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ON LINEAR TOPOLOGICAL AND TOPOLOGICAL CLASSIFICATION OF SPACES $C_p(X)$

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Abstract. Let $C_p(X)$ denote the space of all real-valued continuous functions on a space X in the topology of pointwise convergence (see [3],[4]). We investigate when $C_p(X)$ and $C_p(Y)$ are linearly homeomorphic or just homeomorphic.

1. Notations and terminology

In what follows X,Y,Z,X',Y' are always understood to be non-empty Tychonoff spaces, I is the unit segment $[0,1]$ with the usual topology, N^+ is the set of all positive integers, $N = N^+ \cup \{0\}$, K_o Cantor's perfect set, E^n n-dimensional euclidean space, $S = \{0, 1/n : n \in N^+\}$ is the simplest infinite compact space - converging sequence, $\oplus$ stands for the operation of the free topological sum.

If A is a closed subset of X, then X/A is the quotient space in the category of Tychonoff spaces obtained from X by identifying A to a point. Thus X/A is endowed with the strongest Tychonoff topology with respect to which the identification mapping $X \to X/A$ is continuous.

We also use the following notations: $C_p(X;A) = \{f \in C_p(X) : f|A \equiv 0\}$ and $C_p(X/A)_o = C_p(X/A;\{A\}) = \{f \in C_p(X/A) : f(A)=0\}$. Whenever A denotes a set, this set is assumed to be non-empty.

If L and M are linearly homeomorphic linear topological spaces, we write: $L \simeq M$. If X and Y are homeomorphic topological spaces we write $X \approx Y$. Topological spaces X and Y are said to be l-equivalent (notation: $X \overset{l}{\sim} Y$) if $C_p(X)$ is linearly homeomorphic to $C_p(Y)$. If $C_p(X)$ is just homeomorphic to $C_p(Y)$ we say X and Y are t-equivalent (notation: $X \overset{t}{\sim} Y$). Adding one new isolated point to a space X we obtain a space denoted by X^+.

If Y is a locally compact non-compact space we denote by αY the Alexandroff one-point compactification of Y. If Y is compact we put $\alpha Y = Y^+$.

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A subspace $A\subset X$ is said to be t-embedded (l-embedded) in the space X if there exists a continuous (a linear continuous) mapping $\varphi: C_p(A)\to C_p(X)$ such that $g=\varphi(g)|A$ for every $g\in C_p(A)$. Such mapping φ is called a continuous extender (a linear continuous extender) from A to X. It is not difficult to show that if A is t-embedded into X then A is closed in X.

2. Extenders and l- and t-equivalences

We show how extenders can be used to construct non-trivial pairs of t-equivalent (of l-equivalent) spaces and derive some corollaries.

PROPOSITION 2.1. If A is t-embedded in X, then $C_p(X)\approx C_p(X/A)_o\times C_p(A)$ and $X^+\overset{t}{\sim} X/A\oplus A$.

PROOF. Fix a continuous extender $\varphi: C_p(A)\to C_p(X)$ and define mappings $\psi: C_p(X)\to C_p(A)\times C_p(X;A)$ and $\psi': C_p(A)\times C_p(X;A)\to C_p(X)$ by the following rules: $\psi(f)=(f|A, f-\varphi(f|A))$ and $\psi'((g,h)=\varphi(g)+h$. The mappings ψ and ψ' are obviously continuous and ψ' is the inverse to ψ. Hence $C_p(X)\approx C_p(A)\times C_p(X;A)$. Clearly $C_p(X;A)\simeq C_p(X/A)_o$. It follows that $C_p(X)\approx C_p(X/A)_o\times C_p(A)$. Taking into account that $C_p(X/A)\simeq C_p(X/A)_o\times E^1$ and $C_p(X^+)=C_p(X)\times E^1$ we obtain: $C_p(X^+)\approx C_p(X/A)\times C_p(A)$, which implies that X^+ is t-equivalent to $X/A\oplus A$.

In the same way we get

PROPOSITION 2.2. If A is l-embedded in X then $C_p(X)\simeq C_p(X/A)_o\times C_p(A)$ and $X^+\overset{l}{\sim} X/A\oplus A$.

