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SOME REMARKS ON CURVILINEAR SYMMETRY

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Connections between antisymmetry, (3)-symmetry, (32)-symmetry groups and curvilinear symmetry groups, defined by P.L.Dubov, are established. Curvilinear symmetry groups of categories $\tilde{G}_{20}$, $\tilde{G}_{21}$, $\tilde{G}_2$, are derived. By applying, respectively, curvilinear symmetry groups of categories $\tilde{G}_{20}$, $\tilde{G}_{21}$ on finite segment patterns LPF and strip segment patterns LPS, corresponding curvilinear finite segment patterns LPFC and curvilinear strip segment patterns LPSC, are enumerated and visually interpreted.

1. INTRODUCTION

After introducing the notion of a strictly convex, stright and concave open segment (Figure 1), by P.L.Dubov [1,2] are defined the five transformations, denoted by them as "0", "1", "2", "3" and "*". If the properties of convexity, strightness and concavity of an open segment are denoted, respectively, by 1,2,3, the curvilinear-identity transformations mentioned can be represented by permutation cycles:

"0"=E, "1"=(23)=e_0, "2"=(13)=e_1, "3"=(12)=e_2, "*"=(123)=e_3.

The additional symbols e_i (i=0,1,2,3) point to their relationships with corresponding antisymmetry, (3)-symmetry and (32)-symmetry color-identity transformations [3,4,5].

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(a) (b) (c)

Figure 1: (a) strictly convex, (b) stright, (c) concave open segment.

Hence, "0"$=E$ is the identity transformation, "1"$=e_0$, "2"$=e_1$, "3"$=e_2$ are the curvilinear-antiidentity transformations, defining the groups C_2^i (i=0,1,2) of the structure C_2, given by presentation:

$C_2{}^i$ $\{e_i\}$ $e_i{}^2=E$ (i=0,1,2),

while "*"$=e_3$ is the transformation defining the group C_3^3 of the structure C_3, given by presentation:

C_3^3 $\{e_3\}$ $e_3{}^3=E$.

The same group can be generated by the transformation $e_3{}^{-1}$. Every two of the transformations e_i (i=0,1,2,3) generate the irregular permutation group D_3^{32} of the structure D_3, given by presentation:

D_3^{32} $\{e_i, e_j\}$ $e_i{}^2=e_j{}^2=(e_ie_j)^3=E$ (i,j=0,1,2; $i\neq j$),

or by presentation:

D_3^{32} $\{e_i, e_3\}$ $e_3{}^3=e_i{}^2=(e_3e_i)^2=E$ (i=0,1,2).

Let a discrete symmetry group G be given. Every transformation $S^j=c_jS=Sc_j$, where $S\in G$ and c_j is a transformation e_i (i=0,1,2,3) or product of these transformations different from the identical transformation E, is called a curvilinear symmetry transformation. In such a case, the symmetry group G is called generating curvilinear symmetry group. Every curvilinear symmetry group derived from G and isomorphic to it, is called a junior curvilinear symmetry group. In order to difer between curvilinear symmetry groups derived from G, according to the groups C_2^0, C_2^1, C_2^2, C_3^3, D_3^{32}, the symbols $\tilde{G}^0$, $\tilde{G}^1$, $\tilde{G}^2$, $\tilde{G}^3$, $\tilde{G}^{32}$ will be used. Since in this work only junior curvilinear symmetry groups will be discussed, we can call them simply curvilinear symmetry groups. Also, we will restrict our study to curvilinear symmetry groups with a generating symmetry group G belonging to the category G_{20}, G_{21} or G_2 [6], that means, to curvilinear symmetry groups of the categories $\tilde{G}_{20}$, $\tilde{G}_{21}$ and $\tilde{G}_2$.

2. CURVILINEAR SYMMETRY GROUPS

From the afore mentioned properties of curvilinear symmetry transformations and curvilinear symmetry groups, we can conclude that between curvilinear symmetry groups $\tilde{G}^i$ (i=0,1,2) and junior antisymmetry groups G'derived from the same group G, curvilinear symmetry groups $\tilde{G}^3$ and junior (3)-symmetry groups $G^{(3)}$ as well as curvilinear symmetry groups $\tilde{G}^{32}$ and junior (32)-symmetry groups $G^{(32)}$, there is the isomorphism transforming curvilinear symmetry transformations onto corresponding antisymmetry, (3)-symmetry, or (32)-symmetry transformations, acting upon elements of the symmetry group G as the identical automorphism. Therefore, the complete curvilinear symmetry theory, introduced by P.L.Dubov, can be reduced to the well known antisymmetry, (3)-symmetry and (32)-symmetry theory. Moreover, for obtaining all the curvilinear symmetry groups of any category $\tilde{G}_{n...}$, it is enough to use antisymmetry, (3)-symmetry and (32)-symmetry groups already derived from symmetry groups of the same category $G_{n...}$. The main source of that informations are the works on the general P-symmetry theory, or their particular cases: antisymmetry, (3)-symmetry and (32)-symmetry [3,4,5,7,8,9,10,11,12,13]. Such a treatment of curvilinear symmetry groups is in full agreement with an interpretation of P-symmetry transformations as regular changes of some geometrical property, commuting with symmetries of generating symmetry group.

3. CURVILINEAR SYMMETRY GROUPS OF CATEGORIES $\tilde{G}_{20}$, $\tilde{G}_{21}$, $\tilde{G}_{2}$

For denoting antisymmetry, (3)-symmetry and (32)-symmetry groups as well as corresponding curvilinear symmetry groups, group/subgroup symbols G/H or $G/H_1/H$, will be used. In order to differ curvilinear symmetry groups $\tilde{G}^i$ (i=0,1,2) possessing the same symbol G/H, additional symbols $(G/H)^i$ are used.

