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Extensions of n-ary prime hyperideals via an n-ary multiplicative subset in a Krasner (m,n)-hyperring

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Abstract. Let R be a Krasner (m,n)-hyperring and S be an n-ary multiplicative subset of R. The purpose of this paper is to introduce the notion of n-ary S-prime hyperideals as a new expansion of n-ary prime hyperideals. A hyperideal I of R disjoint with S is said to be an n-ary S-prime hyperideal if there exists $s \in S$ such that whenever $g(x_1^n) \in I$ for all $x_1^n \in R$, then $g(s, x_i, 1^{(n-2)}) \in I$ for some $1 \leq i \leq n$. Several properties and characterizations concerning n-ary S-prime hyperideals are presented. The stability of this new concept with respect to various hyperring-theoretic constructions are studied. Furthermore, the concept of n-ary S-primary hyperideals is introduced. Several properties of them are provided.

1. Introduction

Prime and primary ideals which are quite important in commutative rings have been studied by many authors. In 2019, Hamed and Malek [17] introduced the notion of S-prime ideal which is a generalization of prime ideals. Suppose that R is a commutative ring with identity and $S \subseteq R$ a multiplicative subset. A proper ideal P of R disjoint from S is called an S-prime of R if there exists an $s \in S$ such that for all $x, y \in R$ if $xy \in P$, then $sx \in P$ or $sy \in P$. In [25] , Massaoud defined and investigated the concept of S-primary ideals of a commutative ring in a way that generalizes essentially all the results concerning primary ideals. A proper ideal Q of R disjoint from S is called an S-primary of R if there exists an $s \in S$ such that for all $x, y \in R$ if $xy \in P$, then $sx \in P$ or $sy \in \sqrt{Q}$. Furthermore, some results on S-primary ideals of a commutative ring were studied by Visweswaran in [32].

Hyperstructures represent a natural extension of classical algebraic structures and they were defined by the French mathematician F. Marty. In 1934, Marty [24] defined the concept of a hypergroup as a generalization of groups during the 8^{th} Congress of the Scandinavian Mathematicians. A comprehensive review of the theory of hyperstructures can be found in [2, 10–12, 29, 33–35]. The simplest algebraic hyperstructures which possess the properties of closure and associativity are called semihypergroups. n-ary semigroups and n-ary groups are algebras with one n-ary operation which is associative and invertible in a generalized sense. The notion of investigations of n-ary algebras goes back to Kasner's lecture [19] at a scientific meeting in 1904. In 1928, Dorente wrote the first paper concerning the theory of n-ary groups [15]. Later on, Crombez and Timm [8, 9] defined and described the notion of the (m,n)-rings and their quotient structures. Mirvakili and Davvaz [20] defined (m,n)-hyperrings and obtained several results in this respect.

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In [13], they introduced and illustrated a generalization of the notion of a hypergroup in the sense of Marty and a generalization of an n-ary group, which is called n-ary hypergroup. The n-ary structures has been studied in [20–23, 30]. Mirvakili and Davvaz [27] defined (m, n)-hyperrings and obtained several results in this respect.

It was Krasner, who introduced one important class of hyperrings, where the addition is a hyperoperation, while the multiplication is an ordinary binary operation, which is called Krasner hyperring. In [26], a generalization of the Krasner hyperrings, which is a subclass of (m, n)-hyperrings, was defined by Mirvakili and Davvaz. It is called Krasner (m, n)-hyperring. Ameri and Norouzi in [1] introduced some important hyperideals such as Jacobson radical, n-ary prime and primary hyperideals, nilradical, and n-ary multiplicative subsets of Krasner (m, n)-hyperrings. Afterward, the notions of (k, n)-absorbing hyperideals and (k, n)-absorbing primary hyperideals were studied by Hila et al. [18]. Norouzi et al. proposed and analysed a new definition for normal hyperideals in Krasner (m, n)-hyperrings, with respect to that one given in [26] and they showed that these hyperideals correspond to strongly regular relations [28]. Asadi and Ameri introduced and studied direct limit of a direct system in the category of Krasner (m, n)-hyperrigs [7]. Dongsheng defined the notion of δ-primary ideals in a commutative ring where δ is a function that assigns to each ideal I an ideal $\delta(I)$ of the same ring [14]. Moreover, in [16] he and his colleague investigated 2-absorbing δ-primary ideals which unify 2-absorbing ideals and 2-absorbing primary ideals. Ozel Ay et al. generalized the notion of δ-primary on Krasner hyperrings [31]. The concept of δ-primary hyperideals in Krasner (m, n)-hyperrings, which unifies the prime and primary hyperideals under one frame, was defined in [4].

In this paper, the author aims to complete this circle of ideas. Motivated by the research works on S-prime ideals and S-primary ideals of commutative rings, the notions of n-ary S-prime and n-ary S-primary hyperideals are defined and investigated in a commutative Krasner (m, n)-hyperring.

2. Preliminaries

Recall first the definitions and basic terms from the hyperrings theory.
Let H be a non-empty set and $P^*(H)$ be the set of all the non-empty subsets of H. Then the mapping f from H^n into $P^*(H)$ is called an n-ary hyperoperation and the algebraic system (H, f) is called an n-ary hypergroupoid. Define $f(A_1^n) = f(A_1, ..., A_n) = \bigcup\{f(x_1^n) \mid x_i \in A_i, i = 1, ..., n\}$ for non-empty subsets $A_1, ..., A_n$ of H. The sequence $x_i, x_{i+1}, ..., x_j$ will be denoted by x_i^j and this is the empty symbol for $j < i$. Using this notation, $f(x_1, ..., x_i, y_{i+1}, ..., y_j, z_{j+1}, ..., z_n)$ will be written as $f(x_1^i, y_{i+1}^j, z_{j+1}^n)$. The expression will be written in the form $f(x_1^i, y^{(j-i)}, z_{j+1}^n)$ if $y_{i+1} = ... = y_j = y$. If for every $1 \le i < j \le n$ and all $x_1, x_2, ..., x_{2n-1} \in H$,

$$f\Big(x_1^{i-1}, f(x_i^{n+i-1}), x_{n+i}^{2n-1}\Big) = f\Big(x_1^{j-1}, f(x_j^{n+j-1}), x_{n+j}^{2n-1}\Big)$$

then the n-ary hyperoperation f is called associative. An n-ary hypergroupoid with the associative n-ary hyperoperation is called an n-ary semihypergroup. An n-ary hypergroupoid (H, f) in which the equation $b \in f(a_1^{i-1}, x_i, a_{i+1}^n)$ has a solution $x_i \in H$ for every $a_1^{i-1}, a_{i+1}^n, b \in H$ and $1 \le i \le n$, is called an n-ary quasihypergroup, when (H, f) is an n-ary semihypergroup, (H, f) is called an n-ary hypergroup. An n-ary hypergroupoid (H, f) is commutative if for all $\sigma \in \mathbb{S}_n$, the group of all permutations of $\{1, 2, 3, ..., n\}$, and for every $a_1^n \in H$ we have $f(a_1, ..., a_n) = f(a_{\sigma(1)}, ..., a_{\sigma(n)})$. If $a_1^n \in H$, then $(a_{\sigma(1)}, ..., a_{\sigma(n)})$ is denoted by $a_{\sigma(1)}^{\sigma(n)}$. If f is an n-ary hyperoperation and $t = l(n-1) + 1$, then t-ary hyperoperation $f_{(l)}$ is given by

$$f_{(l)}(x_1^{l(n-1)+1}) = f\Big(f(..., f(f(x_1^n), x_{n+1}^{2n-1}), ...), x_{(l-1)(n-1)+1}^{l(n-1)+1}\Big).$$

Definition 2.1. *[26] A non-empty subset K of an n-ary hypergroup (H, f) is said to be an n-ary subhypergroup of H if (K, f) is an n-ary hypergroup. An element $e \in H$ is called a scalar neutral element if $x = f(e^{(i-1)}, x, e^{(n-i)})$, for every $1 \le i \le n$ and for every $x \in H$.*

An element 0 of an n-ary semihypergroup (H, g) is called a zero element if for every $x_2^n \in H$, $g(0, x_2^n) = g(x_2, 0, x_3^n) = ... = g(x_2^n, 0) = 0$. If 0 and 0′ are two zero elements, then $0 = g(0', 0^{(n-1)}) = 0'$ and so the zero element is unique.

Definition 2.2. *[20] Let (H, f) be a n-ary hypergroup. (H, f) is called a canonical n-ary hypergroup if:*
(1) There exists a unique $e \in H$, such that for every $x \in H$, $f(x, e^{(n-1)}) = x$;
(2) For all $x \in H$ there exists a unique $x^{-1} \in H$, such that $e \in f(x, x^{-1}, e^{(n-2)})$;
(3) If $x \in f(x_1^n)$, then for all i, $x_i \in f(x, x^{-1}, ..., x_{i-1}^{-1}, x_{i+1}^{-1}, ..., x_n^{-1})$.
e is said to be the scalar identity of (H, f) and x^{-1} is the inverse of x. Notice that the inverse of e is e.

