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A CLASS OF MAPPINGS AND CARDINAL FUNCTIONS

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Abstract. Some results concerning σ -spaces and (strong) Σ -spaces are extended to higher cardinals.

0. Introduction and definitions

Several cardinal functions were defined and studied as extensions of some classes of generalized metric spaces to higher cardinalities (see [4], [7], [8], [11], [13], for example). In this paper we present some results of the same type.

In 1970 (see [10]), E. Michael introduced the concept of a σ -locally finite mapping and announced some results regarding such mappings. In [10] he proved, among other, that a regular space X is a σ -space (resp., strong Σ -space) if and only if it is the image of a metrizable space (resp., para-compact p -space) under a σ -locally finite mapping.

Here we define τ -locally finite mappings, τ an infinite cardinal, and extend these results to higher cardinalities. We also give some other results concerning cardinal functions connected with τ -locally finite mappings; those functions extend some classes of generalized metric spaces to higher cardinalities.

We begin with the definitions and notations which we will need.

0.1. Notations and terminology in this paper are standard as in [2], [3]. All spaces are regular T_1 , all mappings are continuous surjections and all cardinals are infinite. $St(x, \mathcal{U})$ denotes the star of a point $x \in X$ with respect to a cover \mathcal{U} of X . For a cover \mathcal{U} of X and $x \in X$ $ord(x, \mathcal{U}) \leq \tau$ means that x is in at most τ many members of \mathcal{U} . A family \mathcal{F} of subsets of X is said

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to be τ -discrete (τ -locally finite) if it is the union of τ many discrete (locally finite) collections.

0.2. ([4]) The metrizability degree $m(X)$ of a space X is the smallest cardinal τ such that there exists a τ -discrete (or equivalently, τ -locally finite) base for X .

0.3. ([13]) The σ -degree of a space X , denoted by $\sigma(X)$, is the smallest cardinal τ such that X has a τ -locally finite network.

0.4. The diagonal degree $\Delta(X)$ (resp., the star-diagonal degree $\Delta^*(X)$) of a space X is the smallest cardinal τ such that there exists a family $\{\mathcal{U}_\alpha : \alpha \in \tau\}$ of open covers of X so that $\{x\} = \cap \{St(x, \mathcal{U}_\alpha) : \alpha \in \tau\}$ (resp., $\{x\} = \cap \{St(x, \mathcal{U}_\alpha) : \alpha \in \tau\}$) for every $x \in X$.

0.5. ([11]) The Moore degree $dv(X)$ of a space X is the smallest cardinal τ for which there is a family $\{\mathcal{U}_\alpha : \alpha \in \tau\}$ of open covers of X such that the collection $\{St(x, \mathcal{U}_\alpha) : \alpha \in \tau\}$ is a local base for each $x \in X$.

0.6. ([13]) The subparacompactness degree $spa(X)$ of a space X is the smallest cardinal τ such that every open cover \mathcal{U} of X has a τ -discrete open refinement.

0.7. If τ is a regular cardinal, then a Tychonoff space X is said to be τ -metrizable, or linearly uniformizable, if there is a uniformity generating the topology of X and having a well-ordered base of order type τ .

0.8. ([8]) The Arhangel'skii number $A(X)$ of a space X is the smallest cardinal τ such that there exist a τ -metrizable space Y and a perfect mapping f from X onto Y .

0.9. DEFINITION. Let τ be a cardinal. A mapping $f: X \rightarrow Y$ is said to be τ -locally finite if every τ -locally finite (not necessarily open or closed) cover \mathcal{P} of X has a refinement \mathcal{R} such that $f(\mathcal{R})$ is a τ -locally finite collection. ■

Note that for $\tau = \omega$ we obtain Michael's notion of a σ -locally finite mapping.

0.10. DEFINITION. A family \mathcal{F} of subsets of X is called a Q_τ -network (resp., K -network) for X if for every $x \in X$ there exists a closed initially τ -compact (resp., compact) subset $C_x \subset X$ such that for any neighbourhood U of C_x there is a member $F \in \mathcal{F}$ such that $C_x \subset F \subset U$. ■

The notion of a K -network was introduced by Michael (see [31]) under the name "(modk)-network"; I. Juhasz calls this a K -network.

