Filomat 37:17 (2023), 5567–5580
https://doi.org/10.2298/FIL2317567Z

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Consensus of NMASs with MSTs subjected to DoS attacks under event-triggered control

Xia Zhou[a,b,*], Chunya Huang[a], Jinde Cao[c,d], Wanbing Liu[a], Meixuan Xi[a]

[a]*School of Mathematics and Computing Science, Guangxi Colleges and Universities Key Laboratory of Data Analysis and Computation, Guilin University of Electronic Technology, Guilin 541004, China*
[b]*Center for Applied Mathematics of Guangxi (Guilin University of Electronic Technology), Guilin 541002, China*
[c]*School of Mathematics, Southeast University, Nanjing 210096, China*
[d]*Yonsei Frontier Lab, Yonsei University, Seoul 03722, South Korea*

Abstract. The leader-following consensus (LFCs) of nonlinear multi-agent systems (NMASs) with Markov switching topologies (MSTs) subjected to denial-of-service (DoS) attacks under event-triggered (ET) control is studied. An ET strategy is applied to reduce unnecessary signal transmission among agents, save network resources, and ensure systems performance. As a result of the open communication network among agents, it is inevitably subjected to attacks that leads to changing in the communication topologies. The communication topologies among agents are modeled as MSTs, and the transfer rates (TRs) are assumed to be partially unknown. DoS attacks are the most common attacks due to their destructive, stealthy, and easy implementation, so the network attacks considered in this paper are DoS attacks. Based on the distributed control theory and Lyapunov stability theory, the Lyapunov direct method and stochastic analysis method are used to explore sufficient conditions for the systems to achieve LFCs. Finally, an example is provided to verify the effectiveness of the methods and the correctness of the results.

1. Introduction

Multi-agent systems (MASs) have the abilities of autonomy, coordination and reasoning, so they have been widely used in intelligent traffic control, unmanned aerial vehicle formation control, sensor networks and other fields [1–3]. As one of the research hotspots in multi-agent cooperative control, the consensus problem, especially the LFCs, has attracted much attention [4].

Because of the open nature of networks, network attacks have become the main factor affecting systems security, destroying the stability of the systems, which will inevitably lead to systems paralysis. Among the common cyber attacks, there are main replay attacks (RAs)[5, 6], deception attacks(DAs)[7, 8] and DoS

2020 *Mathematics Subject Classification*. Primary 93A16; 93D50
Keywords. MASs; DoS attacks; ET scheme; Consensus; Markov random switching topologies.
Received: 26 October 2022; Accepted: 27 December 2022
Communicated by Miljana Jovanović
Research supported by Guangxi Province Natural Science Foundation (No.2021GXNSFAA196045), National Natural Science Foundation of China (No.12161024), Training Program for 1, 000 Young and Middle-aged Cadre Teachers in Universities of Guangxi Province, National College Students' Innovation and Entrepreneurship Training Program (No.202110595049; No.202210595044), Innovation Project of GUET Graduate Education (No.2023YCXS111).
* Corresponding author: Xia Zhou
Email addresses: xiazhou201612@guet.edu.cn (Xia Zhou), hcy97120518@163.com (Chunya Huang), jdcao@seu.edu.cn (Jinde Cao), kaoyankaobo2022@163.com (Wanbing Liu), ximeixuan2022@163.com (Meixuan Xi)

attacks[9–11]. The RAs are that the attacker records a transmitted data sequence and repeatedly transmits this data to overwrite the new data, thus attacking the systems [5]. DAs degrade the performance of the systems by tampering with the transmitted data and injecting false data into the systems[7]. DoS attacks mainly attack the systems by disrupting or interrupting the signal transmission in the communication network [9]. In [6], Wang et al. researched the security consensus issue of discrete MASs under RAs, and gave sufficient conditions for LFCs of MASs under RAs. Wen et al. [8] investigated the fault-tolerant secure consensus tracking issue of time-lagged NMASs, which have DAs, parameter uncertainty and actuator failures, and proposed a distributed impulsive control protocol to obtain sufficient conditions of mean-square bounded consensus. In [10], Feng et al. researched the consensus among followers and the LFCs of the linear MASs with DoS attacks. Compared with RAs and DAs, DoS attacks are more destructive, stealthy, simpler and easier to implement. Therefore, DoS attacks are commonly used in cyber attacks.

Network attacks are inevitable, which leads to randomly changing in communication topologies among agents, deterministic communication topologies can no longer meet the demand, and random communication topologies can describe the actual situation more accurately. Markov chains are discrete-time stochastic processes with Markov properties, which have been widely used in manufacturing, aerospace, and network control [12, 13]. Due to the good properties of the Markov chain, it can model the communication topologies as MSTs. The LFCs of NMASs with MSTs under DoS attacks have been reported [14–16]. In [14], Li et al. investigated the LFCs issue of cyber-physical systems under energy-limited DoS attacks, modeled the irregular communication topologies caused by DoS attacks as MSTs, and proposed a new distributed resilient control strategy to achieve LFCs. Wang et al. [15] researched the LFCs issue for nonlinear MASs under uncertain nonhomogeneous MSTs and DoS attacks, and obtained sufficient conditions of LFCs in mean-square sense. Wang et al. [16] delved into the LFCs issue of NMASs with time delay under uncertain nonhomogeneous MSTs and DoS attacks. In fact, the TRs in Markov switching are not easily or completely available when uncertainties and some technical limitations are taken into account. Therefore, suppose that the TRs are partially unknown in this paper. There were also literature reports that the Markov TRs were partially unknown [15–18].

In addition, due to the bandwidth of the communication networks among agents and their energy is limited, in order to reduce the bandwidth pressure and avoid resource waste, ET conditions are designed to trigger conditions and send data when there is a communication demand among agents, which can not only ensure systems performance but also save network resources[19]. At present, some researchers have introduced the ET scheme into the consensus issue of MASs[20, 21]. He et al. [20] studied the security consensus of MASs under DAs by using static and dynamic ET strategies, respectively, and obtained consensus sufficient conditions. Chen et al. [21] studied the dynamic ET output feedback control of power systems load frequency under multiple network attacks, proposed a new dynamic ET load frequency regulation scheme, and obtained the mean-square exponential stability and robustness conditions of the systems.

Based on previous works, we study the LFCs problem of NMASs with MSTs subjected to DoS attacks under ET control. The principal innovations are as bellow:

(1) The communication topologies considered in this paper are random Markov switching, and TRs are assumed to be partially unknown. In order to reduce bandwidth pressure and saving resources, the ET scheme is adopted. At the same time, considering the open nature of the network, MASs could suffer from network attacks, which are assumed to be damaging and easy to implement DoS attacks.

(2) Diverse from the deterministic topologies in literature [9, 14, 20], the communication topologies of the MASs considered in this paper are random MSTs.

(3) Compared with [15, 16], in this paper, an ET condition is designed, an ET strategy is introduced, and the data are sent when needed according to the trigger condition rather than transmitting all the data as in [15, 16]. The introduction of an ET strategy can effectively save network resources.

Notations: $\mathbb{R}^n$ is the n-dimensional real vector space. $*$ denotes a block caused by symmetry in the matrix. I_n stands for the n order identity matrix. The sign $\otimes$ indicates the Kronecker product. $\mathbb{E}(\cdot)$ is the expected value for a stochastic variable. The Euclidean vector norm is represented as $\|\cdot\|$. ι stands for the time variable.

2. Problem formulations

Graph Theory: $\mathcal{G} = (\varsigma, \chi, C)$ represents the network communication topology of the agents, where ς is the nodes set with $\varsigma = (\varsigma_1, \varsigma_2, \cdots, \varsigma_N)$, χ is the edges set with $\chi \subseteq \varsigma \times \varsigma$, and $C = [a_{ij}] \in \mathbb{R}^{N\times N}$ with $a_{ii} = 0$ stands for the adjacency matrix. It is assumed that a direct edge from agent j to agent i is available, that is, $(j, i) \in \chi$, $a_{ij} > 0$, otherwise, $a_{ij} = 0$. $L = [l_{ij}] \in \mathbb{R}^{N\times N}$ is the Laplacian matrix and satisfies $l_{ij} = -a_{ij}(j \neq i)$ and $l_{ii} = \sum_{j=i} a_{ij}$. In directed graph $\mathcal{G}$, a directed path from node ς_j to ς_i is a finite ordered sequence of edges $(\varsigma_j, \varsigma_{l_1})$, $(\varsigma_{l_1}, \varsigma_{l_2})$, $\cdots$, $(\varsigma_{l_{æ}}, \varsigma_j)$ with distinct nodes $\varsigma_{l_\wp}, \wp = 1, 2, \cdots, æ$. A graph is said to include a directed spanning tree if at least one directed path from one node to all other nodes in the graph.