All the discrete symmetry groups of rosettes G_{20} belong to one of the infinite classes: $C_n(n)$ or $D_n(nm)$ ($n \in N$). From them, the three infinite classes of antisymmetry groups [7]:

$C_{2n}/C_n((2n)/n)$, $D_n/C_n(nm/n)$, $D_{2n}/D_n((2n)m/nm)$,

the one infinite class of (3)-symmetry groups:

$C_{3n}/C_n((3n)/n)$,

as well as the one infinite class of (32)-symmetry groups:

$D_{3n}/D_n((3n)m/nm)$,

are derived. Since to every antisymmetry group G' they correspond three curvilinear symmetry groups $\tilde{G}^0$, $\tilde{G}^1$, $\tilde{G}^2$, we have the amounts of curvilinear symmetry groups, given in Table 1:

Table 1

G	n	$\tilde{G}^i$ (i=0,1,2)	$\tilde{G}^3$	$\tilde{G}^{32}$	$\tilde{G}$
$C_n(n)$	$6k$	3	1		4
	$6k\pm1$				0
	$6k\pm2$	3			3
	$6k-3$		1		1

$D_n(nm)$	$6k$	6	1	7
	$6k\pm1$	3		3
	$6k\pm2$	6		6
	$6k-3$	3	1	4

According to this, there are the 11 curvilinear symmetry groups of rosettes at every $n=6k$, 3 curvilinear symmetry groups at $n=6k\pm1$, 9 curvilinear symmetry groups at $n=6k\pm2$ and 5 curvilinear symmetry groups at $n=6k-3$ ($n\in N$). Between them they are the 37 curvilinear symmetry groups satisfying the crystallographic restriction (n=1,2,3,4,6), derived by P.L.Dubov [1,2].

From the seven discrete symmetry groups of friezes G_{21} are derived the following 17 antisymmetry groups [3,8,10]:

p11/p11, p1g/p11, p1m/p1m, p1m/p1g, p1m/p11, p12/p12, p12/p11, pm1/pm1, pm1/p11, pmg/pm1, pmg/p12, pmg/p1g, pmm/pmm, pmm/pmg, pmm/pm1, pmm/p12, pmm/p1m,

the three (3)-symmetry groups:

p11/p11, p1g/p1g, p1m/p1m,

as well as the four (32)-symmetry groups:

p12/p12/p11, pm1/pm1/p11, pmg/pmg/p1g, pmm/pmm/p1m.

To them correspond the 51 curvilinear symmetry group $\tilde{G}^i$ (i=0,1,2), 3 curvilinear symmetry groups $\tilde{G}^3$ and 4 curvilinear symmetry groups $\tilde{G}^{32}$, that means, in total, the 58 curvilinear symmetry groups of the category $\tilde{G}_{21}$.

From the symmetry groups of ornaments G_2 there are derived the 46 antisymmetry groups:

p1/p1, p2/p2, p2/p1, pg/pg, pg/p1, pm/cm, pm/pm1, pm/p1m, pm/pg, pm/p1, cm/pm, cm/pg, cm/p1, pgg/pg, pgg/p2, pmg/pmg, pmg/pgg,

pmg/pm, pmg/pg, pmg/p2, pmm/pmm, pmm/cmm, pmm/pmg, pmm/pm, pmm/p2, cmm/pmm, cmm/pmg, cmm/pgg, cmm/cm, cmm/p2, p4/p4, p4/p2, p4g/p4, p4g/cmm, p4g/pgg, p4m/p4m, p4m/p4g, p4m/p4, p4m/cmm, p4m/pmm, p3m1/p3, p31m/p3, p6/p3, p6m/p6, p6m/p31m, p6m/p3m,

the eight (3)-symmetry groups:

p1/p1, pg/pg, pm/pm, cm/cm, p3/p3, p3/p1, p31m/p3m1, p6/p2,

as well as the fifteen (32)-symmetry groups:

p2/p2/p1, pg/pg/p1, pm/pm/p1, cm/cm/p1, pgg/pgg/pg, pmg/pmg/pm, pmg/pmg/pg, pmm/pmm/pm, cmm/cmm/cm, p3m1/p31m/p3, p3m1/cm/p1, p31m/cm/p1, p6/p6/p3, p6m/p6m/p3m1, p6m/cmm/p2.

To them correspond the 138 curvilinear symmetry groups $\tilde{G}^i$ (i=0,1,2), 8 curvilinear symmetry groups $\tilde{G}^3$ and 15 curvilinear symmetry groups $\tilde{G}^{32}$, that means, in total, the 161 curvilinear symmetry groups of the category $\tilde{G}_2$.

4. CURVILINEAR SEGMENT PATTERNS

In a search for plane figures possessing a curvilinear symmetry group of the category $\tilde{G}_{20}$, $\tilde{G}_{21}$ or $\tilde{G}_2$, the natural choice will be curvilinear segment patterns obtained from the finite segment patterns LPF, strip segment patterns LPS and periodic segment patterns LPP [14].

They are the 10 homeomeric types of finite segment patterns: LPF1$_n$-1 (n≥2), LPF1$_n$-2 (n≥3), LPF2$_n$-1 (n≥1), LPF2$_n$-2 (n≥2), LPF2$_n$-3 (n≥1), LPF2$_n$-4 (n≥2), LPF3$_n$-1 (n≥2), LPF3$_n$-2 (n≥2), LPF3$_n$-3 (n≥2), LPF3$_n$-4 (n≥3).

