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Soft expandable spaces

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Abstract. In this study, we first introduce a new class of spaces, namely soft expandable spaces, which is a generalization of soft paracompact and countably soft compact spaces. Some properties of these spaces that are discussed in this paper the soft expandable space is equivalent to every A_σ-soft cover has soft$_{\mathcal{LF}}$ soft open refinement. Also, we discuss under what conditions that countably soft expandable space is soft paracompact. With the help of interesting examples, we elucidate there is no relationship between soft topology and its parametric topologies with respect to possession the property of being a soft expandable space. In this regards, we discuss the role of extended soft topology to inherited this property to classical topology. Second, we define the concept of s-expandable spaces which is stronger than soft expandable spaces. We give some characterizations of this concept, and investigate some equivalent concepts to s-expandable spaces. In the end, we study the behaviours of soft s-expandable spaces under some types of soft mappings.

1. Introduction

Many practical problems in various scientific fields such as economics, engineering, environment, and medical science need solutions by technical methods rather than dealing with classical methods. Molodtsov [32] introduced soft set theory as a new mathematical tool to deal with problems involving uncertainties, after that many researchers developed it, the first attempt was made by Maji et al.[31] in 2003, he defined null and absolute soft sets, complement of a soft set and the operators of soft intersection and union between two soft sets. In 2009, some soft operators were redefined and new soft operators were proposed by Ali et al. [2].

In 2011, Shabir and Naz [36] introduced the analysis of soft topological spaces, they used the soft sets to describe soft topology and they developed fundamental notions of soft topological spaces such as soft open, soft closed, soft closure, soft neighbourhood of a point, soft subspace, soft T_i-spaces, for $i = 1, 2, 3, 4$, soft regular and normal spaces. Min [29] made further investigations on soft T_i-spaces and demonstrated that a soft T_3-space is soft T_2. Hussain and Ahmad [21] established the main properties of soft interior and soft closure operators.

The authors of [16, 33] created brilliant concept known as soft point, which was used to analysis some properties of soft interior points and soft neighborhood systems. On the other hand, soft mapping

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were identified and its master properties were created by Kharal and Ahmad [25], where the idea of soft continuous mapping was discussed by Zorlutuna and Çakir [42]. Aygünoğlu and Aygün [13] introduced the concept of soft compact spaces and Al-shami et al. [6] familiarized the concepts of almost soft compact and approximately soft Lindelöf spaces. Hida [20] provided another description for soft compact spaces, namely SCPT1 which is stronger than soft compact spaces that defined in [13]. Two techniques of displaying soft separation axioms were followed, one of them using distinct ordinary points [35, 39] and the other using distinct soft points [14, 22, 38]. The role enriched (or extended) soft topological spaces in verifying some results related to soft compact spaces, soft continuous mappings and soft separation axioms in [8]. The concepts of soft Menger, nearly soft Menger and almost soft Menger spaces were explored and discussed in [26], [9], and [10], respectively. There are many other researchers have contributed towards covering property as [3, 6, 11], in particular, the authors in [28] and [40] presented soft paracompact spaces and soft s-paracompact spaces and studied some of their properties. Alcantud et al. [1] instigated the notions of caliber and chain conditions via soft topologies.

The rest of this manuscript is divided into four sections as follows. Section 2 recalls the concepts and findings that assist to understand and discuss our new sequels. The first main section of this work is Section 3 which is devoted to introducing the concept of soft expandable spaces and studying some basic properties of these spaces. Section 4 is the second main section, it is devoted to presenting the concepts of soft$_{\mathcal{SLF}}$ collections and soft s-expandable spaces. Some descriptions for these concepts are investigated and some explicatory counterexamples are supplied. Finally, some conclusions and upcoming works are given in Section 5.

2. Preliminaries

Herein, we draw the reader's attention to the notions and conclusions that are mentioned in the various previous studies with respect to a fixed parameters set to explore and discuss our new sequels.

Definition 2.1. ([32]) If $\mathcal{G}$ is a mapping from a parameters set Υ to $2^{\mathcal{S}}$, then the pair $(\mathcal{G},\Upsilon)$ is called a soft set over $\mathcal{S}$ and it can be written as follows: $(\mathcal{G},\Upsilon) = \{(a,\mathcal{G}(a)) : a \in \Upsilon \text{ and } \mathcal{G}(a) \in 2^{\mathcal{S}}\}$.

Definition 2.2. ([4, 16]) Let $(\mathcal{G},\Upsilon)$ and $(\mathcal{F},\Upsilon)$ be two soft sets over $\mathcal{S}$. Then:

(i) $(\mathcal{G},\Upsilon)$ is an absolute soft set if $\mathcal{G}(a) = \mathcal{S}$ for each $a \in \Upsilon$ and it is denoted by $\widetilde{\mathcal{S}}$.

(ii) $(\mathcal{G},\Upsilon)$ is a null soft set if $\mathcal{G}(a) = \emptyset$ for each $a \in \Upsilon$ and it is denoted by Φ.

(iii) $(\mathcal{G},\Upsilon)$ is a soft subset of $(\mathcal{F},\Upsilon)$ if $\mathcal{G}(a) \subseteq \mathcal{F}(a)$ for each $a \in \Upsilon$ and it is denoted by $(\mathcal{G},\Upsilon) \sqsubseteq (\mathcal{F},\Upsilon)$.

(iv) $(\mathcal{G},\Upsilon) \sqcup (\mathcal{F},\Upsilon) = (\mathcal{H},\Upsilon)$, where $\mathcal{H}(a) = \mathcal{G}(a) \bigcup \mathcal{F}(a)$ for each $a \in \Upsilon$.

(v) $(\mathcal{G},\Upsilon) \sqcap (\mathcal{F},\Upsilon) = (\mathcal{H},\Upsilon)$, where $\mathcal{H}(a) = \mathcal{G}(a) \bigcap \mathcal{F}(a)$ for each $a \in \Upsilon$.

(vi) The complement of a soft set $(\mathcal{G},\Upsilon)$, denoted by $(\mathcal{G},\Upsilon)^c$, is defined by $(\mathcal{G},\Upsilon)^c = (\mathcal{G}^c,\Upsilon)$, where a mapping $\mathcal{G}^c : \Upsilon \to 2^{\mathcal{S}}$ is given by $\mathcal{G}^c(a) = \mathcal{S} - \mathcal{G}(a)$ for each $a \in \Upsilon$.

Definition 2.3. ([17, 36]) Let $(\mathcal{G},\Upsilon)$ be a soft set over $\mathcal{S}$. Then:

(i) $t \in (\mathcal{G},\Upsilon)$ if $t \in \mathcal{G}(a)$ for each $a \in \Upsilon$.

(ii) $t \Subset (\mathcal{G},\Upsilon)$ if $t \in \mathcal{G}(a)$ for some $a \in \Upsilon$.

Definition 2.4. ([33]) A collection $\mathfrak{T}$ of soft sets over $\mathcal{S}$ with a fixed parameters set Υ is called a soft topology on $\mathcal{S}$ if it satisfies the following:

(i) The null soft set Φ and the absolute soft set $\widetilde{\mathcal{S}}$ are members of $\mathfrak{T}$.

(ii) The soft union of an arbitrary number of soft sets in $\mathfrak{T}$ is also a member of $\mathfrak{T}$.

(iii) The soft intersection of a finite number of soft sets in $\mathfrak{T}$ is also a member of $\mathfrak{T}$.

The triple $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is called a soft topological space and it is denoted by soft$_{\mathcal{TS}}$. Each soft set in $\mathfrak{T}$ is called soft open and its complement is called soft closed.

Definition 2.5. ([33]) Let $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ be a a soft topological space and $(\mathcal{G}, \Upsilon)$ be a non-null soft subset of $\widetilde{\mathcal{S}}$. Then $\mathfrak{T}_{(\mathcal{G},\Upsilon)} = \{(\mathcal{G}, \Upsilon) \sqcap (\mathcal{O}, \Upsilon) : (\mathcal{O}, \Upsilon) \in \mathfrak{T}\}$ is called a relative soft topology on $(\mathcal{G}, \Upsilon)$ and $((\mathcal{G}, \Upsilon), \mathfrak{T}_{(\mathcal{G},\Upsilon)}, \Upsilon)$ is called a soft subspace of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$.

Proposition 2.6. *([33]) Let $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ be a soft$_{\mathcal{TS}}$. Then $\mathfrak{T}_a = \{\mathcal{G}(a) : (\mathcal{G}, \Upsilon) \in \mathfrak{T}\}$ defines a topology on $\mathcal{S}$ for each $a \in \Upsilon$.*

Definition 2.7. ([33]) Let $(\mathcal{G}, \Upsilon)$ be a soft subset of a soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$. Then $(cl(\mathcal{G}), \Upsilon)$ is defined by $cl(\mathcal{G})(a) = cl(\mathcal{G}(a))$, where $cl(\mathcal{G}(a))$ is the closure of $\mathcal{G}(a)$ in $(\mathcal{S}, \mathfrak{T}_a)$ for each $a \in \Upsilon$.

Proposition 2.8. ([33]) *Let $(\mathcal{G}, \Upsilon)$ be a soft subset of a soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$. Then:*

(i) $(cl(\mathcal{G}), \Upsilon) \sqsubseteq cl(\mathcal{G}, \Upsilon)$.

(ii) $(cl(\mathcal{G}), \Upsilon) = cl(\mathcal{G}, \Upsilon)$ *iff* $(cl(\mathcal{G}), \Upsilon)^c$ *is soft closed.*

Lemma 2.9. ([4]) *If $(\mathcal{O}, \Upsilon)$ is a soft open subset of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$, then $(\mathcal{O}, \Upsilon) \sqcap cl(\mathcal{G}, \Upsilon) \sqsubseteq cl((\mathcal{O}, \Upsilon) \sqcap (\mathcal{G}, \Upsilon))$ for each $(\mathcal{G}, \Upsilon) \sqsubseteq \widetilde{\mathcal{S}}$.*

Definition 2.10. A soft set $(\mathcal{P}, \Upsilon)$ over $\mathcal{S}$ is called:

(i) Soft point [9, 19] if there is $a \in \Upsilon$ and $s \in \mathcal{S}$ with $\mathcal{P}(a) = \{s\}$ and $\mathcal{P}(q) = \emptyset$ for each $q \in \Upsilon - \{a\}$. A soft point is denoted by $\mathcal{P}^s_a$.

(ii) Pseudo constant [20] provided that $\mathcal{P}(a) = \mathcal{S}$ or $\emptyset$ for each $a \in \Upsilon$. A family of all pseudo constant soft sets is denoted by $CS(\mathcal{S}, \Upsilon)$.

Definition 2.11. ([12]) Let $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ be a soft$_{\mathcal{TS}}$. Then $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is called a soft extremally disconnected, denoted by soft$_{\mathcal{E}.\mathcal{D}.}$, if the soft closure of every soft open set is soft open; equivalently, if the soft interior of every soft closed set is soft closed.

Definition 2.12. ([15]) A soft subset $(\mathcal{G}, \Upsilon)$ of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is called soft semi open if $(\mathcal{G}, \Upsilon) \sqsubseteq cl(int((\mathcal{G}, \Upsilon))$. The complement of soft semi open is called a soft semi closed set.

The family of all soft semi open (resp. soft semi closed) subset of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ will be denoted $\mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$ (resp. $\mathcal{SSC}(\mathcal{S}, \mathfrak{T}, \Upsilon)$). The soft union of soft semi open set is soft semi open set. The soft semi closure of $(\mathcal{G}, \Upsilon)$ is the soft intersection of all soft semi closed sets of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ containing $(\mathcal{G}, \Upsilon)$ and is denoted by $scl(\mathcal{G}, \Upsilon)$ [15] .

Lemma 2.13. ([12]) *Let a soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ be soft$_{\mathcal{E}.\mathcal{D}}$. If $(\mathcal{G}, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$, then $scl(\mathcal{G}, \Upsilon) = cl(\mathcal{G}, \Upsilon)$.*

Definition 2.14. ([24]) A soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is called a soft semi regular space if for each $(\mathcal{G}, \Upsilon) \in \mathcal{SSC}(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with $\mathcal{P}^s_a \notin (\mathcal{G}, \Upsilon)$ there are $(\mathcal{H}_1, \Upsilon), (\mathcal{H}_2, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$ such that $\mathcal{P}^s_a \in (\mathcal{H}_1, \Upsilon)$, $(\mathcal{G}, \Upsilon) \sqsubseteq (\mathcal{H}_2, \Upsilon)$ and $(\mathcal{H}_1, \Upsilon) \sqcap (\mathcal{H}_2, \Upsilon) = \Phi$.

Definition 2.15. ([6, 28]) A collection $\{(\mathcal{G}_\gamma, \Upsilon) : \gamma \in \Delta\}$ of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is called soft locally finite, denoted by soft$_{\mathcal{LF}}$, if for each $\mathcal{P}^s_a \in \widetilde{\mathcal{S}}$ there is a soft open set $(\mathcal{O}, \Upsilon)$ satisfies that $\mathcal{P}^s_a \in (\mathcal{O}, \Upsilon)$ and a set $\{\mathfrak{m} : (\mathcal{O}, \Upsilon) \sqcap (\mathcal{G}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is finite.

