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PSEUDOCOMPACT TOPOLOGIZATIONS OF GROUPS

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Dedicated to the memory of Eric Karel van Douwen

Abstract. We investigate the following question: When a group G admits a pseudocompact Hausdorff group topology? E.K. van Douwen was the first who found same constraints for the existence of such a topology. We find some other constraints and also a lot of sufficient conditions under which a group *does* admit a pseudocompact Hausdorff group topology. These sufficent conditions almost coincide with necessary ones for four classes of groups, free groups, free Abelian groups, torsion Abelian groups and divisable Abelian groups. The gap here depends often on some extra set-theoretic assumption which is known to be equiconsistent with the existence of large cardinals.

1. Introduction

A topology τ on a group G is a **group topology** if the group operations $x, y \longmapsto x \cdot y$ and $x \longmapsto x^{-1}$ are continuous with respect to τ, Given a topological property Q, we say that a group G is **Q-topologizable** (or **admits a group topology with the property** Q) provided that there exist a group topology on G satisfying property Q. It is clear that each group admits both discrete and anti-discrete group topologies. In 1946. Markov [17] asked whether every infinite group admits a non-discrete Hausdorff group topology. In 1953 Kertész and Szele [14] gave a positive answer to this question for Abelian groups, and in 1980 Ol'shanskii [18] found out

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a contable group which does not admit any Hausdorff group topology beyond the discrete one.

Recall that a topological group is **precompact** (or **totally bounded**) if it is (algebraically and topologically) isomorphic to a subgroup of some compact Hausdorff group, or equivalently, if its Weil completion is compact [21]. While each Abelian group admits a precompact Hausdorff group topology (this immediately follows from the fact that the set of all characters of an Abelian group separates points of it), not every Abelian group admits a compact Hausdorff group topology: In fact, answering a questoin of Halmos [10], Hulanicki [12] gave the complete description of Abelian groups admitting compact Hausdorff group topologies. This led the authors to the following problem: Which (Abelian) groups are Q-topologisable for topological properties Q occupying the intermediate position between compactness and precompactness? Our favourite candidate for such a Q is pseudocompactness which was introduced by Hewitt [11]. Recall that a topological space X is:

countably compact [9] if every infinite subset of X has an accumulation point in X,

strongly pseudocompact [2] if X contains a dence subset D such that every infinite subset of D has an accumulation point in X,

pseudocompact [11] if every real-valued continuous function defined on X is bounded.

For every space we have an obvious chain of implications:

compact => countably compact => strongly pseudocompact => pseudocompact

Neither of these implication is reversable. Indeed, the space of all countable ordinals with the interval topology is countably compact and is not compact; the Isabell-Mróvka space is strongly pseudocompact without being countably compact; and there is also an infinite pseudocompact Tychonoff space X all countable subsets of wich are closed [19] (clearly, X cannot be strongly pseudocompact). Since pseudocompact groups are precompact [4], all properties defined above lie in the intermediate position between compactness and precompactness in the class of topological groups.

Recall that a cardinal τ is **strong limit** if $2^{\sigma} < \tau$ for each cardinal $\sigma < \tau$. If τ is a cardinal, then $cf(\tau)$ denotes the **cofinality** of τ, i.e. the smallest cardinal λ for which there is a representation $\tau = \sum \{\tau_{\alpha}: \alpha < \lambda\}$, where $\tau_{\alpha}<\tau$ for all $\alpha < \lambda$. A cardinal τ is **regular** if $\tau = cf(\tau)$ and is **singular** otherwise.

In 1980 van Douwen [7] fuond the following constraints on the cardinality of an infinite pseudocompact Hausdorff group G:

(i) $|G| \geq 2^{\omega}$, and

(ii) $|G|$ cannot be a strong limit cardinal of countable cofinality.

Furthermore, van Douwen also showed that these are the only restrictions on the cardinality of a group admitting a pseudocompact Hausdorff group topology. This justifies the following

1.1. **DEFINITION.** *We say that a cardinal τ is* **admissible** *provided that $\tau \geq 2^{\omega}$ and if τ is strong limit, then $cf(\tau) > \omega$.*

We introduce also another class of cardinal closely related to admissible cardinals.