The first two of them possess the symmetry group C_n(n), and the others the symmetry group D_n(nm). In order to derive all the different curvilinear symmetry segment patterns LPFC, we can start with three possible trivial curvilinear segment patterns:

the strictly convex, stright and concave segment pattern, satisfying the generating symmetry group G, to subject them under the action of all the curvilinear symmetry groups derived from G. Since the curvilinear symmetry groups $\tilde{G}^i$ (i=0,1,2) derived from symmetry group G, transforme a convex, stright and concave segment pattern onto the same series of LPFC, if we denote by $\tilde{N}(G)$ the number of curvilinear symmetry groups derived from G, and by $\tilde{N}$(LPF) the number of different LPFC obtained from an LPF with the symmetry group G, we can conclude that, in all the non-exceptional cases, holds the relationship:

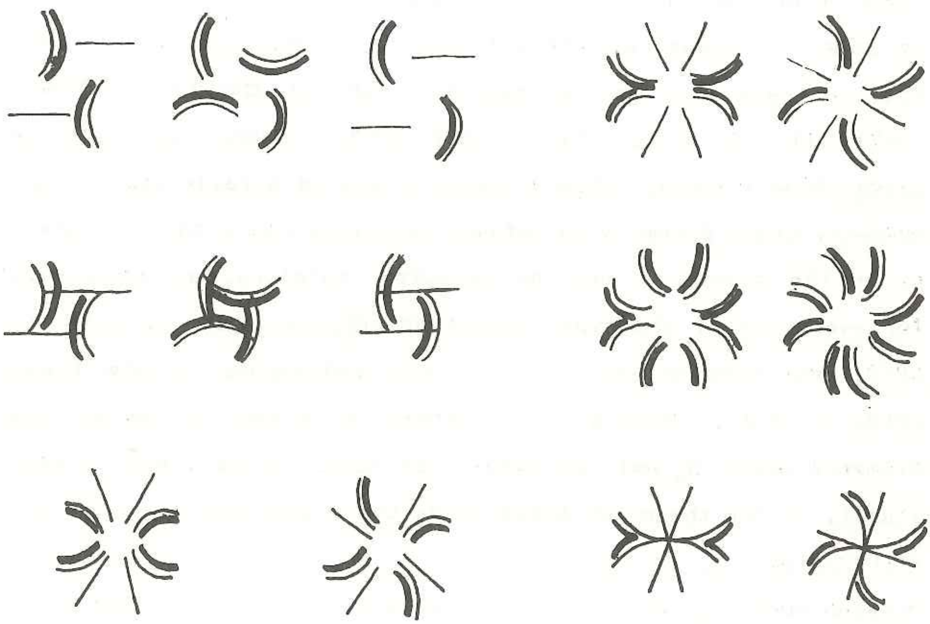
$\tilde{N}(\mathrm{LPF})=3+\tilde{N}(G)$.

Between the LPFC derived, there are $\tilde{N}(G)$ non-trivial LPFC, that means, LPFC with a curvilinear symmetry group derived from G. There are two possibilities for exceptions. The first exceptional cases occur when the stright open segment L of LPF is transformed by some non-identical element of the symmetry group G onto itself. There will be the case with LPF of the types LPF3n-1 (n≥2), LPF3n-2 (n≥2), LPF3n-3 (n≥2), LPF3n-4 (n≥3). At LPF of the types LPF3n-1 (n≥2), LPF3n-3 (n≥2), there is a reflection of the symmetry group D_n(nm) with reflection line containing the segment L, so the segment L must be stright. Therefore, at every n≥2, for every LPF of the type LPF3n-1 or LPF3n-3, there is only one LPFC, and none non-trivial. At LPF belonging to the types LPF3n-2 (n≥2), LPF3n-4 (n≥3), there is a reflection of the symmetry group D_n(nm), perpendicular to L, transforming L onto itself, so for these two types holds the relationship:

$\tilde{N}(\mathrm{LPF})=\tilde{N}(G)$.

Between such the LPFC obtained there are $\tilde{N}(G)$-3 non-trivial LPFC. The second exceptional cases occur when reflections of the symmetry groups there are not mutually equivalent. There will be, at n- an even natural number, with LPF of the type LPF$2n$-2, where reflections of the symmetry group $D_n(nm)$ can be divided into that with reflection lines containing segment endpoints, and the others not containing them. This non-equivalence of the reflections results in 12 (9) LPFC at $n=6k\pm2$ and 13 (10) at $n=6k$.

From the 10 homeomeric types of finite segment patterns, they are obtained the 12 (6) LPFC at $n=1$, 53 (33) LPFC at $n=2$, 46 (20) LPFC at $n=6k-3$, 65 (39) LPFC at $n=6k\pm2$, 38 (12) LPFC at $n=6k\pm1$ and 73 (47) LPFC at $n=6k$ ($k\in N$), where in parentheses are the corresponding numbers of non-trivial LPFC. All the non-trivial LPFC obtained at $n=4$ from LPF [14, Figure 7.4.2] are visually illustrated by Figure 2.