The communication topologies consist of m graphs, $\bigcup_{i=1}^m \mathcal{G}_i = \{\varsigma, \bigcup_{i=1}^m \chi_i, \sum_{i=1}^m C_i\}$ denote the concatenation of m graphs $\mathcal{G}_1 = \{\varsigma, \chi_1, C_1\}$, $\cdots$, $\mathcal{G}_m = \{\varsigma, \chi_m, C_m\}$. Let $\mathcal{G}_{\nu(\iota)} = (\varsigma, \chi_{\nu(\iota)}, C_{\nu(\iota)})$ be the network communication topology of the agents at time ι, where the set of edges $\chi_{\nu(\iota)}$ and the adjacency matrix $C_{\nu(\iota)}$ are time-varying. The graph $\mathcal{G}_{\nu(\iota)}$ is stochastic time-varying and is controlled by the Markov process $\nu(\iota)$.

2.1. System model description

The MASs consist of a leader and N followers. The dynamic model of the ith follower is described as

$$\dot{x}_i(\iota) = Ax_i(\iota) + Bg(\iota, x_i(\iota)) + Cu_i(\iota),\ i = 1, 2, \cdots, N, \tag{1}$$

where $x_i(\iota) \in \mathbb{R}^n$ and $u_i(\iota) \in \mathbb{R}^n$ stand for the state vector and control input vector of the ith follower, respectively. A, B and C denote constant matrices of known suitable dimensions. $g(\cdot, \cdot)$ is continuous nonlinear vector function satisfying the Assumption 3.

Accordingly, the dynamic model of the leader is described as

$$\dot{x}_0(\iota) = Ax_0(\iota) + Bg(\iota, x_0(\iota)), \tag{2}$$

where $x_0(\iota) \in \mathbb{R}^n$ represents the state vector of the leader.

To design the conformance control protocol, in what follows, the combination threshold is defined as follows

$$w_i(\iota) = \sum_{j=1}^{N} a_{ij,\nu(\iota)}(x_j(\iota) - x_i(\iota)) + d_{i0,\nu(\iota)}(x_0(\iota) - x_i(\iota)), \tag{3}$$

where $\nu(\iota)$ stands for a discrete-time Markov process, which can be assumed to take value in the finite set $S = \{1, 2, \cdots, m\}$. The switching of the network communication topology is determined by the Markov process $\{\nu(\iota), \iota \geq 0\}$ on the probability space $(\Omega, \mathcal{F}, P)$, and the transition probability satisfies

$$P\{\nu(\iota + \Delta) = s|\nu(\iota) = \nu\} = \begin{cases} \pi_{\nu s}(\ell)\Delta + o(\Delta), & \nu \neq s, \\ 1 + \pi_{\nu\nu}(\ell)\Delta + o(\Delta), & \nu = s, \end{cases} \tag{4}$$

where $\pi_{\nu s}(\ell) \geq 0$ represents the TRs from mode ν to mode s. If $\nu = s$, then it satisfies $\pi_{\nu\nu}(\ell) = -\sum_{s=1,s\neq\nu}^m \pi_{\nu s}(\ell)$, then $\sum_{s=1}^m \pi_{\nu s}(\ell) = 0$. If $\pi_{\nu s}(\ell)$ is known, $\underline{\pi}_{\nu s}(\ell)$ and $\overline{\pi}_{\nu s}(\ell)$ stand for the lower bound and upper bound of $\pi_{\nu s}(\ell)$, spectively. ℓ denotes the duration between two consecutive jumps. $o(\Delta)$ denotes an infinitesimal of higher order than Δ, i.e. $\lim\limits_{\Delta\to 0} \frac{o(\Delta)}{\Delta} = 0$. Define the TRs matrix of Markov chain as

$$\Pi = \begin{pmatrix} \pi_{11}(\ell) & ? & ? & \cdots & \pi_{1m}(\ell) \\ ? & ? & \pi_{23}(\ell) & \cdots & ? \\ \pi_{31}(\ell) & ? & \pi_{33}(\ell) & \cdots & ? \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ ? & ? & ? & \cdots & \pi_{mm}(\ell) \end{pmatrix},$$

where "?" stands for the unknown element, defining $S_k^\nu = \{s|\pi_{\nu s}(\ell) \text{ is known }\}$ and $S_{uk}^\nu = \{s|\pi_{\nu s}(\ell) \text{ is unknown }\}$, it is clear that $S = S_k^\nu + S_{uk}^\nu$. Also, one obtains $S_k^\nu = \{k_1^\nu, k_2^\nu, \cdots, k_\hbar^\nu\}$, where $k_\varpi^\nu(\varpi = 1, \cdots, \hbar)$ denotes the element of the ϖth column in the νth row of the matrix Π.

The $u_i(\iota)$ is designed as

$$u_i(\iota) = \gamma K_{\nu(\iota)} w_i(\iota_k^i),\ \iota \in [\iota_k^i, \iota_{k+1}^i), \tag{5}$$

where $K_{\nu(\iota)}$ is the control gain matrix. ι_k^i denotes the kth trigger instant of the ith agent, and the set of trigger instants is $\{\iota_0^i, \iota_1^i, \cdots\}$. $\gamma > 0$ is the coupling strength.

According to the definition of combination threshold, its error is

$$\xi_i(\iota) = w_i(\iota_k^i) - w_i(\iota). \tag{6}$$

Let $w(\iota) = [w_1^T(\iota_k^1), w_2^T(\iota_k^2), \cdots, w_N^T(\iota_k^N)]^T$, $x(\iota) = [x_1^T(\iota), x_2^T(\iota), \cdots, x_N^T(\iota)]^T$. Substitute (5) into (1), then (1) can be rewritten as

$$\dot{x}_i(\iota) = (I_N \otimes A)x_i(\iota) + (I_N \otimes B)g(\iota, x_i(\iota)) + \gamma(I_N \otimes CK_{\nu(\iota)})w_i(\iota_k). \tag{7}$$

Define the state vector error as $\eta_i(\iota) = x_i(\iota) - x_0(\iota)$, and let $\eta(\iota) = [\eta_1^T(\iota), \eta_2^T(\iota), \cdots, \eta_N^T(\iota)]$, then $\eta(\iota)$ is derived for time to give

$$\dot{\eta}(\iota) = (I_N \otimes A)\eta(\iota) + (I_N \otimes B)G(\eta(\iota), \iota) + \gamma(I_N \otimes CK_{\nu(\iota)})w(\iota_k). \tag{8}$$

Let $\xi(\iota) = [\xi_1^T(\iota), \xi_2^T(\iota), \cdots, \xi_N^T(\iota)]^T$, according to (3), (6) and $\eta(\iota)$, then one obtains

$$w(\iota) = -(H_{\nu(\iota)} \otimes I_N)\eta(\iota), \tag{9}$$

where $H_{\nu(\iota)} = L_{\nu(\iota)} + D_{\nu(\iota)}$.

Definition 1[15]NMASs (1) and (2) are able to implement LFCs under random MSTs and DoS attacks, if NMASs satisfy

$$\mathbb{E}\{\|\eta_i(\iota)\|^2\} \leq \gamma \exp(-\vartheta\iota)\mathbb{E}\{\|\eta_i(0)\|^2\}, \tag{10}$$

where γ and $\vartheta > 0$ represent the positive scalar and decay rate, respectively.

Assumption 1[16] For MASs (1) and (2), the graphs $\mathcal{G}_{\nu(\iota)}$ are strongly connected.

Assumption 2[16] (A, B) is stabilizable.

Assumption 3[15] For any $x_1, x_2 \in \mathbb{R}^n$, there exists a positive constant ρ such that

$$\| g(x_1) - g(x_2) \| \leq \rho \| x_1 - x_2 \| . \tag{11}$$

2.2. *Description of the attack model*

Assuming that the attackers have finite amount of energy, the attackers enter the sleep zone after the last attack to recharge for the next attack. Therefore, the whole time interval can be divided into a communication interval and an attack interval. During the communication interval, the agent updates the controller at the trigger instant $\{\iota_k^i\}$. During the attack interval, communication among agents is interrupted, and no ET instant.