Theorem 2.16. ([6, 28]) *Let $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ be a collection of subsets of a soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$. Then:*

(i) $\mathfrak{L}$ *is soft$_{\mathcal{LF}}$ iff* $\{cl(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ *is soft$_{\mathcal{LF}}$.*

(ii) $cl(\bigsqcup_{\gamma\in\Delta}(\mathcal{L}_\gamma,\Upsilon)) = \bigsqcup_{\gamma\in\Delta} cl(\mathcal{L}_\gamma,\Upsilon)$ *if* $\mathfrak{L}$ *is soft*$_{\mathcal{LF}}$.

Definition 2.17. ([23]) A collection $\mathfrak{L}$ of subsets of a soft$_{\mathcal{TS}}$ $(\mathcal{S},\mathfrak{T},\Upsilon)$ is called σ-soft locally finite, denoted by σ-soft$_{\mathcal{LF}}$, iff $\mathfrak{L} = \bigsqcup_{k\in\mathbb{N}} \mathfrak{L}_k$ with each $\mathfrak{L}_k$ is soft$_{\mathcal{LF}}$ collection.

Definition 2.18. Let $(\mathcal{S},\mathfrak{T},\Upsilon)$ be a soft$_{\mathcal{TS}}$. Then:

(i) A soft open cover of $\widetilde{\mathcal{S}}$ [13] is a collection $\{(O_\gamma,\Upsilon) : \gamma \in \Delta\}$ of soft open subset of $(\mathcal{S},\mathfrak{T},\Upsilon)$ with $\widetilde{\mathcal{S}} = \bigsqcup_{\gamma\in\Delta}(O_\gamma,\Upsilon)$. A soft open cover is countable if $|\Delta| \leqslant \omega_0$, where ω_0 is the first infinite ordinal.

(ii) $(\mathcal{S},\mathfrak{T},\Upsilon)$ is soft compact [13](resp. countable soft compact [34]) if every soft open (resp. countable soft open) cover of $(\mathcal{S},\mathfrak{T},\Upsilon)$ contains a finite subcover of $(\mathcal{S},\mathfrak{T},\Upsilon)$.

(iii) $(\mathcal{S},\mathfrak{T},\Upsilon)$ is soft paracompact [28] (resp., soft s-paracompact [40]) if every soft open cover $\mathfrak{L}$ of $(\mathcal{S},\mathfrak{T},\Upsilon)$ has a soft$_{\mathcal{LF}}$ soft open (resp., soft$_{\mathcal{LF}}$ soft semi open) refinement $\mathfrak{H}$ that covers $(\mathcal{S},\mathfrak{T},\Upsilon)$.

(iv) $(\mathcal{S},\mathfrak{T},\Upsilon)$ is countably soft paracompact [28] if every countable soft open cover $\mathfrak{L}$ of $(\mathcal{S},\mathfrak{T},\Upsilon)$ has a soft$_{\mathcal{LF}}$ soft open refinement $\mathfrak{H}$ that covers $(\mathcal{S},\mathfrak{T},\Upsilon)$.

Definition 2.19. ([41]) Let $(\mathcal{S},\mathfrak{T}_\mathcal{S},\Upsilon)$ and $(\mathcal{Z},\mathfrak{T}_\mathcal{Z},\Upsilon^*)$ be two soft$_{\mathcal{TS}}$. If $h : \mathcal{S} \to \mathcal{Z}$ and $\varphi : \Upsilon \to \Upsilon^*$ then for each $(\mathcal{G},\Upsilon) \in (\mathcal{S},\mathfrak{T}_\mathcal{S},\Upsilon)$ and $(\mathcal{F},\Upsilon^*) \in (\mathcal{Z},\mathfrak{T}_\mathcal{Z},\Upsilon^*)$ a soft mapping $h_\varphi : (\mathcal{S},\mathfrak{T}_\mathcal{S},\Upsilon) \to (\mathcal{Z},\mathfrak{T}_\mathcal{Z},\Upsilon^*)$ is defined by:

(i) $h_\varphi(\mathcal{G},\Upsilon)(b) = (h_\varphi(\mathcal{G}),\Upsilon^*) = h(\bigcup_{a\in\varphi^{-1}(b)}\mathcal{G}(a))$, where $b \in \Upsilon^*$.

(ii) $h_\varphi^{-1}(\mathcal{F},\Upsilon^*)(a) = (h_\varphi^{-1}(\mathcal{F}),\Upsilon) = h^{-1}((\mathcal{F},\Upsilon^*)(\varphi(a)))$, where $a \in \Upsilon$.

The above definition was improved in [5].

Definition 2.20. ([30, 41]) A soft mapping $h_\varphi : (\mathcal{S},\mathfrak{T}_\mathcal{S},\Upsilon) \to (\mathcal{Z},\mathfrak{T}_\mathcal{Z},\Upsilon)$ is called:

(i) Soft continuous (resp. soft irresolute) if $h_\varphi^{-1}(C,\Upsilon)$ is soft open (resp. soft semi open) set for every soft open (resp. soft semi open) subset (C,Υ) of $(\mathcal{Z},\mathfrak{T}_\mathcal{Z},\Upsilon)$.

(ii) Soft closed (resp. soft semi closed) if $h_\varphi(C,\Upsilon)$ is a soft closed (resp. soft semi closed) set in $(\mathcal{Z},\mathfrak{T}_\mathcal{Z},\Upsilon)$ for every soft closed subset (C,Υ) of $(\mathcal{S},\mathfrak{T}_\mathcal{S},\Upsilon)$.

Definition 2.21. ([13]) Let $(\mathcal{S},\mathfrak{T},\Upsilon)$ be a soft$_{\mathcal{TS}}$. A soft topology $\mathfrak{T}$ on $\mathcal{S}$ is called an extended soft topology if $\mathfrak{T} = \{(O,\Upsilon) : O(\mathfrak{a}) \in \mathfrak{T}_\mathfrak{a}$ for each $\mathfrak{a} \in \Upsilon\}$ where $\mathfrak{T}_\mathfrak{a}$ is a parametric topology on S.

3. Soft Expandable Spaces

In this section, we define the concept of soft expandable spaces and revel its relationships with some soft spaces such as countably soft compact, soft paracompact and countably soft paracompact. We study main properties and determine under what conditions a soft expandable space is equivalent to some soft spaces. Also, we investigate the behaviour of soft expandable spaces under soft closed (soft continuous) mappings. Finally, we discuss the role of extended soft topology to inherited this property to classical topology.

Definition 3.1. A soft$_{\mathcal{TS}}$ $(\mathcal{S},\mathfrak{T},\Upsilon)$ is called soft expandable if for each soft$_{\mathcal{LF}}$ $\mathfrak{L} = \{(\mathcal{L}_\gamma,\Upsilon) : \gamma \in \Delta\}$ of subsets of $\mathcal{S}$, there is a soft$_{\mathcal{LF}}$ $\mathfrak{G} = \{(\mathcal{G}_\gamma,\Upsilon) : \gamma \in \Delta\}$ of soft open subsets of $\mathcal{S}$ satisfying for each $\gamma \in \Delta$ that $(\mathcal{L}_\gamma,\Upsilon) \sqsubseteq (\mathcal{G}_\gamma,\Upsilon)$.

If $|\Delta| \leqslant \omega_0$, we call a soft$_{\mathcal{TS}}$ $(\mathcal{S},\mathfrak{T},\Upsilon)$ countably soft expandable.

Example 3.2. Let $(\mathbb{R},\mathfrak{T},\Upsilon)$ be a soft$_{\mathcal{TS}}$ with $\Upsilon = \{a_1,a_2\}$ and $\mathfrak{T} = \{\mathbb{R},(O,\Upsilon) : (O,\Upsilon) \sqsubseteq \widetilde{\mathbb{Q}}\}$. Note that every soft$_{\mathcal{LF}}$ collection is finite. Hence, $(\mathbb{R},\mathfrak{T},\Upsilon)$ is soft expandable.

Example 3.3. Let $(\mathbb{N}, \mathfrak{T}, \Upsilon)$ be a soft$_{\mathcal{TS}}$ with $\Upsilon = \{a_1, a_2\}$ and $\mathfrak{T} = \{\Phi, (O, \Upsilon) \sqsubseteq \widetilde{\mathbb{N}} : 1 \in (O, \Upsilon)\}$. Then $(\mathbb{N}, \mathfrak{T}, \Upsilon)$ is not soft expandable.

In classical topological property reports that every finite topology is expandable need not be true in soft topology as the following example elucidates. This property is one of the diverges between classical topology and soft topology.

Example 3.4. Let $(S, \mathfrak{T}, \mathbb{R})$ be a soft$_{\mathcal{TS}}$ with $S = \{1, 2, 3, 4\}$ and $\mathfrak{T} = \{\Phi, (O, \mathbb{R}) \sqsubseteq \widetilde{S} : 1 \in (O, \mathbb{R})\}$. Now, $\{\mathcal{P}_a^s \in \widetilde{S} : \forall a \in \mathbb{R}$ such that $s \neq 1\}$ forms a soft$_{\mathcal{LF}}$ collection. It can be noted that $(S, \mathfrak{T}, \mathbb{R})$ is not soft expandable.

Proposition 3.5. *A soft$_{\mathcal{TS}}$ is soft expandable iff for each soft$_{\mathcal{LF}}$ collection $\{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ of subsets of $\mathcal{S}$, there is a soft$_{\mathcal{LF}}$ collection $\{(\mathcal{G}_\gamma, \Upsilon) : \gamma \in \Delta\}$ of soft open subsets of $\mathcal{S}$ satisfying for each $\gamma \in \Delta$ that $(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (\mathcal{G}_\gamma, \Upsilon)$.*

Proof. The necessary part is obvious. Conversely, let $\{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ be a soft$_{\mathcal{LF}}$ collection of subsets of $\mathcal{S}$. It is well known that $\{cl(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ be also a soft$_{\mathcal{LF}}$ collection of closed subsets of $\mathcal{S}$. By hypothesis, there is a soft$_{\mathcal{LF}}$ collection $\{(\mathcal{G}_\gamma, \Upsilon) : \gamma \in \Delta\}$ of soft open subsets of $\mathcal{S}$ satisfying for each $\gamma \in \Delta$ that $cl(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (\mathcal{G}_\gamma, \Upsilon)$. Hence, the proof is complete. □

It is easy to prove the following result.

Proposition 3.6. (*i*) *If $\mathcal{S}$ and Υ be finite sets, then a soft$_{\mathcal{TS}}$ is soft expandable.*

(*ii*) *Every discrete soft$_{\mathcal{TS}}$ is soft expandable.*

(*iii*) *A soft$_{\mathcal{TS}}$ is soft expandable if every soft$_{\mathcal{LF}}$ collection is finite.*

The following theorem shows that the soft expandable space is a generalization for soft paracompact and countably soft compact space.

Theorem 3.7. *A soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft expandable if one of the following holds:*

(*i*) *$(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft paracompact.*

(*ii*) *$(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is countably soft compact.*

Proof. (i) Let $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ be a soft$_{\mathcal{LF}}$ of soft closed subsets of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$. Assume that all finite subsets of Δ is the collection Δ'. For each $\alpha \in \Delta'$, set $(O_\alpha, \Upsilon) = \widetilde{\mathcal{S}} - \sqcup\{(\mathcal{L}_\gamma, \Upsilon) : \gamma \notin \alpha\}$ and $\mathfrak{G} = \{(O_\alpha, \Upsilon) : \alpha \in \Delta'\}$. Then:

(1) For each $\alpha \in \Delta'$, (O_α, Υ) is soft open.

(2) For each $\alpha \in \Delta'$, $\{\mathfrak{m} : (O_\alpha, \Upsilon) \sqcap (\mathcal{L}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is finite.

(3) The collection $\mathfrak{G}$ is a soft open cover of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$. For each $\mathcal{P}_a^s \in \widetilde{\mathcal{S}}$, there is a soft open set $(O_{\mathcal{P}_a^s}, \Upsilon)$ with $\{\mathfrak{m} : (O_{\mathcal{P}_a^s}, \Upsilon) \sqcap (\mathcal{L}_\mathfrak{m}, \Upsilon) \neq \Phi\} = \{\gamma_1, \gamma_2, ..., \gamma_k\}$. Take $\alpha = \{\gamma_1, \gamma_2, ..., \gamma_k\}$, then $\mathcal{P}_a^s \in (O_\alpha, \Upsilon)$ and, by assumption, $\mathfrak{G}$ has a soft$_{\mathcal{LF}}$ soft open refinement $\mathfrak{H} = \{(\mathcal{H}_\beta, \Upsilon) : \beta \in \Delta\}$. For each $\gamma \in \Delta$, define $(\mathcal{G}_\gamma, \Upsilon) = \sqcup\{(\mathcal{H}, \Upsilon) \in \mathfrak{H} : (\mathcal{H}, \Upsilon) \sqcap (\mathcal{L}_\gamma, \Upsilon) \neq \Phi\}$. Then for each $\gamma \in \Delta$, the set $(\mathcal{G}_\gamma, \Upsilon)$ is soft open and $(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (\mathcal{G}_\gamma, \Upsilon)$. Now, we need only to prove that $\mathfrak{C} = \{(\mathcal{G}_\gamma, \Upsilon) : \alpha \in \Delta\}$ is soft$_{\mathcal{LF}}$. Assume that $\mathcal{P}_a^s \in \widetilde{\mathcal{S}}$. Then there is a soft open set $(\mathcal{G}_{\mathcal{P}_a^s}, \Upsilon)$ that contains $\mathcal{P}_a^s$ and the set $\{\mathfrak{m} : (\mathcal{G}_{\mathcal{P}_a^s}, \Upsilon) \sqcap (\mathcal{H}_\beta, \Upsilon) \neq \Phi\}$ is finite. Therefore, $(\mathcal{G}_{\mathcal{P}_a^s}, \Upsilon) \sqcap (\mathcal{G}_\gamma, \Upsilon) \neq \Phi$ iff $(\mathcal{G}_{\mathcal{P}_a^s}, \Upsilon) \sqcap (\mathcal{H}_\beta, \Upsilon) \neq \Phi$ and $(\mathcal{H}_\beta, \Upsilon) \sqcap (\mathcal{L}_\gamma, \Upsilon) \neq \Phi$ for some $\beta \in \Delta$. By the refinement of $\mathfrak{H}$ for $\mathfrak{G}$ and from (2), we obtain $\mathcal{G}$ is soft$_{\mathcal{LF}}$ and hence $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft expandable.