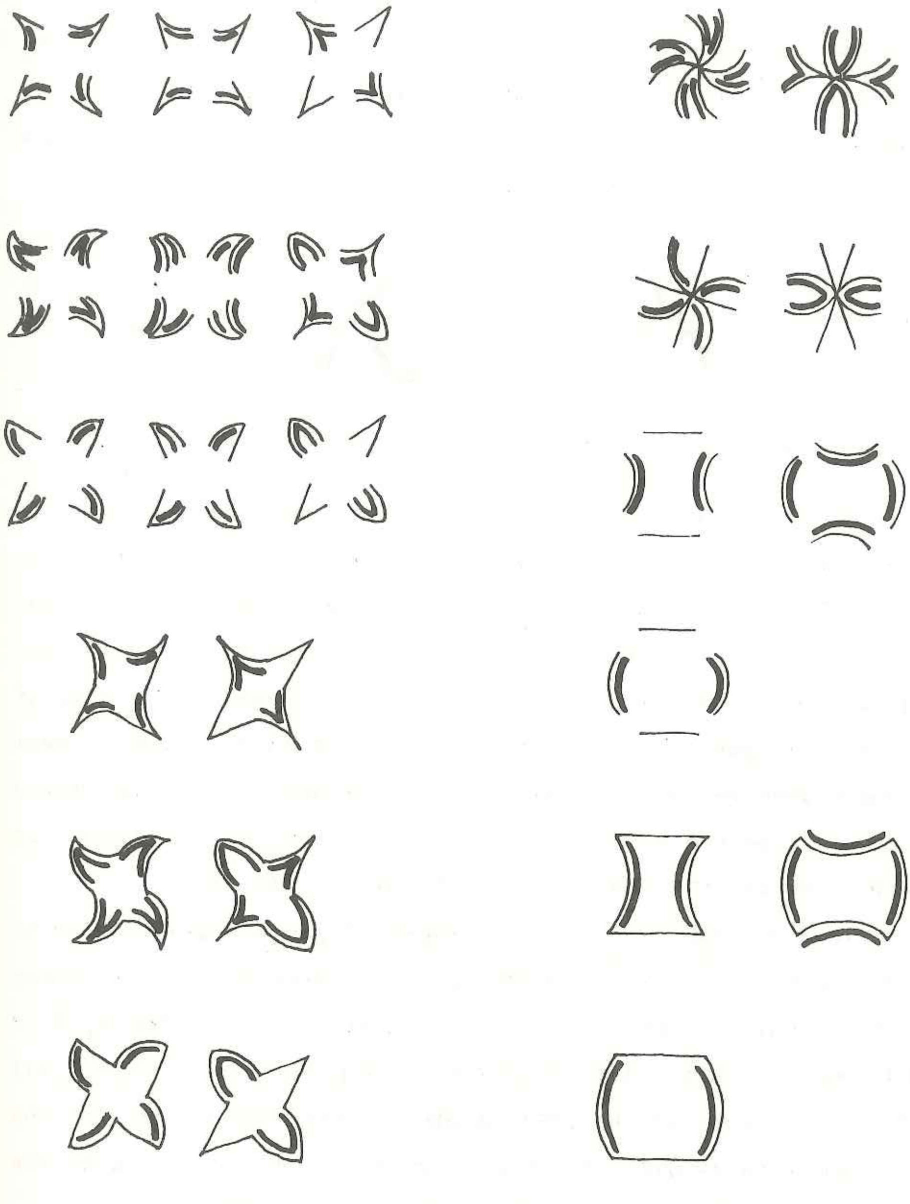


Figure 2

It is also possible to differ between LPFC with curvilinear symmetry groups $\tilde{G}^3$, derived from the LPF of the type LPF1$_n$-2, at $n=6k-6$ or $n=6k$ ($k\in N$), by the use of the transformations e_3 and e_3^{-1}. In that case, there will be one more LPFC at $n=6k-3$ and $n=6k$ (Figure 3).

Figure 3: (a) LPFC1$_3$-2 $(3/1)^3$, (b) LPFC1$_3$-2 $(3/1)^{-3}$.

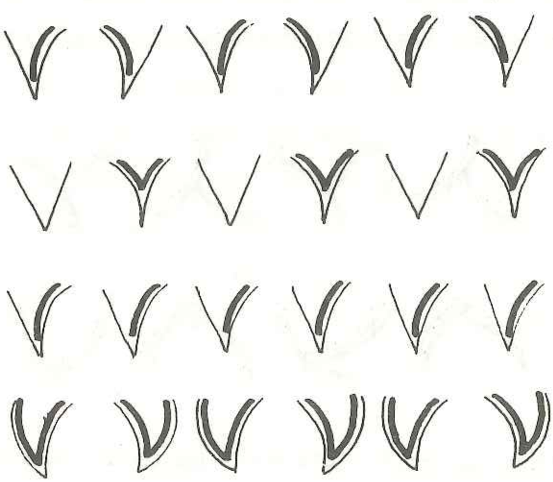
By the similar approach to the 27 flat edge simple forms FESF [15] satisfying the crystallographic restriction (n=1,2,3,4,6), by P.L.Dubov [1,2] are derived the 284 curvilinear flat edge simple forms FESFC, including at $n=4$ the 67 (42) FESFC illustrated in the article [1]. Since possible non-equivalency of reflections mentioned, it is not concerned by P.L.Dubov, these numbers deserve the correction. Let it be noticed that at $n=6k-3$ there will be the 51 (26) FESFC, at $n=6k\pm2$ the 73 (48) FESFC, at $n=6k\pm1$ the 43 (18) FESFC and at $n=6k$ the 81 (56) FESFC.

They are the 37 homeomeric types of strip segment patterns LPS occuring in one diffeomeric type, except for the types LPS9-2, LPS12-2, LPS12-3, ocurring, respectively, in the 2, 3, 2 diffeomeric types [14, Figure 7.4.3]. The symmetry group of all the LPS2 is **p1g**, of the LPS3 is **p1m**, of the LPS5 is **pm1**, of the LPS7 and LPS8 is **p12**, of the LPS9 and LPS10 is **pmg**, while of the LPS12, LPS13, LPS14 and LPS15 is **pmm**. The relationship:

