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Bounds on graviton mass and constraining Yukawa-like gravitational potential from planetary motion in the Solar System

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Abstract. In this study we used the observed perihelion precession of planetary orbits in the Solar System in order to constrain theory of Yukawa-type gravity, and to bound mass of graviton. For that purpose we assumed that the precession angles of the planets in Yukawa-type gravity should be equal to their observed values, as well as to the corresponding predictions of General Relativity (GR). Starting from these requests we constrain Yukawa-like gravitational potential. The obtained results showed that our estimates for the range of Yukawa interaction Λ at the Solar System scales are in good agreement with recent experimental constrains. Assuming that the estimated value of the range of Λ corresponds to the Compton wavelength of graviton λ_g, we then estimate the upper bound for its mass m_g. We found that these estimates were in agreement with our previous results obtained from the observed stellar orbits around the Galactic Center (GC).

1. Introduction

Numbers of modified gravity theories, as possible extensions of Einstein's theory of gravity, have been proposed to explain cosmological and astrophysical data without introducing dark energy and dark matter [1, 2]. These theories have to explain astrophysical and cosmological observations at different scales: Solar system, binary pulsars, spiral and elliptical galaxies, clusters of galaxies and cosmological scales [2–13]. Theories of "massive gravity" have also attracted a lot of attention (see e.g.[14–19] and references therein). In these theories gravity is propagated by a massive field, i.e. by a graviton with some small, nonzero mass m_g [16, 17]. Graviton is supposed to be a carrier of the gravitational interaction, to be spin-2 (tensor) boson and electrically uncharged. According to GR, graviton is massless since, it travels along null geodesics at the speed of light c, but according to "massive gravity" theories, the graviton has small, nonzero mass m_g [16, 17]. Fierz and Pauli [20] were first who introduced this approach in 1939. After that, Boulware and Deser [21] found a presence of ghosts in massive gravity theories, and these theories were considered as

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non-realistic. Latter, a ghost-free massive gravity theory were derived [19], and an interest to these theories was significantly increased again. One of important feature of these theories of massive gravity is their ability to provide a possible explanation for the accelerated expansion of the Universe without dark energy (DE) hypothesis. In these theories, the velocity v_g of gravitational waves (gravitons) depends on their frequency f through relation: $v_g^2/c^2 = 1 - c^2/(f\lambda_g)^2$. Also, the effective gravitational potential has correction of Yukawa form: $\propto r^{-1}\exp(-r/\lambda_g)$, depending on the Compton wavelength of graviton in the following way: $\lambda_g = h/(m_g\, c)$ [22, 23].

The experimental limits for constraining the mass of graviton are given in [24] and references therein. The estimate obtained by LIGO Scientific and Virgo Collaborations, from their analysis of the first Gravitation Wave (GW) signal GW150914, is $m_g \leq 1.2 \times 10^{-22}$ eV$/c^2$ [25]. Observations of gravitational waves by Advanced LIGO and Advanced Virgo detectors update significantly the graviton mass bound to $m_g \leq 1.27 \times 10^{-23}$ eV$/c^2$ [26]. Analysis of the observed stellar orbits around the central SMBH of the Milky Way in the frame of Yukawa gravity, was used for constraining the mass of graviton [27–30].

In this study we use the observed additional perihelion precession of the planets [31–36], which is available for six planets up to Saturn, as well as for Pluto dwarf planet. In Section 2 we presented basic properties of Yukawa-like corrections. We assumed several values of universal constant in order to estimate the range of interaction Λ. In case $\delta = 1$ we obtain upper bound for mass of graviton. These results are presented in Section 3. Also, in this section we provide comparison between our numerical results with astronomical observations. Section 4 is devoted to concluding remarks.

