	17136
15*a*	30	192	288	288	1248	2016
16*a*	30	144	288	288	960	2016
17*a*	18	72	72	72		
18*a*	42	336	504	504	2688	4032
19*a*	42	264	504	504	2112	4032
20*a*	22	120	252	252	960	2016
21*a*	10	48	96	96	384	672
22*a*	17	72	150	150	432	1008
23*a*	30	160	288	288	1056	2016
24*a*	17	108	150	150	624	1008
25*a*	10	36	36	36		
26*a*	18	56	72	72		
27*a*	10	28	36	36		
28*a*	18	56	72	72		
29*a*	6	16	24	24		
33*a*		16				
36*a*		40				
37*a*		32				
38*a*		16				
40*a*	6	16	24	24		
41*a*	4	56	12	12		
42*a*	3	8	6	6		
43*a*	4	32	12	12		
44*a*	4	24	12	12		

45*a*	4	24	12	12		
46*a*	6	40	24	24		
47*a*	7	88	54	54	480	336
48*a*	2					
49*a*	2					
50*a*	4	16	12	12		
51*a*	2					
52*a*	4	24	12	12		
53*a*	2					
54*a*	10	144	96	96	960	672
55*a*	10	192	96	96	1248	672
56*a*	6	56	24	24		
57*a*	6	64	24	24		
58*a*	6	48	24	24		
59*a*	6	48	24	24		
60*a*	10	288	96	96	2016	672
61*a*	10	256	96	96	1728	672
62*a*	8	120	60	60	480	336
63*a*	6	80	24	24		
64*a*	6	80	24	24		
65*a*	6	80	24	24		
66*a*	6	56	24	24		
67*a*	6	80	24	24		
70*a*	1					
71*a*	1					
72*a*	2					
73*a*	2					
79*a*	4					
80*a*	4					
81*a*	6					
82*a*	4					
83*a*	4					
84*a*	8	8	24	24		
85*a*	8	8	24	24		
86*a*	10					
87*a*	18	16	72	72		
88*a*	18	16	72	72		
96*a*	2					
98*a*	1					
99*a*	2					
100*a*	2					
101*a*	2					
102*a*	2					
103*a*	4		12	12		

	(32, 3)	(42, 3)	(62, 3)	(32, 4)
2*a*	112			
4*a*	504			
9*a*	672			
14*a*	17136	96768	241920	241920
15*a*	2016			
16*a*	2016			
18*a*	4032			
19*a*	4032			
20*a*	2016			
21*a*	672			
22*a*	1008			
23*a*	2016			
24*a*	1008			
47*a*	336			

$54a$	672
$55a$	672
$60a$	672
$61a$	672
$62a$	336

For the complete $(p2,l)$-symmetry junior asymmorphic three-dimensional groups of the M^m-type, the numbers $N_m{}^{p2}$ are the following:

$N_0{}^{p2} = 138^{32} + 785^{42} + 725^{62} = 1648;$
$N_1{}^{p2} = 725^{32} + 4467^{42} + 5184^{62} = 10376;$
$N_2{}^{p2} = 5184^{32} + 25216^{42} + 40600^{62} = 71000;$
$N_3{}^{p2} = 40600^{32} + 96768^{42} + 241920^{62} = 379288;$
$N_4{}^{p2} = 241920^{32} = 241920.$

6. $(p2,l)$-symmetry three-dimensional space groups $G_3{}^{l,p2}$ $(p = 3,4,6)$

As the final result, for the complete $(p2,l)$-symmetry junior three-dimensional space groups of the M^m-type the numbers $N_m{}^{p2}$ $(p = 3,4,6)$ are the following:

$N_0{}^{p2} = 309^{32} + 1667^{42} + 1634^{62} = 3610;$
$N_1{}^{p2} = 1634^{32} + 11365^{42} + 13391^{62} = 26390;$
$N_2{}^{p2} = 13391^{32} + 94281^{42} + 150197^{62} = 257869;$
$N_3{}^{p2} = 150197^{32} + 809088^{42} + 1888320^{62} = 2847605;$
$N_4{}^{p2} = 1888320^{32} + 3870720^{42} + 19998720^{62} = 25757760;$
$N_5{}^{p2} = 19998720^{32} = 19998720.$

References

[1] YU. S. Karpova, *Vyvod simmorfnih mladshih prostranstvennyh grupp (p2)-simmetrii*, KGU, Kishinev , 1980 (Deposed in VINITI, No. 855 - 80 Dep.)

[2] YU. S. Karpova, *Vyvod gemisimmorfnih i asimmorfnih mladshih prostranstvennyh grupp (p2)-simmetrii*, KGU, Kishinev , 1980 (Deposed in VINITI, No. 3173 - 80 Dep.)

[3] YU. S. Chubarova, *K vyvodu mladshih prostranstvennyh grupp p-simmetrii*, Avtoref. dis. ... kand. fiz.-mat. nauk, Shtiintsa, Kishinev, 1983.

[4] A. M. Zamorzaev, YU. S. Karpova, A. P. Lungu, A. F. Palistrant, *p-simmetriya i eyo dal'nejshee razvitie*, Shtiintsa, Kishinev, 1986.

[5] S. V. Jablan, *Algebra of Antisymmetric Characteristics*, Publ. Inst. Math. 47 (61), 1990, 39-55.

[6] S. V. Jablan, *Plane and Layer $(p2,2^l)$- symmetry Groups*, Mat. Vesnik 42 (1990), 87-95.

[7] A. M. Zamorzaev, E. I. Galyarskii, A. F. Palistrant, *Tsvetnaya simmetriya, eyo obobscheniya i prilozeniya*, Shtiintsa, Kishinev , 1978.

[8] A. M. Zamorzaev, *Teoriya prostoj i kratnoj antisimmetrii*, Shtiintsa, Kishinev , 1976.

[9] A. M. Zamorzaev, A. F. Palistrant, *Antisymmetry, its Generalizations and Applications*, Z.Kristall. 151 (1980), 231-248.

[10] S. V. Jablan, *A New Method of Deriving and Cataloguing Simple and Multiple Antisymmetry $G_3{}^l$ Space Groups*, Acta Cryst., A43 (1987), 326-337.

Department of Mathematics
18000 Nis
Cirila i Metodija 2
Yugoslavia