(ii) Let $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ be a soft$_{\mathcal{LF}}$ of subsets of $\mathcal{S}$. If $|\Delta| < \omega_\circ$, then the proof is obvious, so we need to prove the result in case $|\Delta| \geqslant \omega_\circ$. Say $|\Delta| = \omega_\circ$, which means $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma = 1, 2, ...\}$. For $k \in \mathbb{N}$, define $(\mathcal{G}_k, \Upsilon) = \bigsqcup_{\gamma=k}^{\infty} cl(\mathcal{L}_\gamma, \Upsilon)$. Then:

(1) For each $k \in \mathbb{N}$, the set $(\mathcal{G}_k, \Upsilon)$ is soft closed with $(\mathcal{G}_{k+1}, \Upsilon) \sqsubseteq (\mathcal{G}_k, \Upsilon)$.

(2) $\bigsqcap_{k=1}^{\infty}(\mathcal{G}_k,\Upsilon) \neq \Phi$, because the collection $\mathfrak{G} = \{(\mathcal{G}_\gamma,\Upsilon) : \gamma = 1,2,...\}$ is decreasing of soft closed subset of countably soft compact $(\mathcal{S},\mathfrak{T},\Upsilon)$ and hence there is $\mathcal{P}^s_a \in \bigsqcap_{k=1}^{\infty}(\mathcal{G}_k,\Upsilon)$. This means that $\{\gamma : \mathcal{P}^s_a \in cl(\mathcal{L}_\gamma,\Upsilon)\}$ is an infinite set which contradicts with the collection $cl(\mathfrak{L})$ is soft$_{\mathcal{LF}}$. Thus, Δ is finite; by Proposition 3.6, $(\mathcal{S},\mathfrak{T},\Upsilon)$ is soft expandable. □

The converse of the theorem above fails as the next examples clarifies.

Example 3.8. Let $(\mathcal{S},\mathfrak{T},\Upsilon)$ be a discrete soft$_{\mathcal{TS}}$ such that $\mathcal{S}$ is uncountable. Then, according to (ii) of Proposition 3.6, $(\mathcal{S},\mathfrak{T},\Upsilon)$ is soft expandable. On the other hand, $(\mathcal{S},\mathfrak{T},\Upsilon)$ is not countable compact.

Example 3.9. It is well known that the soft topology and classical topology are identical when the set of parameters is a singleton. Therefore, we suffice by the example given in [37] which displays a topology (called order topology or interval topology) that is soft expandable, but not soft paracompact.

We show, in the following result, the equivalence between countably soft expandable and countably soft paracompact spaces.

Theorem 3.10. *A soft$_{\mathcal{TS}}$ $(\mathcal{S},\mathfrak{T},\Upsilon)$ is countably soft expandable iff it is countably soft paracompact.*

Proof. (⇒) Let $\mathfrak{G} = \{(O_\gamma,\Upsilon) : \gamma \in \Delta, |\Delta| \leqslant \omega_\circ\}$ be a soft open cover of a soft$_{\mathcal{TS}}$ $(\mathcal{S},\mathfrak{T},\Upsilon)$. Define $(\mathcal{U}_\gamma,\Upsilon) = \sqcup\{(O_k,\Upsilon) : k = 1,2,...,\gamma\}$ and for each $\gamma \geqslant 2$, set $(\mathcal{H}_\gamma,\Upsilon) = (\mathcal{U}_\gamma,\Upsilon) - (\mathcal{U}_{\gamma-1},\Upsilon)$ where $(\mathcal{H}_1,\Upsilon) = (\mathcal{U}_1,\Upsilon)$. Then:

(1) The collection $\mathfrak{H} = \{(\mathcal{H}_\gamma,\Upsilon) : \gamma \in \Delta\}$ is a soft refinement of $\mathfrak{G}$. Since $\mathcal{P}^s_a \in \widetilde{\mathcal{S}}$, $\mathcal{P}^s_a \in (\mathcal{H}_{\gamma_{\mathcal{P}^s_a}},\Upsilon)$, where $\gamma_{\mathcal{P}^s_a}$ is the smallest γ with $\mathcal{P}^s_a \in (O_\gamma,\Upsilon)$. Also, $(\mathcal{H}_\gamma,\Upsilon) \sqsubseteq (O_\gamma,\Upsilon)$ for each $\gamma \in \Delta$; this means that $\mathfrak{H}$ is a soft refinement of $\mathfrak{G}$.

(2) The collection $\mathfrak{H}$ is soft$_{\mathcal{LF}}$ since $(O_\gamma,\Upsilon) \sqcap (\mathcal{H}_j,\Upsilon) = \Phi$ for $j > \gamma$. By assumption, there is a soft$_{\mathcal{LF}}$ $\mathfrak{A} = \{(\mathcal{G}_\gamma,\Upsilon) : \gamma \in \Delta\}$ of soft open sets with for each $\gamma \in \Delta$, $(\mathcal{H}_\gamma,\Upsilon) \sqsubseteq (\mathcal{G}_\gamma,\Upsilon)$. Define the collection $\mathfrak{L} = \{(\mathcal{L}_\gamma,\Upsilon) : \gamma \in \Delta\}$, where $(\mathcal{L}_\gamma,\Upsilon) = (O_\gamma,\Upsilon) \sqcap (\mathcal{G}_\gamma,\Upsilon)$. To show that $\mathfrak{L}$ is soft$_{\mathcal{LF}}$ soft open refinement of $\mathfrak{G}$, let $\mathcal{P}^s_a \in \widetilde{\mathcal{S}}$. Then $\mathcal{P}^s_a \in (\mathcal{H}_\gamma,\Upsilon)$ for some $\gamma \in \Delta$; so that $\mathcal{P}^s_a \in (\mathcal{L}_\gamma,\Upsilon)$. Thus, $\mathfrak{L}$ is a soft open refinement of $\mathfrak{G}$. $\mathfrak{L}$ is soft$_{\mathcal{LF}}$ because $\mathfrak{A}$ is soft$_{\mathcal{LF}}$. This implies that $\mathfrak{L}$ is a soft$_{\mathcal{LF}}$ soft open refinement of $\mathfrak{G}$. Hence, $(\mathcal{S},\mathfrak{T},\Upsilon)$ is countably soft paracompact.

(⇐) It can be proved the sufficient part following similar technique offered in Theorem 3.7. □

Corollary 3.11. *If a soft$_{\mathcal{TS}}$ $(\mathcal{S},\mathfrak{T},\Upsilon)$ is soft expandable space, then it is countably soft paracompact.*

Theorem 3.12. *A soft$_{\mathcal{TS}}$ $(\mathcal{S},\mathfrak{T},\Upsilon)$ is soft paracompact iff $(\mathcal{S},\mathfrak{T},\Upsilon)$ is countably soft expandable and every soft open cover has a σ-soft$_{\mathcal{LF}}$ soft open refinement.*

Proof. (⇒) It comes from Theorem 3.10 and Definition 2.18.

(⇐) If $\mathfrak{G} = \{(\mathcal{G}_\gamma,\Upsilon) : \Upsilon \in \Delta\}$ is a soft open cover of $(\mathcal{S},\mathfrak{T},\Upsilon)$. Then:

(1) There is $\mathfrak{L} = \bigsqcup_{k=1}^{\infty}(\mathfrak{L}_k,\Upsilon)$ as a σ-soft$_{\mathcal{LF}}$ soft open refinement of $\mathfrak{G}$, where $(\mathfrak{L}_k,\Upsilon) = \{(\mathcal{L}_{(\gamma,k)},\Upsilon) : \gamma \in \Delta_k\}$.

(2) The countable collection $\mathfrak{L}^* = \{(\mathfrak{L}^*{}_k,\Upsilon) : k = 1,2,...\}$ where $(\mathfrak{L}^*{}_k,\Upsilon) = \sqcup\{(\mathcal{L}_{(\gamma,k)},\Upsilon) : \gamma \in \Delta_k\}$ is a soft open cover of countably soft expandable space $(\mathcal{S},\mathfrak{T},\Upsilon)$. By Theorem 3.10, we find that $(\mathcal{S},\mathfrak{T},\Upsilon)$ is countably soft paracompact. So that $\mathfrak{L}^*$ has a soft$_{\mathcal{LF}}$ soft open refinement $\mathfrak{H} = \{(\mathcal{H}_k,\Upsilon) : k = 1,2,3,...\}$. Therefore, $\{(\mathcal{H}_k,\Upsilon) \sqcap (\mathcal{L}_{(\gamma,k)},\Upsilon) : \gamma \in \Delta_k,\ k = 1,2,3,...\}$ is a soft$_{\mathcal{LF}}$ soft open refinement of $\mathfrak{G}$; hence $(\mathcal{S},\mathfrak{T},\Upsilon)$ is soft paracompact. □

Proposition 3.13. *A soft$_{\mathcal{TS}}$ $(\mathcal{S},\mathfrak{T},\Upsilon)$ is soft expandable if for every soft$_{\mathcal{LF}}$ $\mathfrak{L} = \{(\mathcal{L}_\gamma,\Upsilon) : \gamma \in \Delta\}$ there is a soft$_{\mathcal{LF}}$ $\mathfrak{H} = \{(\mathcal{H}_\alpha,\Upsilon) : \alpha \in \Delta^*\}$ of soft open sets with*

(*i*) $\widetilde{\mathcal{S}} = \bigsqcup_{\alpha\in\Delta^*}(\mathcal{H}_\alpha,\Upsilon)$.

(*ii*) *For each $\alpha \in \Delta^*$, the set $\{\mathfrak{m} : (\mathcal{H}_\alpha,\Upsilon) \sqcap (\mathcal{L}_\mathfrak{m},\Upsilon) \neq \Phi\}$ is finite.*

Proof. Let $\mathfrak{H} = \{(\mathcal{H}_\alpha, \Upsilon) : \alpha \in \Delta^*\}$ be a collection of soft open subsets of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$. Suppose that $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is a soft$_{\mathcal{LF}}$ collection of subsets of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$. Define for each $\gamma \in \Delta$, $(\mathcal{G}_\gamma, \Upsilon) = \sqcup\{(\mathcal{H}_\alpha, \Upsilon) \in \mathfrak{H} : (\mathcal{H}_\alpha, \Upsilon) \sqcap (\mathcal{L}_\gamma, \Upsilon) \neq \Phi\}$. For each $\mathcal{P}^s_a \in \widetilde{\mathcal{S}}$ there is a soft open set $(O_{\mathcal{P}^s_a}, \Upsilon)$ with $\mathcal{P}^s_a \in (O_{\mathcal{P}^s_a}, \Upsilon)$ and the set $\{\mathfrak{m} : (O_{\mathcal{P}^s_a}, \Upsilon) \sqcap (\mathcal{H}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is finite. Since $(O_{\mathcal{P}^s_a}, \Upsilon) \sqcap (\mathcal{H}_\alpha, \Upsilon) \neq \Phi$ and $(\mathcal{H}_\alpha, \Upsilon) \sqcap (\mathcal{L}_\gamma, \Upsilon) \neq \Phi$ iff $(O_{\mathcal{P}^s_a}, \Upsilon) \sqcap (\mathcal{G}_\gamma, \Upsilon) \neq \Phi$ for some $\alpha \in \Delta^*$, the collection $\mathfrak{G} = \{(\mathcal{G}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is soft$_{\mathcal{LF}}$. Also, for each $\gamma \in \Delta$, $(\mathcal{G}_\gamma, \Upsilon)$ is soft open set with $(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (\mathcal{G}_\gamma, \Upsilon)$, which implies $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft expandable. □

Definition 3.14. A soft open cover $\mathfrak{O} = \{(O_\gamma, \Upsilon) : \gamma \in \Delta\}$ that has σ-soft$_{\mathcal{LF}}$ refinement is called A_σ-soft cover.