2. Yukawa-like gravitation potential

Yukawa-like gravitation potentials differ from the Newtonian gravitational potential due to the presence of decreasing exponential terms [27–30, 32, 37–46]. In the case of the short range parameters of Yukawa gravity potential, reviewed experiments and constraints on the Yukawa term are given in paper by Adelberger et al. [47] and references therein. In the case of the longer range parameters, constraints are given for clusters of galaxies [48, 49] and for rotation curves of spiral galaxies [39]. Also, other investigations of long-range Yukawa-like modifications of gravity can be found in [38, 50–55] and references therein. The state-of-art of Yukawa-like potentials is given in Table 1 of Ref. [56] by Capozziello et al. The authors reported various Extended Theories of Gravity where Yukawa-like corrections in the post-Newtonian limit are the general feature. More details of these results can be found in the following references [57–61]. Yukawa like gravitation potential can be derived in the frame of $f(R)$ theories of gravity. Action of $f(R)$ gravity with a Yukawa correction is given by [62]:

$$\mathcal{S} = \int d^4x\,\sqrt{-g}\,[f(R) + \mathcal{X}\mathcal{L}_m]\,, \quad \mathcal{X} = \frac{16\pi G}{c^4}, \tag{1}$$

where f is a generic function of Ricci scalar curvature R and $\mathcal{X}$ is the coupling constant. The resulting 4$^{\text{th}}$-order field equations are:

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - f'(R)_{;\mu\nu} + g_{\mu\nu}\Box f'(R) = \frac{\mathcal{X}}{2}T_{\mu\nu}, \tag{2}$$

which trace is given by:

$$3\Box f'(R) + f'(R)R - 2f(R) = \frac{\mathcal{X}}{2}T. \tag{3}$$

Yukawa-like corrections in the weak field limit in the case of analytic Taylor expandable $f(R)$ functions can be obtained with respect to the value $R = 0$ (i.e. around the Minkowskian background):

$$f(R) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}R^n = f_0 + f_1 R + \frac{f_2}{2}R^2 + \ldots \tag{4}$$

If we adopt the spherical symmetry, metric of $f(R)$ gravity in the Newtonian limit can be given by following equation [63]:

$$ds^2 = \left[1 + \frac{2\Phi(r)}{c^2}\right] c^2 dt^2 - \left[1 - \frac{2\Psi(r)}{c^2}\right] dr^2 - r^2 d\Omega^2, \tag{5}$$

where $\Phi(r)$ and $\Psi(r)$ are two potentials. The gravitational potential $\Phi(r)$ in the weak field limit from g_{00} component of the metric tensor has a Yukawa-like nonlinear correction [62, 63]:

$$\Phi(r) = \Phi_Y(r) = -\frac{GM}{(1+\delta)r}\left(1 + \delta e^{-\frac{r}{\Lambda}}\right), \tag{6}$$

where parameter $\Lambda^2 = -f_1/f_2$ is the range of interaction and parameter $\delta = f_1 - 1$ is a strength of interaction. The second potential is given by:

$$\Psi(r) = \frac{GM}{(1+\delta)r}\left[\left(1 + \frac{r}{\Lambda}\right)\delta e^{-\frac{r}{\Lambda}} - 1\right]. \tag{7}$$

A general expression for apocenter shifts for Newtonian potential and small perturbing potential are given in the Landau & Lifshitz (L & L) textbook [64] and in paper [65].

Orbital precession $\Delta\varphi$ per orbital period, induced by small perturbations to the Newtonian gravitational potential $\Phi_N(r) = -\frac{GM}{r}$, could be evaluated as [65]:

$$\Delta\varphi^{rad} = \frac{-2L}{GMe^2} \int_{-1}^{1} \frac{z \cdot dz}{\sqrt{1-z^2}} \frac{dV(z)}{dz}, \tag{8}$$

while in the textbook [64] it was given in the form

$$\Delta\varphi^{rad} = \frac{2}{GMe} \int_{0}^{\pi} \cos\varphi r^2 \frac{dV(r)}{dr} d\varphi, \tag{9}$$

where $V(z)$ is the perturbing potential, r is related to z via: $r = \frac{L}{1+ez}$ in Eq. (8) (and $r = \frac{L}{1+e\cos\varphi}$ in Eq. (9)), and L being the semilatus rectum of the orbital ellipse with semi-major axis a and eccentricity e:

$$L = a\left(1 - e^2\right), \tag{10}$$

In this paper we want to compare the orbital precession of Solar System planets in General Relativity (GR) and Yukawa gravity.