1.2. **DEFINITION.** *A cardinal τ is* **good** *if there exist a cardinal σ satisfying*

$$\sigma^{\omega} \leq \tau \leq 2^{\sigma}.$$

1.3. **DEFINITION.** *We fix the symbol SCH for denoting the following condition on cardinals:*

(*) *if $\tau \geq 2^{\omega}$ is a cardinal and $cf(\tau) \neq \omega$, then $\tau^{\omega} = \tau$.*

The condition (*) is known to be equivalent to the so called Singular Cardinals Hypotesis (which says that, for every singular cardinal K, if $K > 2^{cf(K)}$, then $K^{cf(K)} = K^{+}$, see [13, Chapter 8]), and this is the reason why we denote (*) by SCH. One can easily show that SCH follows from the Generalized Continuum Hypothesis GCH, saying that $2^{K} = K^{+}$ for all cardinals K, which is known to be consistent with the usual Zermelo-Fraenkel axioms ZFC of set theory. Nevertheless, SCH is much more weaker than GCH: Easton [8] has shown that any "rule" that is not obviously false can be used to tell what 2^{K} is for regular K, and his models satisfy SCH. Magidor [15,16] has shown that the consistency of the exestence of large cardinals implies the consistency of ZFC+$\rceil$SCH, and Dodd and Jensen [5,6] have shown that one needs a large cardinal to destroy SCH, more precisely, then have shown that the consistency of ZFC+$\rceil$SCH implies the existence of inner submodel with a measurable cardinal.

The following easy proposition clarifies the interrelation between admissible and good cardinals.

1.4. **PROPOSITION.**

(i) Good cardinals are admissible.

(ii) If SCH holds, then each admissible cardinal is good.

In particular, both notions coincide under SCH.

2. Definitions, notations and terminology

2.1. DEFINITION. *We fix the simbol $\mathcal{P}$ ($\mathcal{P}^*$) for denoting the class of Abelian groups admitting a pseudocompact (pseudocompact and connected) Hausdorff group topology.*

In wath follows $\mathbb{N}$ denotes the set of all natural numbers, $\mathbb{P}$ the set of all prime numbers, $\mathbb{Z}$ the group of integer numbers and $\mathbb{Q}$ the group of rational numbers. If X is a subset of group G, then $\langle X \rangle$ denotes the smallest subgroup of G that contains X. An Abelian group G is **torsion** if for every $g \in G$ there is $n \in \mathbb{N}$ with $ng = 0$, and G is a **bounded torsion** group if there is $n \in \mathbb{N}$ so that $ng = 0$ for all $g \in G$.

Let G be an Abelian group. For a cardinal τ we denote by $G^{(\tau)}$ the direct sum of τ copies of the group G. For $n \in \mathbb{N}$ we set

$G[n] = \{ g \in G : ng = 0 \}$ and $nG = \{ ng : g \in G \}$.

One can easily see that both G [n] and nG are subgroups of G, and the quotient group G/G[n] is isomorphic to the group nG. For $p \in \mathbb{P}$ the subgroup

$G_p = \{ g \in G : p^n g = 0 \text{ for some } n \in \mathbb{N} \}$

is called the **p-torsion part** of G. If $G = G_p$, then we say that G is a **p-group.**

By a **variety** of groups we mean, as usual, a class of groups closed under taking Cartesian products, subgroups and quotient groups. We denote by $\mathcal{G}$ and $\mathcal{A}$ the verieties of all groups and all Abelian groups respectively. It is well-known that any proper subvariety of $\mathcal{A}$ coincedes with some of the varieties

$\mathcal{A}_n = \{ G \in \mathcal{A} : G = G[n] \}$

for appropriate $n \in \mathbb{N}$.