Definition 2.3. *[26] (R, f, g), or simply R, is said to be a Krasner (m, n)-hyperring if:*
(1) (R, f) is a canonical m-ary hypergroup;
(2) (R, g) is a n-ary semigroup;
(3) The n-ary operation g is distributive with respect to the m-ary hyperoperation f, i.e., for every $a_1^{i-1}, a_{i+1}^n, x_1^m \in R$, and $1 \leq i \leq n$, $g\Big(a_1^{i-1}, f(x_1^m), a_{i+1}^n\Big) = f\Big(g(a_1^{i-1}, x_1, a_{i+1}^n), ..., g(a_1^{i-1}, x_m, a_{i+1}^n)\Big)$;
(4) 0 is a zero element (absorbing element) of the n-ary operation g, i.e., for every $x_2^n \in R$, $g(0, x_2^n) = g(x_2, 0, x_3^n) = ... = g(x_2^n, 0) = 0$.

Throughout this paper, (R, f, g) denotes a commutative Krasner (m, n)-hyperring.

A non-empty subset S of R is called a subhyperring of R if (S, f, g) is a Krasner (m, n)-hyperring. The non-empty subset I of (R, f, g) is a hyperideal if (I, f) is an m-ary subhypergroup of (R, f) and $g(x_1^{i-1}, I, x_{i+1}^n) \subseteq I$, for every $x_1^n \in R$ and $1 \leq i \leq n$.

Definition 2.4. *[1] Let x be an element in a Krasner (m, n)-hyperring R. The hyperideal generated by x is denoted by $< x >$ and defined as $< x >= g(R, x, 1^{(n-2)}) = \{g(r, x, 1^{(n-2)}) \mid r \in R\}$.*

Definition 2.5. *[1] An element $x \in R$ is said to be invertible if there exists $y \in R$ such that $1_R = g(x, y, 1_R^{(n-2)})$. Also, the subset U of R is invertible if and only if every element of U is invertible.*

Definition 2.6. *[1] Let $P \neq R$ be a hyperideal of a Krasner (m, n)-hyperring R. P is a prime hyperideal if for hyperideals $X_1, ..., X_n$ of R, $g(X_1^n) \subseteq P$ implies that $X_i \subseteq P$ for some $1 \leq i \leq n$.*

Lemma 2.7. *(Lemma 4.5 in [1]) Let $P \neq R$ be a hyperideal of a Krasner (m, n)-hyperring R. Then P is a prime hyperideal if for all $x_1^n \in R$, $g(x_1^n) \in P$ implies that $x_1 \in P$ or ... or $x_n \in P$.*

Definition 2.8. *[1] Let I be a hyperideal in a Krasner (m, n)-hyperring R with scalar identity. The radical (or nilradical) of I, denoted by $\sqrt{I}^{(m,n)}$ is the hyperideal $\bigcap P$, where the intersection is taken over all prime hyperideals P which contain I. If the set of all prime hyperideals containing I is empty, then $\sqrt{I}^{(m,n)}$ is defined to be R.*

Ameri and Norouzi showed that if $x \in \sqrt{I}^{(m,n)}$ then there exists $t \in \mathbb{N}$ such that $g(x^{(t)}, 1_R^{(n-t)}) \in I$ for $t \leq n$, or $g_{(l)}(x^{(t)}) \in I$ for $t = l(n-1) + 1$ [1].

Definition 2.9. *[1] Let I be a proper hyperideal of a Krasner (m, n)-hyperring R with the scalar identity 1_R. I is a primary hyperideal if $g(x_1^n) \in I$ and $x_i \notin I$ implies that $g(x_1^{i-1}, 1_R, x_{i+1}^n) \in \sqrt{I}^{(m,n)}$ for some $1 \leq i \leq n$.*

If I is a primary hyperideal in a Krasner (m, n)-hyperring R with the scalar identity 1_R, then $\sqrt{I}^{(m,n)}$ is prime. (Theorem 4.28 in [1])

Definition 2.10. *[1] Let S be a non-empty subset of a Krasner (m, n)-hyperring R. S is called an n-ary multiplicative if $g(s_1^n) \in S$ for $s_1, ..., s_n \in S$.*

Definition 2.11. *[26] Let (R_1, f_1, g_1) and (R_2, f_2, g_2) be two Krasner (m, n)-hyperrings. A mapping $h : R_1 \longrightarrow R_2$ is called a homomorphism if for all $x_1^m \in R_1$ and $y_1^n \in R_1$ we have*

$$h(f_1(x_1, ..., x_m)) = f_2(h(x_1), ..., h(x_m))$$
$$h(g_1(y_1, ..., y_n)) = g_2(h(y_1), ..., h(y_n)).$$

3. *n*-Ary S-prime hyperideals

In this section, the concept of n-ary S-prime hyperideals is introduced in a Krasner (m, n)-hyperring R such that S is an n-ary multiplicative subset of R. The following definition constitutes the S-version of n-ary prime hyperideals.

Definition 3.1. *Let S be an n-ary multiplicative subset of a Krasner (m, n)-hyperring R and I be a hyperideal of R with $I \cap S = \emptyset$. I refers to an n-ary S-prime hyperideal if there exists an $s \in S$ such that for all $x_1^n \in R$ with $g(x_1^n) \in I$, we get $g(s, x_i, 1^{(n-2)}) \in I$ for some $1 \leq i \leq n$.*

Example 3.2. *The set $R = \{0, 1, 2\}$ with the following 3-ary hyperoperation f and 3-ary operation g is a Krasner $(3, 3)$-hyperring such that f and g are commutative.*

$$f(0,0,0) = 0, \quad f(0,0,1) = 1, \quad f(0,1,1) = 1, \quad f(1,1,1) = 1, \quad f(1,1,2) = R,$$

$$f(0,1,2) = R, \quad f(0,0,2) = 2, \quad f(0,2,2) = 2, \quad f(1,2,2) = R, \quad f(2,2,2) = 2,$$

$$g(1,1,1) = 1, \quad g(1,1,2) = g(1,2,2) = g(2,2,2) = 2,$$

and for $x_1, x_2 \in R$, $g(0, x_1, x_2) = 0$. Consider 3-ary multiplicative subset $S = \{1, 2\}$ of Krasner $(3, 3)$-hyperring (R, f, g). Then hyperideal $P = \{0, 2\}$ is a 3-ary S-prime hyperideal of R.

The following example shows that an n-ary S-prime hyperideal may not be an n-ary prime hyperideal of R.

Example 3.3. *The set $R = \{0, 1, 2, 3\}$ with following 2-hyperoperation "$\oplus$" is a canonical 2-ary hypergroup.*

$\oplus$	0	1	2	3
0	0	1	2	3
1	1	A	3	B
2	2	3	0	1
3	3	B	1	A

In which $A = \{0, 1\}$ and $B = \{2, 3\}$. Define a 4-ary operation g on R as follows:

$$g(a_1^4) = \begin{cases} 2 & \text{if } a_1, a_2, a_3, a_4 \in B \\ 0 & \text{otherwise.} \end{cases}$$

It follows that $(R, \oplus, g)$ is a Krasner (2,4)-hyperring. $S = \{2, 3\}$ is a 4-ary multiplicative subset of R. In the hyperring, $I = \{0\}$ is a 4-ary S-prime hyperideal of R but it is not prime, because $g(1, 2, 2, 3) = 0 \in I$ while $1, 2, 3 \notin I$.

The first theorem gives a characterization of n-ary S-prime hyperideals.

Theorem 3.4. *Let S be an n-ary multiplicative subset of a Krasner (m, n)-hyperring R and I be a hyperideal of R with $I \cap S = \emptyset$. Then I is n-ary S-prime if and only if $(I : s) = \{r \in R \mid g(r, s, 1^{(n-2)}) \in I\}$ is an n-ary prime hyperideal of R for some $s \in S$.*

Proof. $\Longrightarrow$ Let I be is an n-ary S-prime hyperideal of R. Then there exists $s \in S$ such that for all $x_1^n \in R$ with $g(x_1^n) \in I$, we get $g(s, x_i, 1^{(n-2)}) \in I$ for some $1 \leq i \leq n$. Suppose that $g(y_1^n) \in (I : s)$ for $y_1^n \in R$. Then $g(s, g(y_1^n), 1^{(n-2)}) = g(g(s, y_1, 1^{(n-2)}), y_2^n) \in I$ and so $g(s, g(s, y_1, 1^{(n-2)}), 1^{(n-2)}) = g(s^2, y_1, 1^{(n-3)}) \in I$ or $g(s, y_i, 1^{(n-2)}) \in I$ for some $2 \leq i \leq n$. Since $I \cap S = \emptyset$, then we conclude that $g(s, y_1, 1^{(n-2)}) \in I$ or $g(s, y_i, 1^{(n-2)}) \in I$ for some $2 \leq i \leq n$. So, $g(s, y_i, 1^{(n-2)}) \in I$ for some $1 \leq i \leq n$ which implies $y_i \in (I : s)$ for some $1 \leq i \leq n$. Consequently, $(I : s)$ is an n-ary prime hyperideal of R.
$\Longleftarrow$ Let $(I : s)$ be an n-ary prime hyperideal of R for some $s \in S$. Suppose that $g(x_1^n) \in I$ for $x_1^n \in R$. Since $I \subseteq (I : s)$, then $g(x_1^n) \in (I : s)$. Since $(I : s)$ is an n-ary prime hyperideal of R, then $x_i \in (I : s)$ for some $1 \leq i \leq n$. This implies that $g(s, x_i, 1^{(n-2)}) \in I)$ for some $1 \leq i \leq n$ which means I is an n-ary S-prime hyperideal of R. $\square$