2.1. *Orbital precession in General Relativity and in Yukawa gravity*

In the case of GR the well known expression for Schwarzschild precession [23, 34, 67] is given by the following relation:

$$\Delta\varphi^{rad}_{GR} \approx \frac{6\pi GM}{c^2 a(1-e^2)}. \tag{11}$$

In case of Yukawa gravity we assumed the gravitational potential given by Eq. (6) [27].

Yukawa gravity induces a perturbation to the Newtonian gravitational potential. That perturbation is described by the following perturbing potential:

$$V_Y(r) = \Phi_Y(r) + \frac{GM}{r} = \frac{\delta}{1+\delta}\frac{GM}{r}\left[1 - e^{-\frac{r}{\Lambda}}\right] \tag{12}$$

The exact analytical expression for orbital precession in the case of the above perturbing potential could be presented in the integral form given by Eqs. (8) and (9). In this study we will calculate the approximate expression for $\Delta\varphi$ using power series expansion of $V_Y(r)$, assuming that $r \ll \Lambda$:

$$V_Y(r) \approx -\frac{\delta GMr}{2(1+\delta)\Lambda^2}\left[1 - \frac{r}{3\Lambda} + \frac{r^2}{12\Lambda^2} - \ldots\right], \quad r \ll \Lambda, \tag{13}$$

where we neglected the constant term since it does not affect $\Delta\varphi$. We obtain the following approximation for the angle of orbital precession in Yukawa gravity:

$$\Delta\varphi_Y^{rad} \approx \frac{\pi\delta\sqrt{1-e^2}}{1+\delta}\left(\frac{a^2}{\Lambda^2} - \frac{a^3}{\Lambda^3} + \frac{4+e^2}{8}\frac{a^4}{\Lambda^4} - \ldots\right). \tag{14}$$

The right-hand side in Eq. (14) could be presented as series of Gauss's hypergeometric function ${}_2F_1$ with different arguments [65].

Since $r \ll \Lambda$ also implies that $a \ll \Lambda$, we can neglect higher order terms in the above expansion and keep only the first order term. We obtain the following approximate formula for orbital precession in Yukawa gravity:

$$\Delta\varphi_Y^{rad} \approx \frac{\pi\delta\sqrt{1-e^2}}{1+\delta}\frac{a^2}{\Lambda^2}, \quad a \ll \Lambda. \tag{15}$$

Both, $\Delta\varphi_{GR}$ (Eq. (11)) and $\Delta\varphi_Y$ (Eq. (15)) represent the angles of orbital precession per orbital period in the orbital plane.

2.2. *Mass bounds of graviton in Yukawa gravity*

We made two predictions regarding mass bounds of graviton in Yukawa gravity and compared it. First, we assume that Yukawa orbital precession of Solar System planets is equal to the astronomical observation and make prediction of mass bounds. Second prediction of mass bounds is made under assumption that the orbital precession of Solar System planets in Yukawa gravity is equal to the corresponding Schwarzschild precession in GR.

In order to determine mass bounds, we suppose that graviton is massive and start from the simplest model [34], i.e. from Yukawa potential in the Newtonian limit (Eq. (6)) where we put $\delta = 1$. The potential will be the following:

$$\Phi_Y(r) = -\frac{GM}{2r}\left[1 + e^{-\frac{r}{\Lambda}}\right], \tag{16}$$

The approximate formula in case $\delta = 1$ for the orbital precession in Yukawa gravity and for $a \ll \Lambda$ is:

$$\Delta\varphi_Y^{rad} \approx \frac{\pi\sqrt{1-e^2}}{2}\frac{a^2}{\Lambda^2}, \tag{17}$$

where e is orbital eccentricity and a is the semi-major axis of the orbital ellipse.

By equating the above expression with the formula for Schwarzschild precession (see Eq. (11)) [23, 34, 66, 67], it was found that Λ has to satisfy the following condition:

$$\Lambda \approx \sqrt{\frac{c^2(a\sqrt{1-e^2})^3}{12GM}}. \tag{18}$$

The obtained estimates of Λ are then used to find the corresponding constraints for the graviton mass m_g according to $m_g = hc/\lambda_g$ [28, 34], where we assumed that the Compton wavelength λ_g of graviton is equal to the obtained values of Λ ($\lambda_g = \Lambda$).

