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A study on q-analogue of Catalan sequence spaces

Taja Yaying[a], Merve İlkhan Kara[b], Bipan Hazarika[c], Emrah Evren Kara[b]

[a] *Department of Mathematics, Dera Natung Govt. College, Itanagar 791113, Arunachal Pradesh, India*
[b] *Department of Mathematics, Düzce University, Düzce, Turkey*
[c] *Department of Mathematics, Gauhati University, Gauhati 781014, Assam, India*

Abstract. In this study, we construct q-analog $\mathsf{C}(q)$ of Catalan matrix and study the sequence spaces $c_0(\mathsf{C}(q))$ and $c(\mathsf{C}(q))$ defined as the domain of q-Catalan matrix $\mathsf{C}(q)$ in the spaces c_0 and c, respectively. We exhibit some topological properties, obtain Schauder bases and determine α-, β-, and γ-duals of the spaces $c_0(\mathsf{C}(q))$ and $c(\mathsf{C}(q))$. Finally, we characterize certain class of matrix mappings from the spaces $c_0(\mathsf{C}(q))$ and $c(\mathsf{C}(q))$ to the space $\mu = \{\ell_\infty, c_0, c, \ell_1\}$ and give the necessary and sufficient conditions for a matrix operator to be compact.

1. Introduction and preliminaries

The q-analog of a mathematical expression means the generalization of that expression using the parameter q. The generalized expression returns the original expression when q approaches 1. The study of q-calculus dates back to the time of Euler. It is a wide and an interesting area of research in recent times. Several researchers are engaged in the field of q-calculus due to its vast applications in mathematics, physics and engineering sciences. In the field of mathematics, it is widely used by researchers in approximation theory, combinatorics, hypergeometric functions, operator theory, special functions, quantum algebras, etc.

Let $q \in (0,1)$. Then, the q-number (cf. [28]) is defined by

$$[r]_q = \begin{cases} \sum\limits_{v=0}^{r-1} q^v & (r = 1,2,3,\ldots), \\ 0 & (r = 0). \end{cases}$$

One may notice that, when $q \to 1^-$ then $[r]_q \to r$.

The q-analog of binomial coefficient or q-binomial coefficient is defined by

$$\binom{r}{s}_q = \begin{cases} \frac{[r]_q!}{[r-s]_q![s]_q!} & (r \geq s), \\ 0 & (s > r), \end{cases}$$

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Email addresses: tajayaying20@gmail.com (Taja Yaying), merveilkhan@gmail.com (Merve İlkhan Kara), bh_rgu@yahoo.co.in; bh_gu@gauhati.ac.in (Corresponding author) (Bipan Hazarika), karaeevren@gmail.com (Emrah Evren Kara)

where q-factorial $[r]_q!$ of r is given by

$$[r]_q! = \begin{cases} \prod_{v=1}^{r} [v]_q & (r = 1, 2, 3, \ldots), \\ 1 & (r = 0). \end{cases}$$

We stricly refer to [17, 28] for basic terminologies in q-calculus.

1.1. Sequence space

A linear subspace of w, the set of all real-valued sequences, is called a sequence space. Few examples of classical sequence spaces are ℓ_k (k-absolutely summable sequences, $1 \leq k < \infty$), ℓ_∞ (bounded sequences), c_0 (null sequences), c (convergent sequences), etc. Further the spaces of all bounded, null and convergent series are denoted by bs, cs_0 and cs, respectively. Also, ψ denotes the space of all finite sequences. A Banach sequence space having continuous coordinates is called a BK space. The spaces c_0 and c are BK spaces endowed with the supremum norm $\|x\|_\infty = \sup_{r \in \mathbb{N}_0} |x_r|$, where $\mathbb{N}_0$ is the set of natural numbers including zero.

It is well known that the matrix mappings between between BK-spaces are continuous. Because of this celebrated property, the theory of matrix mappings has an important place in the study of sequence spaces. Let λ and μ be two sequence spaces and $\mathsf{A} = (\mathsf{a}_{rs})$ be an infinite matrix of real entries. Further let A_r denote the r^{th} row of the matrix A. The sequence $\mathsf{A}x = \{(\mathsf{A}x)_r\} = \left\{ \sum_{s=0}^{\infty} \mathsf{a}_{rs} x_s \right\}$ is called A-transform of the sequence $x = (x_s)$, provided that the series $\sum_{s=0}^{\infty} \mathsf{a}_{rs} x_s$ converges for each $r \in \mathbb{N}_0$. Further, if $\mathsf{A}x \in \mu$ for every sequence $x \in \lambda$, then the matrix A is said to define a matrix mapping from λ to μ. The notation (λ, μ) represents the family of all matrices that map from λ to μ. Furthermore, the matrix $\mathsf{A} = (\mathsf{a}_{rs})$ is called a triangle if $\mathsf{a}_{rr} \neq 0$ and $\mathsf{a}_{rs} = 0$ for $r < s$.

The matrix domain λ_{A} of the matrix A in the space λ is defined by

$$\lambda_{\mathsf{A}} = \{x \in w : \mathsf{A}x \in \lambda\}. \tag{1}$$

The set λ_{A} itself is a sequence space. This property plays a significant role in constructing new sequence spaces. Additionally, if A is a triangle and λ is a BK-space then the sequence space λ_{A} is also a BK-space equipped with the norm $\|x\|_{\lambda_{\mathsf{A}}} = \|\mathsf{A}x\|_\lambda$. Several authors applied this celebrated theory in the past to construct new Banach (or BK) sequence spaces using some special triangles. For relevant literature, we refer the papers [11, 13, 19, 20, 23, 26, 31–33, 35].

The construction of sequence spaces using q-analog $C(q)$ of Cesàro matrix has been studied recently by Demiriz and Şahin [12], where $C(q) = (c^q_{rs})$ [1] is defined by

$$c^q_{rs} = \begin{cases} \frac{q^s}{(r+1)[q]} & (0 \leq s \leq r), \\ 0 & (s > r). \end{cases}$$

The authors studied the domains $X_0(q) = (c_0)_{C(q)}$ and $X_c(q) = (c)_{C(q)}$. More recently Yaying et al. [33] studied Banach spaces $X^q_k = (\ell_k)_{C(q)}$ and $X^q_\infty = (\ell_\infty)_{C(q)}$, and studied associated operator ideals. Besides, Yaying et al. [34] studied (p, q)-Euler matrix and its domain in the spaces ℓ_k and ℓ_∞. For studies in q-Hausdorff matrices, we refer [1, 2, 7, 27].