2.2. DEFINITION. *Let V be a variety of groups.*

*(i) A subset X of a group G is said to be V-**free** provided that $\langle X \rangle \in V$ and for every map $f: X \to H \in V$ there exists the unique homomorphism $\bar{f}: \langle X \rangle \to H$ extending f.*

*(ii) A subset X of a group G is a V-**base** (of G) if X is V-free and $\langle X \rangle = G$.*

(iii) A group G is $\mathcal{V}$**-free** *if G contains a* $\mathcal{V}$*-base.*

(iv) For a group G we set

$r_{\mathcal{V}}(G) = \sup \{ |X| : X \text{ is a } \mathcal{V}\text{-free subset of } G \}$

and call $r_{\mathcal{V}}(G)$ *a* $\mathcal{V}$*-rank of G.*

(v) For shortness' sake we will use $r(G)$ *instead of* $r_{\mathcal{A}}(G)$ *and* $r_p(G)$ *instead of* $r_{\mathcal{A}_p}(G)$, $p \in \mathbb{P}$.

If $\mathcal{V}$ is a non-trivial variety, then for every set X there is the unique group $F_{\mathcal{V}}(X)$ such that $X \subset F_{\mathcal{V}}(X)$ and X is a $\mathcal{V}$-base of $F_{\mathcal{V}}(X)$.

2.3. DEFINITION. *For every (possibly finite) cardinal* τ, $F_{\mathcal{V}}(\tau)$ *is the (unique up to isomorphism)* $\mathcal{V}$*-free group having a* $\mathcal{V}$*-base of cardinality* τ.

3. Additional constraints for pseudocompact topologizations

Our first proposition presents categorial properties of the class $\mathcal{P}$.

3.1. PROPOSITION.

(i) The class $\mathcal{P}$ *is closed with respect to taking arbitrary Cartesian products.*

(ii) If $G \in \mathcal{P}$, *then* $nG \in \mathcal{P}$ *for all* $n \in \mathbb{N}$ *(or equivalently,* $G/G[n] \in \mathcal{P}$ *for all* $n \in \mathbb{N}$*).*

3.2. COROLLARY. *If* $G \in \mathcal{P}$, *then for every* $n \in \mathbb{N}$ *the cardinal* $|nG| = |G/G[n]|$ *is admissible.*

3.3. PROPOSITION. *If* $G \in \mathcal{P}$ *and* $r(G) < 2^{\omega}$, *then G is a bounded torsion group.*

Proposition 3.3. was announced also in [3].

3.4. COROLLARY. *If* $G \in \mathcal{P}$ *is torsion, then G is a bounded torsion group.*

3.5. THEOREM. *If* $G \in \mathcal{P}$, *then* $|nG| = |G/G[n]| \le 2^{2^{r(G)}}$ *for some* $n \in \mathbb{N}$.

3.6. COROLLARY. *If* $G \in \mathcal{P}$ *is divisable, then* $|G| \le 2^{2^{r(G)}}$.

4. Pseudocompact topologizations of $\mathcal{V}$-free groups

4.1. THEOREM. *Assume that* $\mathcal{V}$ *is a variety of groups,* τ, σ *and* λ *are cardinals and H is a group satisfying the following conditions:*

(i) $\sigma = \sigma^{\omega} \leq \tau \leq 2^{\sigma}$ and $\sigma = \sigma^{\omega} \leq \lambda \leq 2^{\sigma}$,

(ii) $r_{\mathcal{V}}(H) \geq \omega$,

(iii) H admits a compact Hausdorff group topology τ *of weight* $\leq \lambda$.

Let X be a set with $|X| = \tau$. *Then there exists a Hausdorff group topology* τ^* *on* $F_{\mathcal{V}}(X)$ *such that:*

(iv) the weight of τ^* *is equal to* λ,

(v) X is dence in $(F_{\mathcal{V}}(X), \tau^*)$ *and every infinite subset of X has an accumulation point in* $(F_{\mathcal{V}}(X), \tau^*)$ *(in particular,* τ^* *is strongly pseudocompact).*

Moreover, if τ *is connected (connected and locally connected), then so is* τ^*.