Theorem 3.5. *Let S be an n-ary multiplicative subset of a Krasner (m,n)-hyperring R and $R \subseteq G$ be an extension of R. If I is an n-ary S-prime hyperideal of G, then $I \cap R$ is an n-ary S-prime hyperideal of R.*

Proof. Let I be an n-ary S-prime hyperideal of G. Then there exist $s \in S$ such that for all $x_1^n \in R$ with $g(x_1^n) \in I$, we get $g(s, x_i, 1^{(n-2)}) \in I$ for some $1 \leq i \leq n$. Let $g(x_1^n) \in I \cap R$ for $x_1^n \in R$. Since $g(x_1^n) \in I$, then $g(s, x_i, 1^{(n-2)}) \in I$ for some $1 \leq i \leq n$ which means $g(s, x_i, 1^{(n-2)}) \in I \cap R$. Thus, $I \cap R$ is an n-ary S-prime hyperideal of R. □

Theorem 3.6. *Let S be an n-ary multiplicative subset of a Krasner (m,n)-hyperring R and I be a hyperideal of R. If $I \subseteq \cup_{i=1}^n I_i$ for some n-ary S-prime hyperideals I_1^n of R, then there exists $s \in S$ such that $g(s, I, 1^{(n-2)}) \subseteq P_i$ for some $1 \leq i \leq n$.*

Proof. Let $I \subseteq \cup_{i=1}^n I_i$ for some n-ary S-prime hyperideals I_1^n of R. For each $1 \leq i \leq n$, we get $s_i \in S$ such that $(I_i : s_i)$ is an n-ary prime hyperideal of R, by Theorem 3.4. Since $I \subseteq \cup_{i=1}^n I_i \subseteq \cup_{i=1}^n (I_i : s_i)$, we have $I \subseteq (I_i : s_i)$ for some $1 \leq i \leq n$, by Theorem 5.1 in [5]. Thus $g(s_i, I, 1^{(n-2)}) \subseteq I_i$. □

Theorem 3.7. *Let S be an n-ary multiplicative subset of a Krasner (m,n)-hyperring R and I be a hyperideal of R with $I \cap S = \varnothing$. Then I is n-ary S-prime if and only if there exists $s \in S$, for all hyperideals I_1^n of R, if $g(I_1^n) \subseteq I$, then $g(s, I_i, 1^{(n-1)}) \subseteq I$ for some $1 \leq i \leq n$.*

Proof. $\Longrightarrow$ Let I be an n-ary S-prime hyperideal of R. Then there exists $s \in S$ such that for all $x_1^n \in R$ with $g(x_1^n) \in I$ we have $g(s, x_i, 1^{(n-2)}) \in I$ for some $1 \leq i \leq n$. Let for some hyperideals I_1^n of R with $g(I_1^n) \in I$ we have $g(s, I_i, 1^{(n-2)}) \nsubseteq I$ for all $1 \leq i \leq n$. This means $g(s, a_i, 1^{(n-2)}) \notin I$ for some $a_i \in I_i$ and $1 \leq i \leq n$ which is a contradiction, since I is an n-ary S-prime hyperideal of R and $g(a_1^n) \in g(I_1^n) \subseteq I$.
$\Longleftarrow$ Let $g(x_1^n) \in I$ for $x_1^n \in R$. Then $g(\langle x_1 \rangle, \cdots, \langle x_n \rangle) \subseteq I$. Hence we have $g(s, \langle x_i \rangle, 1^{(n-2)}) \subseteq I$ for some $1 \leq i \leq n$ which implies $g(s, x_i, 1^{(n-2)}) \in I$. Thus, I is an n-ary S-prime hyperideal of R. □

In view of Theorem 3.7, the following result is obtained.

Corollary 3.8. *Let I be a proper hyperideal of a Krasner (m,n)-hyperring R. Then I is an n-ary prime hyperideal if and only if for all hyperideals I_1^n of R, If $g(I_1^n) \subseteq I$, then $I_i \subseteq I$ for some $1 \leq i \leq n$.*

Proof. Consider $S = \{1\}$. Then we are done, by Theorem 3.7. □

Theorem 3.9. *Let S be an n-ary multiplicative subset of a Krasner (m,n)-hyperring R and I be an n-ary S-prime hyperideal of R. If J be a hyperideal of R with $J \subseteq I$, then $g(s, \sqrt{J}^{(m,n)}, 1^{(n-2)}) \subseteq I$ for some $s \in S$.*

Proof. Let $a \in \sqrt{J}^{(m,n)}$. Then there exists $t \in \mathbb{N}$ such that $g(a^{(t)}, 1^{(n-t)}) \in J$ for $t \leq n$ or $g_{(l)}(a^{(t)}) \in I$ for $t = l(n-1)+1$. If $g(a^{(t)}, 1^{(n-t)}) \in J \subseteq I$, then $g(\langle a \rangle^{(t)}, 1^{(n-t)}) \subseteq I$. By Theorem 3.7, we get $g(s, \langle a \rangle, 1^{(n-2)}) \subseteq I$ for some $s \in S$ which implies $g(s, a, 1^{(n-2)}) \in I$. Consequently, $g(s, \sqrt{J}^{(m,n)}, 1^{(n-2)}) \subseteq I$. If $t = l(n-1)+1$, then by using a similar argument, one can easily complete the proof. □

Theorem 3.10. *Let S be an n-ary multiplicative subset of a Krasner (m,n)-hyperring R and I_1^n be n-ary S-prime hyperideals of R. Then there exists $s \in S$ such that $g(s, \sqrt{\cap_{i=1}^n I_i}^{(m,n)}, 1^{(n-2)}) \subseteq \cap_{i=1}^n I_i$.*

Proof. Let I_1^n be n-ary S-prime hyperideals of R. Then for each $1 \leq i \leq n$ we have $s_i \in S$ such that for all x_1^n of R, if $g(x_1^n) \in I_i$, then $g(s_i, x_j, 1^{(n-2)}) \in I_i$ for some $1 \leq j \leq n$. By Theorem 3.9, we get $g(s_i, \sqrt{I_i}^{(m,n)}, 1^{(n-2)}) \subseteq I_i$ for each $1 \leq i \leq n$. Put $s = g(s_1^n)$. Hence we obtain $g(s, \sqrt{\cap_{i=1}^n I_i}^{(m,n)}, 1^{(n-2)}) = g(s, \cap_{i=1}^n \sqrt{I_i}^{(m,n)}, 1^{(n-2)}) \subseteq \cap_{i=1}^n I_i$. □

Theorem 3.11. *Let S and T be two n-ary multiplicative subsets of a Krasner (m,n)-hyperring R with $S \subseteq T$ such that for each $t \in T$, there is $t' \in T$ with $g(t, t'^{(n-1)}) \in S$. If I is an n-ary T-prime hyperideal of R, then I is an n-ary S-prime hyperideal of R.*

Proof. Let $g(x_1^n) \in I$ for some $x_1^n \in R$. Since I is an n-ary T-prime hyperideal of R, then there exists $t \in T$ such that $g(t, x_i, 1^{(n-2)}) \in I$ for some $1 \leq i \leq n$. By the hypothesis, there is $t' \in T$ such that $g(t, t'^{(n-1)}) \in S$. Put $s = g(t, t'^{(n-1)})$. Hence we have $g(s, x_i, 1^{(n-2)}) = g(g(t, t'^{(n-1)}), x_i, 1^{(n-2)}) = g(g(t'^{(n-1)}, 1), g(t, x_i, 1^{(n-2)}), 1^{(n-2)}) \in I$, as required. □

Theorem 3.12. *Let $(R_1, f_1, g_1), (R_2, f_2, g_2)$ be two Krasner (m, n)-hyperrings and $h : R_1 \longrightarrow R_2$ be a homomorphism such that $0 \notin h(S)$. If I_2 is an n-ary $h(S)$-prime hyperideal of R_2, then $h^{-1}(I_2)$ is an n-ary S-prime hyperideal of R_1.*

Proof. Suppose that I_2 is an n-ary $h(S)$-prime hyperideal of R_2. Then there exists $s \in S$ such that for all $y_1^n \in R_2$ with $g_2(y_1^n) \in I_2$ we have $g_2(h(s), y_i, 1^{(n-2)}) \in I_2$ for some $1 \leq i \leq n$. Put $I_1 = h^{-1}(I_2)$. It is easy to see that $I_1 \cap S = \varnothing$. Let $g_1(x_1^n) \in I_1$ for $x_1^n \in R_1$. Then $h(g_1(x_1^n)) = g_2(h(x_1), ..., h(x_n)) \in I_2$. So, we have $g_2(h(s), h(x_i), 1^{(n-2)}) = h(g_1(s, x_i, 1^{(n-2)})) \in I_2$ for some $1 \leq i \leq n$ which implies $g_1(s, x_i, 1^{(n-2)}) \in h^{-1}(I_2) = I_1$. Consequently, $h^{-1}(I_2)$ is an n-ary S-prime hyperideal of R_1. □

The concept of Krasner (m, n)-hyperring of fractions was introduced in [6].