3. Numerical results and discussion

In this section we compare our calculations with some astronomical observations for the Solar System planets.

In our previous studies regarding the Yukawa gravity, using the orbit of the S2 star around the Galactic Center we test and constrain Yukawa gravity theory [27–30, 62]. In paper [27] we investigated parameters δ and Λ, and in paper [62] we investigated values of f_1, f_2. In this paper, we assumed a few different values of universal constant δ and estimate the range of interaction Λ, under two requests: 1) the orbital precession in Yukawa gravity is equal to the observed one (available for 6 planets up to Saturn); 2) the orbital precession in Yukawa gravity is equal to the Schwarzschild precession in GR (calculated for all 9 studied objects). After that, assuming that the estimated value of the range of Yukawa interaction Λ corresponded to the Compton wavelength of graviton λ_g, we estimated the upper bound for graviton mass m_g.

The observed orbital elements and their uncertainties are taken from Planetary Fact Sheet table, published by NASA Space Science Data Coordinated Archive (NSSDCA): `https://nssdc.gsfc.nasa.gov/planetary/factsheet`. The values of observed orbital precession for 6 planets up to Saturn are taken from Table 1 in [31].

Table 1: The estimated orbital precession in Yukawa gravity and mass bounds m_g, for value of $\delta = 1$, in the case when the orbital precession in Yukawa gravity is equal to the observed one, as well as to the corresponding GR prediction for Solar System planets.

No. Name of planet	Mass and orbital elements			Precession ($\Delta\varphi$) (in $''/cy$) and m_g (eV)			
	M (M_{sun})	a (AU)	e	$\Delta\varphi_{GR}$	$\Delta\varphi_{obs}$	$m_{g,GR}$	$m_{g,obs}$
1 Mercury	1.66×10^{-7}	0.3871	0.2056	42.9841	43.1	0.12×10^{-19}	0.12×10^{-19}
2 Venus	2.45×10^{-6}	0.7233	0.0068	8.6253	8.0	0.46×10^{-20}	0.45×10^{-20}
3 Earth	3.003×10^{-6}	1.0000	0.0167	3.8391	5.0	0.29×10^{-20}	0.33×10^{-20}
4 Mars	3.25×10^{-7}	1.5237	0.0934	1.3510	1.36	0.15×10^{-20}	0.15×10^{-20}
5 Jupiter	9.55×10^{-4}	5.2034	0.0484	0.0624	0.07	0.24×10^{-21}	0.26×10^{-21}
6 Saturn	2.86×10^{-4}	9.5371	0.0542	0.0137	0.014	0.97×10^{-22}	0.98×10^{-22}
7 Uranus	4.366×10^{-5}	19.1913	0.0473	0.0024	-	0.34×10^{-22}	-
8 Neptune	5.15×10^{-5}	30.0690	0.0086	0.0008	-	0.17×10^{-22}	-
9 Pluto	6.55×10^{-9}	39.4817	0.2488	0.0004	-	0.12×10^{-22}	-

Table 1 presents mass and the corresponding orbital elements (semi-major axes a and eccentricities e) of Solar System planets, values of orbital precession $\Delta\varphi_{GR}$, $\Delta\varphi_{obs}$, and the upper graviton mass bounds in (eV) obtained with assumption that Yukawa orbital precession of Solar System planets is equal to the Schwarzschild precession in GR $m_{g,GR}$ and to the astronomical observation $m_{g,obs}$. The observed data are available only for 6 planets up to Saturn.

Fig. 1 presents comparison between the observed orbital precession of the selected Solar System bodies and the corresponding GR predictions. Agreement between GR prediction and observations are very good, like was expected. Using data from Figure 1, we estimated the range of interaction Λ under request 1) that the orbital precession in Yukawa gravity is equal to the observed one and 2) that the orbital precession in Yukawa gravity is equal to the GR prediction, respectively. Results are shown on following Figures.

