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On the topological locality of antisymmetric connectedness

Filiz Yıldız[a], Nezakat Javanshir[b]

[a]*Department of Mathematics, Hacettepe University, Ankara, Turkey*
[b]*Department of Mathematics, Yıldırım Beyazıt University, Ankara, Turkey*

Abstract. The theory of antisymmetric connectedness for a T_0-quasi-metric space was established in terms of graph theory lately, as corresponding counterpart of the connectedness for the complement of a graph. Following that in the current study, a topological localized version of the antisymmetrically connected spaces is described and studied through a variety of approaches in the context of T_0-quasi-metrics.

Within the framework of this, we examine the cases under which conditions a T_0-quasi-metric space would become locally antisymmetrically connected as well as some topological characterizations of locally antisymmetrically connected T_0-quasi-metric spaces are presented, especially via metrics.

1. Introduction

The theory of antisymmetric connectedness for a T_0-quasi-metric space was introduced and investigated in detail, for the first time in [11]. This theory was especially discussed in terms of graph theory [2, 9, 10] as corresponding counterpart of the connectedness for the complement of a graph. In particular, by describing the notion of symmetry graph in [11], it is also observed that there were natural relations between the theory of antisymmetrically connected T_0-quasi-metric spaces and the theory of connected complementary graphs.

On the other hand, as is well-known from topology, it is useful and customary to localize (see [7, 12]) some topological properties in a natural way. Therefore, following the theory of antisymmetrically connected T_0-quasi-metric spaces constructed in recent years, in the present investigation the authors introduce and discuss the localized version of the antisymmetric connectedness theory as a new idea in the context of asymmetric topology. Indeed, the locality status of antisymmetrically connected spaces can be also considered as another alternative method in order to approach to the asymmetry of non-metric T_0-quasi-metrics.

Accordingly, some necessary background material for this study is presented in Section 2. In particular, it mostly consists of the required information about antisymmetrically connected T_0-quasi-metric spaces as well as antisymmetric T_0-quasi-metric spaces which were treated for the first-time in [11] as a kind of opposite to metric spaces. Therefore, as far as these types of T_0-quasi-metric spaces are concerned, we will conclude Section 2 by recalling some crucial definitions, propositions and results that will enable a casual reader to follow the general ideas presented in this paper.

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Email addresses: yfiliz@hacettepe.edu.tr (Filiz Yıldız), nezakat.javanshir@aybu.edu.tr (Nezakat Javanshir)

After giving the preliminaries, as the main purpose of the paper; in Section 3, we will construct the locality form of antisymmetric connectedness theory [11] under the name *locally antisymmetrically connected*, as a new alternative method for determining the degree of asymmetry of non-metric T_0-quasi-metrics (see also [3, 8] for the other different methods). Following that, some relationships between antisymmetric connectedness and locally antisymmetric connectedness are investigated via some theorems and (counter)examples, in the context of T_0-quasi-metric spaces.

Specifically, it is proved that all finite T_0-quasi-metric spaces are locally antisymmetrically connected. In addition, the property of local antisymmetric connectedness is characterized with the help of various topological structures and conditions.

As another discussion in this framework, it is also natural to ask that under which conditions a T_0-quasi-metric space will become locally antisymmetrically connected. Regarding this question, in Section 3, some answers are presented and studied under certain conditions. In the end of this section, many topological characterizations of locally antisymmetrically connected T_0-quasi-metric spaces are also observed, especially via metrics.

Finally, Section 4 as the last part of the paper gives a conclusion about the whole of the work.

2. Background

The basic information presented in this section is taken from [11]. Now let us start by recalling some crucial notions and examples, especially related to the theory of "antisymmetrically connected T_0-quasi-metric spaces".

Definition 2.1. Let X be a set and $q : X \times X \to [0, \infty)$ a function. Then q is called a *quasi-pseudometric* on X if

(a) $q(x, x) = 0$ whenever $x \in X$,
(b) $q(x, z) \leq q(x, y) + q(y, z)$ whenever $x, y, z \in X$.