Theorem 3.15. *A soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft expandable iff every A_σ-soft cover has soft$_{\mathcal{LF}}$ soft open refinement.*

Proof. (⇒) Let $\mathfrak{O} = \{(O_\gamma, \Upsilon) : \gamma \in \Delta\}$ be an $\mathcal{A}_\sigma$-soft cover. Since $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft expandable, then $\mathfrak{O}$ has σ-soft$_{\mathcal{LF}}$ soft open refinement. Also:

(1) $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is countably soft paracompact, by Corollary 3.11.

(2) Every σ-soft$_{\mathcal{LF}}$ soft open cover of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ has soft$_{\mathcal{LF}}$ soft open refinement. To show that, let $\mathfrak{L} = \bigsqcup_{k\in\mathbb{N}} \mathfrak{L}_k$ be a σ-soft$_{\mathcal{LF}}$ soft open cover of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ where $\mathfrak{L}_k = \{(\mathcal{L}_{k_\gamma}, \Upsilon) : \gamma \in \Delta_k\}$. For each $k \in \mathbb{N}$, set $(\mathcal{L}_k, \Upsilon) = \bigsqcup_{\gamma\in\Delta_k} (\mathcal{L}_{k_\gamma}, \Upsilon)$. Then $\mathfrak{L}^* = \{(\mathcal{L}_k, \Upsilon) : k \in \mathbb{N}\}$ is a countable soft open cover of a countable soft paracompact $(\mathcal{S}, \mathfrak{T}, \Upsilon)$. Therefore, there is a soft$_{\mathcal{LF}}$ soft open refinement, say, $\mathfrak{G}^* = \{(\mathcal{G}_k, \Upsilon) : k \in \mathbb{N}\}$. Now, for each $(k, \gamma) \in \Delta^* = \{(k, \gamma) : k \in \mathbb{N}, \gamma \in \Delta_k\}$ define $(\mathcal{G}_{(k,\gamma)}, \Upsilon) = (\mathcal{G}_k, \Upsilon) \sqcap (\mathcal{L}_{k_\gamma}, \Upsilon)$. Therefore, the collection $\mathfrak{G} = \{(\mathcal{G}_{(k,\gamma)}, \Upsilon) : (k, \gamma) \in \Delta^*\}$ is soft$_{\mathcal{LF}}$ soft open refinement of $\mathfrak{L}$.

(⇐) Assume that $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is a soft$_{\mathcal{LF}}$ of soft closed subsets of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$. For $k \in \mathbb{N}$, set $\Delta_k = \{\Delta^* \subseteq \Delta : |\Delta^*| = k\}$, $\mathfrak{O}_k = \{\widetilde{\mathcal{S}} - \bigsqcup_{\gamma\notin\Delta^*} (\mathcal{L}_\gamma, \Upsilon) : \Delta^* \in \Delta_k\}$ and $\mathfrak{O}_0 = \widetilde{\mathcal{S}} - \bigsqcup_{\gamma\in\Delta} (\mathcal{L}_\gamma, \Upsilon)$. Then, by Theorem 2.16, $\mathfrak{O} = \bigsqcup_{k=0} \mathfrak{O}_k$ is a soft open cover of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with for each $(O, \Upsilon) \in \mathfrak{O}$ the set $\{\mathfrak{m} : (O, \Upsilon) \sqcap (\mathcal{L}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is finite. Define $(\mathcal{H}_k, \Upsilon) = \{\mathcal{P}^s_a \in \widetilde{\mathcal{S}} : ord(\mathcal{P}^s_a, \mathfrak{L}) = k\}$ where $ord(\mathcal{P}^s_a, \mathfrak{L}) = \left|\{(\mathcal{L}_\gamma, \Upsilon) \in \mathfrak{L} : \mathcal{P}^s_a \in (\mathcal{L}_\gamma, \Upsilon)\}\right|$. For each $k \in \mathbb{N}$, set $\mathfrak{V}_\circ = \mathfrak{O}_\circ$, and $\mathfrak{V}_k = \{(\sqcap\{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta^*\}) \sqcap (\mathcal{H}_k, \Upsilon) : \Delta^* \in \Delta_k\}$. Now, the collection $\mathfrak{O}$ is an $\mathcal{A}_\sigma$- soft cover since it has σ-soft$_{\mathcal{LF}}$ refinement $\mathfrak{V} = \bigsqcup_{k\in\mathbb{N}} \mathfrak{V}_k$ and hence $\mathfrak{O}$ has soft$_{\mathcal{LF}}$ soft open refinement. Hence, by Proposition 3.13, $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft expandable. □

Definition 3.16. A soft subset $(\mathcal{G}, \Upsilon)$ of a soft expandable space $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is called soft expandable if the subspace $((\mathcal{G}, \Upsilon), \mathfrak{T}_{(\mathcal{G},\Upsilon)}, \Upsilon)$ is soft expandable.

Theorem 3.17. *Every soft closed subset of a soft expandable space is soft expandable.*

Proof. Let $(\mathcal{G}, \Upsilon)$ be a soft closed subset of a soft expandable space $(\mathcal{S}, \mathfrak{T}, \Upsilon)$. Assume that $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is a soft$_{\mathcal{LF}}$ collection of subsets of $(\mathcal{G}, \Upsilon)$ in $((\mathcal{G}, \Upsilon), \mathfrak{T}_{(\mathcal{G},\Upsilon)}, \Upsilon)$. Now:

(1) For each $\mathcal{P}^s_a \in \widetilde{\mathcal{S}}$ with $\mathcal{P}^s_a \notin (\mathcal{G}, \Upsilon)$ the set $\{\mathfrak{m} : (\widetilde{\mathcal{S}} - (\mathcal{G}, \Upsilon)) \sqcap (\mathcal{L}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is empty. If $\mathcal{P}^s_a \in (\mathcal{G}, \Upsilon)$, then $\{\mathfrak{m} : (O^*, \Upsilon) \sqcap (\mathcal{L}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is a finite set, where (O^*, Υ) is a soft open set in $((\mathcal{G}, \Upsilon), \mathfrak{T}_{(\mathcal{G},\Upsilon)}, \Upsilon)$ contains $\mathcal{P}^s_a$. This automatically means that there is a soft open set $(O\Upsilon)$ in $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with $(O^*, \Upsilon) = (O, \Upsilon) \sqcap (\mathcal{G}, \Upsilon)$; therefore, $\{\mathfrak{m} : (O, \Upsilon) \sqcap (\mathcal{L}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is finite. Thus, $\mathfrak{L}$ is a soft$_{\mathcal{LF}}$ collection in $(\mathcal{S}, \mathfrak{T}, \Upsilon)$.

(2) Since $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft expandable, there is a soft$_{\mathcal{LF}}$ $\mathfrak{H} = \{(\mathcal{H}_\gamma, \Upsilon) : \gamma \in \Delta\}$ of soft open subsets of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with for each $\gamma \in \Delta$, $(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (\mathcal{H}_\gamma, \Upsilon)$.

(3) Define $\mathfrak{H}^* = \{(\mathcal{H}_\gamma, \Upsilon) \sqcap (\mathcal{G}, \Upsilon) : \gamma \in \Delta\}$. Then $\mathfrak{H}^*$ is a soft$_{\mathcal{LF}}$ collection of soft open subsets of $(\mathcal{G}, \Upsilon)$ with for each $\gamma \in \Delta$, $(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (\mathcal{G}, \Upsilon) \sqcap (\mathcal{H}_\gamma, \Upsilon)$. Therefore, $((\mathcal{G}, \Upsilon), \mathfrak{T}_{(\mathcal{G},\Upsilon)}, \Upsilon)$ is soft expandable. □

Theorem 3.18. *A soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is hereditarily soft expandable iff every soft open subset is soft expandable.*

Proof. (⇒) It is obvious.

(⇐) Let $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ be a soft$_{\mathcal{LF}}$ collection of subsets of $(\mathcal{G}, \Upsilon)$ in $((\mathcal{G}, \Upsilon), \mathfrak{T}_{(\mathcal{G},\Upsilon)}, \Upsilon)$ where $(\mathcal{G}, \Upsilon) \sqsubseteq \widetilde{\mathcal{S}}$. Set $\mathfrak{G} = \{(O^*, \Upsilon) \in (\mathcal{S}, \mathfrak{T}, \Upsilon)$ with $\{\mathfrak{m} : (O^*, \Upsilon) \sqcap (\mathcal{L}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is finite and define $(O, \Upsilon) = \sqcup\{\mathcal{P}^s_a \in \widetilde{\mathcal{S}} : \mathcal{P}^s_a \in (O^*, \Upsilon)$ for some $(O^*, \Upsilon) \in \mathfrak{G}\}$. Then (O, Υ) is soft open with $(\mathcal{G}, \Upsilon) \sqsubseteq (O, \Upsilon)$ and by assumption $((O, \Upsilon), \mathfrak{T}_{(O,\Upsilon)}, \Upsilon)$ is soft expandable. Since $\mathfrak{L}$ is a soft$_{\mathcal{LF}}$ in $((O, \Upsilon), \mathfrak{T}_{(O,\Upsilon)}, \Upsilon)$, there is a soft$_{\mathcal{LF}}$

$\mathfrak{H} = \{(\mathcal{H}_\gamma, \Upsilon) : \gamma \in \Delta\}$ of soft open subsets of $((O, \Upsilon), \mathfrak{T}_{(O,\Upsilon)}, \Upsilon)$, with for each $\gamma \in \Delta$, $(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (\mathcal{H}_\gamma, \Upsilon)$. Therefore, the collection $\{(\mathcal{H}_\gamma, \Upsilon) \sqcap (\mathcal{G}, \Upsilon) : \gamma \in \Delta\}$ is a soft$_{\mathcal{LF}}$ of soft open subsets of $(\mathcal{G}, \Upsilon)$ with for each $\gamma \in \Delta$, $(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (\mathcal{H}_\gamma, \Upsilon) \sqcap (\mathcal{G}, \Upsilon)$. Thus, $((\mathcal{G}, \Upsilon), \mathfrak{T}_{(\mathcal{G},\Upsilon)}, \Upsilon)$ is soft expandable. □

We will now analysis the image and preimage of soft expandable space using some types of soft mappings.

Lemma 3.19. *Let $h_\varphi : (\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon) \to (\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$ be a soft continuous surjective mapping. If $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is a soft$_{\mathcal{LF}}$ collection of subsets of $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$, then $h_\varphi^{-1}(\mathfrak{L}) = \{h_\varphi^{-1}(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is a soft$_{\mathcal{LF}}$ collection in $(\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon)$.*

Proof. Let $\mathcal{P}_a^s \in \widetilde{\mathcal{S}}$. Then $h_\varphi(\mathcal{P}_a^s) \in (O, \Upsilon)$ for some soft open sets (O, Υ) in $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$ with $\{\mathfrak{m} : (O, \Upsilon) \sqcap (\mathcal{L}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is finite set. Then $h_\varphi^{-1}(\mathfrak{L})$ is a soft$_{\mathcal{LF}}$ of subsets of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ because $h_\varphi^{-1}(O, \Upsilon)$ is a soft open set with $\mathcal{P}_a^s \in h_\varphi^{-1}(h_\varphi(\mathcal{P}_a^s)) \sqsubseteq h_\varphi^{-1}(O, \Upsilon)$ and the set $\{\mathfrak{m} : h_\varphi^{-1}(O, \Upsilon) \sqcap h_\varphi^{-1}(\mathcal{L}_{\mathfrak{m},\Upsilon}) \neq \Phi\}$ is finite. □

Proposition 3.20. *A surjective mapping $h_\varphi : (\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon) \to (\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$ is soft closed iff for each $\mathcal{P}_a^t \in \widetilde{\mathcal{Z}}$ and each soft open set (O, Υ) in $(\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon)$ with $h_\varphi^{-1}(\mathcal{P}_a^t) \sqsubseteq (O, \Upsilon)$ there is a soft open set (O^*, Υ) in $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$ with $h_\varphi^{-1}(O^*, \Upsilon) \sqsubseteq (O, \Upsilon)$ and $\mathcal{P}_a^t \in (O^*, \Upsilon)$.*

Proof. ($\Rightarrow$) Let $\mathcal{P}_a^t \in \widetilde{\mathcal{Z}}$ and $h_\varphi^{-1}(\mathcal{P}_a^t) \sqsubseteq (O, \Upsilon)$ for some soft open sets (O, Υ) of $(\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon)$. Define the soft open set $(O^*, \Upsilon) = \widetilde{\mathcal{Z}} - h_\varphi(\widetilde{\mathcal{S}} - (O, \Upsilon))$. Then $h_\varphi^{-1}(O^*, \Upsilon) \sqsubseteq (O, \Upsilon)$ with $\mathcal{P}_a^t \in (O^*, \Upsilon)$.