4.2. COROLLARY. *If* τ *is a good cardinal, then* $F_{\mathcal{A}}(\tau)$ *admits a strongly pseudocompact, connected, locally connected, Hausdorff group topology.*

Proof. Take the unit circle group $\mathbb{T}$ as H in theorem 4.1.

4.3. COROLLARY. *If* τ *is a good cardinal, then* $F_{\mathcal{G}}(\tau)$ *admits a strongly pseudocompact, connected, locally connected, Hausdorff group topology.*

Proof. Take the group $SO(3,\mathbb{R})$ as H in Theorem 4.1. From [1] it follows that $r_{\mathcal{G}}(SO(3,\mathbb{R})) = 2^{\omega} > \omega$, so Theorem 4.1. can be applied.

4.4. COROLLARY. *Assume SCH. Let* $\mathcal{V} \in \{\mathcal{G}, \mathcal{A}\}$ *and* τ *be a cardinal. Then the following are equivalent:*

(i) $F_{\mathcal{V}}(\tau)$ *admits a pseudocompact Hausdorff group topology,*

(ii) $F_{\mathcal{V}}(\tau)$ *admits a strongly pseudocompact, connected, locally connected, Hausdorff group topology,*

(iii) τ *is admissible.*

4.5. DEFINITION. *Let* $\mathcal{V}$ *be a variety of groups. We say that an infinite cardinal* τ *is* $\mathcal{V}$-**admissible** *if* $F_{\mathcal{V}}(\tau) \in \mathcal{P}$.

It is clear that, for any non-trivial $\mathcal{V}$, infinite $\mathcal{V}$-admissible cardinals are admissible.

4.6. THEOREM. *(i) For* $p \in \mathbb{P}$ *and* $n \in \mathbb{N}$ *fixed, a cardinal is* $\mathcal{A}_{p^n}$*-admissible if and only if it is* $\mathcal{A}_p$*-admissible.*

(ii) If a cardinal is $\mathcal{A}_p$*-admissible for some* $p \in \mathcal{P}$*, then it is also* $\mathcal{A}$*-admissible.*

(iii) If SCH holds, then $\mathcal{A}$*-admissible cardinal are* $\mathcal{A}_p$*-admissible whenever* $p \in \mathcal{P}$.

Our next result shows that in Corollary 4.3. strong pseudocompactness cannot be strengthened to countable compactness.

4.7. THEOREM. *No non-trivial $\mathcal{G}$-free group admits a countably compact, Hausdorff group topology.*

Proof. Assume the contrary, and let G be a non-trivial $\mathcal{G}$-free group equipped with a countably compact, Hausdorff group topology. Fix $x \in G$, $x \neq e$. Let H be the smallest closed subgroup of G containing x. Since H has a dense cyclic (hence Abelian) subgroup, H is Abelian. On the other hand, H is $\mathcal{G}$-free as a subgroup of the $\mathcal{G}$-free group G. Therefore, $H = \langle\{x\}\rangle$ and so $|H| = \omega$. Being closed in a countably compact space, H is countably compact. Since H is countable, from the Baire category theorem it follows that H contains an isolated point. Since H is a topological group, H is discrete. Being countably compact, H must be finite, a contradiction.

In the contrast to the last theorem Tkačenko [20] showed that, under CH, the group $F_{\mathcal{A}}(2^{\omega})$ *does* admit a countably compact, Hausdorff group topology. This naturally leads to the following

4.8. QUESTION. *For which cardinals τ the group $F_{\mathcal{A}}(\tau)$ admits a countably compact, Hausdorff group topology?*

Now we will return back from the special results which hold for particular varieties to a general case. If $\mathcal{V}$ is a variety of groups, then we say that a $\mathcal{V}$-free group is **finitely genereted** provided that it has a finite $\mathcal{V}$-basis.

4.9. THEOREM. *For an arbitrary variety $\mathcal{V}$ of groups the following conditions are equivalent:*

(i) every finitely generated $\mathcal{V}$-free group admits a precompact, Hausdorff group topology,

(ii) $F_{\mathcal{V}}(\tau) \in \mathcal{P}$ whenever τ is a good cardinal,

(iii) if τ is a good cardinal, then the group $F_{\mathcal{V}}(\tau)$ admits a strongly pseudocompact, Hausdorff group topology.