In Figure 1, $\{h_n\}_{n \in N}$, $h_0 \geq 0$ represents the time sequence of attacks, ∂_n indicates the duration of the attacks. From Figure 1, when the DoS attacks stop, the agents need to recover time Δ_* before it can return to the control state, so the time period of the nth attack is $\hat{\mathcal{H}}_n = [h_n, h_n + \partial_n + \Delta_*)$. For any time period $[\iota_0, \iota)$, there is $[\iota_0, \iota) = \hat{\Pi}_a(\iota_0, \iota) \bigcup \hat{\Pi}_s(\iota_0, \iota)$, where $\hat{\Pi}_a(\iota_0, \iota) = \bigcup \hat{\mathcal{H}}_n \bigcap [\iota_0, \iota]$ indicates the total time that the systems are affected by the attacks; $\hat{\Pi}_s(\iota_0, \iota) = [\iota_0, \iota] \backslash \hat{\Pi}_a(\iota_0, \iota)$ stands for the total time that the systems are not affected by the attacks. Therefore, it can be deduced that

$$|\hat{\Pi}_s(\iota_0, \iota)| = \iota - \iota_0 - |\hat{\Pi}_a(\iota_0, \iota)|, \tag{12}$$

$$|\hat{\Pi}_a(\iota_0, \iota)| \leq |\Pi_a(\iota_0, \iota)| + (1 + N_a(\iota_0, \iota))\Delta_*, \tag{13}$$

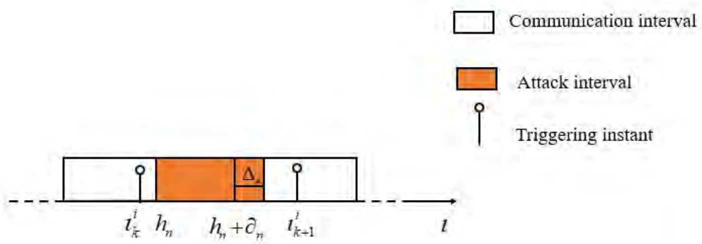

Figure 1 Time series of DoS attacks

where $N_a(\iota_0, \iota)$ stands for the total number of attacks. $\Pi_a(\iota_0, \iota) = \bigcup \mathcal{H}_n \bigcap [\iota_0, \iota], \mathcal{H}_n = [h_n, h_n + \partial_n)$.

For the MASs under DoS attacks, the attack frequency and the attack duration are introduced in this paper to prevent the attackers from launching the attack continuously so as to ensure systems stability.

Assumption 4[22] (Attack Frequency) For any $\mathcal{T}_2 > \mathcal{T}_1 > t_0$, the attack frequency $F_a(\cdot)$ within $[\mathcal{T}_1, \mathcal{T}_2)$ satisfies

$$F_a(\mathcal{T}_1, \mathcal{T}_2) \le \frac{N_a(\mathcal{T}_1, \mathcal{T}_2)}{\mathcal{T}_2 - \mathcal{T}_1}, \tag{14}$$

where $N_a(\mathcal{T}_1, \mathcal{T}_2)$ denotes the number of DoS attacks on $[\mathcal{T}_1, \mathcal{T}_2)$.

Assumption 5[22] (Attack Duration) For any $\mathcal{T}_2 > \mathcal{T}_1 > t_0$, let $\Pi_a(\mathcal{T}_1, \mathcal{T}_2)$ denote the total time interval which the systems are subject to DoS attacks in interval $[\mathcal{T}_1, \mathcal{T}_2)$, the attack duration over $[\mathcal{T}_1, \mathcal{T}_2)$ is defined as: there exist scalars $\mathcal{T}_0 \ge 0$ and $\zeta_a > 1$, such that

$$\Pi_{\mathrm{a}}(\mathcal{T}_1, \mathcal{T}_2) \le \mathcal{T}_0 + \frac{\mathcal{T}_2 - \mathcal{T}_1}{\zeta_a}, \tag{15}$$

where $\mathcal{T}_0$ and ζ_a represent the energy and attack strategy of attacks, respectively.

3. Main results

Next, the consensus of MASs under the ET strategy will be studied. The ET function is

$$\xi_i^T(\iota)\Lambda\xi_i(\iota) - \sigma w_i^T(\iota)\Lambda w_i(\iota) < 0, \tag{16}$$

where $\Lambda = P_\nu CC^T P_\nu$, $0 < \sigma < 1$.

Remark 3.1. *In ET control, a behavior where numerous triggers occur in a finite time is Zeno behavior. Due to the time interval between any two adjacent ET considered in this paper being greater than 0, i.e.,* $\iota_{k+1}^i - \iota_k^i > 0$. *Therefore, the MASs do not have Zeno behavior under the ET control protocol.*

Theorem 3.2. *For given positive scalars* ρ, ϵ, α_1, α_2, β, μ *and* $0 < \hbar_1 < \epsilon$ *and* $0 < \sigma < 1$, *the MASs* (1) *and* (2) *are able to achieve LFCs under the control protocol* (5), *if there exist appropriate dimension positive definite matrices* Q_ν, Q *and* $\mathcal{X} = \mathcal{X}_\nu^T$, *and the Assumptions 1-5 hold, for any* $\nu \in S$, *satisfying*

$$\begin{bmatrix} \Xi_{\nu,11} & Q_\nu & \Xi_{\nu,13} \\ * & -\rho^{-2} I_N & 0 \\ * & * & \Xi_{\nu,33} \end{bmatrix} < 0, \ \nu \in S_k^\nu, \tag{17}$$

$$\begin{bmatrix} \Theta_{\nu,11} & Q_\nu & \Theta_{\nu,13} \\ * & -\rho^{-2} I_N & 0 \\ * & * & \Theta_{\nu,33} \end{bmatrix} < 0, \ \nu \in S_{uk}^\nu, \tag{18}$$

$$\begin{pmatrix} \mathcal{X}_\nu & Q_\nu \\ * & -Q_s \end{pmatrix} \leq 0, \ \forall s \in S^\nu_{uk}, \ s \neq \nu, \tag{19}$$

$$Q_s + \mathcal{X}_\nu \geq 0, \ \forall s \in S^\nu_{uk}, \ s = \nu, \tag{20}$$

$$\begin{bmatrix} QA^T + AQ + BB^T - Q\beta & Q \\ * & -\rho^{-2} I_N \end{bmatrix} < 0, \tag{21}$$

$$F_a = \frac{\epsilon + \beta}{\iota - \iota_0} \leq \frac{\hbar_1}{\ln(u) + \Delta_*(\epsilon + \beta)}, \tag{22}$$

$$\zeta_a > \frac{\epsilon + \beta}{\epsilon + \hbar_1}, \tag{23}$$

where

$$\begin{aligned}
\Xi_{\nu,11} =& Q_\nu A^T + AQ_\nu + BB^T + \pi_{\nu\nu}(\ell) Q_\nu + \sum\nolimits_{s \in S^\nu_k} \pi_{\nu s}(\ell) \mathcal{X}_\nu + Q_\nu \alpha_1 \\
&+ \left(-2\gamma \lambda_{\min}(H_\nu) + \gamma \sqrt{\sigma} \left(\lambda_{\min}(H_\nu) + \lambda_{\max}(H_\nu)\right)\right) CC^T, \\
\Xi_{\nu,13} =& \left(\sqrt{\pi_{\nu k^\nu_1}(\ell)} Q_\nu, \sqrt{\pi_{\nu k^\nu_2}(\ell)} Q_\nu, \cdots, \sqrt{\pi_{\nu k^\nu_{\varpi-1}}(\ell)} Q_\nu, \sqrt{\pi_{\nu k^\nu_{\varpi+1}}(\ell)} Q_\nu, \cdots, \sqrt{\pi_{\nu k^\nu_\hbar}(\ell)} Q_\nu\right), \\
\Xi_{\nu,33} =& -\mathrm{diag}\{Q_{k^\nu_1}, Q_{k^\nu_2}, \cdots, Q_{k^\nu_{\varpi-1}}, Q_{k^\nu_{\varpi+1}}, \cdots, Q_{k^\nu_\hbar}\}, \\
\Theta_{\nu,11} =& Q_\nu A^T + AQ_\nu + BB^T + \sum\nolimits_{s \in S^\nu_k} \pi_{\nu s}(\ell) \mathcal{X}_\nu + Q_\nu \alpha_2 + \left(-2\gamma \lambda_{\min}(H_\nu) + \gamma \sqrt{\sigma} \left(\lambda_{\min}(H_\nu) + \lambda_{\max}(H_\nu)\right)\right) CC^T, \\
\Theta_{\nu,13} =& \left(\sqrt{\pi_{\nu k^\nu_1}(\ell)} Q_\nu, \sqrt{\pi_{\nu k^\nu_2}(\ell)} Q_\nu, \cdots, \sqrt{\pi_{\nu k^\nu_\hbar}(\ell)} Q_\nu\right), \\
\Theta_{\nu,33} =& -\mathrm{diag}\{Q_{k^\nu_1}, Q_{k^\nu_2}, \cdots, Q_{k^\nu_\hbar}\}, \\
Q_\nu =& P_\nu^{-1}, \ Q = P^{-1}, \ \mathcal{X}_\nu = Q_\nu \mathcal{Q}_\nu Q_\nu, \ K_\nu = C^T P_\nu.
\end{aligned}$$

