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SPLITTING RANDOM NUMBERS IN MONTE-CARLO METHOD

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Abstruct. When slitting random numbers with finite number of digits into few 'shorter' ones, the error of Monte-Carlo method can be smaller. Here are the considerations about that error.

1. Intoduction

For the simplicity, we shall consider the case when the second moment of the random variable, which expectation we calculate, is finite, or the problem A, as decrabed in [1].

Then, with n random numbers, the error of the method is of order $n^{-1/2}$, but this assumes that we have an ideal random number generator. In fact, we have pseudo-random numbers only, which are not tru random, and which have limited lenght, i.e. the number of signaficant digits. Here, we shall consider the influence of limited lenght.

Because of that, there is an additional error of order 2^{-k}, where k is the number of binary digits in pseudo-random numbers. Then, the total error for the Monte-Carlo method is

$$e(k) = A\cdot n^{-1/2} + B\cdot 2^{-k} \tag{1.1}$$

Of course, the second term is usually neglected, but, for our purpose we need them.

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2. Problems

Let us suppose that we have a simple algorithm which splits a k-digit binary number into k/k' k'-digit binary numbers. Then, instead of (1.1), the total error is

$$e(k') = A\cdot(nk/k')^{-1/2} + B\cdot 2^{-k'}$$

since n k-digits numbers give nk/k' k'-digits 'short' numbers. Two problems arrive:

Problem 1. For given k and n, find k' such that the error e(k') is minimal.

Problem 2. For given error e(k")=e and k, find k" such that n is minimal.

It is easy to see that those minimums exist, but for k' and k" we have transcendental equations

$$(2.1) \qquad k'\cdot 2^{-2k'} = A^2\cdot(B \ln 4)^{-2}\cdot(nk)^{-1}$$

for the problem 1, and

$$(2.2) \qquad 2^{-k''}\cdot(1 + k'' \ln 4) = e/B$$

for the problem 2.

The solutions we shall give later, but the gains for the error and for n follow immediately

$$e(k')/e(k) = (k'/k)^{1/2}\,(1 + (k' \ln 4)^{-1})$$

$$n''/n = (k''/k)\,(1 + (k'' \ln 4)^{-1})$$

Since k' and k" should be much greater then 1,

$$e(k')/e(k) = (k'/k)^{1/2}$$

$$n''/n = k''/k$$

approximately.

3. Solutions

Let us rewrite (2.1) as

$$k' = c + \log_4 k'$$

where $c = \log_4(nk) + 2\log_4(B\cdot\ln 4/A)$. Then, since $c \geq 4$ for sufficiently large n, an iteration

$$k'_0 = c, \quad k'_1 = c + \log_4 c, \quad k'_2 = c + \log_4(c + \log_4 c)$$

converges. Since

$$k'_{m+1} - k' = \log_4(k'_m/k') < (k'_m - k')/(4\ \ln 4)$$

the convergence is very fast. As a rough approximaton, we shall take k'_1. Then, with a future approximation, the solution of the problem 1 is.

$$(3.1) \qquad k' = (1.66)(x + \log_{10}x) + 0.84$$

where $x = \log_{10}(nk) + 2\log_{10}(B/A)$.

Similarly, for the problem 2, we have an approximation for the solution:

$$(3.2) \qquad k'' = (3.32)(y + \log_{10}y) + 2.20$$

where $y = -\log_{10}(e/B)$.

Since (3.1) and (3.2) are very rough approximations, we need numeric solutions for (2.1) and (2.2). First, we shall search for solutions in the form $k'=k'(x)$ and $k''=k''(y)$ for $0.5 \leq x \leq 9.5$ and $0.5 \leq y \leq 6$ with steps 0.5. An unexpected fact is the linearity of those functions:

$$(3.3) \qquad k' = (1.82)\,x + 1.24$$

$$(3.4) \qquad k'' = (3.71)\,y + 3.05$$

with the mean-square errors 0.05 and 0.17 respectively, and the maximum errors in the given ranges 0.48 and 0.38 respectively

In the second try, we shall search for the solutions in the forms $k' = k'(x + \log_{10}x)$ and $k'' = k''(y + \log_{10}y)$, as in (3.1) and (3.2). For that case, the lines of best fit are

$$k' = (1.63)(x + \log_{10}x) + 1.22$$

$$k'' = (3.16)(y + \log_{10}y) + 3.50$$

with the mean-square errors 0.05 and 0.16 respectively, and the maximum errors 0.13 and 0.39 respectively.

As we see, the errors for both cases are of the same order, but (3.3) and (3.4) are more practical.

4. Example

The values of A and B depend on concrete problems, but as an illustration, we can take $A = B = 1$. Then $x = \log_{10}(nk)$, $y = -\log_{10}e$. With $k=32$, what is usual in meny calculations, from (3.3) and (3.4) it follows

$$k' = (1.82)\log_{10}n + 3.98$$
$$k'' = (3.71)\log_{10}(1/e) + 3.05$$

For $n = 10^2, 10^3, 10^4$ this gives $k = 7.62, 9.44, 11.26$. Also, for $e = 10^{-1}, 10^{-2}$ we have $k'' = 6.76, 10.47$. Those values are not too far from 8, so we can set $k'=k''=8$. Since the splitting of a 32-bits number into four 8-bits number is a relative simple operation, such choice seems to be reasonable. Then, $e(k')/e(k) = 1/2$ and $n''/n = 1/4$, so we can make the error about two times less then usual, or the number of generated random numbers about four times less.

REFERENCES

[1] Ermakov S. M., Metod Monte-Karlo i smežn'e vopros'. Moskva, Nauka, 1971.

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DEOBA SLUČAJNIH BROJEVA U METODI MONTE-KARLO

Razmatran je uticaj dužine (broj cifara) na grešku u metodi Monte-Carlo. Na osnovu osnovne formule (1.1) za ukupnu grešku, razmatrana su dva problema i dati predlozi za njihovo rešavanje: prvi, minimiziranje ukupne greške i drugi, minimiziranje broja slučajnih brojeva pri zadanoj grešci. Dat je i numerički primer da bi se videla efikasnost.

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