($\Leftarrow$) Let $(\mathcal{G}, \Upsilon)$ be a soft closed subset of $(\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon)$. For each $\mathcal{P}_a^t \in \widetilde{\mathcal{Z}} - h_\varphi(\mathcal{G}, \Upsilon)$, $h_\varphi^{-1}(\mathcal{P}_a^t) \sqsubseteq \widetilde{\mathcal{S}} - (\mathcal{G}, \Upsilon) = (O, \Upsilon)$, there is a soft open set $(O^*_{\mathcal{P}_a^t}, \Upsilon)$ with $h_\varphi^{-1}(O^*_{\mathcal{P}_a^t}, \Upsilon) \sqsubseteq (O, \Upsilon)$ and $\mathcal{P}_a^t \in (O^*_{\mathcal{P}_a^t}, \Upsilon)$. Now, if $(O^*, \Upsilon) = \sqcup\{(O^*_{\mathcal{P}_a^t}, \Upsilon) : \mathcal{P}_a^t \in \widetilde{\mathcal{Z}} - h_\varphi(\mathcal{G}, \Upsilon)\}$, then $(O^*, \Upsilon) = \widetilde{\mathcal{Z}} - h_\varphi(\mathcal{G}, \Upsilon)$ is soft open with $h_\varphi^{-1}(O^*, \Upsilon) \sqsubseteq (O, \Upsilon)$ and $\mathcal{P}_a^t \in (O^*, \Upsilon)$. Hence, $h_\varphi(\mathcal{G}, \Upsilon)$ is soft closed. □

Lemma 3.21. *Let $h_\varphi : (\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon) \to (\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$ be a soft closed surjective mapping with for each $\mathcal{P}_a^t \in \widetilde{\mathcal{Z}}$, $h_\varphi^{-1}(\mathcal{P}_a^t)$ is soft compact. If $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is a soft$_{\mathcal{LF}}$ collection of subsets of $(\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon)$, then $h_\varphi(\mathfrak{L}) = \{h_\varphi(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is a soft$_{\mathcal{LF}}$ collection in $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$.*

Proof. Let $\mathcal{P}_a^s \in h_\varphi^{-1}(\mathcal{P}_a^t)$. Then $\mathcal{P}_a^s \in (O_{\mathcal{P}_a^s}, \Upsilon)$ for some soft open sets $(O_{\mathcal{P}_a^s}, \Upsilon)$ in $(\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon)$ with $\{\mathfrak{m} : (O_{\mathcal{P}_a^s}, \Upsilon) \sqcap (\mathcal{L}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is a finite set. Therefore, $h_\varphi^{-1}(\mathcal{P}_a^t) \sqsubseteq \bigsqcup_{\mathcal{P}_a^s \in h_\varphi^{-1}(\mathcal{P}_a^t)} (O_{\mathcal{P}_a^s}, \Upsilon)$ and so $h_\varphi^{-1}(\mathcal{P}_a^t) \sqsubseteq \bigsqcup_{k=1}^{n} (O_{\mathcal{P}_a^{s_k}}, \Upsilon) = (O, \Upsilon)$ where $\mathcal{P}_a^{s_1}, \mathcal{P}_a^{s_2}, ...\mathcal{P}_a^{s_n}$ belongs to $h_\varphi^{-1}(\mathcal{P}_a^t)$. By Proposition 3.20, there is soft open set $(O^*, \Upsilon) \sqsubseteq \widetilde{\mathcal{S}}$ with $\mathcal{P}_a^t \in (O^*, \Upsilon)$ and $h_\varphi^{-1}(O^*, \Upsilon) \sqsubseteq (O, \Upsilon)$. Therefore, the set $\{\mathfrak{m} : (O^*, \Upsilon) \sqcap h_\varphi(\mathcal{L}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is finite; hence, $h_\varphi(\mathfrak{L})$ is soft$_{\mathcal{LF}}$ in $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$. □

Theorem 3.22. *Let $h_\varphi : (\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon) \to (\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$ be soft closed soft continuous surjective mapping, and $h_\varphi^{-1}(\mathcal{P}_a^s)$ be soft compact for each $\mathcal{P}_a^s \in \widetilde{\mathcal{Z}}$. Then $(\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon)$ is soft expandable iff $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$ is soft expandable.*

Proof. ($\Rightarrow$) Let $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ be soft$_{\mathcal{LF}}$ of subsets of $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$. Then:

(1) The collection, by Lemma 3.19, $h_\varphi^{-1}(\mathfrak{L}) = \{h_\varphi^{-1}(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is a soft$_{\mathcal{LF}}$ of subsets of a soft expandable space $(\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon)$, so there is a soft$_{\mathcal{LF}}$ collection $\mathfrak{G} = \{(\mathcal{G}_\gamma, \Upsilon) : \gamma \in \Delta\}$ of soft open subsets of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with for each $\gamma \in \Delta$, $h_\varphi^{-1}(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (\mathcal{G}_\gamma, \Upsilon)$.

(2) For each $\gamma \in \Delta$, define $(\mathcal{H}_\gamma, \Upsilon) = \widetilde{\mathcal{Z}} - h_\varphi(\widetilde{\mathcal{S}} - (\mathcal{G}_\gamma, \Upsilon))$. Since h_φ is soft closed mapping and the collection, by Lemma 3.21, $\{h_\varphi(\mathcal{G}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is soft$_{\mathcal{LF}}$ with $(\mathcal{H}_\gamma, \Upsilon) \sqsubseteq h_\varphi(\mathcal{G}_\gamma, \Upsilon)$, we obtain $\mathfrak{H} = \{(\mathcal{H}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is soft$_{\mathcal{LF}}$ of soft open subsets of $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$. Now, $\widetilde{\mathcal{S}} - (\mathcal{G}_\gamma, \Upsilon) \sqsubseteq \widetilde{\mathcal{S}} - h_\varphi^{-1}(\mathcal{L}_\gamma, \Upsilon)$ which means that $h_\varphi(\widetilde{\mathcal{S}} - (\mathcal{G}_\gamma, \Upsilon)) \sqsubseteq \widetilde{\mathcal{Z}} - (\mathcal{L}_\gamma, \Upsilon)$. Thus, $(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (\mathcal{H}_\gamma, \Upsilon)$. Hence, $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$ is soft expandable.

($\Leftarrow$) Let $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ be soft$_{\mathcal{LF}}$ of subsets of $(\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon)$. By Lemma 3.21, $h_\varphi(\mathfrak{L}) = \{h_\varphi(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is soft$_{\mathcal{LF}}$ of subsets of a soft expandable space $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$, so there is a soft$_{\mathcal{LF}}$ $\mathfrak{G} = \{(\mathcal{G}_\gamma, \Upsilon) : \gamma \in \Delta\}$ of soft open subsets of $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$ with for each $\gamma \in \Delta$, $h_\varphi(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (\mathcal{G}_\gamma, \Upsilon)$. Therefore, by Lemma 3.19 the collection

$h_{\varphi}^{-1}(\mathfrak{G}) = \{h_{\varphi}^{-1}(\mathcal{G}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is soft open soft$_{\mathcal{LF}}$ with for each $\gamma \in \Delta$, $(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq h_{\varphi}^{-1}(\mathcal{G}, \Upsilon)$. Hence $(\mathcal{S}, \mathfrak{T}_{\mathcal{S}}, \Upsilon)$ is soft expandable. □

We close this section by examining the transmission property with respect to soft expandable spaces.

Recall that a collection $\{\mathcal{L}_\gamma : \gamma \in \Delta\}$ of a topological space $(\mathcal{S}, \mathfrak{T})$ is called locally finite, denoted by $\mathcal{LF}$, if for each $s \in \mathcal{S}$ there is an open set O satisfies that $s \in O$ and a set $\{\mathfrak{m} : O \sqcap \mathcal{L}_{\mathfrak{m}} \neq \Phi\}$ is finite. If for every $\mathcal{LF}$ collection $\mathfrak{L} = \{\mathcal{L}_\gamma : \gamma \in \Delta\}$ of subset of S, there is a $\mathcal{LF}$ collection $\mathfrak{G} = \{\mathcal{G}_\gamma : \gamma \in \Delta\}$ of open subset of S with $\mathcal{L}_\gamma \subseteq \mathcal{G}_\gamma$ for each $\gamma \in \Delta$, then $(S, \mathfrak{T})$ is called expandable [27].

Theorem 3.23. *Let a soft$_{\mathcal{TS}}$ $(S, \mathfrak{T}, \Upsilon)$ be extended. If $(S, \mathfrak{T}, \Upsilon)$ is soft expandable, then a parametric topological space $(S, \mathfrak{T}_\epsilon)$ is expandable for each $\epsilon \in \Upsilon$.*

Proof. Let $(S, \mathfrak{T}, \Upsilon)$ be an extended soft expandable space and $(S, \mathfrak{T}_\epsilon)$ be one of its parametric topological spaces. Suppose that $\mathfrak{G} = \{\mathcal{G}_\alpha : \alpha \in \Delta\}$ is a locally finite collection in $(S, \mathfrak{T}_\epsilon)$. That is, for each $s \in S$ there is an open subset O of $(S, \mathfrak{T}_\epsilon)$ with $\{\mathfrak{m} : O \bigcap \mathcal{G}_{\mathfrak{m}} \neq \emptyset\}$ is finite. Set $\mathfrak{L} = \{(\mathcal{L}_\alpha, \Upsilon) : \alpha \in \Delta$ such that $\mathcal{L}_\alpha(\epsilon) = \mathcal{G}_\alpha$ and $\mathcal{L}_\beta(\epsilon) = \emptyset$ for each $\beta \neq \alpha\}$. Now, for each $\mathcal{P}_a^s \in \widetilde{\mathcal{S}}$ we have two cases.

(1) If $a = \epsilon$, then we take a soft open subset $(\mathcal{F}, \Upsilon)$ of $(S, \mathfrak{T}, \Upsilon)$ with $\mathcal{F}(\epsilon) = O$. Then $|\{\mathfrak{m} : (\mathcal{F}, \Upsilon) \sqcap (\mathcal{L}_{\mathfrak{m}}, \Upsilon) \neq \Phi\}| = |\{\mathfrak{m} : O \cap \mathcal{G}_{\mathfrak{m}} \neq \emptyset\}|$, which means that the $\{\mathfrak{m} : (\mathcal{F}, \Upsilon) \sqcap \}(\mathcal{L}_{\mathfrak{m}}, \Upsilon) \neq \Phi\}$ is a finite.

(2) If $a \neq \epsilon$. Say, $a = \lambda$. Since $(S, \mathfrak{T}, \Upsilon)$ is extended, we can take a soft open subset $(\mathcal{H}, \Upsilon)$ of $(S, \mathfrak{T}, \Upsilon)$ with $\mathcal{H}(\lambda) = S$ and $\mathcal{H}(\epsilon) = \emptyset$ for each $\epsilon \neq \lambda$. It is clear that $\{\mathfrak{m} : (\mathcal{H}, \Upsilon) \sqcap (\mathcal{L}_{\mathfrak{m}}, \Upsilon) \neq \Phi\}$ is an empty set.

From (1) and (2), we find that $\mathfrak{L}$ is a soft$_{\mathcal{LF}}$ collection of subsets of $(S, \mathfrak{T}, \Upsilon)$. By hypothesis, there is a soft$_{\mathcal{LF}}$ collection $\mathfrak{O}^* = \{(O_\gamma^*, \Upsilon) : \gamma \in \Delta\}$ of soft open subsets of $\mathcal{S}$ satisfying for each $\gamma \in \Delta$ that $(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (O_\gamma^*, \Upsilon)$. Thus, $\{O_\gamma^*(\epsilon) : \gamma \in \Delta\}$ is a locally finite collection of open subsets of $(S, \mathfrak{T}_\epsilon)$ with $\mathcal{L}_\gamma(\epsilon) = \mathcal{G}_\gamma \in \mathfrak{O}^* \subseteq O_\gamma^*(\epsilon)$ for each $\gamma \in \Delta$. Therefore, $(S, \mathfrak{T}_\epsilon)$ is expandable. □

Example 3.24. In Example 3.4, we showed that a soft$_{\mathcal{TS}}$ $(S, \mathfrak{T}, \mathbb{R})$ is not soft expandable. On the other hand, a topological space $(S, \mathfrak{T}_\epsilon)$ is expandable for each $\epsilon \in \mathbb{R}$.