If B_λ is the unit sphere in a normed space λ, for a BK-space $\lambda \supset \psi$ and $\varsigma = (\varsigma_s) \in w$, utilize the notation

$$\|\varsigma\|^*_\lambda = \sup_{u \in B_\lambda} \left| \sum_s \varsigma_s u_s \right|$$

which implies $\varsigma \in \lambda^\beta$.

Lemma 1.1. *[21, Lemma 6]* $\ell_\infty^\beta = c^\beta = c_0^\beta = \ell_1$ *and* $\|\varsigma\|_\lambda^* = \|\varsigma\|_{\ell_1}$ *for* $\lambda \in \{\ell_\infty, c, c_0\}$.

The collection of all bounded (continuous) linear operators from λ to μ is denoted by $B(\lambda, \mu)$.

Lemma 1.2. *[22, Theorem 1.23 (a)] Let* λ *and* μ *be BK-spaces. Then, for every* $\mathsf{A} \in (\lambda, \mu)$*, there exists a linear operator* $\mathcal{L}_\mathsf{A} \in B(\lambda, \mu)$ *such that* $\mathcal{L}_\mathsf{A}(x) = \mathsf{A}x$ *for all* $x \in \lambda$.

Lemma 1.3. *[22] Let* $\lambda \supset \psi$ *be a BK-space and* $\mu \in \{c_0, c, \ell_\infty\}$*. If* $\mathsf{A} \in (\lambda, \mu)$*, then*

$$\|\mathcal{L}_\mathsf{A}\| = \|\mathsf{A}\|_{(\lambda,\mu)} = \sup_{r\in\mathbb{N}_0} \|\mathsf{A}_r\|_\lambda^* < \infty.$$

The Hausdorff measure of noncompactness of a bounded set Q in a metric space λ is denoted by $\chi(Q)$ and it is defined as

$$\chi(Q) = \inf\{\varepsilon > 0 : Q \subset \cup_{i=1}^r B(x_i, \delta_i), x_i \in \lambda, \delta_i < \varepsilon, r \in \mathbb{N}\},$$

where $B(x_i, \delta_i)$ is the open ball. For more details about the Hausdorff measure of noncompactness, one can consult [22] and references therein.

Theorem 1.4. *Let* $P_k : c_0 \to c_0$ *be the operator defined by* $P_k(x) = (x_0, x_1, x_2, ..., x_k, 0, 0, ...)$ *for all* $x = (x_s) \in c_0$ *and each* $k \in \mathbb{N}_0$*. Then, for any bounded set* Q *in* c_0*, we have*

$$\chi(Q) = \lim_{k\to\infty} \left(\sup_{x\in Q} \|(I - P_k)(x)\|_{c_0} \right),$$

where I *is the identity operator on* c_0.

A linear operator $\mathcal{L}$ from a Banach space λ into another Banach space μ is called a compact operator if the domain of $\mathcal{L}$ is all of λ and for every bounded sequence $x = (x_r)$ in λ, the sequence $(\mathcal{L}(x_r))$ has a convergent subsequence in μ. The necessary and sufficient condition for an operator to be compact is that the Hausdorff measure of noncompactness of $\mathcal{L}$ is zero defined as $\|\mathcal{L}\|_\chi = \chi(\mathcal{L}(B_\lambda)) = 0$.

In the theory of sequence spaces, the Hausdorff measure of noncompactness of a linear operator plays a role to characterize the compactness of an operator between BK spaces. For the relevant literature, see [5, 6, 16, 18, 25].

The catalan matrix $\mathsf{C} = (\mathsf{c}_{rs})$ [15] is defined by

$$\mathsf{c}_{rs} = \begin{cases} \frac{\tilde{c}_s \tilde{c}_{r-s}}{\tilde{c}_{r+1}} & (0 \le s \le r), \\ 0 & (s > r), \end{cases}$$

where $\tilde{c} = (\tilde{c}_s)$ is the sequence of Catalan numbers defined by

$$\tilde{c}_s = \frac{1}{s+1}\binom{2s}{s}. \tag{2}$$

The above definition is equivalent to the recurrence relation

$$\tilde{c}_{r+1} = \sum_{s=0}^{r} \tilde{c}_s \tilde{c}_{r-s}, \ \tilde{c}_0 = 1. \tag{3}$$

Recently, the domains $c(\mathsf{C})$ and $c_0(\mathsf{C})$ of the matrix C in the spaces c and c_0, respectively are studied by İlkhan [15]. Alp [3] studied the sequence spaces $\ell(\mathsf{C}, k) = (\ell(k))_\mathsf{C}$, where $\ell(k)$ denotes the Maddox's space and $k = (k_s)$ is the bounded sequence of strictly positive real numbers.

Several q-analogs of Catalan sequence can be found in the literature. Let $0 < q < 1$. Define the sequences [9]

$$\tilde{c}_s(q) = \frac{1}{[s+1]_q}\binom{2s}{s}_q, \tag{4}$$

$$\bar{c}_{r+1}(q) = \sum_{s=0}^{r} q^s \bar{c}_s(q)\bar{c}_{r-s}(q),\ \bar{c}_0(q) = 1. \tag{5}$$

It is clear that (4) and (5) are the natural q-analogs of (2) and (3), respectively. However, unlike the ordinary case, the sequences defined by (2) and (3) are not equivalent (see [9]). For some more interesting studies in q-Catalan sequences, we strictly refer to [8–10, 14].

Inspired by the above studies, we construct BK sequence spaces $c(\mathsf{C}(q))$ and $c_0(\mathsf{C}(q))$ defined by the q-analog $\mathsf{C}(q)$ of the matrix C. We exhibit some topological properties and determine the bases for the spaces $c(\mathsf{C}(q))$ and $c_0(\mathsf{C}(q))$. In Section 3, we compute α-, β- and γ-duals of the spaces $c(\mathsf{C}(q))$ and $c_0(\mathsf{C}(q))$. In Section 4, we characterize some matrix mappings from the spaces $c(\mathsf{C}(q))$ and $c_0(\mathsf{C}(q))$ to any one of the spaces ℓ_∞, c, c_0, and ℓ_1. In the final section, compact operators are characterized on the spaces $c_0(\mathsf{C}(q))$.

