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ON μ-BOUNDED AND PRECOMPACT UNIFORM MAPPINGS

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Abstract. We transfer some notions and results concerning topological and uniform spaces to the class of uniformly continuous mappings.

Let f be a mapping from a set X onto a topological space $(Y,\mathcal{T})$. A system $U = \{U_O : O \in \mathcal{T}\}$ of pseudouniformities U_O on the sets $f^{-1}(O)$ is called a **pseudopreuniformity on the mapping f** [1] if for every $O,O' \in \mathcal{T}$ such that $O' \subset O$ the identity embedding of $f^{-1}(O')$ into $f^{-1}(O)$ is a uniformly continuous mapping with respect to the pseudouniformities $U_{O'}$ and U_O. Moreover, if for every two distinct points $x,x' \in f^{-1}(y)$, $y \in Y$, there exist a neighbourhood O of y and a cover $\alpha \in U_O$ such that x' does not belong to $\alpha(x) = \cup\{A \in \alpha : x \in A\}$, then the pseudopreuniformity U on f is called a **preuniformity on f** and the pair (f,U) is called a **preuniform mapping**.

Let X' be another set. Let $U' = \{U'_O : O \in \mathcal{T}\}$ be a preuniformity on a mapping $f' : X' \to (Y,\mathcal{T})$. A mapping $\lambda : X \to X'$ satisfying condition $f = f'\cdot\lambda$ is called a **(U,U')-uniformly continuous morphism of f to f'** [1] if for every $O' \in \mathcal{T}$ and every cover $\alpha' \in U'_{O'}$, there exists $O \in \mathcal{T}$ such that $O \subset O'$ and $\lambda^{-1}(\alpha') \cap f^{-1}(O) \in U_O$.

Let $\alpha = \{A\}$ be a uniform cover of a set $f^{-1}(O)$. The star of a set M in $f^{-1}(O)$ with respect to α is the set $\alpha(M) = \cup\{A \in \alpha : A \cap M \neq \emptyset\}$. A set M is called **$\alpha$-dense** in $(f^{-1}(O),U_O)$ [2] if for every $x \in f^{-1}(O)$ there exists $x' \in M$ such that $x \in \alpha(x')$, i.e. $\alpha(M) = f^{-1}(O)$.

Let μ be a cardinal number. A pseudopreuniformity U on f and pseudo preuniform mapping (f,U) are called **μ-bounded** if for every $O \in \mathcal{T}$ and every $\alpha \in U_O$ there exists a set M whi h is α-dense in $f^{-1}(O)$ and $|M| \leq \mu$; if μ is finite, then (f,U) is called **totally bounded**.

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PROPOSITION 1. *If $\lambda: X \to X'$ is a (U,U')-uniformly continuous morphism of a μ-bounded pseudopreuniform mapping $(f,U): X \to Y$ to a pseudo preuniform mapping $(f',U'): X' \to Y$, then the mapping (f',U') is also μ-bounded.*

PROPOSITION 2. *If $(f,U): X \to Y$ is a μ-bounded pseudopreuniform mapping, then for every $X' \subset X$ the induced pseudopreuniformity $U' \equiv U|f'$ on the mapping $f' = f|X' : X' \to Y$ is μ-bounded.*

PROPOSITION 3. *Let X' be a dense set in X with respect to the topology induced by the psedopreuniformity U on f. If the mapping (f',U'): $X' \to Y$ is μ-bounded, then $(f,U): X \to Y$ is also μ-bounded.*

The proofs of these propositions follow from the following known fact [2]: for every pseudouniform space (X,U) the following conditions are equivalent: (1) the space (X,U) is μ-bounded; (2) there is a base $\mathcal{B}$ of U having cardinality $\leq\mu$; (3) there is a base $\mathcal{B}$ for U such that every uniform cover of X has cardinality $\leq\mu$.

A preuniform mapping $(f^*,U^*): X^* \to Y$ is called a **completion** of a mapping $(f,U): X \to Y$ [3] if :

(a) the mapping (f^*,U^*) is complete ;

(b) if $e_U: X \to X^*$ is a uniform embedding, then $X^* = [e_U(X)]$;

(c) for every (U,U')-uniformly continuous morphism λ of (f,U) to a complete mapping (f',U') there exists a (U^*,U')-uniformly continuous morphism λ^* of f^* to f' such that $\lambda^* \cdot e_U = \lambda$.

PROPOSITION 4. *A completion of a μ-bounded preuniform mapping is also μ-bounded.*

A preuniformity U on f and a preuniform mapping (f,U) are called **precompact** if for every $O \in \mathcal{T}$ the uniformity U_O has a base consisting of finite covers.

PROPOSITION 5. *For any preuniform mapping (f,U) the following conditions are equivalent: (1) (f,U) is a totally bounded mapping ; (2) (f,U) is precompact; (3) the completion (f^*,U^*) of (f,U) is a compact ($\equiv$perfect) mapping.*

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