Proof At first, the Lyapunov function is constructed as follows

$$\mathcal{V}(\eta(\iota), \iota, \nu(\iota)) = \eta^T(\iota)(I_N \otimes P_{\nu(\iota)})\eta(\iota). \tag{24}$$

An infinitesimal operator $\mathcal{A}$ is defined as below

$$\mathcal{A}\mathcal{V}(\eta(\iota), \iota, \nu(\iota)) = \lim_{\triangle \to 0^+} \frac{1}{\triangle} \left\{ \mathbb{E}[\mathcal{V}(\eta(\iota + \triangle), \iota + \triangle, \nu(\iota + \triangle)) | \eta(\iota), \iota, \nu(\iota)] - \mathcal{V}(\eta(\iota), \iota, \nu(\iota)) \right\}. \tag{25}$$

To facilitate the operation, let $\nu(\iota) = \nu$. Since $\sum_{s=1}^m \pi_{\nu s}(\ell) = 0$, for any appropriate dimensions matrices $\mathcal{Q}_\nu, \nu \in S$, it can be deduced that

$$\sum_{s=1}^{m} \pi_{\nu s}(\ell) \eta^T(\iota)(I_N \otimes \mathcal{Q}_\nu)\eta(\iota) = 0. \tag{26}$$

Applying the Total Probability formula as well as the Conditional expectation formula, one has

$$\begin{aligned}
\mathbb{E}\{\mathcal{A}\mathcal{V}(\eta(\iota), \iota, \nu)\} = & 2\eta^T(\iota)(I_N \otimes P_\nu A)\eta(\iota) + 2\eta^T(\iota)(I_N \otimes P_\nu B) G(\eta(\iota), \iota) \\
& + 2\gamma \eta^T(\iota)(I_N \otimes P_\nu C K_\nu) w(\iota_k) + \eta^T(\iota) \sum\nolimits_{s \in S^\nu_k} \pi_{\nu s}(\ell)(I_N \otimes (P_s + \mathcal{Q}_\nu))\eta(\iota) \\
& + \eta^T(\iota) \sum\nolimits_{s \in S^\nu_{uk}} \pi_{\nu s}(\ell)(I_N \otimes (P_s + \mathcal{Q}_\nu))\eta(\iota).
\end{aligned} \tag{27}$$

Based on Assumption 1, it is obtained as

$$2\eta(\iota)(I_N \otimes P_\nu B)G(\eta(\iota),\iota) \leq \eta^T(\iota)(I_N \otimes (P_\nu BB^T P_\nu + \rho^2 I_N))\eta^T(\iota). \tag{28}$$

Combining (6) and (9), then

$$2\gamma\eta^T(\iota)(I_N \otimes P_\nu CK_\nu)w(\iota_k) = 2\gamma\eta^T(\iota)(I_N \otimes P_\nu CK_\nu)\xi(\iota) - 2\gamma\eta^T(\iota)(H_\nu \otimes P_\nu CK_\nu)\eta(\iota). \tag{29}$$

Let $K_\nu = C^T P_\nu$, based on Young's inequality: $2ab \leq \varepsilon a^2 + \frac{1}{\varepsilon}b^2$, where $\varepsilon > 0$, one gets

$$2\gamma\eta^T(\iota)(I_N \otimes P_\nu CC^T P_\nu)\xi(\iota) \leq \gamma\varepsilon\eta^T(\iota)(I_N \otimes P_\nu CC^T P_\nu)\eta(\iota) + \gamma\frac{1}{\varepsilon}\xi^T(\iota)(I_N \otimes P_\nu CC^T P_\nu)\xi(\iota). \tag{30}$$

According to ET strategy (16), one has

$$\xi_i^T(\iota)\Lambda\xi_i(\iota) \leq \sigma w_i^T(\iota)\Lambda w_i(\iota), \tag{31}$$

therefore, combining (9), (30) and (31), one obtains

$$\begin{aligned}2\gamma\eta^T(\iota)(I_N \otimes P_\nu CC^T P_\nu)\xi(\iota) &\leq \gamma\frac{\sigma}{\varepsilon}w^T(\iota)(I_N \otimes P_\nu CC^T P_\nu)w(\iota) + \gamma\varepsilon\eta^T(\iota)(I_N \otimes P_\nu CC^T P_\nu)\eta(\iota)\\ &= \gamma\varepsilon\eta^T(\iota)(I_N \otimes P_\nu CC^T P_\nu)\eta(\iota) + \gamma\frac{\sigma}{\varepsilon}\eta^T(\iota)(H_\nu H_\nu \otimes P_\nu CC^T P_\nu)\eta^T(\iota).\end{aligned} \tag{32}$$

From (27)-(29) and (32), it can be deduced

$$\begin{aligned}\mathbb{E}\{\mathcal{AV}(\eta(\iota),\iota,\nu)\} \leq\ &\eta^T(\iota)[I_N \otimes (A^T P_\nu + P_\nu A + P_\nu BB^T P_\nu + \rho^2 I_N\\ &+ \sum\nolimits_{s\in S_k^\nu}\pi_{\nu s}(\ell)(P_s + \mathcal{Q}_\nu)) - 2\gamma(H_\nu \otimes P_\nu CC^T P_\nu)\\ &+ \gamma\varepsilon(I_N \otimes P_\nu CC^T P_\nu) + \gamma\frac{\sigma}{\varepsilon}(H_\nu H_\nu \otimes P_\nu CC^T P_\nu)]\eta(\iota)\\ &+ \eta^T(\iota)\sum\nolimits_{s\in S_{uk}^\nu}\pi_{\nu s}(\ell)(I_N \otimes (P_s + \mathcal{Q}_\nu))\eta(\iota).\end{aligned} \tag{33}$$

Since $H_\nu > 0$, there exists a nonsingular matrix $\mathcal{R}$ satisfying $\mathcal{R}^T H_\nu \mathcal{R} = diag\{\lambda_{min}(H_\nu), \cdots, \lambda_{max}(H_\nu)\}$. Let $\hat{\eta}(\iota) = (\mathcal{R} \otimes I)\eta(\iota)$, then (33) can become as

$$\begin{aligned}\mathbb{E}\{\mathcal{AV}(\eta(\iota),\iota,\nu)\} \leq\ &\hat{\eta}^T(\iota)[I_N \otimes (A^T P_\nu + P_\nu A + P_\nu BB^T P_\nu + \rho^2 I_N + \sum\nolimits_{s\in S_k^\nu}\pi_{\nu s}(\ell)(P_s + \mathcal{Q}_\nu)\\ &+ (\Psi \otimes P_\nu CC^T P_\nu)]\hat{\eta}(\iota) + \hat{\eta}^T(\iota)\sum\nolimits_{s\in S_{uk}^\nu}\pi_{\nu s}(\ell)(I_N \otimes (P_s + \mathcal{Q}_\nu))\hat{\eta}(\iota),\end{aligned} \tag{34}$$

where $\Psi = \left\{-2\gamma\lambda_{\min}(H_\nu) + \frac{\gamma\sigma}{\varepsilon}\lambda^2_{\min}(H_\nu), \cdots, -2\gamma\lambda_{\max}(H_\nu) + \frac{\gamma\sigma}{\varepsilon}\lambda^2_{\max}(H_\nu)\right\} + \gamma\varepsilon I$.