2. The sequence spaces $c(\mathsf{C}(q))$ and $c_0(\mathsf{C}(q))$

We proceed by introducing q-Catalan matrix $\mathsf{C}(q) = \mathsf{c}_{rs}^q$ defined by

$$\mathsf{c}_{rs}^q = \begin{cases} q^s \frac{\bar{c}_s(q)\bar{c}_{r-s}(q)}{\bar{c}_{r+1}(q)}, & s \le r, \\ 0, & s > r. \end{cases}$$

It is clear that the q-Catalan matrix $\mathsf{C}(q)$ reduces to the Catalan matrix C (cf. [15]), when q tends to 1^-. Now we define the q-Catalan sequence spaces $c(\mathsf{C}(q))$ and $c_0(\mathsf{C}(q))$ by

$$c(\mathsf{C}(q)) = \left\{ x = (x_s) \in w : \lim_{r\to\infty} \sum_{s=0}^{r} q^s \frac{\bar{c}_s(q)\bar{c}_{r-s}(q)}{\bar{c}_{r+1}(q)} x_s \text{ exists} \right\},$$

$$c_0(\mathsf{C}(q)) = \left\{ x = (x_s) \in w : \lim_{r\to\infty} \sum_{s=0}^{r} q^s \frac{\bar{c}_s(q)\bar{c}_{r-s}(q)}{\bar{c}_{r+1}(q)} x_s = 0 \right\}.$$

We emphasize that the spaces $c(\mathsf{C}(q))$ and $c_0(\mathsf{C}(q))$ reduce to the Catalan sequence spaces $c(\mathsf{C})$ and $c_0(\mathsf{C})$, respectively, when $q \to 1^-$ as studied by İlkhan [15]. With the notation of (1), the above sequence spaces may by redefined by

$$c(\mathsf{C}(q)) = (c)_{\mathsf{C}(q)} \quad \text{and} \quad c_0(\mathsf{C}(q)) = (c_0)_{\mathsf{C}(q)}. \tag{6}$$

The sequence $y = (y_r)$ is called $\mathsf{C}(q)$-transform of the sequence $x = (x_s)$. That is

$$y_r = (\mathsf{C}(q)x)_r = \sum_{s=0}^{r} q^s \frac{\bar{c}_s(q)\bar{c}_{r-s}(q)}{\bar{c}_{r+1}(q)} x_s, \tag{7}$$

for each $r \in \mathbb{N}_0$. Define $A_0(q) = 1$ and

$$A_r(q) = \begin{vmatrix} \bar{c}_1(q) & \bar{c}_0(q) & 0 & 0 & 0 & \dots & 0 \\ \bar{c}_2(q) & \bar{c}_1(q) & \bar{c}_0(q) & 0 & 0 & \dots & 0 \\ \bar{c}_3(q) & \bar{c}_2(q) & \bar{c}_1(q) & \bar{c}_0(q) & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{c}_r(q) & \bar{c}_{r-1}(q) & \bar{c}_{r-2}(q) & \bar{c}_{r-3}(q) & \bar{c}_{r-4}(q) & \dots & \bar{c}_1(q) \end{vmatrix}.$$

Then, on using (7), we write

$$x_s = \sum_{v=0}^{s} (-1)^{s-v} \frac{A_{s-v}(q)}{q^s} \frac{\bar{c}_{s+1}(q)}{(\bar{c}_0(q))^{s-v+1}\, \bar{c}_s(q)} y_v, \tag{8}$$

for each $s \in \mathbb{N}_0$. In the rest of the paper, the sequences x and y are related by (7) or equaivalently by (8).

Theorem 2.1. *$c(\mathsf{C}(q))$ and $c_0(\mathsf{C}(q))$ are BK-spaces endowed with the same norm defined by*

$$\|x\|_{c(\mathsf{C}(q))} = \|x\|_{c_0(\mathsf{C}(q))} = \sup_{r \in \mathbb{N}_0} \left| \sum_{s=0}^{r} q^s \frac{\bar{c}_s(q)\bar{c}_{r-s}(q)}{\bar{c}_{r+1}(q)} x_s \right|.$$

Proof. The proof is a routine verification and hence details omitted. □

Theorem 2.2. *$c(\mathsf{C}(q)) \cong c$ and $c_0(\mathsf{C}(q)) \cong c_0$.*

Proof. Let $\lambda \in \{c, c_0\}$. Then, the mapping $\pi : \lambda(\mathsf{C}(q)) \to \lambda$ defined by $\pi x = \mathsf{C}(q)x$ for all $x \in \lambda(\mathsf{C}(q))$ is invertible which implies that π is a norm preserving linear bijection. Hence $\lambda(\mathsf{C}(q)) \cong \lambda$. □

To end this section, we construct bases for the spaces $c(\mathsf{C}(q))$ and $c_0(\mathsf{C}(q))$. We recall that the domain λ_{A} of the triangle A in the space λ has a basis if and only if λ has a basis. This statement together with Theorem 2.2 gives us the following result:

Theorem 2.3. *For every fixed $s \in \mathbb{N}_0$, define the sequence $h^{(s)}(q) = (h_r^{(s)}(q))$ of the elements of the space $c_0(\mathsf{C}(q))$ by*

$$h_r^{(s)}(q) = \begin{cases} (-1)^{s-v} \frac{A_{s-v}(q)}{q^s} \frac{\bar{c}_{s+1}(q)}{(\bar{c}_0(q))^{s-v+1}\bar{c}_s(q)} & (s \le r), \\ 0 & (s > r). \end{cases}$$

Then

(a) *the set $\left\{h^{(0)}(q), h^{(1)}(q), h^{(2)}(q), \ldots\right\}$ forms the basis for the space $c_0(\mathsf{C}(q))$ and every $x \in c_0(\mathsf{C}(q))$ has a unique representation $x = \sum_{s=0}^{\infty} y_s h^{(s)}(q)$.*

(b) *the set $\left\{e, h^{(0)}(q), h^{(1)}(q), h^{(2)}(q), \ldots\right\}$ forms the basis for the space $c(\mathsf{C}(q))$ and every $x \in c(\mathsf{C}(q))$ can be uniquely expressed in the form $x = ze + \sum_{s=0}^{\infty}(y_s - z)h^{(s)}(q)$, where $z = \lim_{s\to\infty} y_s = \lim_{s\to\infty}(\mathsf{C}(q)x)_s$.*

3. $\alpha-, \beta-, \gamma-$duals

In the current section, we determine α-, β-, γ-duals of the spaces $c(\mathsf{C}(q))$ and $c_0(\mathsf{C}(q))$. Since the computation of duals is similar for both the spaces, we shall omit the proof for the space $c(\mathsf{C}(q))$. Before proceeding, we recall the definitions of α-, β-, γ-duals.