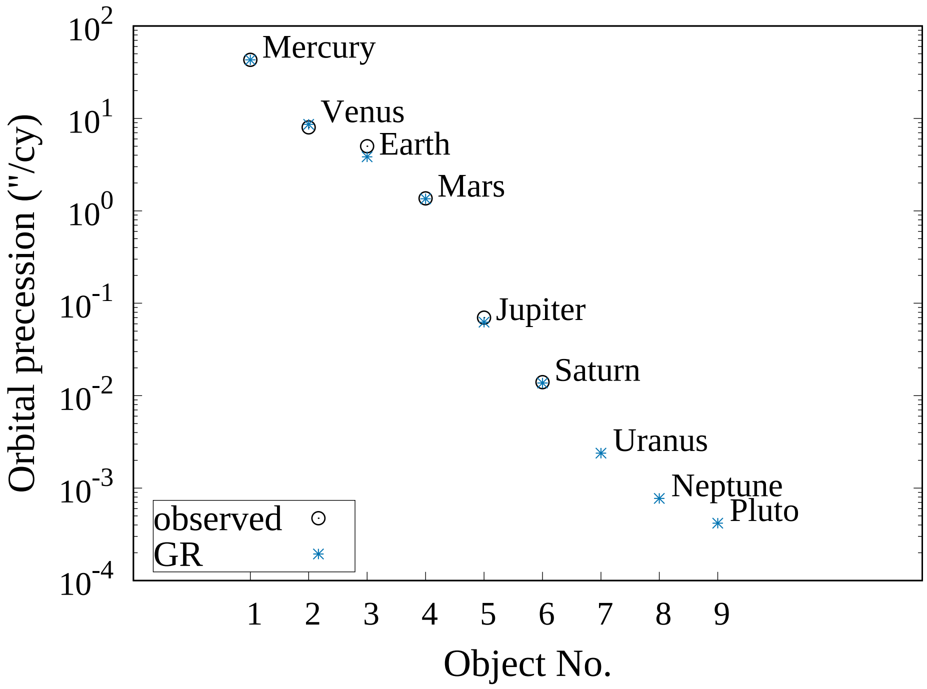

Figure 1: Comparison between the observed orbital precession of the selected Solar System bodies and the corresponding GR predictions.

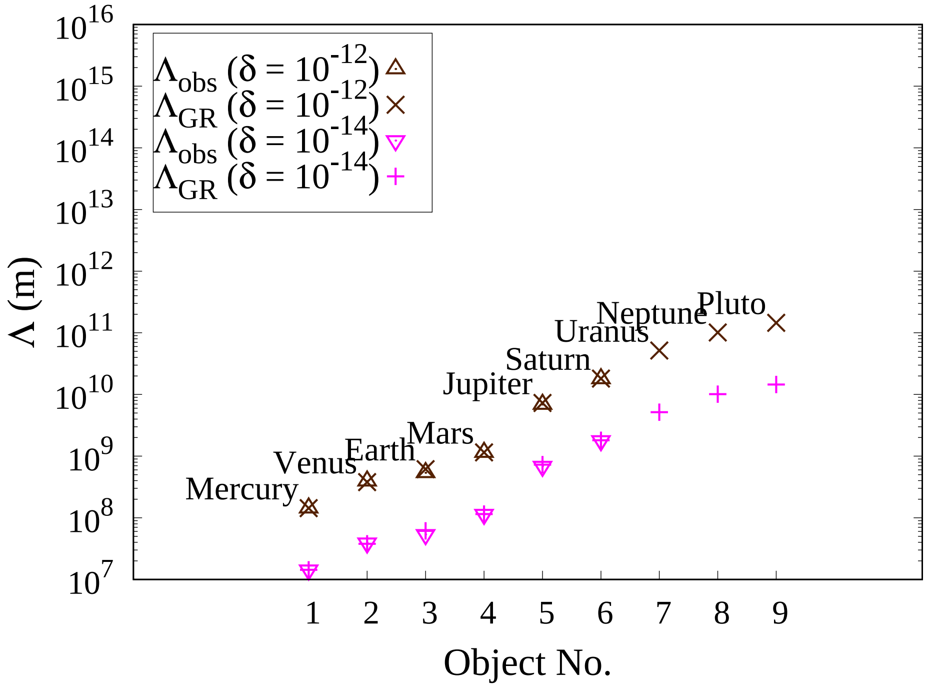

Figure 2: Constraints on range of interaction Λ in the case of $\delta = 10^{-14}$ and 10^{-12}, obtained from the observed orbital precession and its GR prediction.