$\tilde{N}(\mathrm{LPS})=3+\tilde{N}(G)$ $(\tilde{N}(G))$

holds for all the LPS, except for the LPS5-2, LPS7-2, LPS8-1,

LPS10-1, LPS10-2, LPS11-1, LPS11-2, LPS11-3, LPS12-2, LPS12-4, LPS12-5, LPS12-7, LPS13-1, LPS13-2, LPS13-3, LPS13-4, LPS14-1, LPS14-2, LPS14-3, LPS15-1, LPS15-2, LPS15-3. For these exceptional LPS, occuring from the same reason mentioned in the case of the exceptional LPF, there are the following numbers $\tilde{N}$(LPS): $\tilde{N}$(LPS5-2)=13 (10), $\tilde{N}$(LPS7-2)=13 (10), $\tilde{N}$(LPS8-1)=7 (4), $\tilde{N}$(LPS10-1)=7 (4), $\tilde{N}$(LPS10-2)=7 (4), $\tilde{N}$(LPS11-1)=7 (4), $\tilde{N}$(LPS11-2)=1 (0), $\tilde{N}$(LPS11-3)=7 (4), $\tilde{N}$(LPS12-2)=25 (22), $\tilde{N}$(LPS12-4)=25 (22), $\tilde{N}$(LPS12-5)=25 (22), $\tilde{N}$(LPS12-7)=25 (22), $\tilde{N}$(LPS13-1)=13 (10), $\tilde{N}$(LPS13-2)=1 (0), $\tilde{N}$(LPS13-3)=1 (0), $\tilde{N}$(LPS13-4)=13 (10), $\tilde{N}$(LPS14-1)=10 (7), $\tilde{N}$(LPS14-2)=1 (0), $\tilde{N}$(LPS14-3)=1 (0), $\tilde{N}$(LPS15-1)=1 (0), $\tilde{N}$(LPS15-2)=1 (0), $\tilde{N}$(LPS15-3)=1 (0). Knowing that $\tilde{N}$(p1g)=4, $\tilde{N}$(p1m)=10, $\tilde{N}$(pm1)=7, $\tilde{N}$(p12)=7, $\tilde{N}$(pmg)=10, $\tilde{N}$(pmm)=16, and taking in account the exceptional cases, we can conclude that there will be the 397 (302) LPSC occuring in the 479 (372) diffeomeric types. All the non-trivial LPSC5-2 obtained, are visually interpreted by Figure 4.



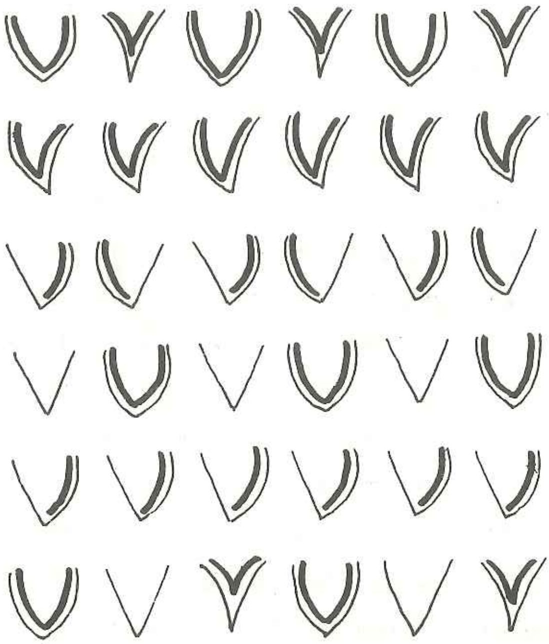


Figure 4

It is also possible to differ between LPSC with curvilinear symmetry groups $\tilde{G}^3$ derived from LPS2-2 and LPS3-2 by use of the transformations e_3 or e_3^{-1}. In that case there will be 399 (304) LPSC occuring in the 479 (372) diffeomeric types (Figure 5).

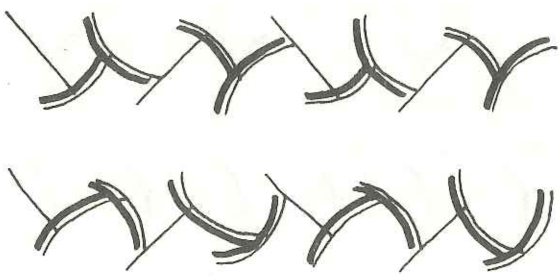


Figure 5: (a) LPSC2-2 $(p1g/p11)^3$, (b) LPSC2-2 $(p1g/p11)^{-3}$.

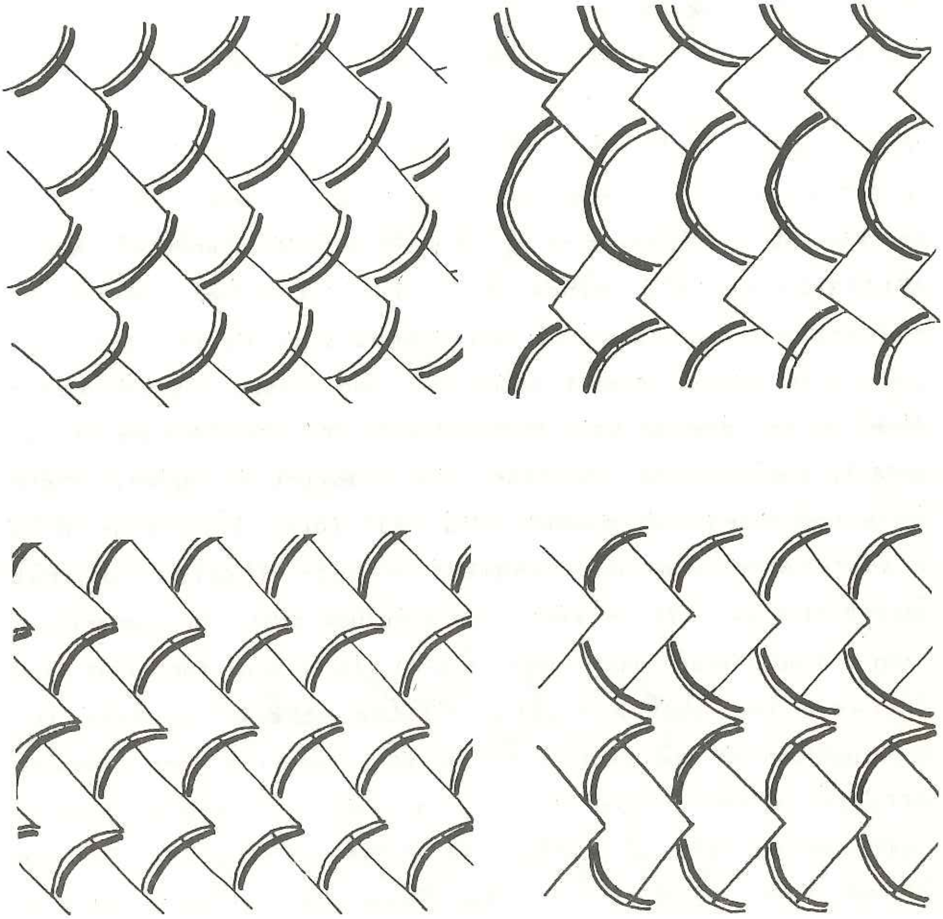
There are the 209 periodic segment patterns LPP [14, Table 7.4.1]. Because of their large number, we will restrict our discussion on the curvilinear periodic segment patterns LPPC to only few examples: the derivation of LPPC from LPP with the symmetry groups **pg**, **pm**, **cm** and **p2**, that means, from the LPP2, LPP3, LPP5, LPP5 and LPP7,LPP8. In all the non-exceptional cases the relationship:

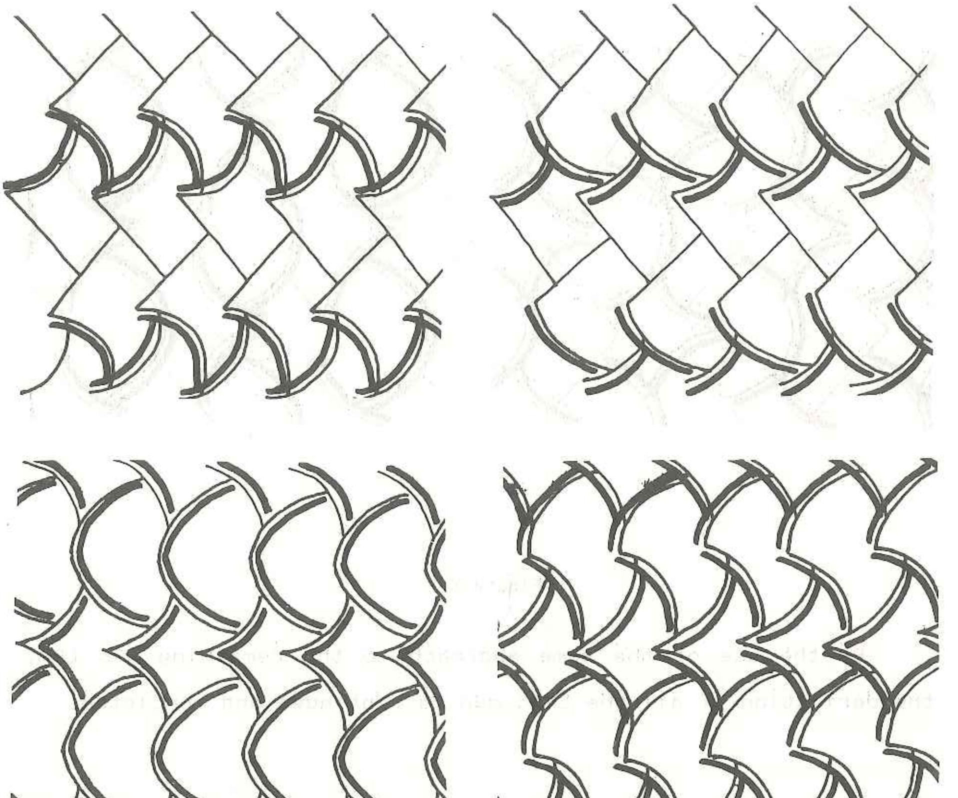
$\tilde{N}(LPP)=3+\tilde{N}(G)$ $(\tilde{N}(G))$,

holds. Hence, $\tilde{N}(LPP2-1)=11$ (8), $\tilde{N}(LPP2-2)=11$ (8), $\tilde{N}(LPP3-1)=20$ (17), $\tilde{N}(LPP5-1)=14$ (11), $\tilde{N}(LPP5-2)=14$ (11), $\tilde{N}(LPP5-3)=14$ (11), $\tilde{N}(LPP5-4)=14$ (11), $\tilde{N}(LPP7-1)=10$ (7), $\tilde{N}(LPP7-3)=10$ (7). The reasons for occurence of exceptions will be the same as in the cases of LPF and LPS already discussed. Hence, $\tilde{N}(LPP3-2)=26$ (23), since there exist the two classes of reflections belonging to the symmetry group **pm**, the ones containing common segment endpoints, and others not containing them. In the similar way, $\tilde{N}(LPP7-2)=19$ (16). In the case of the LPP8-1, there occurs the other kind of exception. Namely, there is a non-indentical symmetry (a half-turn) of the group G transforming the open segment L onto itself, so that $\tilde{N}(LPP8-1)=7$ (4). It is simply to conclude that, in comparition with the non-exceptional number $\tilde{N}(LPP)$, factors of the first kind increase the number $\tilde{N}(LPP)$ and the others decrease it. Combinations of the factors mentioned, producing the opposite effects on the number $\tilde{N}(LPP)$, are also possible. Such a combination in the case of LPP8-2 results in the $\tilde{N}(LPP8-2)=10$ (7), where occurs the compensation of these opposite effects.

There is also the possibility to distinguish LPPC with the same $\tilde{G}^3$, obtained by the use of the transformations e_3 and $e_3{}^{-1}$. In that case, there will be one more LPPC and non-trivial LPPC between the LPPC2-2, LPPC5-3 and LPPC5-4.

As the illustration, the complete survay of LPPC5-4 is given by Figure 6.