We shall say that q is a T_0*-quasi-metric* provided that q also satisfies the following condition:
For each $x, y \in X$,

$$q(x, y) = 0 = q(y, x) \text{ implies that } x = y.$$

Remark 2.2. Let q be a T_0-quasi-metric on a set X. Then the function $q^{-1} : X \times X \to [0, \infty)$ defined by $q^{-1}(x, y) = q(y, x)$ whenever $x, y \in X$, is also a T_0-quasi-metric, called the *conjugate* T_0*-quasi-metric of* q. If $q = q^{-1}$ then q is a *metric*. In line with the usual notational conventions, we write

$$q^s = \sup\{q, q^{-1}\} = q \vee q^{-1}$$

for the symmetrization of q (which obviously is a metric).

The notation τ_{q^s} denotes the topology induced by the symmetrization metric q^s and it is called *symmetrization topology of* q.

An adequate introduction to the theory of T_0-quasi-metrics and the motivation for their study may be obtained from the works [1, 3–6]. Now, let us recall the main structures required for the paper.

Definition 2.3. Let (X, d) be a T_0-quasi-metric space. A pair $(x, y) \in X \times X$ is called

i) an *antisymmetric pair* if it satisfies the condition $d(x, y) \neq d(y, x)$.
ii) a *symmetric pair* if it satisfies the condition $d(x, y) = d(y, x)$.

Definition 2.4. Let (X, d) be a T_0-quasi-metric space. A finite sequence of points in X, starting at x and ending with y, is called a (finite) *antisymmetric path* (*symmetric path*) $P_{x,y} = (x = x_0, x_1, \ldots, x_{n-1}, x_n = y)$, $n \in \mathbb{N}$, from x to y provided that all the pairs (x_i, x_{i+1}) are antisymmetric pairs (symmetric pairs) for $i \in \{0, 1, \ldots, n-1\}$. Note here that no point occurs twice in an antisymmetric (symmetric) path.

Hence, we are in a position to recall the notion of antisymmetric connectedness from [11], as follows:

Definition 2.5. i) In a T_0-quasi-metric space (X, d), two points $x, y \in X$ will be called *antisymmetrically connected* if there is an antisymmetric path $P_{x,y}$ starting at x and ending with y, or $x = y$.

By definition, it is easy to verify that "antisymmetric connectedness" is an equivalence relation on X.

ii) The equivalence class of a point $x \in X$ with respect to the antisymmetric connectedness relation T_d will be called the *antisymmetry component* of x, and it is denoted by

$$T_d(x) = \{y \in X : \text{there is an antisymmetric path from } x \text{ to } y\}.$$

It is clear that $T_d(x)$ is the largest antisymmetrically connected subspace of X containing $x \in X$.

iii) If $T_d = X \times X$, or $T_d(x) = X$ for each $x \in X$, then the T_0-quasi-metric space (X, d) will be called *antisymmetrically connected*.

Now, let us present a well-known antisymmetrically connected T_0-quasi-metric space as follows:

Example 2.6. On the set $\mathbb{R}$ of the reals, take $u(x, y) = max\{x - y, 0\}$ whenever $x, y \in \mathbb{R}$. It is easy to verify that u is a T_0-quasi-metric, called the *standard* T_0-quasi-metric on $\mathbb{R}$. Moreover, the space $(\mathbb{R}, u)$ is antisymmetrically connected since $T_u(x) = \mathbb{R}$ for each $x \in \mathbb{R}$.

At this stage, we can turn our attention to the other notions and details, required for the paper. Then let us recall a notion opposite to that of "metric" , firstly:

Definition 2.7. We shall call a T_0-quasi-metric space (X, d) *antisymmetric*, if

$$d(x, y) \neq d(y, x) \text{ whenever } x \neq y$$

for all $x, y \in X$.

Therefore, by Definition 2.5 iii) we have:

Proposition 2.8. *Each antisymmetric T_0-quasi-metric space is antisymmetrically connected.*

Additionally, now let us recall from [11] the dual counterpart of the antisymmetric connectedness in the context of T_0-quasi-metric spaces.

Definition 2.9. i) Let (X, d) be a T_0-quasi-metric space. We say that $x \in X$ is *symmetrically connected* to $y \in X$ if there is a symmetric path (see Definition 2.4) $P_{x,y}$, starting at the point x and ending at the point y.