Theorem 3.13. *Let R be a Krasner (m, n)-hyperring and S be an n-ary multiplicative subset of R with $1 \in S$. If I is an n-ary S-prime hyperideal of R with $I \cap S = \varnothing$, then $S^{-1}I$ is an n-ary prime hyperideal of $S^{-1}R$.*

Proof. Let $G(\frac{a_1}{s_1}, ..., \frac{a_n}{s_n}) \in S^{-1}I$ for $\frac{a_1}{s_1}, ..., \frac{a_n}{s_n} \in S^{-1}R$. Then we have $\frac{g(a_1^n)}{g(s_1^n)} \in S^{-1}I$. It implies that there exists $t \in S$ such that $g(t, g(a_1^n), 1^{(n-2)}) \in I$. Since I is an n-ary S-prime hyperideal of R and $I \cap S = \varnothing$, then there exists $s \in S$ such that $g(s, a_i, 1^{(n-2)}) \in I$ or $g(s, g(t, a_i, 1^{(n-2)}), 1^{(n-2)}) = g(s, t, a_i, 1^{(n-3)}) \in I$ for some $1 \leq i \leq n$. Hence we conclude that $G(\frac{a_i}{s_i}, \frac{1}{1}^{(n-1)}) = \frac{g(a_i, 1^{(n-1)})}{g(s_i, 1^{(n-1)}))} = \frac{g(s, a_i, 1^{(n-2)})}{g(s, s_i, 1^{(n-2)})} \in S^{-1}I$ or $G(\frac{a_i}{s_i}, \frac{1}{1}^{(n-1)}) = \frac{g(a_i, 1^{(n-1)})}{g(s_i, 1^{(n-1)})} = \frac{g(s, t, a_i, 1^{(n-3)})}{g(s, t, s_i, 1^{(n-2)})} \in S^{-1}I$ for some $1 \leq i \leq n$. Thus $S^{-1}I$ is an n-ary prime hyperideal of $S^{-1}R$. □

Theorem 3.14. *Let R be a Krasner (m, n)-hyperring, S be an n-ary multiplicative subset of R with $1 \in S$ and I be a hyperideal of R with $I \cap S = \varnothing$. If $S^{-1}I$ is an n-ary prime hyperideal of $S^{-1}R$ and $S^{-1}I \cap R = (I : s)$ for some $s \in S$, then I is an n-ary S-prime hyperideal of R.*

Proof. Let $S^{-1}I$ be an n-ary prime hyperideal of $S^{-1}R$ and $S^{-1}I \cap R = (I : s)$ for some $s \in S$. Assume that $g(a_1^n) \in I$ for some $a_1^n \in R$. Then we get $G(\frac{a_1}{1}, \cdots, \frac{a_n}{1}) \in S^{-1}I$. Since $S^{-1}I$ is an n-ary prime hyperideal of $S^{-1}R$, we obtain $\frac{a_i}{1} \in S^{-1}I$ for some $1 \leq i \leq n$ which implies $g(t, a_i, 1^{(n-2)}) \in I$ for some $t \in S$. Hence $a_i = \frac{g(t, a_i, 1^{(n-2)})}{g(t, 1^{(n-1)})} \in S^{-1}I$. This means $a_i \in (I : s)$ for some $s \in S$. Therefore we have $g(s, a_i, 1^{(n-2)}) \in I$. Thus we conclude that I is an n-ary S-prime hyperideal of R. □

Let J be a hyperideal of a Krasner (m, n)-hyperring (R, f, g). Then the set

$$R/J = \{f(x_1^{i-1}, J, x_{i+1}^m) \mid x_1^{i-1}, x_{i+1}^m \in R\}$$

endowed with m-ary hyperoperation f which for all $x_{11}^{1m}, ..., x_{m1}^{mm} \in R$

$$f\Big(f(x_{11}^{1(i-1)}, J, x_{1(i+1)}^{1m}), ..., f(x_{m1}^{m(i-1)}, J, x_{m(i+1)}^{mm})\Big)$$
$$= f\Big(f(x_{11}^{m1}), ..., f(x_{1(i-1)}^{m(i-1)}), J, f(x_{1(i+1)}^{m(i+1)}), ..., f(x_{1m}^{mm})\Big)$$

and with n-ary hyperoperation g which for all $x_{11}^{1m}, ..., x_{n1}^{nm} \in R$

$$g\Big(f(x_{11}^{1(i-1)}, J, x_{1(i+1)}^{1m}), ..., f(x_{n1}^{n(i-1)}, J, x_{n(i+1)}^{nm})\Big)$$
$$= f\Big(g(x_{11}^{n1}), ..., g(x_{1(i-1)}^{n(i-1)}), J, g(x_{1(i+1)}^{n(i+1)}), ..., f(x_{1m}^{nm})\Big)$$

construct a Krasner (m, n)-hyperring, and $(R/J, f, g)$ is called the quotient Krasner (m, n)-hyperring of R by J [1]. Now, it is determined when the hyperideal I/J is n-ary $\bar{S}$-prime in R/J.

Theorem 3.15. *Let S be an n-ary multiplicative subset of a Krasner (m, n)-hyperring R and let I and J be two hyperideals of R such that $J \subseteq I$. Let $J \cap S = \varnothing$ and $I/J \cap \bar{S} = \varnothing$ with $\bar{S} = \{f(s_1^{i-1}, J, s_{i+1}^n) \mid s_1^{i-1}, s_{i+1}^n \in S\}$. If I is an n-ary S-prime hyperideal of R, then I/J is an n-ary $\bar{S}$-prime hyperideal of R/J.*

Proof. Let I be an n-ary S-prime hyperideal of R. Then there exists some $s \in S$ such that if $g(x_1^n) \in I$ for $x_1^n \in R$, then we get $g(s, x_i, 1^{(n-2)}) \in I$ for some $1 \le i \le n$. Let

$$g\Big(f(x_{11}^{1(i-1)}, J, x_{1(i+1)}^{1m}), ..., f(x_{n1}^{n(i-1)}, J, x_{n(i+1)}^{nm})\Big) \in I/J$$

for some $f(x_{11}^{1(i-1)}, J, x_{1(i+1)}^{1m}), ..., f(x_{n1}^{n(i-1)}, J, x_{n(i+1)}^{nm}) \in R/J$.
This means

$$f\Big(g(x_{11}^{n1}), ..., g(x_{1(i-1)}^{n(i-1)}), J, g(x_{1(i+1)}^{n(i+1)}), ..., g(x_{1m}^{nm})\Big) \in I/J.$$

Then

$$f\Big(g(x_{11}^{n1}), ..., g(x_{1(i-1)}^{n(i-1)}), 0, g(x_{1(i+1)}^{n(i+1)}), ..., g(x_{1m}^{nm})\Big) \subseteq I$$

which implies

$$g\Big(f(x_{11}^{1(i-1)}, 0, x_{1(i+1)}^{1m}), ..., f(x_{n1}^{n(i-1)}, 0, x_{n(i+1)}^{nm})\Big) \subseteq I.$$

Since I is an n-ary S-prime hyperideal of R, then for some $1 \le j \le n$, we obtain

$$g\Big(s, f(x_{j1}^{j(i-1)}, 0, x_{j(i+1)}^{jm}), 1^{n-2}\Big) \subseteq I.$$

Therefore

$$f\Big(g(s, f(x_{j1}^{j(i-1)}, 0, x_{j(i+1)}^{jm}), 1^{n-2}), J, 0^{(m-2)}\Big) \in I/J$$

and so

$$f\Big(g(g(s, 1^{(n-2)}), f(x_{j1}^{j(i-1)}, 0, x_{j(i+1)}^{jm}), 1^{n-2}), J, 0^{(m-2)}\Big) \in I/J.$$

Thus we conclude that

$$g\Big(f(s, J, 1^{(n-2)}), f(x_{j1}^{j(i-1)}, J, x_{j(i+1)}^{jm}), 1_{R/J}^{(n-2)}\Big) \in I/J.$$

Consequently, I/J is an n-ary $\bar{S}$-prime hyperideal of R/J. □

Suppose that I is a normal hyperideal of Krasner (m, n)-hyperring (R, f, g). Then the set of all equivalence classes $[R : I^*] = \{I^*[x] \mid x \in R\}$ is a Krasner (m, n)-hyperring with the m-ary hyperoperation f/I and the n-ary operation g/I, defined as follows:

$$f/I(I^*[x_1], \cdots, I^*[x_m]) = \{I^*[z] \mid z \in f(I^*[x_1], \cdots, I^*[x_m])\}, \ \ \forall x_1^m \in R$$
$$g/I(I^*[x_1], \cdots, I^*[x_n]) = I^*[g(x_1^n)], \ \ \forall x_1^n \in R$$

(for more details refer to [26]).