Definition 3.1. *The $\alpha-$, $\beta-$ and $\gamma-$duals of a subset $\lambda \subset w$ are defined by*

$$\lambda^{\alpha} = \{\varsigma = (\varsigma_s) \in w : \varsigma x = (\varsigma_s x_s) \in \ell_1 \text{ for all } x \in \lambda\},$$
$$\lambda^{\beta} = \{\varsigma = (\varsigma_s) \in w : \varsigma x = (\varsigma_s x_s) \in cs \text{ for all } x \in \lambda\} \text{ and}$$
$$\lambda^{\gamma} = \{\varsigma = (\varsigma_s) \in w : \varsigma x = (\varsigma_s x_s) \in bs \text{ for all } x \in \lambda\},$$

respectively.

In the rest of the paper, $\mathcal{N}$ will denote the family of all finite subsets of $\mathbb{N}_0$. First we note down certain lemmas due to Stielglitz and Tietz [29] that are necessary for obtaining the duals:

Lemma 3.2. $\mathsf{A} = (\mathsf{a}_{rs}) \in (c_0, \ell_1)$ *if and only if*

$$\sup_{R \in \mathcal{N}} \left(\sum_{s=0}^{\infty} \left| \sum_{r \in R} \mathsf{a}_{rs} \right| \right) < \infty. \tag{9}$$

Lemma 3.3. $\mathsf{A} = (\mathsf{a}_{rs}) \in (c_0, c)$ *if and only if*

$$\sup_{r \in \mathbb{N}_0} \sum_{s=0}^{r} |\mathsf{a}_{rs}| < \infty, \tag{10}$$

$$\lim_{r \to \infty} \mathsf{a}_{rs} = \alpha_s \text{ for each } s \in \mathbb{N}_0. \tag{11}$$

Lemma 3.4. $\mathsf{A} = (\mathsf{a}_{rs}) \in (c_0, \ell_\infty)$ *if and only if (10) holds.*

Theorem 3.5. *The set $d_1(q)$ defined by*

$$d_1(q) = \left\{ \varsigma = (\varsigma_s) \in w : \sup_{R \in \mathcal{N}} \sum_{s=0}^{\infty} \left| \sum_{r \in R} (-1)^{r-s} \frac{A_{r-s}(q)}{q^r} \frac{\bar{c}_{r+1}(q)}{(\bar{c}_0(q))^{r-s+1} \bar{c}_r(q)} \varsigma_r \right| < \infty \right\}$$

is the α-dual of the spaces $c(\mathsf{C}(q))$ and $c_0(\mathsf{C}(q))$.

Proof. Consider the following equality

$$\begin{aligned} \varsigma_r x_r &= \sum_{s=0}^{r} (-1)^{r-s} \frac{A_{r-s}(q)}{q^r} \frac{\bar{c}_{r+1}(q)}{(\bar{c}_0(q))^{r-s+1} \bar{c}_r(q)} \varsigma_r y_s \\ &= (G(q)y)_r \end{aligned} \tag{12}$$

for all $r \in \mathbb{N}_0$, where the sequence $y = (y_s)$ is the $\mathsf{C}(q)$-transform of the sequence $x = (x_s)$ and the matrix $G(q) = (g^q_{rs})$ is defined by

$$g^q_{rs} = \begin{cases} (-1)^{r-s} \frac{A_{r-s}(q)}{q^r} \frac{\bar{c}_{r+1}(q)}{(\bar{c}_0(q))^{r-s+1} \bar{c}_r(q)} \varsigma_r & (0 \le s \le r), \\ 0 & (s > r). \end{cases}$$

We realize on using Eq. (12) that $\varsigma x = (\varsigma_r x_r) \in \ell_1$ whenever $x \in c_0(\mathsf{C}(q))$ if and only if $G(q)y \in \ell_1$ whenever $y \in c_0$. Thus we deduce that $\varsigma = (\varsigma_r)$ is a sequence in α-dual of $c_0(\mathsf{C}(q))$ if and only the matrix $\mathsf{C}(q)$ belongs to the class (c_0, ℓ_1). Thus we conclude from Lemma 3.2 that $[c_0(\mathsf{C}(q))]^\alpha = d_1(q)$. This completes the proof. □

Theorem 3.6. *Define the sets $d_2(q)$, $d_3(q)$ and $d_4(q)$ by*

$$d_2(q) = \left\{ \varsigma = (\varsigma_r) \in w : \sum_{r=s}^{\infty} (-1)^{r-s} \frac{A_{r-s}(q)}{q^r} \frac{\bar{c}_{r+1}(q)}{(\bar{c}_0(q))^{r-s+1} \bar{c}_r(q)} \varsigma_r \text{ exists for each } s \in \mathbb{N}_0 \right\},$$

$$d_3(q) = \left\{ \varsigma = (\varsigma_r) \in w : \sup_{r \in \mathbb{N}_0} \sum_{s=0}^{r} \left| \sum_{v=s}^{r} (-1)^{v-s} \frac{A_{v-s}(q)}{q^v} \frac{\bar{c}_{v+1}(q)}{(\bar{c}_0(q))^{v-s+1} \bar{c}_v(q)} \varsigma_v \right| < \infty \right\},$$

$$d_4(q) = \left\{ \varsigma = (\varsigma_r) \in w : \lim_{r \to \infty} \sum_{s=0}^{r} \sum_{v=s}^{r} (-1)^{r-s} \frac{A_{v-s}(q)}{q^v} \frac{\bar{c}_{v+1}(q)}{(\bar{c}_0(q))^{v-s+1} \bar{c}_v(q)} \varsigma_v \quad \text{exists} \right\}.$$

Then $[c_0(\mathsf{C}(q))]^\beta = d_2(q) \cap d_3(q)$ and $[c(\mathsf{C}(q))]^\beta = d_2(q) \cap d_3(q) \cap d_4(q)$.