By definition, it is easy to verify that "symmetric connectedness" is an equivalence relation on the set of points in X.

ii) The equivalence class of a point $x \in X$ with respect to the symmetric connectedness relation C_d will be called the *symmetry component* of x, and it is denoted by

$$C_d(x) = \{y \in X : \text{there is a symmetric path from } x \text{ to } y\}.$$

Obviously $C_d(x)$ is the largest symmetrically connected subspace of X containing $x \in X$.

iii) If $C_d = X \times X$, or $C_d(x) = X$ for each $x \in X$, then the T_0-quasi-metric space (X, d) will be called *symmetrically connected*.

In the light of above considerations, the next proposition was established in [11] as Corollary 25, by using the following crucial result well-known from graph theory.

For any graph G, G is connected or $\overline{G}$ the complement of G is connected in the sense of graph theory. (See [2, 9, 10])

Proposition 2.10. *Let (X, d) be a T_0-quasi-metric space. Then (X, d) is symmetrically connected or antisymmetrically connected.*

Incidentally, let us present an antisymmetrically connected T_0-quasi-metric space which is not antisymmetric, as follows:

Example 2.11. The (bounded) Sorgenfrey T_0-quasi-metric space $(\mathbb{R}, s)$, where $s(x,y) = \min\{x - y, 1\}$ if $x \geq y$ and $s(x,y) = 1$ if $x < y$, gives an example of a space which is antisymmetrically connected as well as symmetrically connected, but not antisymmetric.

The next notions will be required for the remainder of paper.

Definition 2.12. Let (X,d) be a T_0-quasi-metric space.

i) $x \in X$ is called an *antisymmetric point* if for each $y \in X \setminus \{x\}$, the pair (x,y) is antisymmetric.
ii) $x \in X$ is called a *symmetric point* if for each $y \in X$, the pair (x,y) is symmetric.

Obviously, $x \in X$ is symmetric point if and only if $T_d(x) = \{x\}$.

Hence, the next proposition which completes Section 2 can be seen easily via Definition 2.12.

Proposition 2.13. *Let (X,d) be a T_0-quasi-metric space.*

i) All points of X are antisymmetric if and only if (X,d) is antisymmetric space.
ii) (X,d) is metric space if and only if all points of X are symmetric.
iii) If X has an antisymmetric point then (X,d) is antisymmetrically connected space which is not symmetrically connected.
iv) If X has a symmetric point then (X,d) is not antisymmetrically connected space which is symmetrically connected.

After presenting the required background information, especially related to antisymmetric connectedness, we are now in a position to describe and study the localized version of the antisymmetrically connected spaces.

3. Local antisymmetric connectedness in T_0-quasi-metric spaces

Definition 3.1. Let (X,d) be a T_0-quasi-metric space and $x_0 \in X$. The space (X,d) is called *locally antisymmetrically connected at $x_0 \in X$* if $T_d(x_0) \in \tau_{d^s}$.

As mentioned in Section 2, τ_{d^s} denotes the topology generated by the metric $d^s = d \vee d^{-1}$.

A T_0-quasi-metric space (X,d) is called *locally antisymmetrically connected space* if (X,d) is locally antisymmetrically connected at each point of X.

That is, a T_0-quasi-metric space (X,d) is locally antisymmetrically connected if and only if $T_d(x)$ is τ_{d^s}-open for each $x \in X$.

Example 3.2. Consider the (bounded) Sorgenfrey T_0-quasi-metric space $(\mathbb{R}, s)$ from Example 2.11. It is known that all antisymmetry components $T_s(x)$ $(x \in \mathbb{R})$ in the space $(\mathbb{R}, s)$ are $\mathbb{R}$, and so they are open w.r.t. the topology τ_{s^s} generated by the symmetrization metric of s. Thus, $(\mathbb{R}, s)$ is locally antisymmetrically connected.

Example 3.3. Take the Star Space (X,d) constructed in [3, Example 2.12], as follows:
On $X = [0, \infty)$ define

$$d(x,y) = \begin{cases} x - y \ ; \ x \geq y \\ x + y \ ; \ x < y \end{cases} \text{ for each } x, y \in X.$$

Trivially, $T_d(0) = \{0\}$ since 0 is symmetric point (that is, $d(x,0) = d(0,x)$ for all $x \in X$), and $\{0\}$ is not τ_{d^s}-open since the topology τ_{d^s} is the usual (Euclidean) topology at 0. (Indeed, the neighborhood of 0 w.r.t the topology τ_{d^s} is the same as the neighborhood in the usual topology.) Therefore, (X,d) is not locally antisymmetrically connected.

