Zbornik radova Filozofskog fakulteta u Nišu
Serija Matematika 4(1990), 113-114

Ž. Pavićević

ON THE LITTLEWOOD THEOREM CONCERNING THE GREEN POTENTIAL

(Received 15.06.1990.)

Abstract. An equivalent of the Littlewood theorem concernig the Green potential is proved.

1. Let

(1)
$$\omega(z) = \iint_{|a|<1} \log\left|\frac{1-\bar{a}z}{z-a}\right| d\mu(a) ,$$

where μ is a positive mass distribution in D: $|z|<1$. $\omega(z)$ is the Green potential connected with mass μ.

Let $d\mu(a) = d\xi d\eta$. For the Green potential

$$\omega^*(z) = \iint_{|a|<1} \log\left|\frac{1-\bar{a}z}{z-a}\right| d\xi d\eta ,$$

in [1], we proved the following lemma:

LEMMA 1. $\omega^*(z) = \frac{\pi}{2}(1-|z|^2)$, $z \in D$.

COROLLARY. *For any* $e^{i\theta} \in \Gamma = \{z: |z| = 1\}$, $\lim_{z\to e^{i\theta}} \omega^*(z) = 0$.

2. **LEMMA 2.** *The following assertions are equivalent:*

(i) $\iint_{|a|<1} (1-|a|)d\mu(a) < +\infty$;

(ii) $\iint_{|z|<1} \omega(z)dxdy < +\infty$, $z=x+iy$.

Proof. Let

(2)
$$\iint_{|a|<1} (1-|a|)d\mu(a) < +\infty .$$

By Lemma 1, we have

(3)
$$1-|a| = \frac{2}{\pi(1+|a|)} \iint_{|a|<1} (\iint_{|z|<1} \log\left|\frac{1-\bar{a}z}{a-z}\right| dxdy , \quad z=x+iy.$$

AMS Subject Classification (1980): 31A15

Putting (3) in (2) we get that (2) holds if and only if

$$(4)\qquad \frac{2}{\pi(1+|a|)} \iint_{|a|<1} \Big(\iint_{|z|<1} \log\Big|\frac{1-\bar{z}a}{a-z}\Big| dxdy\Big) d\mu(a) < +\infty .$$

Changing the order of integration in (4), we obtain that (2) holds iff

$$(5)\qquad \iint_{|z|<1} \Big(\iint_{|a|<1} \log\Big|\frac{1-\bar{z}a}{a-z}\Big| d\mu(a)\Big) dxdy < +\infty , \quad z=x+iy.$$

Since $\log\Big|\frac{1-\bar{z}a}{a-z}\Big| = \log\Big|\frac{1-\bar{a}z}{z-a}\Big|$ for any $a, z \in D$, then by (1), (5) holds iff

$$\iint_{|z|<1} \omega(z)dxdy < +\infty , z=x+iy.$$

3. **THEOREM.** *Let the potential (1) satisfy the condition (ii) in Lemma 2. Then* $\lim_{r\to 1}\omega(re^{i\theta}) = 0$ *for almost all* $e^{i\theta} \in \Gamma$ *and* $\lim_{r\to 1} \int_0^{2\pi} \omega(re^{i\theta})d\theta = 0.$

Since the assertions (i) and (ii) in Lemma 2 are equivalent, the assertion of Theorem is equivalent to the Littlewood theorem (see [2;Th.IV.3.3]).

REFERENCES

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Ž. PAVIĆEVIĆ

O TEOREMI LITTLEWOOD-a O POTENCIJALU GREEN-a

Dokazuje se jedan ekvivalent poznate teoreme Littlewood-a koja se odnosi na potencijal Green-a.

Univerzitet u Titogradu
Prirodno-matematički fakultet
81000 Titograd, Yugoslavia