If SCH holds, then the equivalence of (i) - (iii) remains valid if one replaces in (ii) and (iii) the word "good" by the word "admissible".

Proof. (i) $\Rightarrow$ (iii) By our assumption, for every $n \in \mathbb{N}$ there is a compact (Hausdorff) group H_n with $r_{\mathcal{V}}(H_n) \geq n$. Now $H = \prod \{H_n : n \in \mathbb{N}\}$ satisfies items (ii) and (iii) of Theorem 4.1. for any λ satisfying (i) of Theorem 4.1. This yields (iii).

(iii) ⇒ (ii) is trivial.

(ii) ⇒ (i) By (ii) there is a pseudocompact (Hausdorff) group topology τ on $F_{\mathcal{V}}(2^{\omega})$. Since pseudocompact groups are precompact [4], the completion G of $(F_{\mathcal{V}}(2^{\omega}), \tau)$ is compact and $r_{\mathcal{V}}(G) \geq 2^{\omega}$. Therefore, every finitely generated $\mathcal{V}$-free group admits a precompact Hausdorff group topology.

The last statement of our theorem follows from Proposition 1.4.(ii).

5. Pseudocompact topologization of torsion Abelian groups

Since in this section we are going to consider only torsion Abelian groups admitting a pseudocompact Hausdorff group topology, Corollary 3.6. permits us to restrict our considerations by bounded torsion groups.

5.1. DEFINITION. *Fix $p \in \mathcal{P}$ and let G be a bounded torsion Abelian groups. Then for each $k \in \mathbb{N}$ we set*

$$\beta_{k,p} = |G_p/G_p[p^k]| = |p^k G_p|.$$

Since G is bounded, there are $r_p \in \mathbb{N}$ and a sequence $\alpha_{1,p}, \ldots, \alpha_{r_p,p}$ of cardinals such that

$$G_p = \oplus \{ \mathbb{Z}(p^i)^{(\alpha_{i,p})} : 1 \leq i \leq r_p \}$$

and $\alpha_{r_p,p} \neq 0$.

One can easily see that $\beta_{k,p} = \max \{\alpha_{k+1,p}, \ldots, \alpha_{r_p,p}\}$ for all $p \in \mathbb{P}$ and $k < r_p$.

5.2. THEOREM. *For a bounded torsion Abelian group G fixed, consider the following conditions:*

(a) $G \in \mathcal{P}$,

(b) $G_p \in \mathcal{P}$ for all $p \in \mathbb{P}$,

(c) $|G_p/G_p[p^k]|$ is $\mathcal{A}_p$-admissible whenever $p \in \mathbb{P}$ and $k \in \mathbb{N}$,

(c^*) $|G_p/G_p[p^k]|$ is admissible whenever $p \in \mathbb{P}$ and $k \in \mathbb{N}$,

(d) $|G/G[m]|$ is admissible for each $m \in \mathbb{N}$.

Then (c) ⇒ (b) ⇔ (a) ⇒ (d) ⇔ (c^*). If SCH holds then all conditions are equivalent.

5.3. COROLLARY. *If G is a bounded torsion Abelian group such that $\alpha_{n,p}$ is $\mathcal{A}_p$-admissible whenever $p \in \mathbb{P}$ and $n \leq r_p$, then $G \in \mathcal{P}$.*

For a cardinal τ, log τ denotes the smallest cardinal λ with $2^{\lambda} \geq \tau$.

5.4. THEOREM. *Supose that* $G \in \mathcal{P}$ *is a (bounded) torsion group,* $p \in \mathbb{P}$*, and for each* $k < r_p$ *the condition*

$$\log \log \alpha_{k,p} \le |G_p/G_p[p^k]| < \alpha_{k,p}$$

does not hold. Then the cardinal $|G_p/G_p[p^k]|$ *is* $\mathcal{A}_p$*-admissible for every* $k \in \mathbb{N}$.