Theorem 3.16. *Let S be an n-ary multiplicative subset of a Krasner (m, n)-hyperring (R, f, g) and let I be a normal hyperideal of R. If J is an n-ary S-prime hyperideal of R such that $I \subseteq J$, then $[J : I^*]$ is an n-ary $[S : I^*]$-prime hyperideal of $[R : I^*]$.*

Proof. First of all, notice that $S \cap J = \varnothing$ if and only if $[S : I^*] \cap [J : I^*] = \varnothing$. Let $g/I(I^*[x_1], \cdots, I^*[x_n]) \in [J : I^*]$ for some $x_1^n \in R$. Then $I^*[g(x_1^n)] \in [J : I^*]$. This means $I^*[g(x_1^n)] \subseteq J$. So

$$I^*[g(x_1^n)] = f\Big(I, g(x_1^n), 0^{(m-2)}\Big) = f\Big(I, g(x_1^n), g(0^{(n)})^{(m-2)}\Big)$$
$$= g\Big(f(I, x_1, 0^{(m-2)}), \cdots, f(I, x_n, 0^{(m-2)})\Big) \subseteq J.$$

Since J is an n-ary S-prime hyperideal of R, then there exists $s \in S$ such that $g(s, f(I, x_i, 0^{(m-2)}), 1^{(n-2)}) \subseteq J$ for some $1 \leq i \leq n$ which implies

$$f\Big(I, g(s, x_i, 1^{(n-2)}), 0^{(m-2)}\Big) \subseteq J.$$

Hence $I^*[g(s, x_i, 1^{(n-2)})] \in [J : I^*]$ which means $g/I(I^*[s], I^*[x_i], I^*[1]^{(n-2)}) \in [J : I^*]$. Thus $[J : I^*]$ is an n-ary $[S : I^*]$-prime hyperideal of $[R : I^*]$. $\square$

Let (R_1, f_1, g_1) and (R_2, f_2, g_2) be two Krasner (m, n)-hyperrings such that 1_{R_1} and 1_{R_2} be scalar identitis of R_1 and R_2, respectively. Then the (m, n)-hyperring $(R_1 \times R_2, f_1 \times f_2, g_1 \times g_2)$ is defined by m-ary hyperoperation $f = f_1 \times f_2$ and n-ary operation $g = g_1 \times g_2$, as follows:

$$f_1 \times f_2((a_1, b_1), \cdots, (a_m, b_m)) = \{(a, b) \mid \ a \in f_1(a_1^m), b \in f_2(b_1^m)\}$$
$$g_1 \times g_2((x_1, y_1), \cdots, (x_n, y_n)) = (g_1(x_1^n), g_2(y_1^n)),$$

for all $a_1^m, x_1^n \in R_1$ and $b_1^m, y_1^n \in R_2$ [3]. Suppose that $S = S_1 \times S_2$ such that S_1 and S_2 are n-ary multiplicative subsets of R_1 and R_2, respectively. Assume that I_1 is an n-ary S_1-prime hyperideal of R_1. It is easy to verify that $(I_1 \times R_2) \cap S = \varnothing \Longleftrightarrow I_1 \times S_1 = \varnothing$. Next theorem determines the n-ary S-prime hyperideals in the product of two, and hence any finite number of, Krasner (m, n)-hyperrings.

Theorem 3.17. *Let (R_1, f_1, g_1) and (R_2, f_2, g_2) be two Krasner (m, n)-hyperrings such that 1_{R_1} and 1_{R_2} be two scalar identities of R_1 and R_2, respectively. Suppose that $S = S_1 \times S_2$ such that S_1 and S_2 are n-ary multiplicative subsets of R_1 and R_2, respectively. Then I_1 is an n-ary S_1-prime hyperideal of R_1 if and only if $I_1 \times R_2$ is an n-ary S-prime hyperideal of $R_1 \times R_2$.*

Proof. $\Longrightarrow$ Assume that I_1 is an n-ary S_1-prime hyperideal of R_1. Let $g_1 \times g_2((x_1, y_1), \cdots, (x_n, y_n)) \in I_1 \times R_2$ for some $x_1^n \in R_1$ and $y_1^n \in R_2$. Then we get $g_1(x_1^n) \in I_1$. By the hypothesis, there exists $s_1 \in S_1$ such that $g_1(s_1, x_i, 1^{(n-2)}) \in I_1$ for some $1 \leq i \leq n$. Then we have $g_1 \times g_2((s_1, 1_{R_2}), (x_i, y_i), (1_{R_1}, 1_{R_2})^{(n-2)}) \in I_1 \times R_2$. Thus we conclude that $I_1 \times R_2$ is an n-ary S-prime hyperideal of $R_1 \times R_2$.
$\Longleftarrow$ Let $I_1 \times R_2$ be an n-ary S-prime hyperideal of $R_1 \times R_2$. Assume that $g_1(x_1^n) \in I_1$ for some $x_1^n \in R_1$. Then $g_1 \times g_2((x_1, 1_{R_2}), \cdots (x_n, 1_{R_2})) \in I_1 \times R_2$. Since $I_1 \times R_2$ is an n-ary S-prime hyperideal of $R_1 \times R_2$, then there exists an element (s_1, s_2) in S such that $g_1 \times g_2((s_1, s_2), (x_i, 1_{R_2}), (1_{R_1}, 1_{R_2})^{(n-2)}) \in I_1 \times R_2$ for some $1 \leq i \leq n$. This means $g_1(s_1, x_i, 1_{R_1}^{(n-2)}) \in I_1$. Consequently, I_1 is an n-ary S_1-prime hyperideal of R_1. $\square$

Now, the following result obtained by the previous theorem is given.

Corollary 3.18. *Let (R_i, f_i, g_i) be a Krasner (m, n)-hyperring for each $1 \leq i \leq t$ such that 1_{R_i} is scalar identity of R_i. Assume that $S = S_1 \times \cdots \times S_t$ such that S_i is an n-ary multiplicative subset of R_i for each $1 \leq i \leq t$. If I_i is an n-ary S_i-prime hyperideal of R_i for some $1 \leq i \leq t$, then $R_1 \times \cdots \times R_{i-1} \times I_i \times R_{i+1} \times \cdots \times R_t$ is an n-ary S-prime hyperideal of $R_1 \times \cdots \times R_t$.*

4. *n*-Ary S-primary hyperideals

The aim of this section is to define the notion of n-ary S-primary hyperideals in a Krasner (m, n)-hyperring. The overall framework of the structure is then explained.

Definition 4.1. *Let S be an n-ary multiplicative subset of a Krasner (m, n)-hyperring R and I be a hyperideal of R with $I \cap S = \varnothing$. I refers to an n-ary S-primary hyperideal if there exists an $s \in S$ such that for all $x_1^n \in R$ with $g(x_1^n) \in I$, we have $g(s, x_i, 1^{(n-2)}) \in I$ or $g(x_1^{i-1}, s, x_{i+1}^n) \in \sqrt{I}^{(m,n)}$.*

Example 4.2. *In Example 3.2, the hyperideal $P = \{0\}$ is a 3-ary S-primary hyperideal of R.*

It is clear that any n-ary S-prime hyperideal of a Krasner (m, n)-hyperring R is an n-ary S-primary hyperideal of R. The converse in general is not true. The following is an example of an n-ary S-primary hyperideal that is not an n-ary S-prime hyperideal.

Example 4.3. *Suppose that $G = [0, 1]$ and define a 2-ary hyperoperation " $\boxplus$ " on G as follows:*

$$\alpha \boxplus \beta = \begin{cases} \{max\{\alpha, \beta\}\}, & if\ \alpha \neq \beta \\ [0,\alpha], & if\ \alpha = \beta. \end{cases}$$

Let " $\cdot$ " is the usual multiplication on real numbers and $S = \{1\}$. In the Krasner $(2, 3)$-hyperring G, the hyperideal $I = [0, 0.5]$ is a 3-ary S-primary hyperideal of G.

Example 4.4. *Consider the multiplicative group of units $\mathbb{Z}^*_{12} = \{1, 5, 7, 11\}$ of $\mathbb{Z}_{12} = \{0, 1, 2, ..., 11\}$. The construction $\mathbb{Z}_{12}/\mathbb{Z}^*_{12}$ is a Krasner hyperring. The set $S = \{\mathbb{Z}_{12}, 3\mathbb{Z}_{12}, 9\mathbb{Z}_{12}\}$ is a 3-ary multiplicative subset of $\mathbb{Z}_{12}/\mathbb{Z}^*_{12}$. We can observe that the hyperideal $I = \{0\mathbb{Z}_{12}, 4\mathbb{Z}_{12}\}$ with $\sqrt{I} = \{0\mathbb{Z}^*_{12}, 2\mathbb{Z}^*_{12}, 4\mathbb{Z}^*_{12}, 6\mathbb{Z}^*_{12}\}$ is a 3-ary S-primary hyperideal of $\mathbb{Z}_{12}/\mathbb{Z}^*_{12}$. The hyperideal I is not a 3-ary S-prime hyperideal of $\mathbb{Z}_{12}/\mathbb{Z}^*_{12}$. Because, $g((2\mathbb{Z}^*_{12})^2, 3\mathbb{Z}^*_{12}) \in I$ but neither $g(2\mathbb{Z}^*_{12}, s, 1) \in I$ nor $g(3\mathbb{Z}^*_{12}, s, 1) \in I$ for all $s \in S$.*

The following is a direct consequence and can be proved easily and so the proof is omited.

Theorem 4.5. *Let S be an n-ary multiplicative subset of a Krasner (m, n)-hyperring R such that $S \subseteq U(R)$ and I be a hyperideal of R. Then I is an n-ary S-primary hyperideal of R if and only if I is an n-ary primary hyperideal of R.*

The following theorem offers a characterization of n-ary S-primary hyperideals.