0.11. DEFINITION. The Σ -degree $\Sigma(X)$ and the strong Σ -degree $s\Sigma(X)$ of a space X are defined as follows:

$\Sigma(X) = \min\{\tau: X \text{ has a } \tau\text{-locally finite } (\Leftrightarrow \tau\text{-discrete}) Q_\tau\text{-network}\},$
 $s\Sigma(X) = \min\{\tau: X \text{ has a } \tau\text{-locally finite } (\Leftrightarrow \tau\text{-discrete}) K\text{-network}\}. \blacksquare$

Note that a space X is a Σ -space (strong Σ -space) if and only if $\Sigma(X) = \omega$ ($s\Sigma(X) = \omega$).

1. Characterizations of $\sigma(X)$ and $s\Sigma(X)$

The following assertion can be verified by a straightforward checking.

1.1. PROPOSITION. Let $f: X \rightarrow Y$ be a τ -locally finite mapping. Then:

(i) $\sigma(X) \leq \tau$ implies $\sigma(Y) \leq \tau$; (ii) $s\Sigma(X) \leq \tau$ implies $s\Sigma(Y) \leq \tau$. \blacksquare

We also need the following result.

1.2. PROPOSITION. For every space X we have (i) $\text{spa}(X) \leq s\Sigma(X) \leq \sigma(X)$ and (ii) $s\Sigma(X) \leq A(X)$.

Proof. (i) The inequality $s\Sigma(X) \leq \sigma(X)$ is trivial. Let us prove the first one. Let $s\Sigma(X) = \tau$ and let $\mathcal{F} = \cup\{\mathcal{F}_\alpha: \alpha \in \tau\}$ be a τ -locally finite K -network for X . Take an arbitrary open cover \mathcal{U} of X . Every $F \in \mathcal{F}$ is covered by finitely many elements $U_1(F), \dots, U_k(F)$ of \mathcal{U} ; in this case we put

$$V_{\alpha,i} = \mathcal{F}_\alpha \cap U_i(F) = \{F \cap U_i(F): F \in \mathcal{F}_\alpha\}, \quad (\alpha \in \tau, i \leq k),$$

$$\mathcal{V} = \cup\{V_{\alpha,i}: \alpha \in \tau, i \in \omega\}.$$

Since every $V_{\alpha,i}$, ($\alpha \in \tau, i \leq k$), is locally finite, we have that \mathcal{V} is a τ -locally finite collection. On the other hand, obviously, every $V_{\alpha,i}$ is an open refinement of \mathcal{U} . Let us prove that \mathcal{V} is a cover of X . Let $x \in X$. If C_x is a compact set containing x , then there exists a finite $\mathcal{U}' = \{U_1, \dots, U_k\} \subset \mathcal{U}$ so that $x \in C_x \subset F \subset \cup\mathcal{U}'$ for some $F \in \mathcal{F}$. But, $\cup\mathcal{U}' = \cup\{F \cap U_i: i \leq k\} \subset \cup\{V_{\alpha,i}: \alpha \in \tau, i \in \omega\} = \mathcal{V}$, i.e. \mathcal{V} is a cover for X . This means $\text{spa}(X) \leq \tau$.

(ii) Let $A(X) = \tau$ and let f be a perfect mapping from X onto a τ -metrizable space Y . Since $m(Y) = \tau$ there is a τ -locally finite base \mathcal{B} for Y . We will prove that $f^{-1}(\mathcal{B})$ is a τ -locally finite K -network for X . Clearly, $f^{-1}(\mathcal{B})$ is locally finite and every $x \in X$ belongs to the compact set $f^{-1}f(x) = C_x$. If U is a neighbourhood of C_x , then $Y \setminus f(X \setminus U)$ is a neighbourhood of $f(x)$ so that there is some $B \in \mathcal{B}$ with $f(x) \in B \subset Y \setminus f(X \setminus U)$. Therefore, $C_x \subset f^{-1}(B) \subset U$, i.e. $s\Sigma(X) \leq \tau$. \blacksquare

1.3. THEOREM. For a space X , $\sigma(X) \leq \tau$ if and only if X is the image of a space Y with $m(Y) \leq \tau$ under a τ -locally finite mapping f .