Lemma 3.4. *A T_0-quasi-metric space (X,d) is locally antisymmetrically connected if and only if (X,d^{-1}) is locally antisymmetrically connected.*

Proof. Straightforward by the fact that $\tau_{d^s} = \tau_{(d^{-1})^s}$. □

Proposition 3.5. *Let (X, d) be a T_0-quasi-metric space. If (X, d) is antisymmetrically connected then (X, d) is locally antisymmetrically connected.*

Proof. By the antisymmetric connectedness of (X, d), we have that $T_d(x) = X$ whenever $x \in X$. Hence (X, d) is locally antisymmetrically connected since the sets $T_d(x)$ are τ_{d^s}-open. □

Note that the converse of Proposition 3.5 is not true as it will be seen in the next example. However, it will be true under a specific topological condition (see Proposition 3.9).

Example 3.6. Let us define a T_0-quasi-metric on the set $X = \{1, 2, 3\}$ by the matrix

$$\mathbf{V} = \begin{pmatrix} 0 & 3 & 6 \\ 3 & 0 & 4 \\ 5 & 4 & 0 \end{pmatrix}.$$

That is, $V = (v_{ij})$ where $v(i, j) = v_{ij}$ for $i, j \in X$. It is easy to prove that v is a T_0-quasi-metric on X and the space (X, v) is not antisymmetrically connected since there is no antisymmetric path from 1 to 2. Moreover, the metric topology τ_{v^s} is discrete topology on X, since the unique topology which is T_1 on a finite set is discrete topology. So, all the antisymmetry components are τ_{v^s}-open, and thus (X, v) is locally antisymmetrically connected.

Because of Proposition 2.8 and Proposition 3.5, the next result is straightforward.

Corollary 3.7. *If (X, d) is an antisymmetric space then (X, d) is locally antisymmetrically connected.*

The converse of Corollary 3.7 is not true. Really, in Example 3.6, the T_0-quasi-metric space (X, v) is not antisymmetric since $v(2, 3) = v(3, 2)$. However, it is locally antisymmetrically connected as mentioned there.

Incidentally, the converse of Proposition 3.5 holds only under a specific condition as it will be seen later. Before it, we must give the following required fact:

Proposition 3.8. *In a locally antisymmetrically connected T_0-quasi-metric space (X, d), all the antisymmetry components are τ_{d^s}-clopen.*

Proof. Clearly, all antisymmetry components are τ_{d^s}-open in the locally antisymmetrically connected T_0-quasi-metric space (X, d). Moreover, it is easy to verify that if all the components $T_d(x)$ are τ_{d^s}-open for each $x \in X$, then they are τ_{d^s}-closed as well, for each $x \in X$. Indeed, T_d is an equivalence relation, and X can be written as the union of all the equivalence classes which are pairwise disjoint. Hence, the claim is verified. □

As we promised above, we are now in a position to present the converse part of Proposition 3.5.

Proposition 3.9. *If (X, d) is locally antisymmetrically connected T_0-quasi-metric space and the topological space (X, τ_{d^s}) is connected then (X, d) is antisymmetrically connected.*

Proof. Note that in a locally antisymmetrically connected space (X, d), the set $T_d(x)$ is τ_{d^s}-clopen for $x \in X$, by Proposition 3.8. Thus $T_d(x) = X$ with the help of connectedness of the space (X, τ_{d^s}). Finally, this means that (X, d) is antisymmetrically connected. □

Proposition 3.10. *Each T_0-quasi-metric space (X, d) such that τ_{d^s} is discrete is locally antisymmetrically connected.*

Proof. Let (X, d) be a T_0-quasi-metric space such that τ_{d^s} is discrete. In this case, the proof is seen easily, since each subset of X is τ_{d^s}-open in X. □

The converse of Proposition 3.10 is not true as follows:

Example 3.11. Consider the function on $\mathbb{R}$:

$$e(x,y) = \begin{cases} y-x & ; x < y \\ 2(x-y) & ; x \geq y \end{cases}$$

for every $x, y \in \mathbb{R}$.