Example 3.25. Let $(\mathbb{R}, \mathfrak{T}, \Upsilon)$ be a soft$_{\mathcal{TS}}$ with $\Upsilon = \{a_1, a_2\}$ and $\mathfrak{T} = \{\Phi, (O, \Upsilon) \sqsubseteq \widetilde{\mathbb{R}} : 1 \in O(a_1)$ and $2 \in O(a_2)\} \cup \{\mathcal{P}_{a_1}^1, \mathcal{P}_{a_2}^2\}$. To prove that $(\mathbb{R}, \mathfrak{T}, \Upsilon)$ is soft expandable, let $\mathfrak{O} = \{(O_\alpha, \Upsilon) : \alpha \in \Delta\}$ be a soft$_{\mathcal{LF}}$ collection of subsets of $(\mathbb{R}, \mathfrak{T}, \Upsilon)$. Then for each $\mathcal{P}_a^s \in \widetilde{\mathbb{R}}$, there is a soft open set (O^*, Υ) satisfies that $\mathcal{P}_a^s \in (O^*, \Upsilon)$ and a set $\{m : (O^*, \Upsilon) \sqcap (O_m, \Upsilon) \neq \Phi\}$ is finite. Now, we prove that $\mathfrak{H} = \{(\mathcal{H}_\alpha, \Upsilon) = (O_\alpha, \Upsilon) \sqcup \mathcal{P}_{a_1}^1 \sqcup \mathcal{P}_{a_2}^2 : \alpha \in \Delta\}$ is a soft$_{\mathcal{LF}}$ collection of soft open subsets of $(\mathbb{R}, \mathfrak{T}, \Upsilon)$. Consider $\mathcal{P}_a^s \in \widetilde{\mathbb{R}}$. Then $(O^*, \Upsilon) \sqcap (\mathcal{H}_\alpha, \Upsilon) = (O^*, \Upsilon) \sqcap ((O_\alpha, \Upsilon) \sqcap \mathcal{P}_{a_1}^1 \sqcap \mathcal{P}_{a_2}^2) = ((\mathcal{H}_\alpha, \Upsilon) \sqcap (O_\alpha, \Upsilon)) \sqcup ((\mathcal{H}_\alpha, \Upsilon) \cap \mathcal{P}_{a_1}^1) \cup ((\mathcal{H}_\alpha, \Upsilon) \sqcap \mathcal{P}_{a_2}^2)$. Since $\mathcal{P}_{a_1}^1$ and $\mathcal{P}_{a_2}^2$ are the smallest open sets respectively containing the soft points $\mathcal{P}_{a_1}^1$ and $\mathcal{P}_{a_2}^2$, a set $\{m : (O^*, \Upsilon) \sqcap (\mathcal{H}_\alpha, \Upsilon) \neq \Phi\}$ is finite. It is clear that for each $\alpha \in \Delta$, $(\mathcal{H}_\alpha, \Upsilon)$ is a soft open set and $(O_\alpha, \Upsilon) \sqsubseteq (\mathcal{H}_\alpha, \Upsilon)$. Thus $(\mathbb{R}, \mathfrak{T}, \Upsilon)$ is soft expandable. On the other hand, $\mathfrak{T}_{a_1} = \{\emptyset, \mathcal{G} \subseteq \mathbb{R} : 1 \in \mathcal{G}\}$ and $\mathfrak{T}_{a_2} = \{\emptyset, \mathcal{G} \subseteq \mathbb{R} : 2 \in \mathcal{G}\}$ are the particular point topologies. It is known from the general topology that neither $(\mathbb{R}, \mathfrak{T}_{a_1})$ nor $(\mathbb{R}, \mathfrak{T}_{a_n})$ is expandable.

4. Soft *s*-Expandable Spaces

In this section, we introduce the idea of soft$_{\mathcal{SLF}}$ collections and explore its fundamental features. Then, we define the concept of *s*-expandable spaces which is stronger than soft expandable spaces. We characterize it and reveal its behaviour under some types of soft mappings.

Definition 4.1. A collection $\{(\mathcal{G}_\gamma, \Upsilon) : \gamma \in \Delta\}$ of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is called soft semi-locally finite, denoted by soft$_{\mathcal{SLF}}$, if for each $\mathcal{P}_a^s \in \widetilde{\mathcal{S}}$ there is $(O, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with $\mathcal{P}_a^s \in (O, \Upsilon)$ and the set $\{\mathfrak{m} : (O, \Upsilon) \sqcap (\mathcal{G}_{\mathfrak{m}}, \Upsilon) \neq \Phi\}$ is finite.

Proposition 4.2. *Every soft$_{\mathcal{LF}}$ of subsets of a soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft$_{\mathcal{SLF}}$.*

Proof. It follows from the fact that every soft open set is soft semi open. □

The next example sets forth that the converse of Proposition 4.2 fails.

Example 4.3. Let $\Upsilon = \{a_1, a_2\}$ be a set of parameters, and $\mathfrak{T} = \{(\mathcal{O}, \Upsilon) \sqsubseteq \widetilde{\mathbb{R}} : \{1 \in (\mathcal{O}, \Upsilon)$ and $(\mathcal{O}^c, \Upsilon)$ is finite$\}$ or $1 \notin (\mathcal{O}, \Upsilon)\}$ be a soft topology in $\mathbb{R}$. Then the collection $\{\mathcal{P}^s_{a_1} : s \in \mathbb{R} - \mathbb{Q}\}$ is not soft$_{\mathcal{LF}}$, but it is soft$_{\mathcal{SLF}}$.

Theorem 4.4. *Let a soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ be soft semi regular and soft$_{\mathcal{E.D}}$. Then a collection $\mathfrak{L}$ is soft$_{\mathcal{LF}}$ iff $\mathfrak{L}$ is soft$_{\mathcal{SLF}}$.*

Proof. ($\Rightarrow$) By Proposition 4.2.

($\Leftarrow$) Assume that $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is a soft$_{\mathcal{SLF}}$ collection of subsets of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ and $\mathcal{P}^s_a \in \widetilde{\mathcal{S}}$. Then, there is $(\mathcal{G}, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with $\mathcal{P}^s_a \in (\mathcal{G}, \Upsilon)$ and the set $\{\mathfrak{m} : (\mathcal{G}, \Upsilon) \sqcap (\mathcal{L}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is finite. By assumption, there is $(\mathcal{H}, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with $\mathcal{P}^s_a \in (\mathcal{H}, \Upsilon) \sqsubseteq scl(\mathcal{H}, \Upsilon) \sqsubseteq (\mathcal{G}, \Upsilon)$. Since $(\mathcal{H}, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$, there is a soft open set $(\mathcal{O}, \Upsilon)$ with $(\mathcal{O}, \Upsilon) \sqsubseteq (\mathcal{H}, \Upsilon) \sqsubseteq cl(\mathcal{O}, \Upsilon)$. Now, $cl(\mathcal{O}, \Upsilon) = cl(\mathcal{H}, \Upsilon)$ and by Lemma 2.13, $cl(\mathcal{O}, \Upsilon) \sqcap (\mathcal{L}_\gamma, \Upsilon) = scl(\mathcal{H}, \Upsilon) \sqcap (\mathcal{L}_\gamma, \Upsilon)$. Since the set $\{\mathfrak{m} : cl(\mathcal{O}, \Upsilon) \sqcap (\mathcal{L}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is finite where $cl(\mathcal{O}, \Upsilon)$ is soft open set with $\mathcal{P}^s_a \in cl(\mathcal{O}, \Upsilon)$, $\mathfrak{L}$ is soft$_{\mathcal{LF}}$. □

Theorem 4.5. *Let $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ be a collection of subsets of a soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$. Then:*

(*i*) *Every subcollection of $\mathfrak{L}$ is soft$_{\mathcal{SLF}}$ if $\mathfrak{L}$ is soft$_{\mathcal{SLF}}$.*

(*ii*) *The collection $\mathfrak{L}$ is soft$_{\mathcal{SLF}}$ iff $scl(\mathfrak{L}) = \{scl(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is soft$_{\mathcal{SLF}}$.*

(*iii*) *$scl(\bigsqcup_{\gamma\in\Delta} (\mathcal{L}_\gamma, \Upsilon)) = \bigsqcup_{\gamma\in\Delta} scl(\mathcal{L}_\gamma, \Upsilon)$ if $\mathfrak{L}$ is soft$_{\mathcal{SLF}}$ and $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft$_{\mathcal{E.D}}$.*

Proof. (i) It is obvious.

(ii) The proof is immediately from the fact that for each $\gamma \in \Delta$, $(\mathcal{G}, \Upsilon) \sqcap (\mathcal{L}_\gamma, \Upsilon) \neq \Phi$ iff $(\mathcal{G}, \Upsilon) \sqcap scl(\mathcal{L}_\gamma, \Upsilon) \neq \Phi$ for every $(\mathcal{G}, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$.

(iii) It is clear that $\bigsqcup_{\gamma\in\Delta} scl(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq scl(\bigsqcup_{\gamma\in\Delta} (\mathcal{L}_\gamma, \Upsilon))$. To show $scl(\bigsqcup_{\gamma\in\Delta} (\mathcal{L}_\gamma, \Upsilon)) \sqsubseteq \bigsqcup_{\gamma\in\Delta} scl(\mathcal{L}_\gamma, \Upsilon)$, let $\mathcal{P}^s_a \in scl(\bigsqcup_{\gamma\in\Delta} (\mathcal{L}_\gamma, \Upsilon))$. Then there is $(\mathcal{G}, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with $\mathcal{P}^s_a \in (\mathcal{G}, \Upsilon)$ and $\Delta^* = \{\mathfrak{m} : (\mathcal{G}, \Upsilon) \sqcap (\mathcal{L}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is a finite set. Since $scl(\bigsqcup_{\gamma\in\Delta} (\mathcal{L}_\gamma, \Upsilon)) = scl(\bigsqcup_{\gamma\in\Delta^*} (\mathcal{L}_\gamma, \Upsilon) \sqcup \bigsqcup_{\gamma\in\Delta-\Delta^*} (\mathcal{L}_\gamma, \Upsilon)) = scl(\bigsqcup_{\gamma\in\Delta^*} (\mathcal{L}_\gamma, \Upsilon)) \sqcup scl(\bigsqcup_{\gamma\in\Delta-\Delta^*} (\mathcal{L}_\gamma, \Upsilon)$, then $\mathcal{P}^s_a \in scl(\bigsqcup_{\gamma\in\Delta^*} (\mathcal{L}_\gamma, \Upsilon)) = \bigsqcup_{\gamma\in\Delta^*} scl(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq \bigsqcup_{\gamma\in\Delta} scl(\mathcal{L}_\gamma, \Upsilon)$. □

Definition 4.6. A soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is said to be soft s-expandable if for each soft$_{\mathcal{SLF}}$ $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ of subsets of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$, there is a soft$_{\mathcal{LF}}$ $\mathfrak{G} = \{(\mathcal{G}_\gamma, \Upsilon) : \gamma \in \Delta\}$ of soft open subsets of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with for each $\gamma \in \Delta$, $(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (\mathcal{G}_\gamma, \Upsilon)$.

We call $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ a countably soft s-expandable space if $|\Delta| \leqslant \omega_0$.

Proposition 4.7. *If a soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft s-expandable (resp. countably soft s-expandable), then it is soft expandable (resp. countably soft expandable) space.*

Example 4.3 illustrates that there exists a soft$_{\mathcal{TS}}$ which is soft expandable but not soft s-expandable.

Proposition 4.8. *A soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft s-expandable iff it is soft expandable and every soft$_{\mathcal{SLF}}$ is soft$_{\mathcal{LF}}$.*

Proof. ($\Rightarrow$) Suppose that $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is a soft$_{\mathcal{SLF}}$ of subsets of a soft s-expandable space $(\mathcal{S}, \mathfrak{T}, \Upsilon)$. Then, by assumption, there is a soft$_{\mathcal{LF}}$ $\mathfrak{G} = \{(\mathcal{G}_\gamma, \Upsilon) : \gamma \in \Delta\}$ of soft open subsets of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with for each $\gamma \in \Delta$, $(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (\mathcal{G}_\gamma, \Upsilon)$. Therefore, $\mathfrak{L}$ is soft$_{\mathcal{LF}}$. By Proposition 4.7, the proof is complete.

($\Leftarrow$) It is obvious. □

Theorem 4.9. *A soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft s-expandable if one of the following hold:*

(*i*) *Every soft semi open cover of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ has a soft$_{\mathcal{LF}}$ soft open refinement.*

(*ii*) *$(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft$_{\mathcal{E.D}}$ and every soft semi open cover $\mathfrak{G}$ of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ has a soft$_{\mathcal{LF}}$ soft semi open refinement.*

Proof. (i) It can be proved following the same technique of Theorem 3.7.

(ii) Let $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ be a soft$_{S\mathcal{LF}}$ collection with for each $\gamma \in \Delta$, $(\mathcal{L}_\gamma, \Upsilon) \in \mathcal{SSC}(\mathcal{S}, \mathfrak{T}, \Upsilon)$. Define a soft$_{\mathcal{LF}}$ collection $\mathfrak{C} = \{(\mathcal{G}_\gamma, \Upsilon) : \gamma \in \Delta\}$ as the proof of Theorem 3.7 where for each $\gamma \in \Delta$, $(\mathcal{G}_\gamma, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$ and $(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (\mathcal{G}_\gamma, \Upsilon)$. Now, there is a soft open subset $(\mathcal{O}_\gamma, \Upsilon)$ with $(\mathcal{O}_\gamma, \Upsilon) \sqsubseteq (\mathcal{G}_\gamma, \Upsilon) \sqsubseteq cl(\mathcal{O}_\gamma, \Upsilon)$. Since $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft$_{\mathcal{E}.\mathfrak{D}}$, then the collection $\{cl(\mathcal{O}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is soft$_{\mathcal{LF}}$ of soft open subsets in $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with for each $\gamma \in \Delta$, $(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq cl(\mathcal{O}_\gamma, \Upsilon)$. Thus, $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft s-expandable. □

Lemma 4.10. *If $(\mathcal{O}, \Upsilon)$ is a soft open set in $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ and $(\mathcal{G}, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$, then $(\mathcal{O}, \Upsilon) \sqcap (\mathcal{G}, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$.*

Proof. By Lemma 2.9, $(\mathcal{O}, \Upsilon) \sqcap (\mathcal{G}, \Upsilon) \sqsubseteq (\mathcal{O}, \Upsilon) \sqcap cl(int(\mathcal{G}, \Upsilon)) \sqsubseteq cl((\mathcal{O}, \Upsilon) \sqcap int(\mathcal{G}, \Upsilon)) = cl(int(\mathcal{O}, \Upsilon) \sqcap int(\mathcal{G}, \Upsilon)) \sqsubseteq cl(int((\mathcal{O}, \Upsilon) \sqcap (\mathcal{G}, \Upsilon)))$. □

Theorem 4.11. *Let a soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ be soft$_{\mathcal{E}.\mathfrak{D}}$. Then $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is countably soft s-expandable iff every countable soft semi open cover $\mathfrak{G}$ of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ has a soft$_{\mathcal{LF}}$ soft semi open refinement.*

Proof. The sufficient part follows from Theorem 4.9.