Proof. Let $\sigma(X) = \tau$ and let $\mathcal{A} = \cup\{\mathcal{A}_\alpha: \alpha \in \tau\}$ be a τ -locally finite closed network for X . Define a new topology on X by taking \mathcal{A} to be a base for this topology and denote by Y the set X with that topology. Y is a regular space

since every member of \mathcal{A} is open and closed in Y . Since Y has a τ -locally finite base, we have that $m(Y) \leq \tau$. The identity mapping $f \equiv \text{id}_X: X \rightarrow Y$ is continuous, because \mathcal{A} is a network for X . It remains to prove that f is τ -locally finite. Let $\mathcal{P} = \cup\{\mathcal{P}_\alpha: \alpha \in \tau\}$ be a τ -locally finite cover for Y . Let \mathcal{U} be an open cover of Y such that every $U \in \mathcal{U}$ intersects only finitely many members of \mathcal{P}_α , $\alpha \in \tau$. It is understood that one can take $\mathcal{U} \subset \mathcal{A}$. Put $\mathcal{S} = \{P \cap U: P \in \mathcal{P}, U \in \mathcal{U}\}$. We have that \mathcal{S} is a refinement of both \mathcal{P} and \mathcal{U} . So, $f(\mathcal{S})$ is τ -locally finite in X , because \mathcal{U} is τ -locally finite.

Conversely, let $f: Y \rightarrow X$ be a τ -locally finite mapping and $m(Y) \leq \tau$. Since $\sigma(Y) \leq m(Y) \leq \tau$, then, by Proposition 1.1, $\sigma(X) \leq \tau$. ■

The method used in the proof of the following theorem is a higher cardinality version of Michael's method from [9]; the same method was used in the author's papers [5] and [6].

1.4. THEOREM. *Let τ be a regular cardinal. Then $s\Sigma(X) = \tau$ if and only if it is a τ -locally finite image of a space Z such that $A(Z) = \tau$.*

Proof. Let X be a τ -locally finite image of a space Z with $A(Z) = \tau$. According to Proposition 1.2, $s\Sigma(Z) \leq A(Z) = \tau$ and $s\Sigma(X) \leq s\Sigma(Z) \leq \tau$.

Let $s\Sigma(X) = \tau$ and let $\mathcal{F} = \cup\{\mathcal{F}_\alpha: \alpha \in \tau\}$ be a τ -locally finite closed K-network for X (we assume that the intersections of $< \tau$ many members of \mathcal{F} are in \mathcal{F}). It is convenient to write $\mathcal{F} = \{F_\lambda: \lambda \in \Lambda\}$. Let Λ be topologized by the discrete topology. Consider the set Λ^τ and its subset Y consisting of all $y = (\lambda_\alpha: \alpha \in \tau)$ for which $\{F_{\lambda_\alpha}: \alpha \in \tau\}$ is a (decreasing) K-network for some compact $C_y \subset X$. (Let us note that such a K-network has cardinality $\leq \tau$, as can be easily seen.) The topology on Y is generated by the "natural topology" (see, for example, [6]) on Λ^τ defined by the base \mathcal{B} consisting of the sets

$$B_\alpha(y) = \{p = (\mu_\alpha: \alpha \in \tau) \in Y: \mu_\beta = \lambda_\beta \text{ for } \beta \in \alpha, y \in Y, \alpha \in \tau\}.$$

Denote by \mathcal{U} the collection of all sets $U_\alpha \subset Y \times Y$ of the form:

$$U_\alpha = \{((\mu_\alpha), (\lambda_\alpha)) \in Y \times Y: \mu_\beta = \lambda_\beta \text{ for } \beta \in \alpha, \alpha \in \tau\}.$$

This collection is a well-ordered base of a uniformity \mathcal{U} on Y generating the "natural topology"; since $|\mathcal{U}| = \tau$, Y is a τ -metrizable space. Put now

$$Z = \{(x, y) \in X \times Y: x \in C_y \text{ for some compact set } C_y \subset X\},$$

and denote by f and g the projections of Z onto X and Y , respectively.

Claim A. $fg^{-1}(B_\alpha(y)) = F_{\lambda_\alpha}$, where $y = (\lambda_\alpha: \alpha \in \tau) \in Y$, $\alpha \in \tau$.

If $p \in B_\alpha(y)$, then $fg^{-1}(p) = C_p \subset F_{\mu_\alpha} = F_{\lambda_\alpha}$, i.e. $B_\alpha(y) \subset F_{\lambda_\alpha}$. Conversely, let $p \in F_{\lambda_\alpha}$. Then $p \in K$ for some compact set $K \subset X$ with $p \in K \subset F_{\lambda_\alpha}$.