It is easy to show that $(\mathbb{R}, e)$ is a T_0-quasi-metric space. Moreover, it is antisymmetric by the fact that $e(x,y) = e(y,x)$ if and only if $x = y$. Thus, $(\mathbb{R}, e)$ is locally antisymmetrically connected by Corollary 3.7.

On the other hand, note that $m \leq e \leq 2m$, where $m(x,y) = |x-y|$. Thus, $e^s = 2m$ and so, τ_{e^s} is the usual (Euclidean) topology on $\mathbb{R}$, not discrete.

In particular, as the results of Proposition 3.10 we have:

Corollary 3.12. *(a) Each finite T_0-quasi-metric space is locally antisymmetrically connected.*
(b) Let $\leq$ be a partial order on a set X and $d_{\leq}$ its natural T_0-quasi-metric on X defined by

$$d_{\leq}(x,y) = \begin{cases} 0 \ ; & x \leq y \\ 1 \ ; & x > y \end{cases} .$$

In this case, the natural T_0-quasi-metric space $(X, d_{\leq})$ is locally antisymmetrically connected.

Proof. (a) Let (X,d) be a finite T_0-quasi-metric space. Thus, (X, τ_{d^s}) is discrete space since the unique topology which is T_1 on a finite set is discrete. So, by Proposition 3.10, (X,d) is locally antisymmetrically connected.

(b) Let $\leq$ be a partial order on X. So, according to the definition of $d_{\leq}$ on X, it is easy to verify that the symmetrization metric $d^s_{\leq} = d_{\leq} \vee d^{-1}_{\leq}$ is discrete, and so the topology $\tau_{d^s_{\leq}}$ is discrete. Finally, the space $(X, d_{\leq})$ is locally antisymmetrically connected by Proposition 3.10. $\square$

Now, if we turn our attention to the metric spaces, particulary we have:

Remark 3.13. Let (X,q) be any discrete metric space. It is not antisymmetrically connected since $T_q(x) = \{x\} \neq X$ (all points of X are symmetric), even if it is locally antisymmetrically connected via Proposition 3.10, by the fact that the topology $\tau_{q^s} = \tau_q$ is discrete.

The following characterization shows that the non-discrete metric spaces cannot be locally antisymmetrically connected.

Proposition 3.14. *A metric space (X,d) is locally antisymmetrically connected if and only if its induced topology τ_d is discrete.*

Proof. Let (X,d) be a metric space. Thus $d^s = d$ and also $T_d(x) = \{x\}$ for each $x \in X$, since all the points in X are symmetric (see Proposition 2.13 ii)). Now, suppose that (X,d) is locally antisymmetrically connected. In this case, for each $x \in X$, $T_d(x)$ is τ_{d^s}-open, that is τ_d-open. Therefore, for each $x \in X$, $\{x\}$ is τ_d-open. This means that τ_d is discrete topology.

In order to establish the converse, assume that (X,d) is a metric space and τ_d is discrete. Since $d^s = d$, we have $\tau_{d^s} = \tau_d$. Hence, by Proposition 3.10 the space (X,d) is locally antisymmetrically connected. $\square$

Example 3.15. Recall the standard T_0-quasi-metric space $(\mathbb{R}, u)$ given in Example 2.6, where $u(x,y) = max\{x-y, 0\}$. If we consider the usual metric space $(\mathbb{R}, u^s)$, then $(\mathbb{R}, u^s)$ is not locally antisymmetrically connected by Proposition 3.14.

The next proposition is obtained easily, by virtue of Proposition 2.10 and Proposition 3.5.

Proposition 3.16. *Let (X,d) be a T_0-quasi-metric space. Then (X,d) is locally antisymmetrically connected or symmetrically connected.*

Example 3.17. Consider the (unbounded) Sorgenfrey T_0-quasi-metric space $(\mathbb{R}, t)$, where

$$t(x,y) = \begin{cases} x-y\,; & x \geq y \\ 1\,; & x < y \end{cases}$$

is defined on $\mathbb{R}$.