Let $\varepsilon = \sqrt{\sigma}\lambda_{\max}(H_\nu)$, and taking it into (34), one can introduce

$$\begin{aligned}\mathbb{E}\{\mathcal{AV}(\eta(\iota),\iota,\nu)\} \leq\ &\hat{\eta}^T(\iota)[I_N \otimes (A^T P_\nu + P_\nu A + P_\nu BB^T P_\nu + \rho^2 I_N + \sum\nolimits_{s\in S_k^\nu}\pi_{\nu s}(\ell)(P_s + \mathcal{Q}_\nu)\\ &+ (-2\gamma\lambda_{\min}(H_\nu) + \gamma\sqrt{\sigma}(\lambda_{\min}(H_\nu) + \lambda_{\max}(H_\nu)))P_\nu CC^T P_\nu)]\hat{\eta}(\iota)\\ &+ \hat{\eta}^T(\iota)\sum\nolimits_{s\in S_{uk}^\nu}\pi_{\nu s}(\ell)(I_N \otimes (P_s + \mathcal{Q}_\nu))\hat{\eta}(\iota).\end{aligned} \tag{35}$$

Let $Q_\nu = P_\nu^{-1}$, multiplying the left and right sides of (35) by Q_ν, one gets

$$\begin{aligned}\mathbb{E}\{\mathcal{AV}(\eta(\iota),\iota,\nu)\} \leq\ &\hat{\eta}^T(\iota)[I_N \otimes (Q_\nu A^T + AQ_\nu + BB^T + \rho^2 Q_\nu Q_\nu + \sum\nolimits_{s\in S_k^\nu}\pi_{\nu s}(\ell)Q_\nu P_s Q_\nu\\ &+ \sum\nolimits_{s\in S_k^\nu}\pi_{\nu s}(\ell)Q_\nu\mathcal{Q}_\nu Q_\nu + \pi_{\nu\nu}(\ell)Q_\nu + (-2\gamma\lambda_{\min}(H_\nu) + \gamma\sqrt{\sigma}(\lambda_{\min}(H_\nu)\\ &+ \lambda_{\max}(H_\nu)))CC^T)]\hat{\eta}(\iota) + \hat{\eta}^T(\iota)\sum\nolimits_{s\in S_{uk}^\nu}\pi_{\nu s}(\ell)(I_N \otimes (P_s + \mathcal{Q}_\nu))\hat{\eta}(\iota).\end{aligned} \tag{36}$$

For any $\nu \in S_k^\nu$, and according to (17), if the following inequality

$$P_s + \mathcal{Q}_\nu \leq 0, \forall s \in S_{uk}^\nu, s \neq \nu \tag{37}$$

holds, then it yields

$$\mathbb{E}\{\mathcal{A}\mathcal{V}(\eta(\iota), \iota, \nu)\} \leq -\alpha_1 \mathbb{E}\{\mathcal{V}(\eta(\iota), \iota, \nu)\}.$$

At the same time, for any $\nu \in S_{uk}^\nu$, one gets

$$\begin{aligned}\mathbb{E}\{\mathcal{A}\mathcal{V}(\eta(\iota), \iota, \nu)\} \leq \hat{\eta}^T(\iota)[I_N \otimes (Q_\nu A^T + AQ_\nu + BB^T + \rho^2 Q_\nu Q_\nu + \sum\nolimits_{s \in S_k^\nu} \pi_{\nu s}(\ell) Q_\nu P_s Q_\nu \\ + \sum\nolimits_{s \in S_k^\nu} \pi_{\nu s}(\ell) Q_\nu \mathcal{Q}_\nu Q_\nu + \pi_{\nu\nu}(\ell) Q_\nu + (-2\gamma \lambda_{\min}(H_\nu) + \gamma \sqrt{\sigma}(\lambda_{\min}(H_\nu) \\ + \lambda_{\max}(H_\nu)))CC^T)]\hat{\eta}(\iota) + \hat{\eta}^T(\iota) \sum\nolimits_{s \in S_{uk}^\nu} \pi_{\nu s}(\ell)(I_N \otimes (P_s + \mathcal{Q}_\nu))\hat{\eta}(\iota).\end{aligned} \tag{38}$$

Combining (18), if the following inequalities

$$P_s + \mathcal{Q}_\nu \geq 0, \forall s \in S_{uk}^\nu, s = \nu, \tag{39}$$

$$P_s + \mathcal{Q}_\nu \leq 0, \forall s \in S_{uk}^\nu, s \neq \nu \tag{40}$$

hold, therefore one has

$$\mathbb{E}\{\mathcal{A}\mathcal{V}(\eta(\iota), \iota, \nu)\} \leq -\alpha_2 \mathbb{E}\{\mathcal{V}(\eta(\iota), \iota, \nu)\}.$$

Let $\epsilon = min(\alpha_1, \alpha_2)$, one gets

$$\mathbb{E}\{\mathcal{V}(\eta(\iota), \iota, \nu)\} \leq \exp(-\epsilon(\iota - h_{n-1} - \partial_{n-1}))\mathbb{E}\{\mathcal{V}(\eta(h_{n-1} + \partial_{n-1}), h_{n-1} + \partial_{n-1}, \nu)\}, \iota \in [h_{n-1} + \partial_{n-1}, h_n). \tag{41}$$

In the attack interval, the Lyapunov function is constructed as

$$\mathcal{V}(\eta(\iota), \iota, \nu(\iota)) = \eta^T(\iota)(I_N \otimes P)\eta(\iota). \tag{42}$$

Due to the communication network among agents suffering from DoS attacks, causing communication to be interrupted, i.e., control input $u_i(\iota) = 0$, then

$$\mathbb{E}\{\mathcal{A}\mathcal{V}(\eta(\iota), \iota, \nu)\} = 2\eta^T(\iota)(I_N \otimes PA)\eta(\iota) + 2\eta^T(\iota)(I_N \otimes PB)G(\eta(\iota), \iota). \tag{43}$$

According to Assumption 1, one has

$$\begin{aligned}2\eta(\iota)(I_N \otimes PB)G(\eta(\iota), \iota) &\leq G^T(\eta_i(\iota), \iota)G(\eta_i(\iota), \iota) + \sum_{i=1}^{N} \eta_i^T(\iota) PBB^T P \eta_i(\iota) \\ &\leq \eta^T(\iota)(I_N \otimes (PBB^T P + \rho^2 I_N))\eta^T(\iota).\end{aligned} \tag{44}$$

Bringing (44) into (43), one gets

$$\mathbb{E}\{\mathcal{A}\mathcal{V}(\eta(\iota), \iota, \nu)\} = \eta^T(\iota)[I_N \otimes (PA + A^T P + PBB^T P + \rho^2 I_N)]\eta^T(\iota). \tag{45}$$

According to the (21), then

$$\mathbb{E}\{\mathcal{A}\mathcal{V}(\eta(\iota), \iota, \nu)\} \leq \beta \mathbb{E}\{\mathcal{V}(\eta(\iota), \iota, \nu)\}, \tag{46}$$

therefore

$$\mathbb{E}\left\{\mathcal{V}(\eta(\iota),\iota,\nu)\right\} \leq \exp(\beta(\iota-h_n))\mathbb{E}\left\{\mathcal{V}(\eta(h_n),h_n,\nu)\right\}, \iota\in[h_n,h_n+\partial_n+\Delta_*). \tag{47}$$

Assume $\mathcal{V}(\iota)=\mathcal{V}_{\phi(\iota)}(\iota)$ are the communication interval and attack interval of the switching systems, respectively, where $\phi(\iota)\in\{1,2\}$, i.e., $\mathcal{V}_1(\iota)$ and $\mathcal{V}_2(\iota)$ corresponding communication interval and attack interval, then

$$\begin{cases} \mathbb{E}\{\mathcal{V}(\iota)\} \leq \exp(-\epsilon(\iota-h_{n-1}-\partial_{n-1}))\mathbb{E}\left\{\mathcal{V}_1(h_{n-1}+\partial_{n-1})\right\}, & \iota\in[h_{n-1}+\partial_{n-1},h_n), \\ \mathbb{E}\{\mathcal{V}(\iota)\} \leq \exp(\beta(\iota-h_n))\mathbb{E}\left\{\mathcal{V}_2(h_n)\right\}, & \iota\in[h_n,h_n+\partial_n+\Delta_*). \end{cases} \tag{48}$$

If $\iota\in[h_{n-1}+\partial_{n-1},h_n)$, according to (48), one has

$$\begin{aligned} \mathbb{E}\{\mathcal{V}(\iota)\} &\leq \exp(-\epsilon(\iota-h_{n-1}-\partial_{n-1}))\mathbb{E}\left\{\mathcal{V}_1(h_{n-1}+\partial_{n-1})\right\} \\ &\leq \mu\exp(-\epsilon(\iota-h_{n-1}-\partial_{n-1}))\mathbb{E}\left\{\mathcal{V}_1(h_{n-1}+\partial_{n-1})\right\} \\ &\leq \mu\exp(-\epsilon(\iota-h_{n-1}-\partial_{n-1}))\left[\exp(\beta(\iota-h_{n-2}-\partial_{n-2}))\mathbb{E}\left\{\mathcal{V}_1(h_{n-2}+\partial_{n-2})\right\}\right] \\ &\quad\cdots \\ &\leq \mu^n\exp(-\epsilon|\hat{\Pi}_s(\iota_0,\iota)|)\exp(\beta|\hat{\Pi}_a(\iota_0,\iota)|)\mathbb{E}\left\{\mathcal{V}_1(\iota_0)\right\}. \end{aligned} \tag{49}$$

If $\iota\in[h_n,h_n+\partial_n+\Delta_*)$, according to the (48), it can be deduced that

$$\begin{aligned} \mathbb{E}\{\mathcal{V}(\iota)\} &\leq \exp(\beta(\iota-h_n))\mathbb{E}\left\{\mathcal{V}_2(h_n)\right\} \\ &\leq \mu\exp(\beta(\iota-h_n))\mathbb{E}\left\{\mathcal{V}_2(h_n^-)\right\} \\ &\leq \mu\exp(\beta(\iota-h_n)\left[\exp(-\epsilon(h_n-h_{n-1}-\partial_{n-1}))\mathbb{E}\left\{\mathcal{V}_2(h_{n-1}+\partial_{n-1})\right\}\right] \\ &\quad\cdots \\ &\leq \mu^{n+1}\exp(-\epsilon|\hat{\Pi}_s(\iota_0,\iota)|)\exp(\beta|\hat{\Pi}_a(\iota_0,\iota)|)\mathbb{E}\left\{\mathcal{V}_1(\iota_0)\right\}, \end{aligned} \tag{50}$$

where $\mu>0$.