Proof. Consider the following equality

$$\begin{aligned}\sum_{s=0}^{r} \varsigma_s x_s &= \sum_{s=0}^{r} \left\{ \sum_{v=0}^{s} (-1)^{s-v} \frac{A_{s-v}(q)}{q^s} \frac{\bar{c}_{s+1}(q)}{(\bar{c}_0(q))^{s-v+1}\, \bar{c}_s(q)} y_v \right\} \varsigma_s \\ &= \sum_{s=0}^{r} \left\{ \sum_{v=s}^{r} (-1)^{v-s} \frac{A_{v-s}(q)}{q^v} \frac{\bar{c}_{v+1}(q)}{(\bar{c}_0(q))^{v-s+1}\, \bar{c}_r(q)} \varsigma_v \right\} y_s \\ &= (H(q)y)_r \end{aligned} \tag{13}$$

for each $r \in \mathbb{N}_0$, where the sequence $y = (y_s)$ is the $\mathsf{C}(q)$-transform of the sequence $x = (x_s)$ and the matrix $H(q) = (h^q_{rs})$ is defined by

$$h^q_{rs} = \begin{cases} \sum_{v=s}^{r} (-1)^{v-s} \frac{A_{v-s}(q)}{q^v} \frac{\bar{c}_{v+1}(q)}{(\bar{c}_0(q))^{v-s+1}\bar{c}_r(q)} \varsigma_v & (0 \le s \le r), \\ 0 & (s > r), \end{cases}$$

for all $r, s \in \mathbb{N}_0$. Thus on using Eq. (13), we realize that $\varsigma x = (\varsigma_r x_r) \in cs$ whenever $x = (x_r) \in c_0(\mathsf{C}(q))$ if and only if $H(q)y \in c$ whenever $y = (y_s) \in c_0$. This yields that $\varsigma = (\varsigma_r)$ is a sequence in β-dual of $c_0(\mathsf{C}(q))$ if and only the matrix $H(q)$ belongs to the class (c_0, c). This in turn implies on using Lemma 3.3 that

$$\sup_{r \in \mathbb{N}_0} \sum_{s=0}^{r} \left| h^q_{rs} \right| < \infty \text{ and } \lim_{r \to \infty} h^q_{rs} \text{ exists for each } s \in \mathbb{N}_0.$$

Thus $c_0(\mathsf{C}(q)) = d_2(q) \cap d_3(q)$. This completes the proof. □

Theorem 3.7. *The γ-dual of the spaces $c(\mathsf{C}(q))$ and $c_0(\mathsf{C}(q))$ is $d_3(q)$.*

Proof. The proof is similar to the previous theorem except that Lemma 3.4 is employed instead of Lemma 3.3. □

4. Matrix mappings

In the present section, we determine necessary and sufficient conditions for a matrix to define mapping from the spaces $c(\mathsf{C}(q))$ and $c_0(\mathsf{C}(q))$ to anyone of the spaces ℓ_∞, c, c_0, and ℓ_1. The following theorem is fundamental in our investigation.

Theorem 4.1. *Let μ be any arbitrary subset of w. Then $\mathsf{A} = (\mathsf{a}_{rs}) \in (c_0(\mathsf{C}(q)), \mu)$ (or respectively $(c(\mathsf{C}(q)), \mu)$) if and only if $\mathsf{B}^{(r)} = (\mathsf{b}^{(r)}_{ms}) \in (c_0, c)$ (or respectively (c, c)) for each $r \in \mathbb{N}_0$, and $\mathsf{B} = (\mathsf{b}_{rs}) \in (c_0, \mu)$ (or respectively (c, μ)) where*

$$\mathsf{b}^{(r)}_{ms} = \begin{cases} 0 & (s > m), \\ \sum\limits_{v=s}^{m} (-1)^{v-s} \frac{A_{v-s}(q)}{q^v} \frac{\bar{c}_{v+1}(q)}{(\bar{c}_0(q))^{v-s+1}\bar{c}_v(q)} \mathsf{a}_{rv} & (0 \le s \le m), \end{cases}$$

and

$$\mathsf{b}_{rs} = \sum_{v=s}^{\infty} (-1)^{v-s} \frac{A_{v-s}(q)}{q^v} \frac{\bar{c}_{v+1}(q)}{(\bar{c}_0(q))^{v-s+1}\, \bar{c}_v(q)} \mathsf{b}_{rv} \textit{ for all } r, s \in \mathbb{N}_0. \tag{14}$$

Proof. The details of the proof are omitted since it is similar to the proof of Theorem 4.1 of [20]. □

Before proceeding further, we list certain conditions:

$$\lim_{m\to\infty} \mathsf{b}_{ms}^{(r)} \text{ exists for each } r,s \in \mathbb{N}_0; \tag{15}$$

$$\sup_{m\in\mathbb{N}_0} \sum_{s=0}^{\infty} \left|\mathsf{b}_{ms}^{(r)}\right| < \infty \text{ for each } r \in \mathbb{N}_0; \tag{16}$$

$$\lim_{m\to\infty} \sum_{s=0}^{\infty} \mathsf{b}_{ms}^{(r)} \text{ exists for each } r \in \mathbb{N}_0; \tag{17}$$

$$\lim_{r\to\infty} \sum_{s=0}^{r} |\mathsf{b}_{rs}| = \beta_s \text{ for each } s \in \mathbb{N}_0. \tag{18}$$

Now, using the results presented in the Stielglitz and Tietz [29] together with Theorem 4.1, we obtain the following results:

Corollary 4.2. *The following statements hold:*

1. $\mathsf{A} \in (c_0(\mathsf{C}(q)), \ell_\infty)$ *if and only if (15) and (16) hold, and (10) also holds with* b_{rs} *instead of* a_{rs}.
2. $\mathsf{A} \in (c_0(\mathsf{C}(q)), c)$ *if and only if (15) and (16) hold, and (10) and (11) also hold with* b_{rs} *instead of* a_{rs}.
3. $\mathsf{A} \in (c_0(\mathsf{C}(q)), c_0)$ *if and only if (15) and (16) hold, and (10) and (11) with* $\alpha_s = 0$ *hold with* b_{rs} *instead of* a_{rs}.
4. $\mathsf{A} \in (c_0(\mathsf{C}(q)), \ell_1)$ *if and only if (15) and (16) hold, and (9) also holds with* b_{rs} *instead of* a_{rs}.