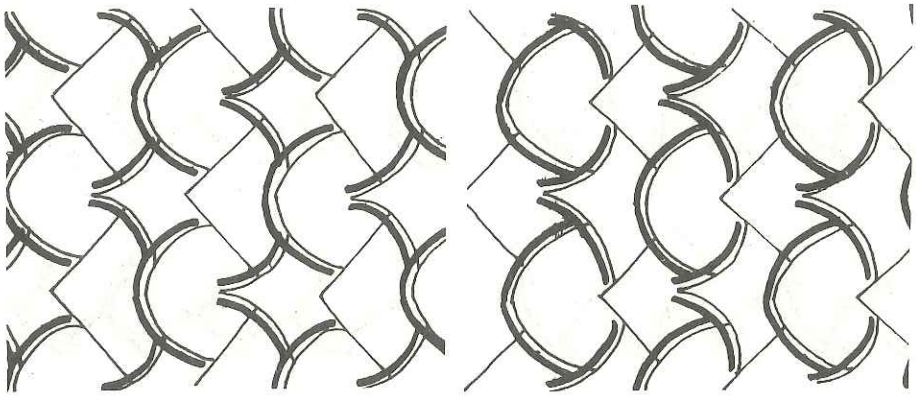


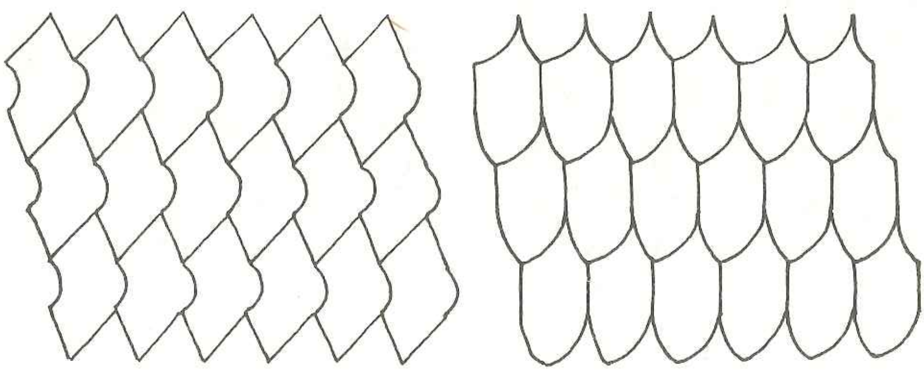
Figure 6

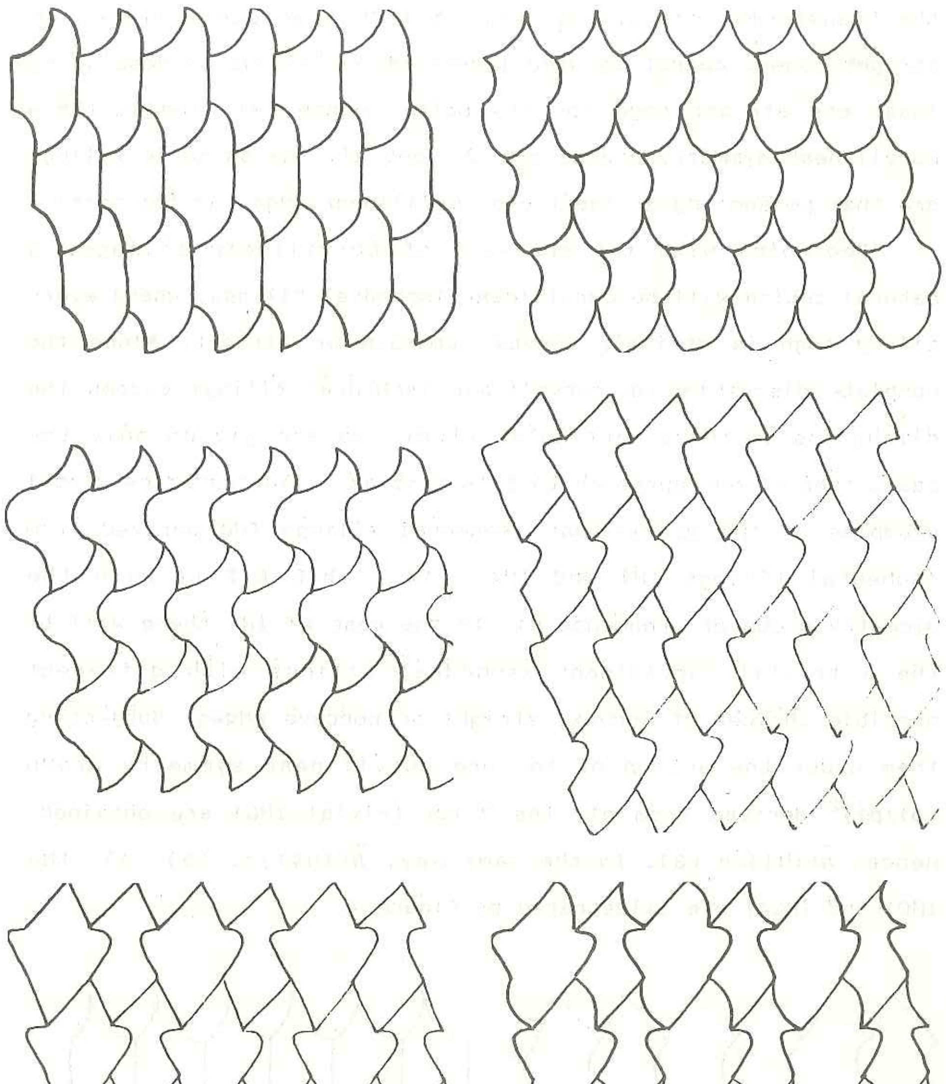
By the use of the same approach to the remaining 196 LPP, the derivation of all the LPPC can be continued and completed.