PROPOSITION 2.3. If A is t-embedded (l-embedded) in X and $A\oplus A\overset{t}{\sim} A$ ($A\oplus A\overset{l}{\sim} A$) then $X\overset{t}{\sim} X\oplus A$ (then $X\overset{l}{\sim} X\oplus A$).

PROOF. $C_p(X)\approx C_p(X/A)_o\times C_p(A)\approx C_p(X/A)_o\times C_p(A)\times C_p(A)\approx C_p(X)\times C_p(A)$. Thus $X\overset{t}{\sim} X\oplus A$.

EXAMPLE 2.1. $S\oplus S\overset{l}{\sim} S$. Indeed, if we take A to be the set consisting of the two non-isolated points in $S\oplus S$, then $(S\oplus S)/A\oplus A$ is homeomorphic to S. Also $(S\oplus S)^+$ is homeomorphic to S. Hence $S\oplus S\overset{l}{\sim} S$.

PROPOSITION 2.4.([9]). If X contains a non-trivial converging sequence S, then $X\overset{l}{\sim} X\oplus S$ and $X^+\overset{l}{\sim} X$.

PROOF. Each metrizable compact subspace of any space X is l-embedded in X (this follows easily from results in [7]). Taking into account example 2.1 we get: $X\overset{l}{\sim} X\oplus S$ and $X^+\overset{l}{\sim}(X\oplus S)^+\overset{l}{\sim} X\oplus S^+\overset{l}{\sim} X\oplus S\overset{l}{\sim} X$.

From Propositions 2.1 and 2.4 we derive

THEOREM 2.1. If X contains a non-trivial converging sequence and A is t-embedded (l-embedded) in X then $X \overset{t}{\sim} X/A \oplus A$ (then $X \overset{l}{\sim} X/A \oplus A$).

PROBLEM 2.1. Can Theorem 2.1 be extended to all infinite spaces X?

It is well known (J.Dugundji [7]) that every closed subspace of any metrizable space X is l-embedded in X. From Theorem 2.1 we get now

THEOREM 2.2. For every infinite metrizable space X and closed subspace $A \subset X$, $X \overset{l}{\sim} X/A \oplus A$.

REMARK 2.1. Proposition 2.2 and Theorem 2.2 provide a broad generalization of Okunev's construction in which retracts were used (see [10], [4]).

COROLLARY 2.1. If A is a closed subspace of a metrizable space X and $A \oplus A \overset{l}{\sim} A$ then $X \oplus A \overset{l}{\sim} X$ (if $A \oplus A \overset{t}{\sim} A$ then $X \oplus A \overset{t}{\sim} X$).

It is also true that every closed subspace of any stratifiable space X is l-embedded in X [6]. But there is no reason why a non-discrete stratifiable space should contain a non-trivial converging sequence. So that applying Proposition 2.2 we only get

COROLLARY 2.2. If A is a closed subspace of a stratifiable space X then $X^+ \overset{l}{\sim} X/A \oplus A$.

PROBLEM 2.2. Is it true that $X^+ \overset{l}{\sim} X$ for every infinite stratifiable space X ?

By Proposition 2.4, the answer to Problem 2.2 is "yes" for all infinite stratifiable k-spaces.

Proposition 2.4 can be obviously generalized into the following assertion

PROPOSITION 2.5. If X contains a non-trivial converging sequence then for every finite discrete space Y we have: $X \overset{l}{\sim} X \oplus Y$.

EXAMPLE 2.2. Let P_n be the n-dimensional euclidean sphere, where $n \geqslant 1$. Fix a point $p^* \in P_n$, take $k \in N^+$, endow the finite set $\{1,\ldots,k\}$ with discrete topology and in the product space $X = P_n \times \{1,\ldots,k\}$ identify to a point the set $A = \{(p^*,i) : i=1,\ldots,k\}$. The quotient space X/A will be denoted by P(n,k). By Theorem 2.1 we have: $X \overset{l}{\sim} X/A \oplus A$, i.e. $X \overset{l}{\sim} P(n,k) \oplus A$. Clearly, the space P(n,k) contains a converging sequence. Also, the space A is finite and discrete. Applying Proposition 2.5, we conclude that $P(n,k) \oplus A \overset{l}{\sim} P(n,k)$ and $X \overset{l}{\sim} P(n,k)$.