Corollary 4.3. *The following statements hold:*

1. $\mathsf{A} \in (c(\mathsf{C}(q)), \ell_\infty)$ *if and only if (15), (16) and (17) hold, and (10) also holds with* b_{rs} *instead of* a_{rs}.
2. $\mathsf{A} \in (c(\mathsf{C}(q)), c)$ *if and only if (15), (16), (17) and (18) hold, and (10) and (11) also hold with* b_{rs} *instead of* a_{rs}.
3. $\mathsf{A} \in (c(\mathsf{C}(q)), c)$ *if and only if (15), (16), (17) and (18) with* $\beta_s = 0$ *hold, and (10) and (11) also hold with* b_{rs} *instead of* a_{rs}.
4. $\mathsf{A} \in (c(\mathsf{C}(q)), \ell_1)$ *if and only if (15), (16) and (17) hold, and (9) also holds with* b_{rs} *instead of* a_{rs}.

We recall a basic lemma due to Başar and Altay [4] that will help in characterizing certain classes of matrix mappings from the spaces $c_0(\mathsf{C}(q))$ and $c(\mathsf{C}(q))$ to any arbitrary space μ.

Lemma 4.4. *[4] Let λ and μ be any two sequence spaces,* A *be an infinite matrix and* B *be a triangle. Then,* $\mathsf{A} \in (\lambda, \mu_{\mathsf{B}})$ *if and only if* $\mathsf{BA} \in (\lambda, \mu)$.

Now, by combining Lemma 4.4 with Corollaries 4.2 and 4.3, we define following classes of matrix mappings:

Corollary 4.5. *Let* $\mathsf{A} = (\mathsf{a}_{rs})$ *be an infinite matrix and define the matrix* $\bar{C}(q) = (\bar{c}_{rs}^q)$ *by*

$$\bar{c}_{rs}^q = \sum_{m=0}^{r} \frac{q^m}{[r+1]_q} \mathsf{a}_{ms}, (0 < q < 1) \text{ for all } r,s \in \mathbb{N}.$$

Then, the necessary and sufficient conditions that A *is in any one of the classes* $(c_0(\mathsf{C}(q)), X_0^q)$, $(c_0(\mathsf{C}(q)), X_c^q)$, $(c(\mathsf{C}(q)), X_0^q)$ *and* $(c(\mathsf{C}(q)), X_c^q)$ *is determined from the respective ones in Corollaries 4.2 and 4.3, by replacing the elements of the matrix* A *by those of matrix* $\bar{\mathsf{C}}(q)$, *where* X_0^q *and* X_c^q *are q-Cesàro sequence spaces defined by Demiriz and Şahin [12].*

Corollary 4.6. *Let* $\mathsf{A} = (\mathsf{a}_{rs})$ *be an infinite matrix and define the matrix* $E = (e_{rs})$ *by*

$$e_{rs} = \sum_{m=0}^{r} \mathsf{a}_{ms}, \ (r,s \in \mathbb{N}_0).$$

Then, the necessary and sufficient conditions that A *is in any one of the classes* $(c_0(\mathsf{C}(q)), bs)$, $(c_0(\mathsf{C}(q)), cs)$, $(c_0(\mathsf{C}(q)), cs_0)$, $(c(\mathsf{C}(q)), bs)$, $(c(\mathsf{C}(q)), cs)$ *and* $(c(\mathsf{C}(q)), cs_0)$ *is determined from the respective ones in Corollaries 4.2 and 4.3, by replacing the elements of the matrix* A *by those of the matrix* E.

Corollary 4.7. *Let* $\mathsf{A} = (\mathsf{a}_{rs})$ *be an infinite matrix and define the matrix* $\mathcal{F} = (f_{rs})$ *by*

$$f_{rs} = \sum_{m=0}^{r} \frac{f_m^2}{f_r f_{r+1}} \mathsf{a}_{ms}, \ (r, s \in \mathbb{N}_0)$$

where (f_r) *are sequence of Fibonacci numbers. Then, the necessary and sufficient conditions that* A *is in any one of the classes* $(c_0(\mathsf{C}(q)), \ell_\infty(\mathcal{F}))$, $(c_0(\mathsf{C}(q)), c(\mathcal{F}))$, $(c_0(\mathsf{C}(q)), c_0(\mathcal{F}))$, $(c(\mathsf{C}(q)), \ell_\infty(\mathcal{F}))$, $(c(\mathsf{C}(q)), c(\mathcal{F}))$ *and* $(c(\mathsf{C}(q)), c_0(\mathcal{F}))$, *is determined from the respective ones in Corollaries 4.2 and 4.3, by replacing the elements of the matrix* A *by those of matrix* $\mathcal{F}$, *where* $\ell_\infty(\mathcal{F})$, $c(\mathcal{F})$ *and* $c_0(\mathcal{F})$ *are Fibonacci sequence spaces defined by Kara and Başarır [19].*

5. Compact operators on the spaces $c_0(\mathsf{C}(q))$

Let $\varsigma = (\varsigma_s) \in \omega$ and define a sequence $\upsilon = (\upsilon_s)$ as

$$\upsilon_s = \sum_{v=s}^{\infty} (-1)^{v-s} \frac{A_{v-s}(q)}{q^v} \frac{\bar{c}_{v+1}(q)}{(\bar{c}_0(q))^{v-s+1} \bar{c}_v(q)} \varsigma_v$$

for all $s \in \mathbb{N}_0$.

Lemma 5.1. *Let* $\varsigma = (\varsigma_s) \in [c_0(\mathsf{C}(q))]^\beta$. *Then* $\upsilon = (\upsilon_s) \in \ell_1$ *and*

$$\sum_s \varsigma_s x_s = \sum_s \upsilon_s y_s \tag{19}$$

for all $x = (x_s) \in c_0(\mathsf{C}(q))$.