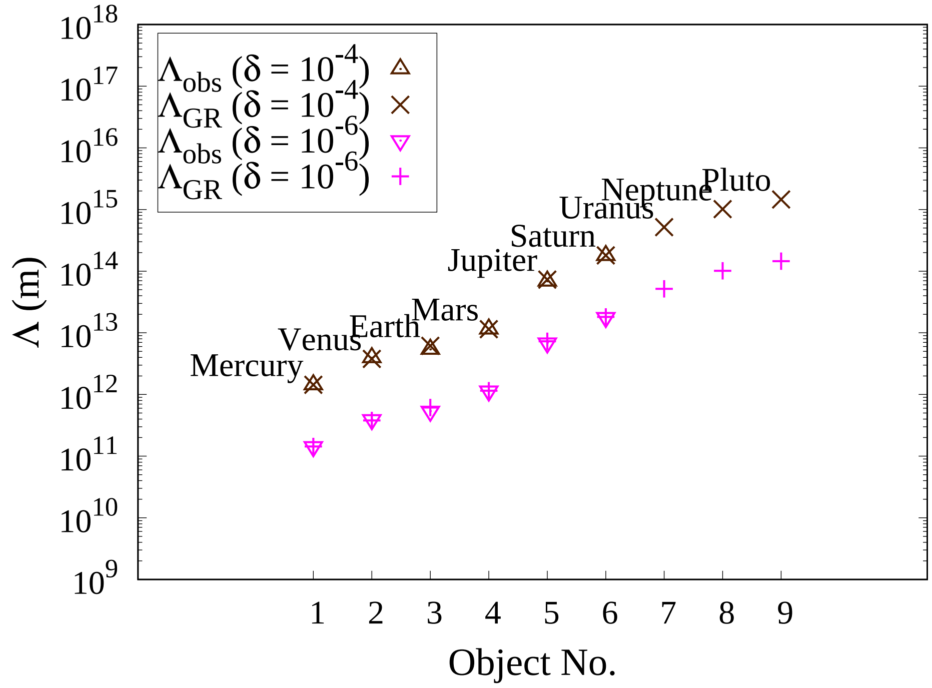

Figure 3: The same as in Fig. 2, but for $\delta = 10^{-10}$ and 10^{-8}.

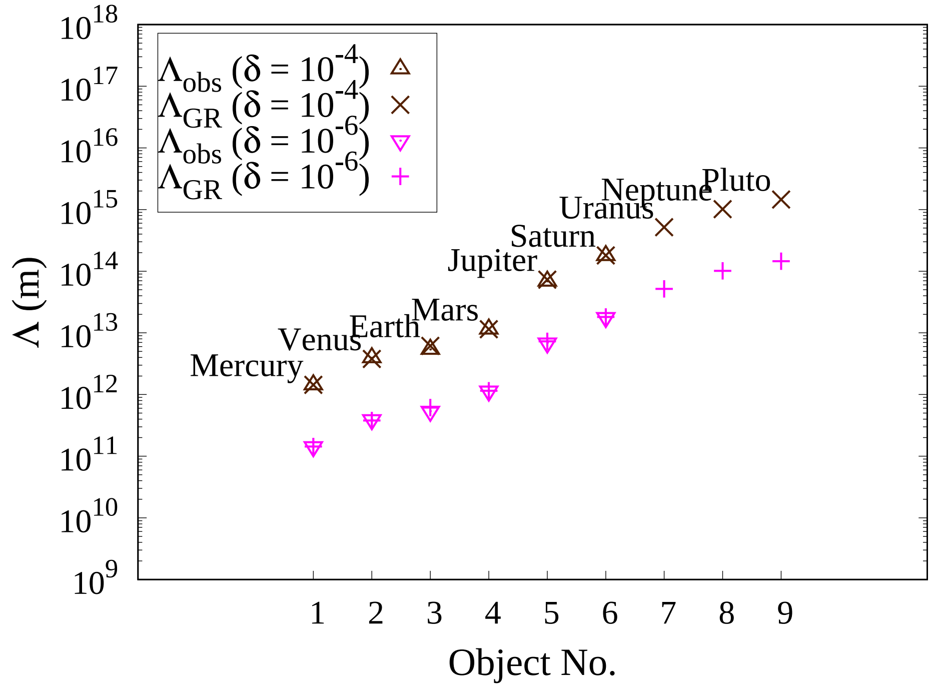

Figure 4: The same as in Fig. 2, but for $\delta = 10^{-6}$ and 10^{-4}.