EXAMPLE 2.3. Let A be the equator of the sphere P_n. Then A is homeomorphic to P_{n-1} and the quotient space P_n/A is homeomorphic to $P(n,2)$. Hence $P_n \overset{l}{\sim} P(n,2) \oplus A = P(n,2) \oplus P_{n-1}$. By Example 2.2, $P(n,2) \overset{l}{\sim} P_n \oplus P_n$. Thus, $P_n \overset{l}{\sim} P_n \oplus P_n \oplus P_{n-1}$. If n=1, then the set $P_{n-1} = P_o$ consists of two points. It follows that $P_1 \overset{l}{\sim} P_1 \oplus P_1$. Arguing by induction let us assume that $P_{n-1} \oplus P_{n-1} \overset{l}{\sim} P_{n-1}$ for some n. Then $P_n \oplus P_{n-1} \overset{l}{\sim} P_n$ by Corollary 2.1 as P_{n-1} is homeomorphic to a subspace of P_n. Hence $P_n \overset{l}{\sim} P_n \oplus P_n \oplus P_{n-1} \overset{l}{\sim} P_n \oplus P_n$ for all $n \in N^+$. The space P(n,2) also contains a topological copy of the space P_{n-1}. From $P_n \overset{l}{\sim} P(n,2) \oplus P_{n-1}$ it follows now by means of Corollary 2.1 that $P_n \overset{l}{\sim} P(n,2)$. From $P_n \overset{l}{\sim} P_n \oplus P_n$ we get by induction : $P_n \overset{l}{\sim} P_n \times \{1,\dots,k\}$ for all $k \in N^+$. Taking into account Example 2.2 we conclude that P(n,k) is l-equivalent to P_n for all $n,k \in N^+$.

EXAMPLE 2.4. Let K be a finite complex of dimension $n \geq 1$ and let $X = |K|$ be the corresponding space. We shall show that $X \overset{l}{\sim} P_n$. Denote by K' the subcomplex of K formed by all simplexes belonging to K the dimension of which is less then n. Put $A = |K'|$. Then $X \overset{l}{\sim} X/A \oplus A$, where X/A is obviously homeomorphic to the space $P_{n,k}$ (see Example 2.3), where k is the number of n-dimensional simplexes in K. By Example 2.3, $P(n,k) \overset{l}{\sim} P_n$ and $X \overset{l}{\sim} P_n \oplus A$. As $\dim A = \dim K' = n-1 < n$ we can argue by induction and assume that it is already established that $A \overset{l}{\sim} P_{n-1}$ if $n \geq 2$. Obviously if n=1 then A is finite and $X \overset{l}{\sim} P_n$ by Proposition 2.5. For $n \geq 2$ we have: $X \overset{l}{\sim} P_n \oplus A \overset{l}{\sim} P_n \oplus P_{n-1} \overset{l}{\sim} P_n$ by Corollary 2.1 — the last equivalence was already established in Example 2.3. The result established in this example was obtained by D. Pavlovskij by a more complicated argument (see [11], [12]).

If X is a compact space and Y is an open subspace of X then $X/(X \setminus Y)$ is αY — the one point compactification of Y. From Theorem 2.2 we now get more explicite result:

THEOREM 2.3. If X is an infinite metrizable compact space and Y is open in X, then $X \overset{l}{\sim} (X \setminus Y) \oplus \alpha Y$.

COROLLARY 2.3. Let X_1 and X_2 be metrizable compact spaces and Y_1, Y_2 open subspaces of X_1 and X_2, respectively, such that Y_1 is homeomorphic to Y_2 and $X_1 \setminus Y_1$ is l-equivalent to $X_2 \setminus Y_2$. Then $X_1 \overset{l}{\sim} X_2$.

The following assertion is technically quite important.

THEOREM 2.4. If compact spaces X and Y are l-equivalent then for any space Z we have: $X \times Z \overset{l}{\sim} Y \times Z$.

An outline of the **proof.** Fix a linear homeomorphism $\varphi : C_p(X) \to C_p(Y)$. Take any $f \in C_p(X \times Z)$. For arbitrary $z \in Z$ define $f_z \in C_p(X)$ by the rule: $f_z(z) = f(x,z)$

for all $x \in X$. Put $g_z = \varphi(f_z) \in C_p(Y)$ and $g(y,z) = g_z(y)$ for all $y \in Y$. Applying some basic results from [1] we can show that the function g is continuous. A linear homeomorphism $\Psi : C_p(X \times Z) \to C_p(Y \times Z)$ can be now defined by the rule: $\Psi(f) = g$.