5.5. COROLLARY. *For* $p \in \mathbb{P}$ *fixed, assume that* $G \in \mathcal{P}$ *is a (bounded) torsion p-group such that for each* $k < r_p$ *the condition*

$$\log \log \alpha_{k,p} \le \beta_{k,p} < \alpha_{k,p}$$

does not hold. Then each $|G_p|$ *is* $\mathcal{A}_p$*-admissible, and so* $|G|$ *is* $\mathcal{A}$*-admissible by Theorem 4.6.(ii).*

5.6. COROLLARY. *Assume that* $p \in \mathbb{P}$ *and G is a bounded torsion Abelian p-group. Suppose also that* α_k *is a strong limit cardinal for every* $k = 1, \ldots, r_p$*. Then the following are equivalent:*

(i) $G \in \mathcal{P}$,

(ii) $\beta_{k,p}$ *is* $\mathcal{A}_p$*-admissible for all* $k = 1, \ldots, r_p$.

5.7. COROLLARY. *For* $n \in \mathbb{N}$*, a cardinal is* $\mathcal{A}_p$*-admissible if and only if it is* $\mathcal{A}_p$*-admissible for each* $p \in \mathbb{P}$ *which divides n.*

6. Miscellanea

6.1. DEFINITION. *For a cardinal* σ *we denote by* $\mathcal{P}_\sigma (\mathcal{P}^*_\sigma)$ *the class of all Abelian groups admitting a pseudocompact (pseudocompact and connected) Hausdorff group topology of weight* σ.

6.2. THEOREM. *For cardinals* τ *and* σ *the following are equivalent:*

(i) there exists a divisible group $G \in \mathcal{P}_\sigma$ *with* $|G| = \tau$,

(ii) there exists a $G \in \mathcal{P}^*_\sigma$ *with* $|G| = \tau$,

(iii) $\mathbb{Q}^{(\tau)} \in \mathcal{P}_\sigma$,

(iv) $F_{\mathcal{A}}(\tau) \in \mathcal{P}^*_\sigma$.

Since (Hausdorff) pseudocompact divisible groups are connected, our next theorem implies Corollary 3.6.

6.3. THEOREM. *If* $G \in \mathcal{P}^*$*, then* $|G| \le 2^{2^{r(G)}}$.

6.4. THEOREM. *Suppose that G is an Abelian group such that*

$$\sigma^\omega \le r(G) \le 2^\sigma \quad \text{and} \quad |G| \le 2^{2^\sigma}$$

for some cardinal σ*. Then* $G \in \mathcal{P}^*_{2^\sigma}$.

6.5. COROLLARY. *If G is an infinite Abelian group such that $r(G) = r(G)^{\omega}$ and $|G| \le 2^{2^{2^{r(G)}}}$, then $G \in \mathcal{P}^*_{2^{r(G)}}$ (in particular, $G \in \mathcal{P}^*$).*

6.6. COROLLARY. *If G is an Abelian group such that $r(G)$ is good and $|G| \le 2^{r(G)}$, then $G \in \mathcal{P}^*$.*

6.7. COROLLARY. *Let G be an Abelian group with $r(G) = |G|$. Then the following conditions are equivalent:*

(a) $|G|$ is admissible;

(b) $G \in \mathcal{P}$;

(c) $G \in \mathcal{P}^*$.

6.8. COROLLARY. *For a divisable Abelian group G with $r(G) = r(G)^{\omega}$ the following are equivalent:*

(i) $G \in \mathcal{P}$,

(ii) $|G| \le 2^{2^{2^{r(G)}}}$.

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PSEUDOKOMPAKTNE TOPOLOGIZACIJE GRUPA

Izučava se problematika kada grupa dopušta pseudokompaktnu T topologizaciju. Nadjeni su dovoljni uslovi pri kojima je to moguće učiniti. U nekim slučajevima razlika izmedju ovih dovoljnih i potrebnih i dovoljnih uslova često zavisi od nekih dodatnih pretpostavki teorije skupova povezanih sa egzistencijom velikih kardinala.

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