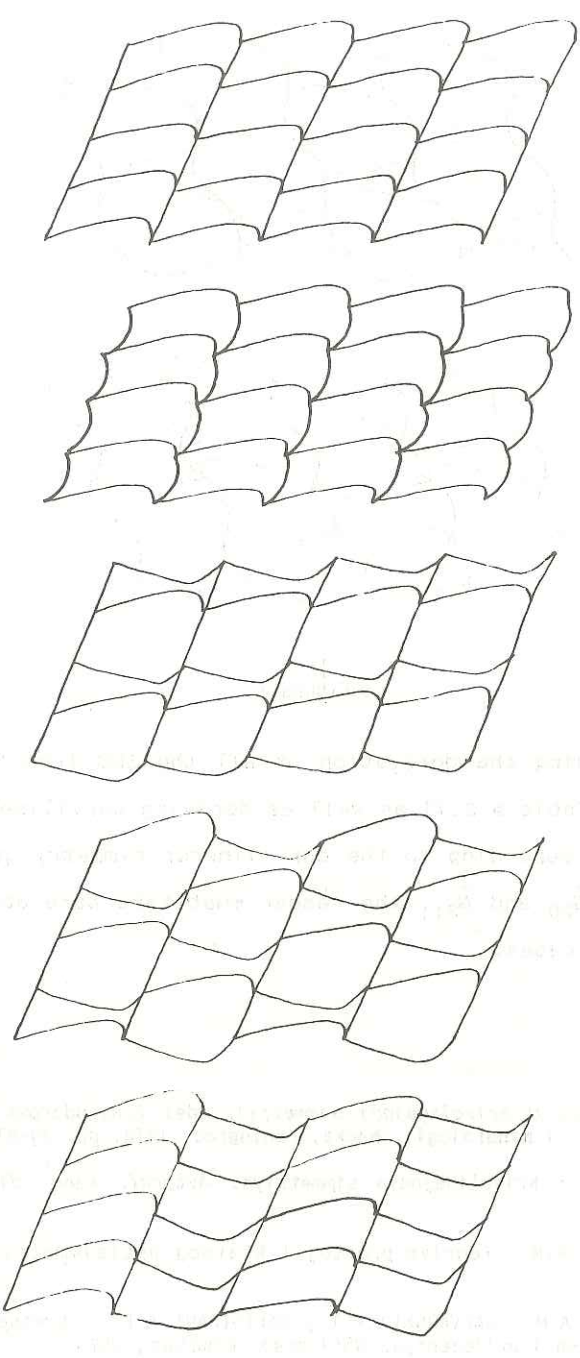
5. CURVILINEAR TILINGS

The next class of figures possesing a curvilinear symmetry group of the category $\tilde{G}_{20}$, $\tilde{G}_{21}$ or $\tilde{G}_{2}$, are curvilinear tilings. There will be one essential difference between them and curvilinear segment patterns. At curvilinear segment patterns every segment keeps its own referent system, making possible differing between convex and concave segments (Figure 1). In the case of curvilinear tilings, it is naturally to define convexity and concavity of a tiling edge in agreement with convexity and concavity of the adjacent tiles. We will discuss curvilinear tilings, with which every edge is strictly convex, stright or concave. Since one edge is the boundary of two adjacent tiles, the transformations e_1, e_3, having a contradictory effect on stright edges, cannot be used transforming tilings possessing at least one stright edge (or its part). Hence, all the suitable curvilinear symmetry groups are $\tilde{G}^2$, and all the suitable tilings are that possessing at least one curvilinear edge (or its part).

Beginning with the analysis of curvilinear tilings, a natural choice will be curvilinear isohedral tilings, where every tiling edge is strictly convex, concave or stright. Since the complete discussion on curvilinear isohedral tilings exceds the dimensions of these particular study, we are giving only the basic idea of our aprroach to this problem, illustrated by visual examples of the curvilinear isohedral tilings IHC derived from isohedral tilings IH1 and IH41 [14, Table 6.2.1] with the symmetry group of ornaments p1. In the case of IH1 there will be the 5 trivial curvilinear isohedral tilings with different possible choices of convex, stright or concave edges. Subjecting them under the action of the one curvilinear symmetry group $(p1/p1)^2$ derived from p1, the 3 non-trivial IHC1 are obtained. Hence, $\tilde{N}$(IH1)=8 (3). In the same way, $\tilde{N}$(IH41)=7 (5). All the IHC1 and IHC41 are illustrated by Figure 7.







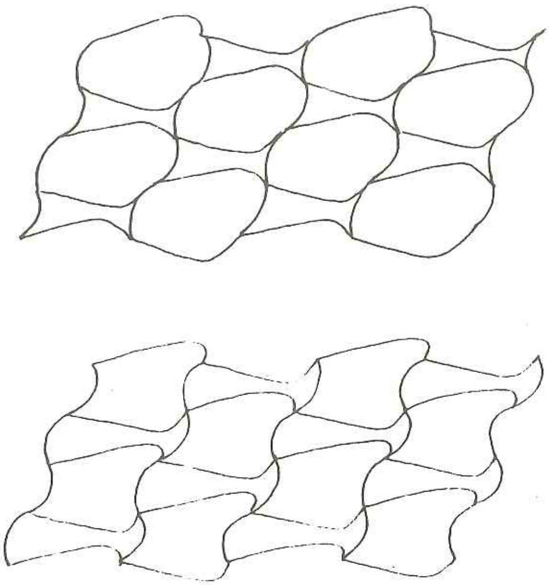


Figure 7

Continuing the derivation of all the IHC from the 93 types of IH [14, Table 6.2.1] as well as deriving curvilinear isohedral tilings corresponding to the curvilinerar symmetry groups of the categories $\tilde{G}_{20}$ and $\tilde{G}_{21}$, the reader must take care about possible exceptional cases.

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NEKE NAPOMENE O KRIVOLINIJSKOJ SIMETRIJI

Uspostavljene su veze izmedju grupa antisimetrije, (3)-simetrije, (32)-simetrije i grupa krivolinijske simetrije koje je definisao P.L.Dubov. Izvedene su grupe krivolinijske simetrije kategorija $\tilde{G}_{20}$, $\tilde{G}_{21}$, $\tilde{G}_{2}$. Primenjujuči, redom, grupe krivilinijske simetrije kategorija $\tilde{G}_{20}$, $\tilde{G}_{21}$ na rozetalne (konačne) segmentne motive LPF i bordurne segmentne motive LPS odredjen je broj odgovarajučih krivolinijskih rozetalnih (konačnih) segmentnih motiva LPFC i krivolinijskih bordurnih segmentnih motiva LPSC i date njihove vizuelne interpretacije.

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