This argument doesn't work for t-equivalence (we cannot refer to [1] in that case). The following question arises:

PROBLEM 2.3. Suppose that X and Y are t-equivalent compact spaces. Is it true then that $X \times Z \overset{t}{\sim} Y \times Z$ for every space Z ?

3. <u>l-equivalence and stability concepts</u>

Now we shall introduce some strong stability concepts related to l-equivalence.

Let us call a space X **l-additive** if $X \oplus X \overset{l}{\sim} X$ and $X^+ \overset{l}{\sim} X$. Let us say that X is **strongly l-additive** if $X \oplus A \overset{l}{\sim} X$ for every l-embedded subspace A of X. Similarly **t-additive** and **strongly t-additive spaces** are defined. It is clear that every strongly l-additive space is l-additive.

Let T be a space. A space X will be called **T-stable** if X is l-equivalent to $X \times T$. Of particular interest to us will be S-stable spaces and N-stable spaces (where S is the converging sequence with the limit and N is the discrete space of natural numbers). The importance of these concepts is based on the following facts.

PROPOSITION 3.1. If a compact space X is S-stable then it is strongly l-additive.

PROOF. Clearly $X^+ \overset{l}{\sim} (X \times S)^+ \overset{l}{\sim} X \times S \overset{l}{\sim} X$. Let A be l-embedded in X. Then $X \overset{l}{\sim} X^+ \overset{l}{\sim} X/A \oplus A$. By Theorem 2.4 we have:

$$X \times S \overset{l}{\sim} (X/A \oplus A) \times S = (X/A \times S) \oplus (A \times S) = (X/A \times S) \oplus (A \times S) \oplus A = ((X/A \oplus A) \times S) \oplus A \overset{l}{\sim} (X \times S) \oplus A \overset{l}{\sim} X \oplus A.$$

From $X \overset{l}{\sim} X \times S$ it follows now that $X \overset{l}{\sim} X \oplus A$.

The following assertion is obvious.

PROPOSITION 3.2. If $X \overset{l}{\sim} Y$ then $X \times N \overset{l}{\sim} Y \times N$.

Arguing as in the proof of Proposition 3.1 and applying Proposition 3.2 we obtain

PROPOSITION 3.3. Every N-stable space is strongly l-additive.

PROPOSITION 3.4. For every space X the space $X \times N$ is N-stable.

PROOF. Obviously $N \times N$ is homeomorphic to N; hence $(X\times N)\times N$ is homemorphic to $X\times N$ and $X\times N \overset{l}{\sim} (X\times N)\times N$.

PROPOSITION 3.5. For every space X the space $X\times S$ is S-stable.

PROOF. To prove that $(X\times S)\times S = X\times(S\times S)$ is l-equivalent to $X\times S$ we need only to show that $S \overset{l}{\sim} S\times S$ — the rest will follow from Theorem 2.4. By Theorem 2.2, $S\times S \overset{l}{\sim} (S\times S)/A \oplus A$, where $A = (S\times S)\setminus\{(1/n,1/m):n,m\in N^+\}$. Clearly, both spaces $(S\times S)/A$ and A are homeomorphic to S. Hence $S\times S \overset{l}{\sim} S \oplus S \overset{l}{\sim} S$.

Let us say that a space Y is l-dominated (t-dominated) by a space X if there exists a subspace $X_1 \subset X$ which is l-embedded (t-embedded) in X and l-equivalent (t-equivalent) to Y. If Y is l-dominated by X and X is l-dominated by Y, the spaces X and Y are called l-equidimensional.

One can find many examples of S-stable spaces with the help of the following assrtion.

PROPOSITION 3.6. If X is compact and the space $X\times S$ is l-dominated by the space X, then $X \overset{l}{\sim} X\times S$, i.e. the space X is S-stable.

To prove this we need one of our basic results:

THEOREM 3.1. If X and Y are l-equidimensional spaces and at least one of them is strongly l-additive, then X and Y are l-equivalent.