Lemma 5.2. $\|\varsigma\|^*_{c_0(\mathsf{C}(q))} = \sum_s |\upsilon_s| < \infty$ *for all* $\varsigma = (\varsigma_s) \in [c_0(\mathsf{C}(q))]^\beta$.

Proof. Choose $\varsigma = (\varsigma_s) \in [c_0(\mathsf{C}(q))]^\beta$. Then, by Lemma 5.1, we have $\upsilon = (\upsilon_s) \in \ell_1$ and (19) holds. Since $\|x\|_{c_0(\mathsf{C}(q))} = \|y\|_{c_0}$ holds, we obtain that $x \in B_{c_0(\mathsf{C}(q))}$ if and only if $y \in B_{c_0}$. Hence, we deduce that $\|\varsigma\|^*_{c_0(\mathsf{C}(q))} = \sup_{x \in B_{c_0(\mathsf{C}(q))}} |\sum_s \varsigma_s x_s| = \sup_{y \in B_{c_0}} |\sum_s \upsilon_s y_s| = \|\upsilon\|^*_{c_0}$. From Lemma 1.1, it follows that $\|\varsigma\|^*_{c_0(\mathsf{C}(q))} = \|\upsilon\|^*_{c_0} = \|\upsilon\|_{\ell_1} = \sum_s |\upsilon_s|$. □

Throughout this section, we use the matrix $\tilde{\mathsf{A}} = (\tilde{\mathsf{a}}_{rs})$ defined by an infinite matrix $\mathsf{A} = (\mathsf{a}_{rs})$ via

$$\tilde{\mathsf{a}}_{rs} = \sum_{v=s}^{\infty} (-1)^{v-s} \frac{A_{v-s}(q)}{q^v} \frac{\bar{c}_{v+1}(q)}{(\bar{c}_0(q))^{v-s+1} \bar{c}_v(q)} \mathsf{a}_{rv}$$

for all $r, s \in \mathbb{N}_0$ under the assumption that the series is convergent.

Lemma 5.3. *Let* $\lambda \in \omega$ *and* $\mathsf{A} = (\mathsf{a}_{rs})$ *be an infinite matrix. If* $\mathsf{A} \in (c_0(\mathsf{C}(q)), \lambda)$, *then* $\tilde{\mathsf{A}} \in (c_0, \lambda)$ *and* $\mathsf{A}x = \tilde{\mathsf{A}}y$ *for all* $x \in c_0(\mathsf{C}(q))$.

Proof. It follows from Lemma 5.1. □

Lemma 5.4. $\|\mathcal{L}_\mathsf{A}\| = \|\mathsf{A}\|_{(c_0(\mathsf{C}(q)),\mu)} = \sup_{r \in \mathbb{N}_0} \left(\sum_s |\tilde{\mathsf{a}}_{rs}|\right) < \infty$ *holds for* $\mathsf{A} \in (c_0(\mathsf{C}(q)), \mu)$, *where* $\mu \in \{c_0, c, \ell_\infty\}$.

Lemma 5.5. *[24, Theorem 3.7] Let* $\lambda \supset \psi$ *be a BK-space. Then, the following statements hold.*

(a) $\mathsf{A} \in (\lambda, \ell_\infty)$, *then* $0 \leq \|\mathcal{L}_\mathsf{A}\|_\chi \leq \limsup_r \|\mathsf{A}_r\|^*_\lambda$.
(b) $\mathsf{A} \in (\lambda, c_0)$, *then* $\|\mathcal{L}_\mathsf{A}\|_\chi = \limsup_r \|\mathsf{A}_r\|^*_\lambda$.
(c) If λ *has AK or* $\lambda = \ell_\infty$ *and* $\mathsf{A} \in (\lambda, c)$, *then* $\frac{1}{2} \limsup_r \|\mathsf{A}_r - \mathsf{a}\|^*_\lambda \leq \|\mathcal{L}_\mathsf{A}\|_\chi \leq \limsup_r \|\mathsf{A}_r - \mathsf{a}\|^*_\lambda$, *where* $\mathsf{a} = (\mathsf{a}_s)$ *and* $\mathsf{a}_s = \lim_r \mathsf{a}_{rs}$ *for each* $s \in \mathbb{N}_0$.

Lemma 5.6. *[24, Theorem 3.11] Let $\lambda \supset \psi$ be a BK-space. If $\mathsf{A} \in (\lambda, \ell_1)$, then*

$$\lim_p \left(\sup_{N \in \mathcal{N}_p} \left\| \sum_{r \in N} \mathsf{A}_r \right\|_\lambda^* \right) \leq \|\mathcal{L}_\mathsf{A}\|_\chi \leq 4 \lim_p \left(\sup_{N \in \mathcal{N}_p} \left\| \sum_{r \in N} \mathsf{A}_r \right\|_\lambda^* \right)$$

and $\mathcal{L}_\mathsf{A}$ is compact if and only if $\lim_p \left(\sup_{N \in \mathcal{N}_p} \| \sum_{r \in N} \mathsf{A}_r \|_\lambda^* \right) = 0$, where $\mathcal{N}_p$ is the sub-collection of $\mathcal{N}$ consisting of subsets of $\mathbb{N}$ with elements that are greater than p.

Theorem 5.7.

(1) If $\mathsf{A} \in (c_0(\mathsf{C}(q)), \ell_\infty)$, then $0 \leq \|\mathcal{L}_\mathsf{A}\|_\chi \leq \limsup_r \sum_s |\tilde{\mathsf{a}}_{rs}|$ holds.

(2) If $\mathsf{A} \in (c_0(\mathsf{C}(q)), c)$, then $\frac{1}{2} \limsup_r \sum_s |\tilde{\mathsf{a}}_{rs} - \tilde{\mathsf{a}}_s| \leq \|\mathcal{L}_\mathsf{A}\|_\chi \leq \limsup_r \sum_s |\tilde{\mathsf{a}}_{rs} - \tilde{\mathsf{a}}_s|$ holds.

(3) If $\mathsf{A} \in (c_0(\mathsf{C}(q)), c_0)$, then $\|\mathcal{L}_\mathsf{A}\|_\chi = \limsup_r \sum_s |\tilde{\mathsf{a}}_{rs}|$ holds.