To prove the necessary part, suppose that $\mathfrak{G} = \{(\mathcal{G}_\gamma, \Upsilon) : \gamma \in \Delta, |\Delta| \leqslant \omega_0\}$ is a countable cover of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with $(\mathcal{G}_\gamma, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$. For each $\gamma \in \Delta$, define $(\mathcal{H}_\gamma, \Upsilon) = \bigsqcup_{\alpha \leq \gamma} (\mathcal{G}_\alpha, \Upsilon)$ and $(\mathcal{L}_\gamma, \Upsilon) = \widetilde{\mathcal{S}} - (\mathcal{H}_\gamma, \Upsilon)$. Then:

(1) The collection $\mathfrak{H} = \{(\mathcal{H}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is an increasing cover of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with for each $\gamma \in \Delta$, $(\mathcal{H}_\gamma, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$.

(2) The collection $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is a soft$_{S\mathcal{LF}}$ of countably soft s-expandable space $(\mathcal{S}, \mathfrak{T}, \Upsilon)$, so there is a soft$_{\mathcal{LF}}$ $\mathfrak{L}^* = \{(\mathcal{L}^*_\gamma, \Upsilon) : \gamma \in \Delta\}$ of soft open subsets of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with for each $\gamma \in \Delta$, $(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (\mathcal{L}^*_\gamma, \Upsilon)$.
(3) For each $\gamma \in \Delta$, define $(\mathcal{O}_\gamma, \Upsilon) = (\mathcal{G}_\gamma, \Upsilon) - \bigsqcup_{\alpha < \gamma} (\widetilde{\mathcal{S}} - (\mathcal{L}^*_\alpha, \Upsilon))$. By Lemma 4.10, for each $\gamma \in \Delta$, $(\mathcal{O}_\gamma, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with $(\mathcal{O}_\gamma, \Upsilon) \sqsubseteq (\mathcal{G}_\gamma, \Upsilon)$. Therefore, the collection $\{(\mathcal{O}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is soft$_{\mathcal{LF}}$ refinement of $\mathfrak{G}$. □

Theorem 4.12. *Let a soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ be soft$_{\mathcal{E}.\mathfrak{D}}$ and countably soft s-expandable space. If every soft open cover of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ has a σ-soft$_{\mathcal{LF}}$ soft semi open refinement, then it is soft s-paracompact.*

Proof. Let $\mathfrak{O} = \{(\mathcal{O}_\gamma, \Upsilon) : \gamma \in \Delta\}$ be a soft open cover. Then, by assumption, $\mathfrak{O}$ has a σ-soft$_{\mathcal{LF}}$ soft semi open refinement, say $\mathfrak{L} = \bigsqcup_{k=1}^{\infty} \mathfrak{L}_k$ where $\mathfrak{L}_k = \{(\mathcal{L}_{\gamma,k}, \Upsilon) : \gamma \in \Delta_k\}$. Now, for each $k \in \mathbb{N}$ define $(\mathcal{L}^*_k, \Upsilon) = \bigsqcup_{\gamma \in \Delta_k} (\mathcal{L}_{\gamma,k}, \Upsilon)$. Then:

(1) $\widetilde{\mathcal{S}} = \bigsqcup_{k \in \mathbb{N}} (\mathcal{L}^*_k, \Upsilon)$, where for each $k \in \mathbb{N}$, $(\mathcal{L}^*_k, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$. By Theorem 4.11, there is a soft$_{\mathcal{LF}}$ $\mathfrak{G} = \{(\mathcal{G}_k, \Upsilon) : k \in \mathbb{N}\}$ with for each $k \in \mathbb{N}$, $(\mathcal{G}_k, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$ and $(\mathcal{G}_k, \Upsilon) \sqsubseteq (\mathcal{L}^*_k, \Upsilon)$.

(2) For each $k \in \mathbb{N}$, $(\mathcal{H}_k, \Upsilon) \sqsubseteq (\mathcal{G}_k, \Upsilon) \sqsubseteq cl(\mathcal{H}_k, \Upsilon)$ for some a soft open set $(\mathcal{H}_k, \Upsilon)$. Since $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft$_{\mathcal{E}.\mathfrak{D}}$ and by Lemma 4.10 the collection $\{cl(\mathcal{H}_k, \Upsilon) \sqcap (\mathcal{L}_{\gamma,k}, \Upsilon) : \gamma \in \Delta_k, k \in \mathbb{N}\}$ is soft$_{\mathcal{LF}}$ soft semi open refinement of $\mathfrak{O}$. Thus $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft s-paracompact. □

Proposition 4.13. *Let a soft$_{\mathcal{TS}}$$(\mathcal{S}, \mathfrak{T}, \Upsilon)$ be soft$_{\mathcal{E}.\mathfrak{D}}$. Then $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft s-expandable if for every soft$_{S\mathcal{LF}}$ $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ there is a soft$_{\mathcal{LF}}$ $\mathfrak{H} = \{(\mathcal{H}_\alpha, \Upsilon) : \alpha \in \Delta\}$ of soft semi open set with:*

(*i*) $\widetilde{\mathcal{S}} = \bigsqcup_{\alpha \in \Delta} (\mathcal{H}_\alpha, \Upsilon)$.

(*ii*) *For each $\alpha \in \Delta$, the set $\{\mathfrak{m} : (\mathcal{H}_\alpha, \Upsilon) \sqcap (\mathcal{L}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is finite.*

Proof. It can be proved following similar procedures of Theorem 3.13. □

Theorem 4.14. *Let a soft$_{\mathcal{TS}}$ $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ be soft$_{\mathcal{E}.\mathfrak{D}}$. Then $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft s-expandable iff every soft semi open cover of $\widetilde{\mathcal{S}}$ with a σ-soft$_{S\mathcal{LF}}$ refinement has soft$_{\mathcal{LF}}$ soft semi open refinement.*

Proof. (⇒) Assume that $\mathfrak{O} = \{(O_\gamma, \Upsilon) : \gamma \in \Delta\}$ where $(O_\gamma, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is a cover of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with a σ-soft$_{\mathcal{SLF}}$ refinement $\mathfrak{G} = \bigsqcup_{k\in\mathbb{N}} \mathfrak{G}_k$ where $\mathfrak{G}_k = \{(\mathcal{G}_{\alpha,k}, \Upsilon) : \alpha \in \Delta_k\}$. Then,

(1) $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft s-expandable so for each $k \in \mathbb{N}$ there is a soft$_{\mathcal{LF}}$ $\mathfrak{H}^* = \{(\mathcal{H}_{\alpha,k}, \Upsilon) : \alpha \in \Delta_k\}$ of soft open with for each $\alpha \in \Delta_k$, $(\mathcal{G}_{\alpha,k}, \Upsilon) \sqsubseteq (\mathcal{H}_{\alpha,k}, \Upsilon)$. Now, for each $(\mathcal{G}_{\alpha,k}, \Upsilon) \in \mathfrak{G}$ there is $\gamma(\alpha, k) \in \Delta$ with $(\mathcal{G}_{\alpha,k}, \Upsilon) \sqsubseteq (O_{\gamma(\alpha,k)}, \Upsilon)$.

(2) Define $\mathfrak{L}_k = \{(\mathcal{L}_{\alpha,k}, \Upsilon) : \alpha \in \Delta_k\}$ where for each $k \in \mathbb{N}$ and $\alpha \in \Delta_k$, $(\mathcal{L}_{\alpha,k}, \Upsilon) = (\mathcal{H}_{\alpha,k}, \Upsilon) \sqcap (O_{\gamma(\alpha,k)}, \Upsilon)$. Then $\mathfrak{L} = \bigsqcup_{k\in\mathbb{N}} \mathfrak{L}_k$ is a σ-soft$_{\mathcal{LF}}$ semi open refinement of $\mathfrak{O}$. Now, the collection $\{(\mathcal{W}_k, \Upsilon) : k \in \mathbb{N}\}$ where $(\mathcal{W}_k, \Upsilon) = \bigsqcup_{\alpha\in\Delta_k} (\mathcal{L}_{\alpha,k}, \Upsilon)$ is a cover of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$. Since $(\mathcal{W}_k, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$ then by Theorem 4.11 there is a soft$_{\mathcal{LF}}$ soft semi open refinement, say $\{(\mathcal{V}_k, \Upsilon)$: $k \in \mathbb{N}\}$ with $(\mathcal{V}_k, \Upsilon) \sqsubseteq (\mathcal{W}_k, \Upsilon)$. For each $k \in \mathbb{N}$, there is a soft open set $(\mathcal{U}_k, \Upsilon)$ with $(\mathcal{U}_k, \Upsilon) \sqsubseteq (\mathcal{V}_k, \Upsilon) \sqsubseteq cl(\mathcal{U}_k, \Upsilon)$. By soft$_{\mathcal{E.D}}$ and Lemma 4.10, the collection $\{cl(\mathcal{U}_k, \Upsilon) \sqcap (\mathcal{L}_{\alpha,k}, \Upsilon) : \alpha \in \Delta_k, k \in \mathbb{N}\}$ is soft$_{\mathcal{LF}}$ soft semi open refinement of $\mathfrak{O}$.

(⇐) Assume that $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is a soft$_{\mathcal{SLF}}$ with $(\mathcal{L}_\gamma, \Upsilon) \in \mathcal{SSC}(\mathcal{S}, \mathfrak{T}, \Upsilon)$. For each $k \in \mathbb{N}$, set $\Delta_k = \{\Delta^* \subseteq \Delta : |\Delta^*| = k\}$, $\mathfrak{O}_k = \{\widetilde{\mathcal{S}} - \bigsqcup_{\gamma\notin\Delta^*} (\mathcal{L}_\gamma, \Upsilon) : \Delta^* \in \Delta_k\}$ and $\mathfrak{O}_0 = \widetilde{\mathcal{S}} - \bigsqcup_{\gamma\in\Delta} (\mathcal{L}_\gamma, \Upsilon)$. Then, by Theorem 4.5, $\mathfrak{O} = \bigsqcup_{k\in\mathbb{N}} \mathfrak{O}_k$ is a soft semi open cover of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with for each $(O, \Upsilon) \in \mathfrak{O}$ the set $\{\mathfrak{m} : (O, \Upsilon) \sqcap (\mathcal{L}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is finite. Now, set $(\mathcal{H}_k, \Upsilon) = \{\mathcal{P}^s_a \in \widetilde{\mathcal{S}} : ord(\mathcal{P}^s_a, \mathfrak{L}) = k\}$. For each $k \in \mathbb{N}$, set $\mathfrak{V}_\circ = \mathfrak{O}_\circ$, and $\mathfrak{V}_k = \{(\sqcap\{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta^*\} \sqcap (\mathcal{H}_k, \Upsilon) : \Delta^* \in \Delta_k\}$. So $\mathfrak{V} = \bigsqcup_{k=0}^{\infty} \mathfrak{V}_k$ is a σ-soft$_{\mathcal{SLF}}$ refinement of $\mathfrak{O}$. Hence $\mathfrak{O}$ has soft$_{\mathcal{LF}}$ soft semi open refinement $\mathfrak{M}$. Therefore, by Proposition 4.13, $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ is soft s-expandable. □

Theorem 4.15. *Every soft clopen subset of a soft s-expandable space is soft s-expandable.*

Proof. Since for any soft subspace $((\mathcal{H}, \Upsilon), \mathfrak{T}_{(\mathcal{H},\Upsilon)}, \Upsilon)$ of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ if $(\mathcal{H}, \Upsilon) \in \mathfrak{T}$ and $(\mathcal{G}, \Upsilon) \in \mathcal{SSO}((\mathcal{H}, \Upsilon), \mathfrak{T}_{(\mathcal{H},\Upsilon)}, \Upsilon)$, then $(\mathcal{G}, \Upsilon) \in \mathcal{SSO}(\mathcal{S}, \mathfrak{T}, \Upsilon)$ and by using the same procedure of Theorem 3.17 the proof is completed. □

Finally, we will use some types of soft mapping to study the image and preimage of soft s-expandable spaces.

Lemma 4.16. *Let* $h_\varphi : (\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon) \to (\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$ *be a soft irresolute surjective mapping. If* $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ *is a soft*$_{\mathcal{SLF}}$ *of subsets of* $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$*, then* $h^{-1}_\varphi(\mathfrak{L}) = \{h^{-1}_\varphi(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ *is a soft*$_{\mathcal{SLF}}$ *in* $(\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon)$.