Based on Assumptions 4 and 5, when $\iota\in[h_{n-1}+\partial_{n-1},h_n)$, $N_a(t_0,\iota)=n$; when $\iota\in[h_n,h_n+\partial_n+\Delta_*)$, $N_a(\iota_0,\iota)=n+1$. According to (49) and (50), the following inequality

$$\mathbb{E}\{\mathcal{V}(\iota)\} \leq \mu^{N_a(\iota_0,\iota)}\exp(-\epsilon|\hat{\Pi}_s(\iota_0,\iota)|)\exp(\beta|\hat{\Pi}_a(\iota_0,\iota)|)\mathbb{E}\left\{\mathcal{V}(\iota_0)\right\} \tag{51}$$

holds. Taking (12) and (13) into (50), one gets

$$\begin{aligned} \mathbb{E}\left\{\mathcal{V}(\iota)\right\} &\leq \mu^{N_a(\iota_0,\iota)}\exp(-\epsilon(\iota-\iota_0-\hat{\Pi}_a(\iota_0,\iota))+(\beta|\hat{\Pi}_a(\iota_0,\iota)|))\mathbb{E}\left\{\mathcal{V}(\iota_0)\right\} \\ &= \mu^{N_a(\iota_0,\iota)}\exp(-\epsilon(\iota-\iota_0)+(\epsilon+\beta)|\hat{\Pi}_a(\iota_0,\iota)|)\mathbb{E}\left\{\mathcal{V}(\iota_0)\right\} \\ &\leq \mu^{N_a(\iota_0,\iota)}\exp(-\epsilon(\iota-\iota_0)+(\epsilon+\beta)(T_0+\frac{\iota-\iota_0}{\zeta_a}+(1+N_a(\iota_0,\iota))\Delta_*))\mathbb{E}\left\{\mathcal{V}(\iota_0)\right\} \\ &= \exp(N_a(\iota_0,\iota)(\ln(u)+\Delta_*(\epsilon+\beta)))\exp(-\epsilon(\iota-\iota_0))\exp((\epsilon+\beta)(T_0+\Delta_*)) \\ &\quad\times\exp(\frac{\epsilon+\beta}{\zeta_a}(\iota-\iota_0))\mathbb{E}\left\{\mathcal{V}(\iota_0)\right\}. \end{aligned} \tag{52}$$

Let $\hat{\hbar}=\epsilon-\frac{(\epsilon+\beta)}{\zeta_a}-\hbar_1$, combining (22) and (23), then the following inequality can be derived from (52)

$$\mathbb{E}\left\{\mathcal{V}(\iota)\right\} \leq \exp((\epsilon+\beta)(T_0+\Delta_*))\exp(-\hat{\hbar}(\iota-\iota_0))\mathbb{E}\left\{\mathcal{V}(\iota_0)\right\}. \tag{53}$$

According to Definition 1, the MASs achieve LFCs under DoS attacks. The proof is completed.

Due to there exists time-varying term $\pi_{\nu s}(\ell)$, Theorem 3.2 cannot be solved by LMI. This problem can be solved by Theorem 3.3.

Theorem 3.3. *For given positive scalars ϵ, α_1, α_2, β,ρ, μ and $0 < \hbar_1 < \epsilon$ and $0 < \sigma < 1$, the MASs* (1) *and* (2) *are able to achieve LFCs under the control protocol* (5)*, if there exist appropriate dimension positive definite matrices Q_ν, Q and $\mathcal{X} = \mathcal{X}_\nu^T$, and the Assumptions 1-5 hold, for any $\nu \in S$, satisfying*

$$\begin{bmatrix} \underline{\Xi}_{\nu,11} & Q_\nu & \underline{\Xi}_{\nu,13} \\ * & -\rho^{-2}I_N & 0 \\ * & * & \Xi_{\nu,33} \end{bmatrix} < 0,\ \nu \in S_k^\nu, \tag{54}$$

$$\begin{bmatrix} \underline{\Theta}_{\nu,11} & Q_\nu & \underline{\Theta}_{\nu,13} \\ * & -\rho^{-2}I_N & 0 \\ * & * & \Theta_{\nu,33} \end{bmatrix} < 0,\ \nu \in S_{uk}^\nu, \tag{55}$$

$$\begin{bmatrix} \overline{\Xi}_{\nu,11} & Q_\nu & \overline{\Xi}_{\nu,13} \\ * & -\rho^{-2}I_N & 0 \\ * & * & \Xi_{\nu,33} \end{bmatrix} < 0,\ \nu \in S_k^\nu, \tag{56}$$

$$\begin{bmatrix} \overline{\Theta}_{\nu,11} & Q_\nu & \overline{\Theta}_{\nu,13} \\ * & -\rho^{-2}I_N & 0 \\ * & * & \Theta_{\nu,33} \end{bmatrix} < 0,\ \nu \in S_{uk}^\nu, \tag{57}$$

$$\begin{pmatrix} \mathcal{X}_\nu & Q_\nu \\ * & -Q_s \end{pmatrix} \leq 0,\ \forall s \in S_{uk}^\nu,\ s \neq \nu, \tag{58}$$

$$Q_s + \mathcal{X}_\nu \geq 0,\ \forall s \in S_{uk}^\nu,\ s = \nu, \tag{59}$$

$$\begin{bmatrix} QA^T + AQ + BB^T - Q\beta & Q \\ * & -\rho^{-2}I_N \end{bmatrix} < 0, \tag{60}$$

$$F_a = \frac{\epsilon + \beta}{t - t_0} \leq \frac{\hbar_1}{\ln(u) + \Delta_*(\epsilon + \beta)}, \tag{61}$$