(4) If $\mathsf{A} \in (c_0(\mathsf{C}(q)), \ell_1)$, then $\lim_p \|\mathsf{A}\|^{(p)}_{(c_0(\mathsf{C}(q)), \ell_1)} \leq \|\mathcal{L}_\mathsf{A}\|_\chi \leq 4 \lim_p \|\mathsf{A}\|^{(p)}_{(c_0(\mathsf{C}(q)), \ell_1)}$ holds, where

$$\|\mathsf{A}\|^{(p)}_{(c_0(\mathsf{C}(q)), \ell_1)} = \sup_{N \in \mathcal{N}_p} \left(\sum_s \Big| \sum_{r \in N} \tilde{\mathsf{a}}_{rs} \Big| \right) (p \in \mathbb{N}_0).$$

Proof. (1) Let $\mathsf{A} \in (c_0(\mathsf{C}(q)), \ell_\infty)$. Since the series $\sum_{s=1}^{\infty} \mathsf{a}_{rs} x_s$ converges for each $r \in \mathbb{N}_0$, we have $\mathsf{A}_r \in [c_0(\mathsf{C}(q))]^\beta$. From Lemma 5.2, we write $\|\mathsf{A}_r\|^*_{c_0(\mathsf{C}(q))} = \|\tilde{\mathsf{A}}_r\|^*_{c_0} = \|\tilde{\mathsf{A}}_r\|_{\ell_1} = \left(\sum_s |\tilde{\mathsf{a}}_{rs}| \right)$ for each $r \in \mathbb{N}_0$. By using Lemma 5.5 (a), we conclude that $0 \leq \|\mathcal{L}_\mathsf{A}\|_\chi \leq \limsup_r \left(\sum_s |\tilde{\mathsf{a}}_{rs}| \right)$.
(2) Let $\mathsf{A} \in (c_0(\mathsf{C}(q)), c)$. By Lemma 5.3, we have $\tilde{\mathsf{A}} \in (c_0, c)$. Hence, from Lemma 5.5 (c), we write

$$\frac{1}{2} \limsup_r \|\tilde{\mathsf{A}}_r - \tilde{\mathsf{a}}\|^*_{c_0} \leq \|\mathcal{L}_\mathsf{A}\|_\chi \leq \limsup_r \|\tilde{\mathsf{A}}_r - \tilde{\mathsf{a}}\|^*_{c_0},$$

where $\tilde{\mathsf{a}} = (\tilde{\mathsf{a}}_s)$ and $\tilde{\mathsf{a}}_s = \lim_r \tilde{\mathsf{a}}_{rs}$ for each $s \in \mathbb{N}_0$. Moreover, Lemma 1.1 implies that $\|\tilde{\mathsf{A}}_r - \tilde{\mathsf{a}}\|^*_{c_0} = \|\tilde{\mathsf{A}}_r - \tilde{\mathsf{a}}\|_{\ell_1} = \left(\sum_s |\tilde{\mathsf{a}}_{rs} - \tilde{\mathsf{a}}_s| \right)$ for each $r \in \mathbb{N}_0$. This completes the proof.

(3) Let $\mathsf{A} \in (c_0(\mathsf{C}(q)), c_0)$. Since we have $\|\mathsf{A}_r\|^*_{c_0(\mathsf{C}(q))} = \|\tilde{\mathsf{A}}_r\|^*_{c_0} = \|\tilde{\mathsf{A}}_r\|_{\ell_1} = \left(\sum_s |\tilde{\mathsf{a}}_{rs}| \right)$ for each $r \in \mathbb{N}_0$, we conclude from Lemma 5.5 (b) that $\|\mathcal{L}_\mathsf{A}\|_\chi = \limsup_r \left(\sum_s |\tilde{\mathsf{a}}_{rs}| \right)$.
(4) Let $\mathsf{A} \in (c_0(\mathsf{C}(q)), \ell_1)$. By Lemma 5.3, we have $\tilde{\mathsf{A}} \in (c_0, \ell_1)$. It follows from Lemma 5.6 that

$$\lim_p \left(\sup_{N \in \mathcal{N}_p} \left\| \sum_{r \in N} \tilde{\mathsf{A}}_r \right\|_{c_0}^* \right) \leq \|\mathcal{L}_\mathsf{A}\|_\chi \leq 4 \lim_p \left(\sup_{N \in \mathcal{N}_p} \left\| \sum_{r \in N} \tilde{\mathsf{A}}_r \right\|_{c_0}^* \right).$$

Moreover, Lemma 1.1 implies that $\| \sum_{r \in N} \tilde{\mathsf{A}}_r \|^*_{c_0} = \| \sum_{r \in N} \tilde{\mathsf{A}}_r \|_{\ell_1} = \left(\sum_s | \sum_{r \in N} \tilde{\mathsf{a}}_{rs} | \right)$ which completes the proof. □

As a consequence of this theorem, we have the following corollary.

Corollary 5.8.

(1) $\mathcal{L}_\mathsf{A}$ is compact for $\mathsf{A} \in (c_0(\mathsf{C}(q)), \ell_\infty)$ if $\lim_r \sum_s |\tilde{\mathsf{a}}_{rs}| = 0$.

(2) $\mathcal{L}_{\mathsf{A}}$ *is compact for* $\mathsf{A} \in (c_0(\mathsf{C}(q)), c)$ *if and only if* $\lim_r \sum_s |\tilde{\mathsf{a}}_{rs} - \tilde{\mathsf{a}}_s| = 0$.

(3) $\mathcal{L}_{\mathsf{A}}$ *is compact for* $\mathsf{A} \in (c_0(\mathsf{C}(q)), c_0)$ *if and only if* $\lim_r \sum_s |\tilde{\mathsf{a}}_{rs}| = 0$.

(4) $\mathcal{L}_{\mathsf{A}}$ *is compact for* $\mathsf{A} \in (c_0(\mathsf{C}(q)), \ell_1)$ *if and only if* $\lim_p \|\mathsf{A}\|^{(p)}_{(c_0(\mathsf{C}(q)),\ell_1)} = 0$, *where* $\|\mathsf{A}\|^{(p)}_{(c_0(\mathsf{C}(q)),\ell_1)} = \sup_{N\in\mathcal{N}_p} \left(\sum_s |\sum_{r\in N} \tilde{\mathsf{a}}_{rs}| \right)$.

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