PROOF. We fix $X_1\subset X$ which is l-equivalent to Y and l-embedded in X and $Y_1\subset Y$ which is l-equivalent to X and l-embedded in Y. Assume that Y is strongly l-additive. Then $Y \oplus Y \overset{l}{\sim} X_1 \oplus X_1$ and it follows from Proposition 2.3 that $X \overset{l}{\sim} X \oplus X_1 \overset{l}{\sim} X \oplus Y$. On the other hand, $Y \overset{l}{\sim} Y \oplus Y_1 \overset{l}{\sim} Y \oplus X$ as Y is strongly l-additive. Hence $X \overset{l}{\sim} Y$.

COROLLARY 3.1. If X and Y are compact l-equidimensional spaces and at least one of them is S-stable, then $X \overset{l}{\sim} Y$.

COROLLARY 3.2. If X and Y are l-equidimensional spaces and at least one of them is N-stable, then $X \overset{l}{\sim} Y$.

PROOF of Proposition 3.6. Obviously X is l-dominated by $X\times S$. By the assumptions, $X\times S$ is also l-dominated by X. The space $X\times S$ is S-stable by Proposition 3.5. It remains to apply Corollary 3.1.

The same argument together with a reference to Proposition 3.4 constitutes the proof of

PROPOSITION 3.7. If X is a space such that the space $X \times N$ is l-dominated by the space X, then $X \overset{l}{\sim} X \times N$, i.e. X is N-stable.

Proposition 3.7 and Proposition 3.6 are quite instrumental in finding strongly l-additive spaces.

REMARK 3.1. A non-empty compact space X is never N-stable. Indeed, $X \times N$ is not compact and it is known [1], [4] that a compact space and a non-compact space cannot be l-equivalent. Thus the concept of N-stability is of no help when we are looking for strongly l-additive compact spaces. We have to rely here on the less elementary concept of S-stability.

EXAMPLE 3.1. The space S is not N-stable. Indded, it is compact (see the argument above).

EXAMPLE 3.2. The discrete space N is not S-stable. Indded, by a result of V.V. Tkačuk [13], [2] , if a discrete space is t-equivalent to a space Y, then Y is also discrete. As the space $N \times S$ is not discrete we conclude that $N \times S$ is not t-equivalent to N.

EXAMPLE 3.3. The Hilbert cube $I^{\aleph_0}$ is S-stable: $I^{\aleph_0} \times S$ can be homeomorphically embedded in $I^{\aleph_0}$ (see [8]). It follows that $I^{\aleph_0}$ is strongly l-additive.

COROLLARY 3.3. (V.Valov [14]) If a space Y which is metrizable compact contains a subspace which is l-equivalent to $I^{\aleph_0}$, then $Y \overset{l}{\sim} I^{\aleph_0}$.

EXAMPLE 3.4. The Cantor perfect set K_0 is S-stable: $K_0 \times S$ is homeomorphic to a subspace of K_0 (see [8]).

Now we can easily prove the following result from [5] :

THEOREM 3.2. Every two uncountable zero-dimensional compact metrizable spaces X and Y are l-equivalent.

PROOF. The space X contains a topological copy of the Cantor perfect set K_0 and K_0 contains a topological copy of the space X. Thus X and K_0 are l-equidimensional spaces. As K_0 is S-stable it follows from Corollary 3.1 that $X \overset{l}{\sim} K_0$. The same is true for Y. Hence $X \overset{l}{\sim} Y$.

Theorem 3.2 cannot be extended to compact metrizable spaces of positive dimension. But we have the following result:

THEOREM 3.3. If X is uncountable locally compact separable metrizable space, then for every zero-dimensional metrizable compact space Y we have $X \oplus Y \overset{l}{\sim} X$.

PROOF. The space X contains a topological copy of K_0 [8]. Thus $X \overset{1}{\sim} X \oplus K_0$ and $X \oplus Y \overset{1}{\sim} X \oplus K_0 \oplus Y$. By Theorem 3.2, $K_0 \oplus Y \overset{1}{\sim} K_0$. It follows that $X \oplus (K_0 \oplus Y) \overset{1}{\sim} X \oplus K_0 \overset{1}{\sim} X$. Hence $X \overset{1}{\sim} X \oplus Y$.

