Filomat 37:23 (2023), 8039–8039
https://doi.org/10.2298/FIL2323039Y

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Erratum: S-Zariski Topology on S-Spectrum of Modules

Eda Yıldız[a], Bayram Ali Ersoy[a], Ünsal Tekir[b]

[a]*Department of Mathematics, Faculty of Arts and Sciences, Yildiz Technical University, Istanbul, Turkey*
[b]*Department of Mathematics, Faculty of Arts and Sciences, Marmara University, Istanbul, Turkey*

Corollary 3.14 in the published version is incorrect. The condition "every S-prime submodule of M is prime" is necessary. Consider the ring $R = M = \mathbb{Z} \times \mathbb{Z}$ and the multiplicatively closed subset $S = \mathbb{Z} \times \mathbb{Z}^*$, where $\mathbb{Z}^* = \mathbb{Z} - \{0\}$. Then $Spec_S(M) = \{n\mathbb{Z} \times \{0\} : n \in \mathbb{Z}\}$. Note that $(2\mathbb{Z} \times 0 : (0,1)) = \mathbb{Z} \times 0 = (3\mathbb{Z} \times 0 : (0,1))$ and $2\mathbb{Z} \times 0, 3\mathbb{Z} \times 0 \in Spec_S(M)$. By [1, Proposition 4], $\overline{\{2\mathbb{Z} \times 0\}} = \overline{\{3\mathbb{Z} \times 0\}}$. As $2\mathbb{Z} \times 0 \neq 3\mathbb{Z} \times 0$, $Spec_S(M)$ is not a T_0-space.

Thus the following is the corrected version of the mentioned result.

Corollary. *Suppose that M is a multiplication module. Then $Spec_S(M)$ is a T_0-space for both S-Zariski topology τ_S and quasi S-Zariski topology τ_S^* iff every S-prime submodule of M is prime.*

Proof. First note that $\tau_S = \tau_S^*$ since M is multiplication. Assume that every S-prime submodule is prime. Suppose $V_S(P) = V_S(Q)$ for $P, Q \in Spec_S(M)$. Then $S^{-1}(P : M) = S^{-1}(Q : M)$ by [2, Lemma 3.5] . As P, Q are S-prime submodules, they are also prime by the assumption. Then $(P : M)$ and $(Q : M)$ are prime ideals. So $S^{-1}(P : M) = S^{-1}(Q : M)$ implies that $(P : M) = (Q : M)$. Hence we obtain $P = (P : M)M = (Q : M)M = Q$ since M is a multiplication module. Thus $Spec_S(M)$ is T_0 by [2, Theorem 3.13]. The rest follows from the fact that $\tau_S \leq \tau_S^*$. Now assume $Spec_S(M)$ is T_0-space. Let $P \in Spec_S(M)$. Then there exists $s_P \in S$ such that $(P :_M s_P)$ is a prime submodule. Also by [2, Theorem 3.11], $V_S(P) = V_S((P :_M s_P))$. Since $Spec_S(M)$ is a T_0-space, we have $P = (P :_M s_P)$ is a prime submodule. □

References

[1] E. Yıldız, B.A. Ersoy, Ü. Tekir and S. Koç, On S-Zariski topology, Communications in Algebra 49(3) (2021) 1212-1224.
[2] E. Yıldız, B.A. Ersoy, Ü. Tekir, S-Zariski Topology on S-Spectrum of Modules, Filomat, 36(20) (2022) 7103-7112.

2020 *Mathematics Subject Classification*. Primary 13A15 ; Secondary 13C13, 54B35
Keywords. Zariski topology, prime spectrum, S-prime spectrum, S-maximal ideal
Received: 17 April 2023; Accepted: 17 April 2023
Communicated by Dragan S. Djordjević
Email addresses: edyildiz@yildiz.edu.tr (Eda Yıldız), ersoya@yildiz.edu.tr (Bayram Ali Ersoy), utekir@marmara.edu.tr (Ünsal Tekir)