$$\zeta_a > \frac{\epsilon + \beta}{\epsilon + \hbar_1}, \tag{62}$$

where

$$\begin{aligned}
\underline{\Xi}_{\nu,11} =& Q_\nu A^T + AQ_\nu + BB^T + \underline{\pi}_{\nu\nu}Q_\nu + \sum\nolimits_{s\in S_k^\nu} \underline{\pi}_{\nu s}\mathcal{X}_\nu + Q_\nu\alpha_1 \\
&+ \left(-2\gamma\lambda_{\min}(H_\nu) + \gamma\sqrt{\sigma}\left(\lambda_{\min}(H_\nu) + \lambda_{\max}(H_\nu)\right)\right)CC^T, \\
\underline{\Xi}_{\nu,13} =& (\sqrt{\underline{\pi}_{\nu k_1^\nu}}Q_\nu, \sqrt{\underline{\pi}_{\nu k_2^\nu}}Q_\nu, \cdots, \sqrt{\underline{\pi}_{\nu k_{\varpi-1}^\nu}}Q_\nu, \sqrt{\underline{\pi}_{\nu k_{\varpi+1}^\nu}}Q_\nu, \cdots, \sqrt{\underline{\pi}_{\nu k_h^\nu}}Q_\nu), \\
\overline{\Xi}_{\nu,11} =& Q_\nu A^T + AQ_\nu + BB^T + \overline{\pi}_{\nu\nu}Q_\nu + \sum\nolimits_{s\in S_k^\nu} \overline{\pi}_{\nu s}\mathcal{X}_\nu + Q_\nu\alpha_1 \\
&+ \left(-2\gamma\lambda_{\min}(H_\nu) + \gamma\sqrt{\sigma}\left(\lambda_{\min}(H_\nu) + \lambda_{\max}(H_\nu)\right)\right)CC^T, \\
\overline{\Xi}_{\nu,13} =& (\sqrt{\overline{\pi}_{\nu k_1^\nu}}Q_\nu, \sqrt{\overline{\pi}_{\nu k_2^\nu}}Q_\nu, \cdots, \sqrt{\overline{\pi}_{\nu k_{\varpi-1}^\nu}}Q_\nu, \sqrt{\overline{\pi}_{\nu k_{\varpi+1}^\nu}}Q_\nu, \cdots, \sqrt{\overline{\pi}_{\nu k_h^\nu}}Q_\nu), \\
\Xi_{\nu,33} =& -\operatorname{diag}\{Q_{k_1^\nu}, Q_{k_2^\nu}, \cdots, Q_{k_{\varpi-1}^\nu}, Q_{k_{\varpi+1}^\nu}, \cdots, Q_{k_h^\nu}\}, \\
\underline{\Theta}_{\nu,11} =& Q_\nu A^T + AQ_\nu + BB^T + \sum\nolimits_{s\in S_k^\nu} \underline{\pi}_{\nu s}\mathcal{X}_\nu + Q_\nu\alpha_2 + \left(-2\gamma\lambda_{\min}(H_\nu) + \gamma\sqrt{\sigma}\left(\lambda_{\min}(H_\nu) + \lambda_{\max}(H_\nu)\right)\right)CC^T, \\
\underline{\Theta}_{\nu,13} =& (\sqrt{\underline{\pi}_{\nu k_1^\nu}}Q_\nu, \sqrt{\underline{\pi}_{\nu k_2^\nu}}Q_\nu, \cdots, \sqrt{\underline{\pi}_{\nu k_h^\nu}}Q_\nu), \\
\overline{\Theta}_{\nu,11} =& Q_\nu A^T + AQ_\nu + BB^T + \sum\nolimits_{s\in S_k^\nu} \overline{\pi}_{\nu s}\mathcal{X}_\nu + Q_\nu\alpha_2 + \left(-2\gamma\lambda_{\min}(H_\nu) + \gamma\sqrt{\sigma}\left(\lambda_{\min}(H_\nu) + \lambda_{\max}(H_\nu)\right)\right)CC^T, \\
\overline{\Theta}_{\nu,13} =& (\sqrt{\overline{\pi}_{\nu k_1^\nu}}Q_\nu, \sqrt{\overline{\pi}_{\nu k_2^\nu}}Q_\nu, \cdots, \sqrt{\overline{\pi}_{\nu k_h^\nu}}Q_\nu), \\
\Theta_{\nu,33} =& -\operatorname{diag}\{Q_{k_1^\nu}, Q_{k_2^\nu}, \cdots, Q_{k_h^\nu}\}, \\
Q_\nu =& P_\nu^{-1},\ Q = P^{-1},\ \mathcal{X}_\nu = Q_\nu\mathcal{Q}_\nu Q_\nu,\ K_\nu = C^T P_\nu.
\end{aligned}$$

Proof Given a particular ℓ, there exist positive scalars θ_1, θ_2 satisfying $\theta_1 + \theta_2 = 1$ such that $\pi_{\nu s}(\ell) = \theta_1\underline{\pi}_{\nu s} + \theta_2\,\overline{\pi}_{\nu s}$. For any $\nu \in S_k^\nu$, according to the formula (35), define

$$\begin{aligned}
\Xi_\nu =& (A^T P_\nu + P_\nu A + P_\nu BB^T P_\nu + \rho^2 I_N + \sum\nolimits_{s\in S_k^\nu} (\theta_1\underline{\pi}_{\nu s} + \theta_2\overline{\pi}_{\nu s})(P_s + \mathcal{Q}_\nu) \\
&+ (-2\gamma\lambda_{\min}(H_\nu) + \gamma\sqrt{\sigma}(\lambda_{\min}(H_\nu) + \lambda_{\max}(H_\nu)))P_\nu CC^T P_\nu. \qquad (63)
\end{aligned}$$

Thus, $\Xi_\nu = \theta_1\underline{\Xi}_\nu + \theta_2\overline{\Xi}_\nu$. According to (54) and (56), by applying a similar proof as in Theorem 3.2, it follows that $\Xi_\nu < 0$. Similarly, for any $\nu \in S_{uk}^\nu$, in accordance to (55) and (57), one has $\Theta_\nu = \theta_1\underline{\Theta}_\nu + \theta_2\overline{\Theta}_\nu < 0$. In summary, by changing the parameters θ_1 and θ_2, all possible $\pi_{\nu s}(\ell) \in [\,\underline{\pi}_{\nu s}, \overline{\pi}_{\nu s}]$ can be achieved. The proof is completed.

4. Numerical example

Consider that MASs have a leader and three followers. The communication topologies without DoS attacks among agents are shown in Figure 2. The communication topologies with DoS attacks among agents are shown in Figure 3. Assume that constant coefficient matrices A, B, C and TRs matrix Π are as follows

$$A = \begin{bmatrix} -0.275 & 1.851 \\ -6.658 & -0.673 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$C = \begin{bmatrix} -1.104 & -0.302 \\ -8.029 & -2.046 \end{bmatrix},\ \Pi = \begin{bmatrix} \pi_{11}(\ell) & ? & ? \\ \pi_{21}(\ell) & ? & \pi_{23}(\ell) \\ \pi_{31}(\ell) & ? & ? \end{bmatrix},$$

where $\pi_{11}(\ell) \in [-0.2, 0.8]$, $\pi_{21}(\ell) \in [0.5, 0.1]$, $\pi_{23}(\ell) \in [0.7, 0.6]$, $\pi_{31}(\ell) \in [0.1, 0.4]$, ”?” denote the unknown elements.

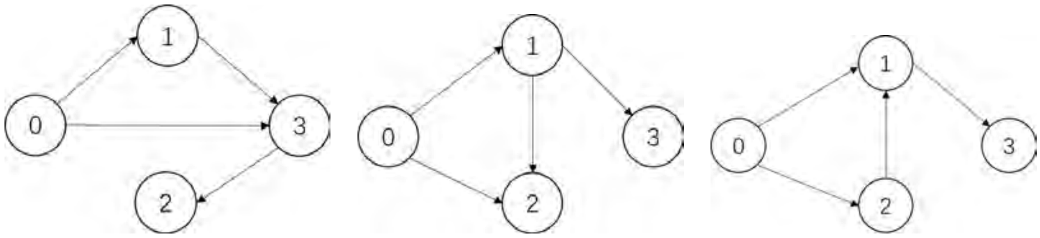

Figure 2 Communication topologies without DoS attacks

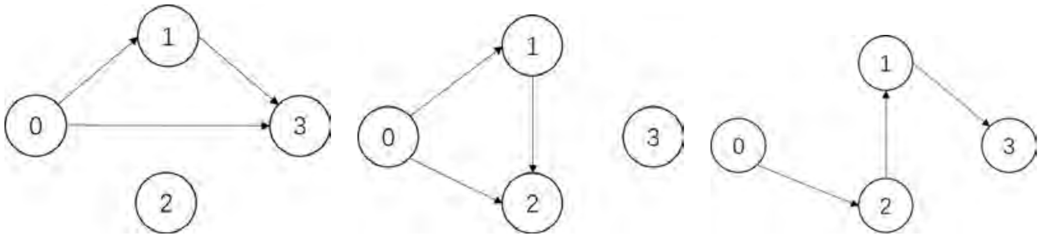

Figure 3 Communication topologies with DoS attacks

Assume that nonlinear functions $g(\iota, x_i(\iota)) = (0.01\sin(x_{i1}(\iota)), 0.01\sin(x_{i2}(\iota)))^T, i = 0,1,2,3$. Choose $\alpha_1 = 0.89$, $\sigma = 0.6$, $\alpha_2 = 0.68$, $\gamma = 0.21$, $\rho = 0.4$. By solving the linear matrix inequalities , it is easy to get the control gain matrices and trigger parameter matrices as below

$$K_1 = \begin{bmatrix} -2.4771 & -1.8088 \\ -0.6558 & -0.4639 \end{bmatrix}, \ K_2 = \begin{bmatrix} -2.2356 & -2.1526 \\ -0.5922 & -0.5512 \end{bmatrix}, \ K_3 = \begin{bmatrix} -0.8709 & -1.0974 \\ -0.2320 & -0.2805 \end{bmatrix},$$

$$\Lambda_1 = \begin{bmatrix} 6.5660 & 4.7849 \\ 4.7849 & 3.4871 \end{bmatrix}, \quad \Lambda_2 = \begin{bmatrix} 5.3485 & 5.1387 \\ 5.1387 & 4.9375 \end{bmatrix}, \quad \Lambda_3 = \begin{bmatrix} 0.8122 & 1.0208 \\ 1.0208 & 1.2830 \end{bmatrix}.$$

Choose $\epsilon = 0.89$, $\beta = 0.7$, $\mu = 2$, $\hbar_1 = 0.03$, $\Delta_* = 0.016$, then one can get $F_a \leq 0.042$, $\zeta_a > 2.2$.