Let $\{F_{\mu_\nu} : \nu \in \tau\}$ be a network for K ; we may assume $\mu_\nu = \lambda_\nu$ for $\nu \in \alpha$. If one takes $q = (\mu_\nu : \nu \in \tau)$ we will have $q \in B_\alpha(y)$, $p \in K = fg^{-1}(q)$ and, therefore, $F_{\lambda_\alpha} \subset fg^{-1}(B_\alpha(y))$. The claim is proved.

Claim B. g is a perfect surjection.

From the definition of g it follows $g^{-1}(y) = C_y \times \{y\}$ so that g is a compact surjection. We are going to prove that g is closed. Let $y \in Y$ and let $W \subset Z$ be a neighbourhood of the compact set $g^{-1}(y) = C_y \times \{y\}$. By the well known theorem of Wallace [2] one can find neighbourhoods U of C_y and $B_\alpha(y)$ of y for which $C_y \times \{y\} \subset (U \times B_\alpha(y)) \cap Z \subset W$. Let $\{F_{\lambda_\alpha} : \alpha \in \tau\} \subset \mathcal{F}$ be a network (in X) around C_y . We can find $\beta \in \tau$, $\beta \geq \alpha$, for which $C_y \subset F_{\lambda_\beta} \subset U$ holds. Using Claim A we obtain

$$g^{-1}(B_\alpha(y)) = (F_{\lambda_\alpha} \times B_\beta(y)) \cap Z \subset (U \times B_\alpha(y)) \cap Z \subset W,$$

which means that g is closed.

Claim C. $g^{-1}(\mathcal{B})$ is a K -network for Z .

Let $z = (x, y) \in Z$. Then z is in the compact set $K_z = g^{-1}(y) = C_y \times \{y\}$. If $W \subset Z$ is a neighbourhood of K_z , then, as in the the proof of Claim B, there is a $B_\beta(y) \in \mathcal{B}$ such that $K_z \subset g^{-1}(B_\beta(y)) \subset W$, which proves the claim.

Claim D. f is a τ -locally finite mapping.

Let $\mathcal{P} = \cup\{\mathcal{P}_\alpha : \alpha \in \tau\}$ be a τ -locally finite cover of Z . Let \mathcal{H} be an open cover of Z such that each $H \in \mathcal{H}$ intersects only finitely many members of \mathcal{P}_α for $\alpha \in \tau$. Denote by \mathcal{H}^* the collection of all finite unions of elements of \mathcal{H} . Since $\mathcal{P} = g^{-1}(\mathcal{B})$ is a K -network for Z and \mathcal{H}^* is closed under finite unions, \mathcal{H}^* has a refinement $\mathcal{R} \subset \mathcal{P}$ and so $f(\mathcal{R})$ is τ -locally finite by Claim A. Put $\mathcal{R}^* = \{P \cap Q : P \in \mathcal{P}, Q \in \mathcal{R}\}$. Then \mathcal{R}^* is a refinement of \mathcal{P} and $f(\mathcal{R}^*)$ is a τ -locally finite collection in X . Hence, f is τ -locally finite.

The proof of the theorem is completed. ■

2. Two factorization theorems

In 1971, T. Shiraki (see [3]) proved that a space X is a σ -space if and only if it is a Σ -space with a G_δ -diagonal. Now we will extend this result to higher cardinality by proving the following factorization theorem.

2.1. THEOREM. *For every space X we have $\sigma(X) = \Sigma(X)\Delta(X)$.*

Proof. Obviously, $\Delta(X) \leq \sigma(X)$ and $\Sigma(X) \leq \sigma(X)$ so that $\Sigma(X)\Delta(X) \leq \sigma(X)$.

Let $\Sigma(X)\Delta(X) = \tau$. Let $\{\mathcal{U}_\alpha : \alpha \in \tau\}$ be as in the definition of $\Delta(X) \leq \tau$ and $\mathcal{F} = \cup\{\mathcal{F}_\alpha : \alpha \in \tau\}$ a τ -discrete Q_τ -network for X . Every member of \mathcal{F} is

initially τ -compact and its diagonal degree is $\leq \tau$. By a result of P.K.Hart [11] (which states that every initially τ -compact space Y with $\Delta(Y) \leq \tau$ is compact), every $F \in \mathcal{F}$ is actually compact so that $s\Sigma(X) \leq \tau$. By Proposition 1.2 we have $\text{spa}(X) \leq \tau$. As was remarked by Unger in [13], then $\Delta^*(X) \leq \text{spa}(X)\Delta(X) \leq \tau$. That says that, without loss of generality, we may assume that $\{\mathcal{U}_\alpha : \alpha \in \tau\}$ satisfies the conditions in the definition of $\Delta^*(X) \leq \tau$. Since $\text{spa}(X) \leq \tau$, let $V_\alpha = \cup\{V_{\alpha\beta} : \beta \in \tau\}$, $\alpha \in \tau$, be a τ -discrete closed refinement of \mathcal{U}_α . Put