Figs. 2-5 presents constraints on range of interaction Λ in the case of several values of parameter δ, i.e. $\delta = 10^{-14}$, 10^{-12}, 10^{-10}, 10^{-8}, 10^{-6}, 10^{-4}, 0.01 and 1 obtained from the observed orbital precession and its GR

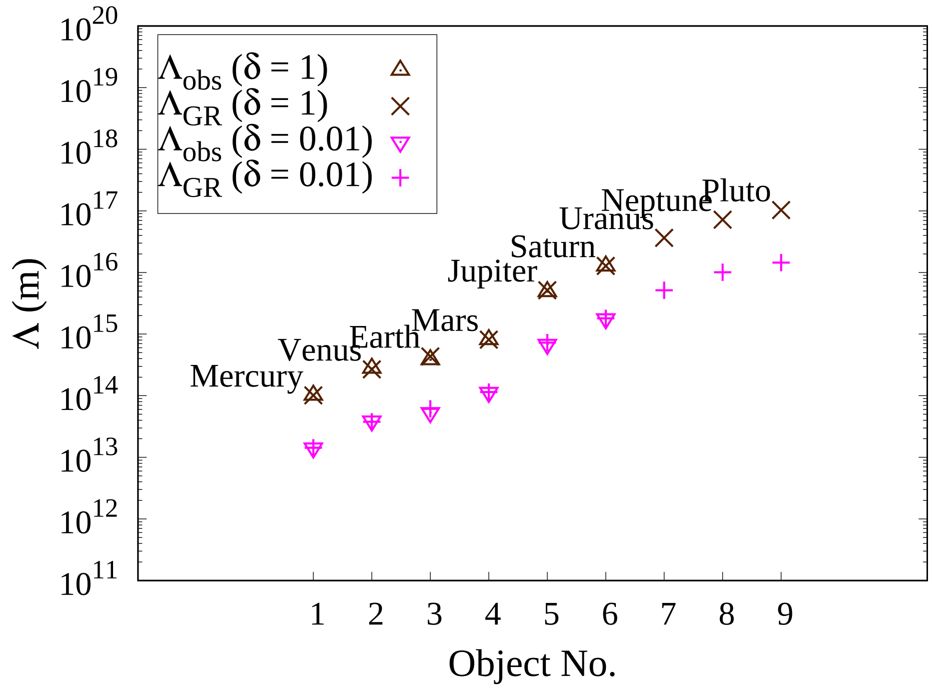

Figure 5: The same as in Fig. 2, but for $\delta = 0.01$ and 1.