Figure 4 shows the Markov jumping with three modes; Figure 5 depicts the ET instant for all the followers, which shows that the ET strategy reduces the number of communications as well as saves resources; Figure 6 and Figure 7 show the trajectories of the error systems η_{i1} and η_{i2} between the leader and the followers, respectively, the analysis of the two figures shows that the error tends to zero, so MASs achieve LFCs.

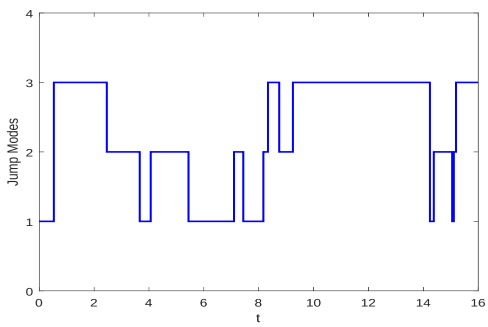

Figure 4 Markov jumping mode

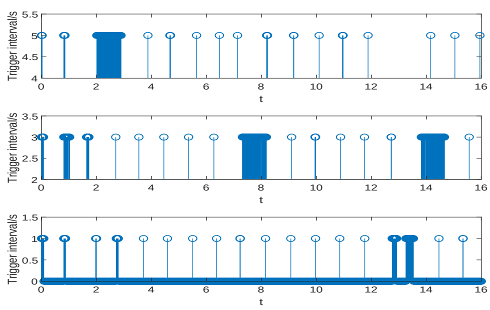

Figure 5 The trigger instants and trigger intervals

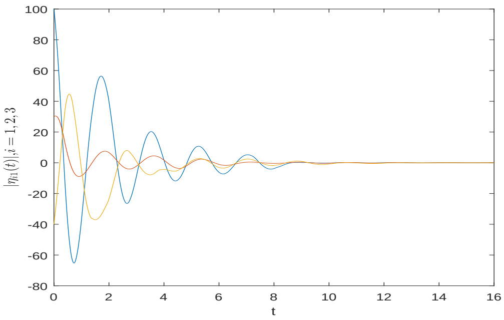

Figure 6 State trajectories of the error systems η_{i1}

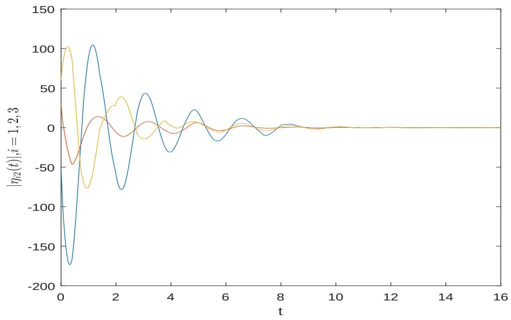

Figure 7 State trajectories of the error systems η_{i2}

5. Conclusion

The issue of LFCs of NMASs with MSTs subjected to DoS attacks under ET control has been addressed. Based on Lyapunov stability theory and Markov theory, using the ET strategy, sufficient conditions of LFCs for MASs have been obtained, which extends the results of some existing literature. An example has been provided to verify the effectiveness of the methods and the correctness of the results. We will investigate the LFCs problem of NMASs with MSTs subjected to multiple network attacks under dynamic ET in the future.

References

[1] N. Morri, S. Hadouaj, L. B. Said, An approach to intelligent control public transportation system using a multi-agent system, in: International Conference on Enterprise Information Systems, Springer, Cham (2020) 242–267.

[2] H. Du, W. Zhu, G. Wen, Z. Duan, J. Lu, Distributed formation control of multiple quadrotor aircraft based on nonsmooth consensus algorithms, IEEE Transactions on Cybernetics 49 (1) (2017) 342–353.

[3] Y. Chen, J. Lu, X. Yu, D. J. Hill, Multi-agent systems with dynamical topologies-consensus and applications, IEEE Circuits and Systems Magazine 13 (3) (2013) 21–34.

[4] W. Xing, R. Dong, Z. Wu, M. Gu, Adaptive event-triggered consensus for Markovain jumping multi-agent systems, Control and Decision 35 (11) (2020) 2780–2786.

[5] J. Liu, M. Yang, E. Tian, J. Cao, S. Fei, Event-based security control for state-dependent uncertain systems under hybrid-attacks and its application to electronic circuits, IEEE Transactions on Circuits and Systems 66 (12) (2019) 4817–4828.

[6] L. Wang, Z. Wu, Distributed secure consensus for first-order multi-agent systems under replay attacks, in: 2021 International Conference on Computer, Control and Robotics (ICCCR), Shanghai, China (2021) 250–255.

[7] D. Wang, Z. Wang, B. Shen, F. E. Alsaadi, T. Hayat, Recent advances on filtering and control for cyber-physical systems under security and resource constraints, Journal of the Franklin Institute 353 (11) (2016) 2451–2466.

[8] G. Wen, X. Zhai, Z. Peng, A. Rahmani, Fault-tolerant secure consensus tracking of delayed nonlinear multi-agent systems with deception attacks and uncertain parameters via impulsive control, Communications in Nonlinear Science and Numerical Simulation 82 (2019) 105043–105043.

[9] S. Yuan, C. Yu, J. Sun, Adaptive event-triggered consensus control of linear multi-agent systems with cyber attacks, Neurocomputing 442 (5) (2021) 1–9.

[10] Z. Feng, G. Hu, Secure cooperative event-triggered control of linear multiagent systems under DoS attacks, IEEE Transactions on Control Systems Technology 28 (3) (2019) 741–752.

[11] J. Liu, X. Wang, H. Li, Observer-based consensus for multi-agent systems subject to consecutive DoS attacks, Journal of Systems Science and Mathematical Sciences 42 (7) (2022) 1715–1726.

[12] X. Zhou, J. Cheng, J. Cao, M. Ragulskis, Asynchronous dissipative filtering for nonhomogeneous Markov switch ingneural networks with variable packet dropouts, Neural Networks 130 (2020) 229–237.

[13] X. Zhou, J. Cheng, J. Cao, J. H. Park, Event-based asynchronous dissipative filtering for fuzzy nonhomogeneous Markov switching systems with variable packet dropouts, Fuzzy Sets and Systems 432 (2022) 50–67.

[14] Z. Li, Q. Li, D. Ding, X. Xie, Event-based secure leader-following consensus for cyber-physical systems under denial-of-service attacks, International Journal of Robust and Nonlinear Control 32 (11) (2022) 6346–6360.

[15] X. Wang, H. Yang, S. Zhong, Improved results on consensus of nonlinear MASs with nonhomogeneous Markov switching topologies and DoS cyber attacks, Journal of the Franklin Institute 358 (14) (2021) 7237–7253.

[16] X. Wang, J. H. Park, H. Yang, An improved protocol to consensus of delayed MASs with UNMS and aperiodic DoS cyber-attacks, IEEE Transactions on Network Science and Engineering 8 (3) (2021) 2506–2516.

[17] X. Zhou, Y. Tang, J. Cheng, J. Cao, C. Xue, D. Yan, Nonstationary quantized control for discrete-time Markov jump singularly perturbed systems against deception attacks, Journal of the Franklin Institute 358 (6) (2021) 2915–2932.

[18] M. He, X. Mu, Z. Hu, Robust H_∞ leader-following consensus of multi-agent systems over Markovian switching topologies with randomly occurring uncertainties of communication links, Journal of Systems Science and Mathematical Sciences 41 (3) (2021) 589–601.

[19] X. Zhou, L. Chen, J. Cao, J. Cheng, Asynchronous filtering of MSRSNSs with the event-triggered try-once-discard protocol and deception attacks, ISA Transactions 131 (2022) 210–221.

[20] W. He, Z. Mo, Secure event-triggered consensus control of linear multiagent systems subject to sequential scaling attacks, IEEE Transactions on Cybernetics 52 (10) (2022) 10314–10327.

[21] P. Chen, D. Zhang, L. Yu, H. Yan, Dynamic event-triggered output feedback control for load frequency control in power systems with multiple cyber attacks, IEEE Transactions on Systems, Man, and Cybernetics: Systems 52 (10) (2022) 6246–6258.

[22] C. Deng, C. Wen, MAS-based distributed resilient control for a class of cyber-physical systems with communication delays under DoS attacks, IEEE Transactions on Cybernetics 51 (5) (2020) 2347–2358.