$$W_{\alpha\beta\gamma} = V_{\alpha\beta} \wedge \mathcal{F}_\gamma = \{V \cap F : V \in V_{\alpha\beta}, F \in \mathcal{F}_\gamma\}, (\alpha, \beta, \gamma \in \tau).$$

Clearly, $W_{\alpha\beta\gamma}$ is discrete for every $\alpha, \beta, \gamma \in \tau$ because $V_{\alpha\beta}$ and \mathcal{F}_γ are. It remains to prove that $W = \cup\{W_{\alpha\beta\gamma} : \alpha, \beta, \gamma \in \tau\}$ is a (τ -discrete) network for X . Let $x \in X$, U a neighbourhood of x . Pick a compact set C_x containing x . But, for the compact set C_x one has $m(C_x) = \Delta(C_x) \leq \Delta(X) \leq \tau$ (see Corollary 1.5 in [11]), so that $\text{dv}(C_x) \leq m(X) \leq \tau$ [11]. Hence, we can assume that $\{\mathcal{U}_\alpha : \alpha \in \tau\}$ satisfies the definition of $\text{dv}(X) \leq \tau$. So, there exists $\alpha \in \tau$ such that $\overline{\text{St}(x, \mathcal{U}_\alpha)} \cap C_x \subset U \cap C_x$. For every $y \in C_x \setminus U$ there exists an open set V_y containing y and satisfying $V_y \cap \text{St}(x, \mathcal{U}_\alpha) = \emptyset$. Take $\gamma \in \tau$ and $F \in \mathcal{F}_\gamma$ such that $C_x \subset F \subset U \cup (\cup\{V_y : y \in C_x \setminus U\})$ and $V \in V_\alpha$ with $x \in V \subset \text{St}(x, \mathcal{U}_\alpha)$. Of course, $V \cap F \in W$ and, as is easy to check, $x \in V \cap F \subset U$. So, $\sigma(X) \leq \tau$. ■

The following is a generalization (to higher cardinals) of the condition (1.5) from [1].

2.2. DEFINITION. ([8]) *The strong separating cover degree of a space X , denoted by $\text{ssc}(X)$, is the smallest cardinal τ such that there exists a (not necessarily open) cover \mathcal{P} of X with $\text{ord}(x, \mathcal{P}) \leq \tau$ for each $x \in X$ and such that if $x, y \in X$, $x \neq y$, then there is a finite $\mathcal{F} \subset \mathcal{P}$ such that $x \in \text{int}(\cup \mathcal{F})$ and $y \notin \cup \mathcal{F}$. ■*

Now we will give the following factorization theorem (which should be compared with Theorem 5.2 in [1]). Recall that a space X is τ -additive if the intersections of less than τ many open sets in X are open.

2.3. THEOREM. *Let τ be a regular cardinal. For a τ -additive space X we have $\sigma(X) = \tau$ if and only if $s\Sigma(X)\text{ssc}(X) = \tau$.*

Proof. (a) $s\Sigma(X)\text{ssc}(X) \leq \sigma(X)$. (For this part we need not the τ -additivity of X .) Let $\sigma(X) = \tau$. Since $s\Sigma(X) \leq \sigma(X)$ we will prove $\text{ssc}(X) \leq \tau$. Let $\mathcal{A} = \cup\{\mathcal{A}_\alpha : \alpha \in \tau\}$ be a τ -locally finite closed network for X , where every \mathcal{A}_α is a cover for X and $X \in \mathcal{A}_\alpha$, $\alpha \in \tau$. For each $\alpha \in \tau$ and each $\mathcal{B} \subset \mathcal{A}$ let

$$P_\alpha(\mathcal{B}) = \cap \mathcal{B} \setminus \cup(\mathcal{A}_\alpha \setminus \mathcal{B}),$$

$$\mathcal{P}_\alpha = \{P_\alpha(\mathcal{B}) : \mathcal{B} \subset \mathcal{A}_\alpha\},$$