EXAMPLE 3.5. The space J of irrational numbers with the usual topology is N-stable: the space $J \times N$ is homeomorphic to a closed subspace of the space J (see [8]). It remains to apply Proposition 3.4 and Corollary 3.2.

THEOREM 3.4. If X is a zero-dimensional complete separable metric space which is not σ-compact, then $X \overset{1}{\sim} J$.

PROOF. The space X contains a closed subspace homeomorphic to J (see [8]). On the other hand, X is homeomorphic to a closed subspace of J. Thus X and J are 1-equidimensional. The space J being N-stable it follows that $X \overset{1}{\sim} J$.

EXAMPLE 3.6. The space Q of rational numbers with usual topology is obviously N-stable and hence this space is strongly 1-additive. Using this fact we can prove

THEOREM 3.5. A closed subspace X of the space Q is 1-equivalent to Q if and only if X contains a closed subspace homeomorphic to Q.

Let $\mathcal{P}$ be a class of topological spaces. A space X will be called $(\mathcal{P},1)$-universal, if $X \in \mathcal{P}$ and every $Y \in \mathcal{P}$ is 1-equivalent to a subspace $X_1 \subset X$ which is 1-embedded in X.

PROPOSITION 3.8. Let $\mathcal{P}$ be a class of spaces such that $X \times N \in \mathcal{P}$ for every $X \in \mathcal{P}$. Then any two $(\mathcal{P},1)$-universal spaces X_o and Y_o are 1-equivalent.

PROOF. As X_o is $(\mathcal{P},1)$-universal, the space $X_o \times N$ is 1-equivalent to a subspace of X_o which is 1-embedded in X_o. It follows from Proposition 3.7 that X_o is N-stable. Being $(\mathcal{P},1)$-universal the spaces X_o and Y_o are obviously 1-equidimensional. Corollary 3. 2 now implies that $X_o \overset{1}{\sim} Y_o$.

PROPOSITION 3.9. Let $\mathcal{P}$ be a class of compact spaces such that $X \times S \in \mathcal{P}$ for every $X \in \mathcal{P}$. Then any two $(\mathcal{P},1)$-universal spaces X_o and Y_o are 1-equivalent.

PROOF. We argue in the same way as in the proof of Proposition 3.8: the only difference is that instead of Corollary 3.2 we invoke Corollary 3.1.

THEOREM 3.6. Let $n \in N$ and let $\mathcal{P}$ be the class of all metrizable compact spaces of dimension $\leq n$. Then any two $(\mathcal{P},1)$-universal spaces X_o and Y_o are 1-equivalent.

PROOF. This follows from Proposition 3.9.

There are many other classes of spaces satisfying the assumptions in Propositions 3.8 or 3.9.

In conclusion let us state three more results the proofs of which will be published elsewhere.

THEOREM 3.7. Every Tychonoff cube I^{τ}, where $\tau \geq \aleph_0$, is S-stable and hence strongly l-additive space.

THEOREM 3.8. For every two non-empty open subspaces X and Y of the euclidean space E^n, where $n \in N^+$, the one-point compactifications αX and αY are l-equivalent.

THEOREM 3.9. If X is the space of a countable CW-complex K such that $\dim X = n \in N^+$ and the set of n-dimensional simplexes of K is infinite, then X is l-equivalent to the euclidean space E^n.

This result can be extebded to polytopes.

I would like to mention that some results of this paper related to S-stability and l-equidimensionality are close to results of V.Valov in [14].

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O LINEARNO TOPOLOŠKOJ I TOPOLOŠKOJ KLASIFIKACIJI PROSTORA $C_p(X)$

Neka $C_p(X)$ označava prostor svih neprekidnih realnoznačnih funkcija definisanih na datom topološkom prostoru X snabdeven topologijom obične konvergencije. U radu se izučava kada su prostori $C_p(X)$ i $C_p(Y)$ linearno homeomorfni — tada se kaže da su X i Y l-ekvivalentni — ili samo topološki homeomorfni — tada se kaže da su X i Y t-ekvivalentni. Daje se niz rezultata i primera u vezi sa ovim ekvilalentnostima. Pokazuje se kako ekstendori mogu biti korišćeni za konstruisanje netrivijalnih parova t-ekvivalentnih i l-ekvivalentnih prostora. Ispituje se i veza izmedu l-ekvivalentnosti i stabilnosti u odnosu na proizvod.

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