It is easy to verify that the space $(\mathbb{R}, t)$ is not symmetrically connected since there is no path consisting of symmetric pairs, between the points 1 and $\frac{5}{2}$. Therefore, $(\mathbb{R}, t)$ will be locally antisymmetrically connected space by Proposition 3.16.

In addition to the above result, it is very natural to consider that some spaces can be both locally antisymmetrically connected and symmetrically connected as follows:

Example 3.18. 1) It is well known from Example 2.11 that the (bounded) Sorgenfrey T_0-quasi-metric space is antisymmetrically connected and symmetrically connected. Moreover, it is also locally antisymmetrically connected by Proposition 3.5.

2) Take a T_0-quasi-metric on the set $X = \{1, 2, 3, 4\}$, described by the matrix

$$\mathbf{W} = \begin{pmatrix} 0 & 8 & 4 & 1 \\ 9 & 0 & 6 & 7 \\ 4 & 6 & 0 & 5 \\ 3 & 7 & 5 & 0 \end{pmatrix}.$$

That is, $W = (w_{ij})$ where $w(i, j) = w_{ij}$ for $i, j \in X$. It is easy to prove that w is a T_0-quasi-metric on X. Note also that the space (X, w) is not antisymmetrically connected since there is no any antisymmetric path from 1 to 3. Despite these, it is locally antisymmetrically connected by Corollary 3.12(a), and symmetrically connected by Proposition 2.10.

Incidentally, we have the next characterization of local antisymmetric connectedness:

Proposition 3.19. *A T_0-quasi-metric space (X, d) is locally antisymmetrically connected if and only if for each symmetric point $x \in X$, $\{x\}$ is τ_{d^s}-open, that is x is a τ_{d^s}-isolated point.*

Proof. If (X, d) is locally antisymmetrically connected then $T_d(x)$ is τ_{d^s}-open whenever $x \in X$. Since $T_d(x) = \{x\}$ for any symmetric point x in X, we see that $\{x\}$ is τ_{d^s}-open whenever x is a symmetric point.

Conversely, suppose that $\{x\}$ is τ_{d^s}-open whenever $x \in X$ is a symmetric point in (X, d). In order to establish the local antisymmetric connectedness of (X, d), take $a \in X$.

Case 1. If a is a symmetric point then clearly, $T_d(a) = \{a\}$ and moreover $\{a\}$ is τ_{d^s}-open by the hypothesis. Thus, $T_d(a)$ is τ_{d^s}-open.

Case 2. If a is not a symmetric point then $T_d(a) \neq \{a\}$ and so, with the help of [11, Corollary 31] $T_d(a)$ is τ_{d^s}-open.

Finally, the space (X, d) is locally antisymmetrically connected. □

Corollary 3.20. *Each T_0-quasi-metric space without symmetric points is locally antisymmetrically connected.*

Proof. The assertion is an immediate consequence of Proposition 3.19. □

4. Conclusion

In the context of asymmetric topology, following the theory of antisymmetrically connected T_0-quasi-metric spaces constructed lately, in this paper the authors introduce and discuss the localized version of antisymmetric connectedness as a different approach to the asymmetry degree of a non-metric T_0-quasi-metric.

In particular, some relationships between the theories of antisymmetric connectedness and locally antisymmetric connectedness are discussed via some theorems and (counter)examples. Moreover, it is proved that these theories are equivalent under the topological connectedness condition. Besides the fact that all finite T_0-quasi-metric spaces are locally antisymmetrically connected, we also showed that the property of local antisymmetric connectedness is characterized with the help of various topological properties, and via metrics.

Additionally, it is also natural to ask that under which conditions a T_0-quasi-metric space will become locally antisymmetrically connected. Hence, as a final observation, some answers in the framework of this question were presented in Section 3.

In the light of the above considerations, we leave as an open problem how local antisymmetric connectedness behaves for subspaces, superspaces and products in the context of T_0-quasi-metrics. Also, it is natural to ask whether the images of locally antisymmetrically connected spaces under an isometric isomorphism have the same property or not. Moreover, the theory of local antisymmetric connectedness can be also investigated in the context of asymmetric norms, as another natural approach in the framework of asymmetric topology.

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