$$\mathcal{P} = \cup \{ \mathcal{P}_\alpha : \alpha \in \tau \}.$$

Let us prove that \mathcal{P} witnesses that $\text{ssc}(X) \leq \tau$. It is easy to see that for every $\alpha \in \tau$, \mathcal{P}_α is a disjoint cover of X , so that $\text{ord}(x, \mathcal{P}_\alpha) \leq \tau$ for each x in X . Suppose $x, y \in X$, $x \neq y$. Choose $\lambda \in \tau$ such that $y \in A$, $x \notin A$ for some $A \in \mathcal{A}_\lambda$. For every $z \in X$ put $\mathcal{B}_z = \{ B \in \mathcal{A}_\lambda : z \in B \}$. It is understood, \mathcal{B}_z is finite. If $\mathcal{F} = \{ \mathcal{P}_\lambda(B) : B \in \mathcal{B}_x \}$, then \mathcal{F} is finite and $\mathcal{F} \subset \mathcal{P}_\lambda \subset \mathcal{P}$. Let us show that \mathcal{F} is the desired collection from the definition of $\text{ssc}(X)$, i.e. that $x \in \text{int}(\cup \mathcal{F})$, $y \notin \cup \mathcal{F}$.

(i) $x \in \text{int}(\cup \mathcal{F})$. Let $U = X \setminus \cup (\mathcal{A}_\lambda \setminus \mathcal{B}_x)$. Then U is open and contains x . So, for the proof of the claim it is enough to show $U = \cup \mathcal{F}$. If $z \in \cup \mathcal{F}$, then $z \in \mathcal{P}_\lambda(B)$ for some $B \in \mathcal{B}_x$, so that $z \in \mathcal{P}_\lambda(B) \subset X \setminus \cup (\mathcal{A}_\lambda \setminus \mathcal{B}_x) \subset X \setminus \cup (\mathcal{A}_\lambda \setminus \mathcal{B}_x) = U$, i.e. $\cup \mathcal{F} \subset U$. If $z \in U$, then $\mathcal{B}_z \subset \mathcal{B}_x$ and so $\mathcal{P}_\lambda(\mathcal{B}_z) \in \mathcal{F}$; as $z \in \mathcal{P}_\lambda(\mathcal{B}_z)$, one has $z \in \cup \mathcal{F}$, i.e. $U \subset \cup \mathcal{F}$. Hence, $U = \cup \mathcal{F}$.

(ii) $y \notin \cup \mathcal{F}$. Since $\mathcal{B} \subset \mathcal{B}_x$ we have $A \in \mathcal{A}_\lambda \setminus \mathcal{B}$ so that $\mathcal{P}_\lambda(\mathcal{B}) \subset X \setminus A$ and thus $y \notin \mathcal{P}_\lambda(\mathcal{B})$ and consequently $y \notin \cup \mathcal{F}$.

So, we have proved $\text{ssc}(X) \leq \tau$, i.e. $s\Sigma(X)\text{ssc}(X) \leq \sigma(X)$.

(b) $\sigma(X) \leq s\Sigma(X)\text{ssc}(X)$. Let $s\Sigma(X)\text{ssc}(X) = \tau$. According to Theorem 1.4 there are a τ -metrizable space Y , a space $Z \subset X \times Y$ with $A(Z) = \tau$ and a τ -locally finite mapping $f: Z \rightarrow X$. As $\text{ssc}(Y) \leq \sigma(Y) \leq m(Y) = \tau$ and $\text{ssc}(X) \leq \tau$, we have $\text{ssc}(X \times Y) \leq \tau$, so $\text{ssc}(Z) \leq \tau$. Moreover, Z is τ -additive because $X \times Y$ is such a space and $A(Z) \leq \tau$. By Theorem 2' in [8] then $A(Z)\text{ssc}(Z) = m(Z) \leq \tau$. According to Theorem 1.3 we have $\sigma(X) \leq \tau$ and the proof of the theorem is completed. ■

The paper [8] contains some other results involving $\text{ssc}(X)$.

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JEDNA KLASA PRESLIKAVANJA I KARDINALNE FUNKCIJE

Za proizvoljan kardinal τ definiše se klasa τ -lokalno konačnih preslikavanja koja predstavlja generalizaciju klase σ -lokalno konačnih preslikavanja koja je uveo E. Michael. Posredstvom takvih preslikavanja definišu se i karakterišu kardinalne funkcije. Koristeći te funkcije dokazuju se dve faktorizacione teoreme za kardinalne invarijante.

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