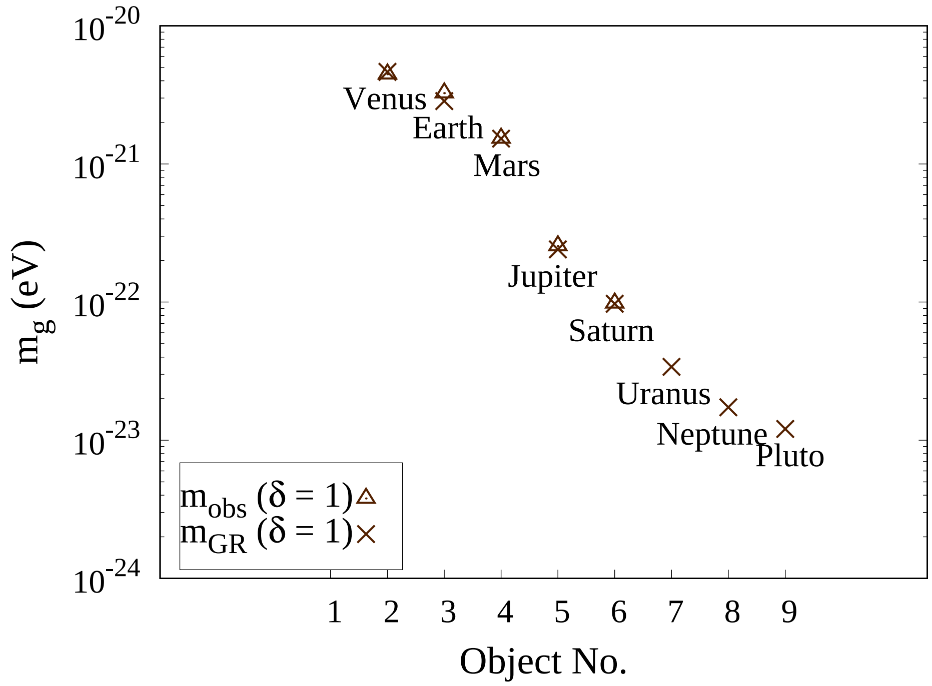

Figure 6: Upper bound for graviton mass m_g in case $\delta = 1$, obtained with assumption that Yukawa orbital precession is equal to the observed orbital precession and its GR prediction.

prediction. From Figs. 2-5 we can notice that parameters Λ and δ have a strong influence on the obtained results. Depending on choice of parameter δ, the range of parameter Λ is in interval from 10^7 to 10^{18} m, which is consistent with findings of other authors [32, 34, 43, 47].

Fig. 6 presents the upper graviton mass bound obtained with assumption that Yukawa orbital precession of Solar System planets is equal to the astronomical observation and Schwarzschild precession in GR. In case of Saturn orbit obtained values for the upper graviton mass bound are: $m_{g,GR}$ = 0.97 $\times 10^{-22}$ eV and $m_{g,obs}$ = 0.98 $\times 10^{-22}$ eV. The stringent constrain according Yukawa precession (assuming to be equal with the GR prediction) is obtained in case of Pluto orbit. In that case graviton mass bound is: $m_{g,GR}$ = 0.12 $\times 10^{-22}$ eV.

We can conclude that our estimates for Λ are in good agreement with the corresponding other experimental constraints given in Fig. 10 from Adelberger et al. [47]. Also, we can conclude that our estimates for $m_{g,GR}$ and $m_{g,obs}$ are in good agreement with the corresponding values given in paper [34].

4. Conclusions

In this study we constrained the Yukawa gravity at the Solar System scales and bound mass of graviton, using the observed and Schwarzschild orbital precessions of the planets and Pluto dwarf planet. In order to constrain the Yukawa gravity at the Solar System scales we investigated a few different values of parameter δ. In order to bound mass of graviton we assume Yukawa-like potential in form that values of universal constant $\delta = 1$ and estimate the range of interaction Λ, so that the resulted orbital precession in Yukawa gravity is equal to the observed one (available for 6 planets up to Saturn), as well as to the Schwarzschild precession in GR (calculated for all 8 planets and Pluto dwarf planet). Our findings are the following:

(1) Our estimates for the range of Yukawa interaction Λ at the Solar System scales are in good agreement with recent experimental constrains [32, 47];

(2) There is a small difference between the values for Λ in the case of the observed orbital precession and the corresponding GR prediction (i.e. Schwarzschild precession);

(3) Assuming such slightly different values of Λ, we found that these types of modified gravity can explain the observed additional perihelion precession of the planets without dark matter hypothesis;

(4) The obtained results show that Yukawa gravity could be used to improve the results for motion of the Solar System bodies (planets, planetary satellites, asteroids and comets).

(5) From 2019, our estimate for bound mass of graviton obtained from the observed stellar orbits around the Galactic Center (GC) is in Gauge and Higgs Boson Particle Listings by PDG (Zyla et al., Particle Data Group, 2020, PTEP, 083C01) [24].

(6) We can conclude that our estimates for $m_{g,GR}$ and $m_{g,obs}$ are in good agreement with the corresponding values for Solar System given in paper [34], and with our previous results [28, 29] obtained for stellar orbit around GC.

We can notice that Yukawa gravity could help us to get more reliable predictions for natural hazards in the Solar System, such as those from near-Earth objects. Also, we can conclude that comparison between different massive gravity models and the observed Solar System planetary and stellar orbits around GC is a powerful tool for constraining the graviton mass and probing the predictions of GR and different gravitational theories.

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