Theorem 4.6. *Let S be an n-ary multiplicative subset of a Krasner (m, n)-hyperring R and I be a hyperideal of R with $I \cap S = \varnothing$. Then I is an n-ary S-primary hyperideal of R if and only if $(I : s) = \{x \in R \mid g(x, s, 1^{(n-2)}) \in I\}$ is an n-ary primary hyperideal of R for some $s \in S$.*

Proof. $\Longrightarrow$ Let I be an n-ary S-primary hyperideal of R. Then there exists $s \in S$ such that if $g(a_1^n) \in I$, then $g(s, a_i, 1^{(n-2)}) \in I$ or $g(a_1^{i-1}, s, a_{i+1}^n) \in \sqrt{I}^{(m,n)}$. Now we show $(I : s) = \{x \in R \mid g(s, x, 1^{(n-2)}) \in I\}$ is an n-ary primary hyperideal of R. Suppose that $g(x_1^n) \in (I : s)$ for some $x_1^n \in R$. This means $g(g(x_1^n), s, 1^{(n-2)}) = g(x_1^{i-1}, g(s, x_i, 1^{(n-2)}), x_{i+1}^n) \in I$. Then we have $g(s, g(s, x_i, 1^{(n-2)}), 1^{(n-2)}) = g(s^{(2)}, x_i, 1^{(n-3)}) \in I$ or $g(x_1^{i-1}, s, x_{i+1}^n) \in \sqrt{I}^{(m,n)}$. In the first case, we have $g(s, x_i, 1^{(n-2)}) \in I$ or $g(s^{(3)}, 1^{(n-3)}) \in \sqrt{I}^{(m,n)}$. From $g(s^{(3)}, 1^{(n-3)}) \in \sqrt{I}^{(m,n)}$, it follows that there exists $t \in \mathbb{N}$ such that $g(g(s^{(3)}, 1^{(n-3)})^{(t)}, 1^{(n-t)}) \in I$ for $t \leq n$ or $g_{(l)}(g(s^{(3)}, 1^{(n-3)})^{(t)}) \in I$ for $t = l(n-1)+1$. In both possibilities, we conclude that $I \cap S \neq \varnothing$ which is a contradiction. Hence get $g(s, x_i, 1^{(n-2)}) \in I$ which implies $x_i \in (I : s)$. In the second case, there exists $t \in \mathbb{N}$ such that $g(g(x_1^{i-1}, s, x_{i+1}^n)^{(t)}, 1^{(n-t)}) \in I$ for $t \leq n$ or $g_{(l)}(g(x_1^{i-1}, s, x_{i+1}^n)^{(t)}) \in I$ for $t = l(n-1)+1$. If $t \leq n$, then we obtain $g(s, g(s^{(t)}, 1^{(n-t)}), 1^{(n-2)}) = g(s^{(t+1)}, 1^{(n-t-1)}) \in \sqrt{I}^{(m,n)}$ or $g(s, g(x_1^{i-1}, 1, x_{i+1}^n)^{(t)}, 1^{(n-t-1)}) \in I$. From $g(s^{(t+1)}, 1^{(n-t-1)}) \in \sqrt{I}^{(m,n)}$, it follows that $I \cap S \neq \varnothing$. Thus we conclude that $g(s, g(x_1^{i-1}, 1, x_{i+1}^n)^{(t)}, 1^{(n-t-1)}) \in I$ which means $g(g(x_1^{i-1}, 1, x_{i+1}^n)^{(t)}, 1^{(n-t)}) \in (I : s)$. Therefore $g(x_1^{i-1}, 1, x_{i+1}^n) \in \sqrt{(I : s)}^{(m,n)}$. Similar for other case. Consequently, $(I : s)$ is an n-ary primary hyperideal of R.

$\Longleftarrow$ Let $(I : s) = \{x \in R \mid g(x, s, 1^{(n-2)}) \in I\}$ be an n-ary primary hyperideal of R for some $s \in S$. Suppose that $g(x_1^n) \in I$ for some $x_1^n \in R$. Then we get $g(x_1^n) \in (I : s)$. Since $(I : s)$ is an n-ary primary hyperideals of R, then $x_i \in (I : s)$ which implies $g(s, x_i, 1^{(n-2)}) \in I$ or $g(x_1^{i-1}, 1, x_{i+1}^n) \in \sqrt{(I : s)}^{(m,n)}$ which means there exists $t \in \mathbb{N}$ such that $g(s, g(x_1^{i-1}, 1, x_{i+1}^n)^{(t)}, 1^{(n-t-1)}) \in I$ for $t \leq n$ or $g(s, g_{(l)}(x_1^{i-1}, 1, x_{i+1}^n)^{(t)}), 1^{(n-2)}) \in I$ for $t = l(n-1)+1$. If $t \leq n$, then we have $g(g(x_1^{i+1}, s, x_{i+1}^n)^{(t)}, 1^{(n-t)}) \in I$ which implies $g(x_1^{i-1}, s, x_{i+1}^n) \in \sqrt{I}^{(m,n)}$. Similar for other case. This means I is an n-ary S-primary hyperideal of R. $\square$

In the following, the relationship between an n-ary S-primary hyperideal and its radical is considered.

Theorem 4.7. *Let S be an n-ary multiplicative subset of a Krasner (m,n)-hyperring R and I be an n-ary S-primary hyperideal of R. Then $\sqrt{I}^{(m,n)}$ is an n-ary S-prime hyperideal of R.*

Proof. Since $I \cap S = \varnothing$ then we conclude that $\sqrt{I}^{(m,n)} \cap S = \varnothing$. Now, let $g(x_1^n) \in \sqrt{I}$ and for all $j \in \{1,...,i-1,i+1,...,n\}$, $g(s,x_j,1^{(n-2)}) \notin \sqrt{I}^{(m,n)}$ for each $s \in S$. Since $g(x_1^n) \in \sqrt{I}^{(m,n)}$, there exists $t \in \mathbb{N}$ such that $g(g(x_1^n)^{(t)},1^{(n-t)}) \in I$ for $t \leq n$ or $g_{(l)}(g(x_1^n)^{(t)}) \in I$ for $t = l(n-1)+1$. If $g(g(x_1^n)^{(t)},1^{(n-t)}) \in I$, then there exists $s \in S$ such that $g(s,x_i^{(t)},1^{(n-t-1)}) \in I$ or $g(s,g(x_1^{i-1},1,x_{i+1}^n)^{(t)},1^{(n-t-1)})$ as I is a n-ary S-primary hyperideal of R. Hence we get $g(g(s,x_i,1^{(n-2)})^{(t)},1^{(n-t)}) \in I$ which implies $g(s,x_i,1^{(n-2)}) \in \sqrt{I}^{(m,n)}$ or $g(g(x_1^{i-1},s,x_{i+1}^n)^{(t)},1^{(n-t)}) \in I$. In the second case, we get

$$
\begin{aligned}
& g\Big(g(x_1^{(t)},g(x_2^{i-1},s,1,x_{i+1}^n)^{(t)},1^{(n-2t)}\Big) \in I \\
\Longrightarrow\ & g\Big(g(x_1^{(t)},1^{(n-t)}),g(x_2^{i-1},s,1,x_{i+1}^n)^{(t)},1^{(n-t-1)}\Big) \in I \\
\Longrightarrow\ & g\Big(s,g(x_1^{(t)},1^{(n-t)}),1^{(n-2)}\Big) \in I \text{ or } g\Big(s,g(x_2^{i-1},s,1,x_{i+1}^n)^{(t)},1^{(n-t-1)}\Big) \in \\
& \sqrt{I}^{(m,n)} \\
\Longrightarrow\ & g\Big(g(s,x_1,1^{(n-2)})^{(t)},1^{(n-t)}\Big) \in I \text{ or } g\Big(g(x_2^{i-1},s^{(2)},x_{i+1}^n)^{(t)},1^{(n-t)}\Big) \in \\
& \sqrt{I}^{(m,n)} \\
\Longrightarrow\ & g(s,x_1,1^{(n-2)}) \in \sqrt{I}^{(m,n)} \text{ or } g\Big(g(x_2^{i-1},s^{(2)},x_{i+1}^n)^{(t)},1^{(n-t)}\Big) \in \sqrt{I}^{(m,n)}.
\end{aligned}
$$

Since $g(s,x_1,1^{(n-2)}) \in \sqrt{I}^{(m,n)}$ is a contradiction, then

$$
\begin{aligned}
& g\Big(g(x_2^{i-1},s^{(2)},x_{i+1}^n)^{(t)},1^{(n-t)}\Big) \in \sqrt{I}^{(m,n)} \\
\Longrightarrow\ & \exists w \in \mathbb{N};\, g\Big(g(g(x_2^{i-1},s^{(2)},x_{i+1}^n),1^{(n-t)})^{(w)},1^{(n-w)}\Big) \in I \\
\Longrightarrow\ & g\Big(g(x_2^{(t+w)},1^{(n-t-w)},g(g(x_3^{i-1},s^{(2)},1,x_{i+1}^n)^{(w)},1^{(n-w-1)}\Big) \in I \\
\Longrightarrow\ & g(s,x_2,1^{(n-2)}) \in \sqrt{I}^{(m,n)} \text{ or } g\Big(g(g(x_3^{i-1},s^{(3)},x_{i+1}^n)^{(t)},1^{(n-t)})^{(w)},1^{(n-w)}\Big) \in \\
& \sqrt{I}^{(m,n)} \\
& \vdots \\
\Longrightarrow\ & \cdots \text{ or } g(s,x_n,1^{(n-2)}) \in \sqrt{I}^{(m,n)}
\end{aligned}
$$

which is contradiction with $g(s,x_j,1^{(n-2)}) \notin \sqrt{I}^{(m,n)}$ for all $j \in \{1,...,i-1,i+1,...,n\}$. Thus we have $g(s,x_i,1^{(n-2)}) \in \sqrt{I}^{(m,n)}$. Consequently, $\sqrt{I}^{(m,n)}$ is an n-ary S-prime hyperideal of R. If $t = l(n-1)+1$, then by using a similar argument, one can easily complete the proof. □

For a hyperideal I of a Krasner (m,n)-hyperring R, we refer to the n-ary S-prime hyperideal $P = \sqrt{I}^{(m,n)}$ as the associated S-prime hyperideal of I and on the other hand I is referred to as an n-ary P-S-primary hyperideal of R. The intersection of n-ary P-S-primary hyperideals is discussed in the next theorem.