Proof. The same technique of Lemma 3.19. □

Proposition 4.17. *A surjective mapping* $h_\varphi : (\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon) \to (\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$ *is soft semi closed iff for each* $\mathcal{P}^t_a \in \widetilde{\mathcal{Z}}$ *and each soft open set* (O, Υ) *in* $(\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon)$ *with* $h^{-1}_\varphi(\mathcal{P}^t_a) \sqsubseteq (O, \Upsilon)$ *there is a soft semi open set* (O^*, Υ) *in* $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$ *with* $h^{-1}_\varphi(O^*, \Upsilon) \sqsubseteq (O, \Upsilon)$ *and* $\mathcal{P}^t_a \in (O^*, \Upsilon)$.

Proof. The same technique of Proposition 3.20. □

Lemma 4.18. *Let* $h_\varphi : (\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon) \to (\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$ *be a soft semi closed surjective mapping with for each* $\mathcal{P}^t_a \in \widetilde{\mathcal{Z}}$, $h^{-1}_\varphi(\mathcal{P}^t_a)$ *is soft compact. If* $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ *is a soft*$_{\mathcal{LF}}$ *of subsets of* $(\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon)$*, then* $h_\varphi(\mathfrak{L}) = \{h_\varphi(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ *is a soft*$_{\mathcal{SLF}}$ *in* $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$.

Proof. The same technique of Lemma 3.21. □

Theorem 4.19. *Let* $h_\varphi : (\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon) \to (\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$ *be a soft semi closed soft irresolute surjective mapping and* $h^{-1}_\varphi(\mathcal{P}^s_a)$ *be soft compact for each* $\mathcal{P}^s_a \in \widetilde{\mathcal{Z}}$*. Then* $(\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon)$ *is soft s-expandable iff* $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$ *is soft s-expandable.*

Proof. (⇒) Let $\mathfrak{L} = \{(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ be a soft$_{\mathcal{SLF}}$ of subsets of $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$. Then:

(1) By Lemma 4.16, the collection $h^{-1}_\varphi(\mathfrak{L}) = \{h^{-1}_\varphi(\mathcal{L}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is a soft$_{\mathcal{SLF}}$ of subsets of a soft s-expandable $(\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon)$, so there is a soft$_{\mathcal{LF}}$ $\mathfrak{G} = \{(\mathcal{G}_\gamma, \Upsilon) : \gamma \in \Delta\}$ of soft open subset of $(\mathcal{S}, \mathfrak{T}, \Upsilon)$ with for each for each $\gamma \in \Delta$, $h^{-1}_\varphi(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (\mathcal{G}_\gamma, \Upsilon)$.

(2) For each $\gamma \in \Delta$ define the soft open set $(\mathcal{H}_\gamma, \Upsilon) = \widetilde{\mathcal{Z}} - h_\varphi(\widetilde{\mathcal{S}} - (\mathcal{G}_\gamma, \Upsilon))$. By Lemma 4.18, the collection $\{h_\varphi(\mathcal{G}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is soft$_{\mathcal{LF}}$ with $(\mathcal{H}_\gamma, \Upsilon) \sqsubseteq h_\varphi(\mathcal{G}_\gamma, \Upsilon)$. Now, to show $\mathfrak{H} = \{(\mathcal{H}_\gamma, \Upsilon) : \gamma \in \Delta\}$ is soft$_{\mathcal{LF}}$ of subsets of $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$, let $\mathcal{P}^s_a \in h^{-1}_\varphi(\mathcal{P}^t_a)$. Then $\mathcal{P}^s_a \in (O_{\mathcal{P}^s_a}, \Upsilon)$ for some soft open set $(O_{\mathcal{P}^s_a}, \Upsilon)$ in $(\mathcal{S}, \mathfrak{T}_\mathcal{S}, \Upsilon)$ with the set $\{\mathfrak{m} : (O_{\mathcal{P}^s_a}, \Upsilon) \sqcap (\mathcal{G}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is finite. Therefore, $h^{-1}_\varphi(\mathcal{P}^t_a) \sqsubseteq \bigsqcup_{\mathcal{P}^s_a \in h^{-1}_\varphi(\mathcal{P}^t_a)} (O_{\mathcal{P}^s_a}, \Upsilon)$ and hence $h^{-1}_\varphi(\mathcal{P}^t_a) \sqsubseteq \bigsqcup_{k=1}^{n} (O_{\mathcal{P}^{s_k}_a}, \Upsilon) = (O, \Upsilon)$ where $\mathcal{P}^{s_1}_a, \mathcal{P}^{s_2}_a, ...\mathcal{P}^{s_n}_a$ belongs to $h^{-1}_\varphi(\mathcal{P}^t_a)$. By Proposition 4.17, there is soft open set $(O^*, \Upsilon) \sqsubseteq \widetilde{\mathcal{S}}$ with $\mathcal{P}^t_a \in (O^*, \Upsilon)$ and $h^{-1}_\varphi(O^*, \Upsilon) \sqsubseteq (O, \Upsilon)$. Therefore, the set $\{\mathfrak{m} : (O^*, \Upsilon) \sqcap (\mathcal{H}_\mathfrak{m}, \Upsilon) \neq \Phi\}$ is finite and since $(\mathcal{L}_\gamma, \Upsilon) \sqsubseteq (\mathcal{H}_\gamma, \Upsilon)$ then the soft$_{\mathcal{TS}}$ $(\mathcal{Z}, \mathfrak{T}_\mathcal{Z}, \Upsilon)$ is soft s-expandable.

($\Leftarrow$) the same technique sufficient part of Theorem 3.22. □

5. Conclusion

Soft topology has gotten a lot of attention from researchers in recent years, and there has been a lot of progress. Because some soft topological concepts and notions, such as soft separation axioms in [7, 17, 18], have no analogues in general topology, soft topologies allow us to examine more concepts and features than classical topologies. Furthermore, the behaviors of many topological concepts may be explored using their soft topological concepts, and vice versa, under specific sorts of soft topologies in [8].

In this manuscript, the concept of soft expandable spaces, a new generalization of soft paracompact spaces, has been described in this work. Then there is the concept of soft s-expandable spaces, which is more stronger than soft expandable spaces. We have given some examples to show how they interact with each other and other soft spaces. These concepts have been defined and main properties have been established. Finally, we have examined the interrelations between soft topology and its parametric topologies in terms of possession an expandable property. In this regards, we have discussed the role of extended soft topology to inherited this property to classical topology.

We intend to investigate the concepts and results presented herein using various soft structures such as supra and infra soft topologies, in future works. Finally, we hope that this work will aid in the study of soft topology and allow for the development of new findings.

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References

[1] J.C.R. Alcantud, T.M. Al-shami, A.A. Azzam, Caliber and chain conditions in soft topologies, Mathematics 9 (2021) 2349, https://doi.org/10.3390/math9192349.
[2] M.I. Ali, F. Feng, X. Liu, M. Shabir, On some new operations in soft set theory, Comput. Math. Appl. 57 (2009) 1547–1553.
[3] H. Al-jarrah, A. Rawshdeh, T. Al-shami, On soft compact and soft Lindelöf spaces via soft regular closed sets, Afr. Mat. 33 (2022) https://doi.org/10.1007/s13370-021-00952-z.
[4] T.M. Al-shami, Soft somewhere dense sets on soft topological spaces, Commun. Korean Math. Soc. 33 (2018) 1341–1356.
[5] T.M. Al-shami, Homeomorphism and quotient mappings in infra soft topological spaces, J. Math. Vol. 2021, Article ID 3388288, 10 pages.
[6] T.M. Al-shami, M.. El-Shafei, M. Abo-Elhamayel, Almost soft compact and approximately soft Lindelöf spaces, J. Taibah Univ. Sci. 12 (2018) 620–630.
[7] T.M. Al-shami, M.E. El-Shafei, Partial belong relation on soft separation axioms and decision-making problem, two birds with one stone, Soft Comput. 24 (2020) 5377–5387.
[8] T.M. Al-shami, Lj.D.R. Kočinac, The equivalence between the enriched and extended soft topologies, Appl. Comput. Math. 18 (2019) 149–162.
[9] T.M. Al-shami, Lj.D.R. Kočinac, Nearly soft Menger spaces, J. Math. 2020 (2020), Article ID 3807418, 9 pages.
[10] T.M. Al-shami, Lj.D.R. Kočinac, Almost soft Menger and weakly soft Menger spaces, Appl. Comput. Math. 21 (2022) 35–51.
[11] T.M. Al-shami, A. Mhemdi, A. Rawshdeh, H. Aljarrah, Soft version of compact and Lindelöf spaces using soft somewhere dense set, AIMS Mathematics 6 (2021) 8064–8077.
[12] B.A. Asaad, Results on soft extremally disconnectedness of soft topological spaces, J. Math. Computer Sci. 17 (2017) 448–464.

[13] A. Aygünoğlu, H. Aygün, Some notes on soft topological spaces, Neural Comput. Applic. 21 (2012) 113–119.
[14] S. Bayramov, C.G. Aras, A new approach to separability and compactness in soft topological spaces, TWMS J. Pure Appl. Math. 9 (2018) 82–93.
[15] B. Chen, Soft semi-open sets and related properties in soft topological spaces, Appl. Math. Inf. Sci. 7 (2013) 287–294.
[16] S. Das, S.K. Samanta, Soft metric, Ann. Fuzzy Math. Inform. 6 (2013) 77–94.
[17] M.E. El-Shafei, M. Abo-Elhamayel, T.M. Al-shami, Partial soft separation axioms and soft compact spaces, Filomat 32 (2018) 4755–4771.
[18] M.E. El-Shafei, T.M. Al-shami, Applications of partial belong and total non-belong relations on soft separation axioms and decision-making problem, Comput. Appl. Math. 39 (2020) https://doi.org/10.1007/s40314-020-01161-3.
[19] F. Feng, Y.M. Li, B. Davvaz, M.I. Ali, Soft sets combined with fuzzy sets and rough sets: a tentative approach, Soft Comput. 14 (2010) 899–911.
[20] T. Hida, A comprasion of two formulations of soft compactness, Ann. Fuzzy Math. Inform. 8 (2014) 511–524.
[21] S. Hussain, B. Ahmad, Some properties of soft topological spaces, Comput. Math. Appl. 62 (2011) 4058–4067.
[22] S. Hussain, B. Ahmad, Soft separation axioms in soft topological spaces, Hacet. J. Math. Stat. 44 (2015) 559–568.
[23] A. Iguda, Soft sets and soft Topological notions, J. New Theory 30 (2020) 21–34.
[24] A. Kandil, O.A.E. Tantawy, S.A. El-Sheikh, A.M. Abd El-latif, Soft semi separation axioms and some types of soft functions, Ann. Fuzzy Math. Inform. 8 (2014) 305–318.
[25] A. Kharal, B. Ahmad, Mappings on soft classes, New Math. Nat. Comput. 7 (2011) 471–481.
[26] Lj.D.R. Kočinac, T.M. Al-shami, V. Çetkin, Selection principles in the context of soft sets: Menger spaces, Soft Comput. 25 (2021) 12693–12702.
[27] L.L. Krajewski, On expanding locally finite collections, Canadian J. Math. 23 (1971) 58–68.
[28] F. Lin, Soft connected and soft paracompact spaces, Int. J. Math. Comput. Sci. Eng. 7 (2013) 37–43.
[29] W.K. Min, A note on soft topological spaces, Comput. Math. Appl. 62 (2011) 3524–3528.
[30] J. Mahanta, P.K. Das, On soft topological space via semiopen and semiclosed soft sets, Kyungpook Math. J. 54 (2014) 221–236.
[31] P.K. Maji, R. Biswas, R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562.
[32] D. Molodtsov, Soft set theory-first results, Comput. Math. Appl. 37 (1999) 19–31.
[33] S. Nazmul, S.K. Samanta, Neighbourhood properties of soft topological spaces, Ann. Fuzzy Math. Inform. 6 (2013) 1–15.
[34] E. Peyghan, B. Samadi, A. Tayebi, Some results related to soft topological spaces, Facta Univ. Ser. Math. Inform. 29 (2014) 325–336.
[35] A. Singh, N.S. Noorie, Remarks on soft axioms, Ann. Fuzzy Math. Inform. 14 (2017) 503–513.
[36] M. Shabir, M. Naz, On soft topological spaces, Comput. Math. Appl. 61 (2011) 1786–1799.
[37] L.A. Steen, J.A. Seebach, Counterexample in Topology, Springer-Verlag, New Yourk, 1978.
[38] O. Tantawy, S.A. El-Sheikh, S. Hamde, Separation axioms on soft topological spaces, Ann. Fuzzy Math. Inform. 11 (2016) 511–525.
[39] M. Terepeta, On separating axioms and similarity of soft topological spaces, Soft Comput. 23 (2019) 1049–1057.
[40] E. Turanli, I. Demirb, O.B. Özbakir, Some types of soft paracompactness via soft ideals, Int. J. Nonlinear Anal. Appl. 10 (2019) 197–211.
[41] I. Zorlutuna, M. Akdag, W.K. Min, S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform. 2 (2012) 171–185.
[42] I. Zorlutuna, H. Çakir, On continuity of soft mappings, Appl. Math. Inf. Sci. 9 (2015) 403–409.