Theorem 4.8. *Let S be an n-ary multiplicative subset of a Krasner (m,n)-hyperring R and I_1^n be n-ary P-S-primary hyperideals of R for some n-ary S-prime hyperideal P of R. Then, $I = \cap_{i=1}^n I_i$ is an n-ary P-S-primary hyperideal of R.*

Proof. Let I_j be an n-ary S-primary hyperideal of R for all $1 \leq j \leq n$. Then there exists $s_j \in S$ such that if $g(x_1^n) \in I_j$ for $x_1^n \in R$, then $g(s_j,x_i,1^{(n-2)}) \in I_j$ or $g(x_1^{i-1},s_j,x_{i+1}^n) \in \sqrt{I_j}^{(m,n)}$ for some $1 \leq i \leq n$. We put $s = g(s_1^n)$. Let $g(a_1^n) \in I$ for some $a_1^n \in R$ and let us assume that $g(s,a_i,1^{(n-2)}) \notin I$ for all $1 \leq i \leq n$. This means $g(s_j,a_i,1^{(n-2)}) \notin I_j$ for some $1 \leq j \leq n$. Since I_j is an S-primary hyperideal of R, we get

$g(a_1^{i-1}, s_j, a_{i+1}^n) \in \sqrt{I_j}^{(m,n)}$. Since I_1^n are n-ary P-S-primary hyperideals of R for some n-ary S-prime hyperideal P of R, then $\sqrt{I_j}^{(m,n)} = \sqrt{I}^{(m,n)}$. Then we obtain $g(a_1^{i-1}, s_j, a_{i+1}^n) \in \sqrt{I}^{(m,n)}$ which implies $g(a_1^{i-1}, s, a_{i+1}^n) \in \sqrt{I}^{(m,n)}$ and the proof is over. □

Theorem 4.9. *Let S be an n-ary multiplicative subset of a Krasner (m,n)-hyperring R and I_1^{n-1} be hyperideals of R such that for each $1 \le i \le n-1$, $I_i \cap S \neq \emptyset$. If I is an n-ary S-primary hyperideal of R, then $g(I_1^{n-1}, I)$ is an n-ary S-primary hyperideal of R.*

Proof. Since $g(I_1^{n-1}, I) \subseteq I$ and $I \cap S = \emptyset$, we get $g(I_1^{n-1}, I) \cap S = \emptyset$. Let I be an n-ary S-primary hyperideal of R. Then there exists $s \in S$ such that if $g(a_1^n) \in I$ for $a_1^n \in R$, then $g(s, a_i, 1^{(n-2)}) \in I$ or $g(a_1^{i-1}, s, a_{i+1}^n) \in \sqrt{I}^{(m,n)}$. Suppose that $g(x_1^n) \in g(I_1^{n-1}, I)$ for some $x_1^n \in R$. As $g(I_1^{n-1}, I) \subseteq I$, then $g(x_1^n) \in I$. Since for each $1 \le i \le n-1$, $I_i \cap S \neq \emptyset$, then we have $u_i \in I_i \cap S$. Put $u = g(u_1^{n-1}, 1)$. If $g(s, x_i, 1^{(n-2)}) \in I$, then $g(g(u, s, 1^{(n-2)}), x_i, 1^{(n-2)}) = g(u, g(s, x_i, 1^{(n-2)}), 1^{(n-2)}) \in g(I_1^{(n-1)}, I)$. Now, let $g(x_1^{i-1}, s, x_{i+1}^n) \in \sqrt{I}^{(m,n)}$. Then there exists $t \in \mathbb{N}$ such that $g(g(x_1^{i-1}, s, x_{i+1}^n)^{(t)}, 1^{(n-t)}) \in I$ for $t \le n$ or $g_{(l)}(g(x_1^{i-1}, s, x_{i+1}^n)^{(t)}) \in I$, for $t = l(n-1)+1$. In the former case, we have

$$
\begin{aligned}
& g\Big(u^{(t)}, g\Big(g(x_1^{i-1}, s, x_{i+1}^n)^{(t)}, 1^{(n-t)}\Big), 1^{(n-t-1)}\Big) \\
&= g\Big(g\Big(x_1^{i-1}, g(u, s, 1^{(n-2)}), x_{i+1}^n\Big)^{(t)}, 1^{(n-t)}\Big) \\
&\in g(I_1^{n-1}, I)
\end{aligned}
$$

which implies $g(x_1^{i-1}, g(u, s, 1^{(n-2)}), x_{i+1}^n) \in \sqrt{g(I_1^{n-1}, I)}^{(m,n)}$. Thus, $g(I_1^{n-1}, I)$ is an n-ary S-primary hyperideal of R. In the second case, a similar argument completes the proof. □
□

Theorem 4.10. *Let $(R_1, f_1, g_1), (R_2, f_2, g_2)$ be two Krasner (m,n)-hyperrings and $h : R_1 \longrightarrow R_2$ be a homomorphism such that $0 \notin h(S)$. If I_2 is an n-ary $h(S)$-primary hyperideal of R_2, then $h^{-1}(I_2)$ is an n-ary S-primary hyperideal of R_1.*

Proof. Suppose that I_2 is an n-ary $h(S)$-primary hyperideal of R_2. Then there exists $s \in S$ such that for all $y_1^n \in R_2$ with $g_2(y_1^n) \in I_2$ we have $g_2(h(s), y_i, 1^{(n-2)}) \in I_2$ or $g_2(y_1^{i-1}, h(s), y_{i+1}^n) \in \sqrt{I_2}^{(m,n)}$ for some $1 \le i \le n$. Put $I_1 = h^{-1}(I_2)$. It is easy to see that $I_1 \cap S = \emptyset$. Let $g_1(x_1^n) \in I_1$ for $x_1^n \in R_1$. Then $h(g_1(x_1^n)) = g_2(h(x_1), ..., h(x_n)) \in I_2$. So, we have $g_2(h(s), h(x_i), 1^{(n-2)}) = h(g_1(s, x_i, 1^{(n-2)})) \in I_2$ or $g_2(h(x_1), ..., h(x_{i-1}), h(s), h(x_{i+1}), ..., h(x_n)) = h(g_1(x_1^{i-1}, s, x_{i+1}^n)) \in \sqrt{I_2}^{(m,n)}$ which implies $g_1(s, x_i, 1^{(n-2)}) \in h^{-1}(I_2) = I_1$ or $g_1(x_1^{i-1}, s, x_{i+1}^n) \in h^{-1}(\sqrt{I_2}^{(m,n)}) = \sqrt{h^{-1}(I_2)}^{(m,n)} = \sqrt{I_1}^{(m,n)}$. Thus $h^{-1}(I_2) = I_1$ is an n-ary S-primary hyperideal of R_1. □

Theorem 4.11. *Let (R_1, f_1, g_1) and (R_2, f_2, g_2) be two Krasner (m,n)-hyperrings such that 1_{R_1} and 1_{R_2} be two scalar identities of R_1 and R_2, respectively. Suppose that $S = S_1 \times S_2$ such that S_1 and S_2 are n-ary multiplicative subsets of R_1 and R_2, respectively. Then I_1 is an n-ary S_1-primary hyperideal of R_1 if and only if $I_1 \times R_2$ is an n-ary S-primary hyperideal of $R_1 \times R_2$.*

Proof. $\Longrightarrow$ Suppose that I_1 is an n-ary S_1-primary hyperideal of R_1. Let $g_1 \times g_2((x_1, y_1), \cdots, (x_n, y_n)) \in I_1 \times R_2$ for some $x_1^n \in R_1$ and $y_1^n \in R_2$. Then we have $g_1(x_1^n) \in I_1$. Then there exists $s_1 \in S_1$ with $g_1(s_1, x_i, 1^{(n-2)}) \in I_1$ or $g_1(x_1^{i-1}, s_1, x_{i+1}^n) \in \sqrt{I_1}^{(m,n)}$. So $g_1 \times g_2((s_1, 1_{R_2}), (x_i, y_i), (1_{R_1}, 1_{R_2})^{(n-2)}) \in I_1 \times R_2$ or $g_1 \times g_2((x_1, y_1), ..., (x_{i-1}, y_{i-1}), (s_1, 1_{R_2}), (x_{i+1}, y_{i+1}), ..., (x_n, y_n)) \in \sqrt{I_1}^{(m,n)} \times R_2 = \sqrt{I_1 \times R_2}^{(m,n)}$. Consequently, $I_1 \times R_2$ is an n-ary S-primary hyperideal of $R_1 \times R_2$.
$\Longleftarrow$ Let $I_1 \times R_2$ be an n-ary S-primary hyperideal of $R_1 \times R_2$. Suppose that $g_1(x_1^n) \in I_1$ for some $x_1^n \in R_1$. Then $g_1 \times g_2((x_1, 1_{R_2}), \cdots (x_n, 1_{R_2})) \in I_1 \times R_2$. Since $I_1 \times R_2$ is an n-ary S-primary hyperideal of

$R_1 \times R_2$, then there exists an element (s_1, s_2) in S such that $g_1 \times g_2((s_1, s_2), (x_i, 1_{R_2}), (1_{R_1}, 1_{R_2})^{(n-2)}) \in I_1 \times R_2$ or $g_1 \times g_2((x_1, 1_{R_2}), ..., (x_{i-1}, 1_{R_2}), (s_1, s_2), (x_{i+1}, 1_{R_2}), ..., (x_n, 1_{R_2}) \in \sqrt{I_1 \times R_2}^{(m,n)} = \sqrt{I_1}^{(m,n)} \times R_2$. This means $g_1(s_1, x_i, 1_{R_1}^{(n-2)}) \in I_1$ or $g_1(x_1^{i-1}, s_1, x_{i+1}^n) \in \sqrt{I_1}^{(m,n)}$. Thus I_1 is an n-ary S_1-primary hyperideal of R_1. □

Theorem 4.12. *Let R be a Krasner (m, n)-hyperring and S be an n-ary multiplicative subset of R with $1 \in S$. If I is an n-ary S-primary hyperideal of R with $I \cap S = \varnothing$, then $S^{-1}I$ is an n-ary primary hyperideal of $S^{-1}R$.*

Proof. Let $\frac{a_1}{s_1}, ..., \frac{a_n}{s_n} \in S^{-1}R$ such that $G(\frac{a_1}{s_1}, ..., \frac{a_n}{s_n}) \in S^{-1}I$. Then we have $\frac{g(a_1^n)}{g(s_1^n)} \in S^{-1}I$. It implies that there exists $t \in S$ such that $g(t, g(a_1^n), 1^{(n-2)}) \in I$ and so $g(a_1^{i-1}, g(t, a_i, 1^{(n-2)}), a_{i+1}^n) \in I$. Without loss of generality, we may assume that $g(a_1^{n-1}, g(t, a_n, 1^{(n-2)})) \in I$. Since I is an n-ary S-primary hyperideal of R, then there exist $s \in S$ such that at least one of the cases hold: $g(s, a_i, 1^{(n-2)}) \in I$ for some $1 \leq i \leq n-1$, $g(s, g(t, a_n, 1^{(n-2)}), 1^{(n-2)}) \in I$, $g(a_1^{i-1}, s, a_{i+1}^{n-1}, g(t, a_n, 1^{(n-2)})) \in \sqrt{I}^{(m,n)}$ for some $1 \leq i \leq n-1$ or $g(a_1^{n-1}, s) \in \sqrt{I}^{(m,n)}$.

If $g(s, a_i, 1^{(n-2)}) \in I$ for some $1 \leq i \leq n-1$, then we get $G(\frac{a_i}{s_i}, \frac{1}{1}^{(n-1)}) = \frac{g(a_i, 1^{(n-1)})}{g(s_i, 1^{(n-1)})} = \frac{g(s, a_i, 1^{(n-2)})}{g(s, s_i, 1^{(n-2)})} \in S^{-1}I$. If $g(s, g(t, a_n, 1^{(n-2)}), 1^{(n-2)}) \in I$, then $G(\frac{a_n}{s_n}, \frac{1}{1}^{(n-1)}) = \frac{g(a_n, 1^{(n-1)})}{g(s_n, 1^{(n-1)})} = \frac{g(s, t, a_n, 1^{(n-3)})}{g(s, t, s_n, 1^{(n-3)})} \in S^{-1}I$. If $g(a_1^{i-1}, s, a_{i+1}^{n-1}, g(t, a_n, 1^{(n-2)}) \in \sqrt{I}^{(m,n)}$ for some $1 \leq i \leq n-1$, then we get $G(\frac{a_1}{s_1}, ..., \frac{a_{i-1}}{s_{i-1}}, \frac{1}{1}, \frac{a_{i+1}}{s_{i+1}}, ..., \frac{a_{n-1}}{s_{n-1}}, \frac{a_n}{s_n}) = \frac{g(a_1^{i-1}, g(s, t, 1^{(n-2)}), a_{i+1}^{n-1}, a_n)}{g(s_1^{i-1}, g(s, t, 1^{(n-2)}), s_{i+1}^{n-1}, a_n)} \in S^{-1}\sqrt{I}^{(m,n)} = \sqrt{S^{-1}I}^{(m,n)}$, by Lemma 4.7 in [6]. If $g(a_1^{n-1}, s) \in \sqrt{I}^{(m,n)}$, then $G(\frac{a_1}{s_1}, ..., \frac{a_{n-1}}{s_{n-1}}, \frac{1}{1}) = \frac{g(a_1^{n-1}, s)}{g(s_1^{n-1}, s)} \in S^{-1}\sqrt{I}^{(m,n)} = \sqrt{S^{-1}I}^{(m,n)}$. Thus $S^{-1}I$ is an n-ary primary hyperideal of $S^{-1}R$. □

The following theorem shows that if $S^{-1}I$ is an n-ary primary hyperideal of $S^{-1}R$ and $S^{-1}I \cap R = (I : s)$ for some $s \in S$, then I is an n-ary S-primary hyperideal of R.

Theorem 4.13. *Let R be a Krasner (m, n)-hyperring, S be an n-ary multiplicative subset of R such that $1 \in S$ and I be a hyperideal of R with $I \cap S = \varnothing$. If $S^{-1}I$ is an n-ary primary hyperideal of $S^{-1}R$ and $S^{-1}I \cap R = (I : s)$ for some $s \in S$, then I is an n-ary S-primary hyperideal of R.*

Proof. Suppose that $S^{-1}I$ is an n-ary primary hyperideal of $S^{-1}R$ and $S^{-1}I \cap R = (I : s)$ for some $s \in S$. Assume that $g(a_1^n) \in I$ for some $a_1^n \in R$. Then we obtain $G(\frac{a_1}{1}, \cdots, \frac{a_n}{1}) \in S^{-1}I$. Since $S^{-1}I$ is an n-ary primary hyperideal of $S^{-1}R$, we get $\frac{a_i}{1} \in S^{-1}I$ or $G(\frac{a_1}{1}, \cdots, \frac{a_{i-1}}{1}, \frac{1}{1}, \frac{a_{i+1}}{1}, \cdots, \frac{a_n}{1}) \in \sqrt{S^{-1}I}^{(m,n)}$ for some $1 \leq i \leq n$. In the former case, we have $a_i = \frac{a_i}{1} \in S^{-1}I \cap R$ which implies $a_i \in (I : s)$ by the hypothesis. This means $g(s, a_i, 1^{(n-2)}) \in I$ and we are done. In the second case, we get $G(G(\frac{a_1}{1}, \cdots, \frac{a_{i-1}}{1}, \frac{1}{1}, \frac{a_{i+1}}{1}, \cdots, \frac{a_n}{1})^{(t)}, \frac{1}{1}^{(n-t)}) \in S^{-1}I$ for $t \leq n$ or $G_{(l)}(G(\frac{a_1}{1}, \cdots, \frac{a_{i-1}}{1}, \frac{1}{1}, \frac{a_{i+1}}{1}, \cdots, \frac{a_n}{1})^{(t)}) \in S^{-1}I$ for $t = l(n-1)+1$. The first possibility follows that $g(g(a_1, \cdots, a_{i-1}, 1, a_{i+1}, \cdots, a_n)^{(t)}, 1^{(n-t)}) = \frac{g(g(a_1, \cdots, a_{i-1}, 1, a_{i+1}, \cdots, a_n)^{(t)}, 1^{(n-t)})}{g(g(1^{(n)})^{(t)}, 1^{(n-t)})} \in S^{-1}I \cap R$. Hence we conclude that $g(s, g(a_1, \cdots, a_{i-1}, 1, a_{i+1}, \cdots, a_n)^{(t)}, 1^{(n-t-1)}) \in I$, by the hypothesis. So $g(g(a_1, \cdots, a_{i-1}, s, a_{i+1}, \cdots, a_n)^{(t)}, 1^{(n-t)}) \in I$ which implies $g(a_1, \cdots, a_{i-1}, s, a_{i+1}, \cdots, a_n) \in \sqrt{I}^{(m,n)}$, as needed. In the second possibility, one can easily complete the proof by using an argument similar. □

5. Conclusion

In this paper, the author extended the study initiated in [17] about S-prime ideals and in [25] about S-primary ideals in a commutative ring. The concepts of n-ary S-prime and n-ary S-primary hyperideals were introduced in a Krasner (m, n)-hyperring. Some of their essential characteristics were investigated and some examples were provided. Moreover, the stabilty of the notions were examined in some hyperring-theoretic constructions.

This study can be continued in several directions, such as: to define graded S-prime and graded S-primary hyperideals, to analyse similar notions in the context of (m, n)-hypermodules.

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