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CLOSURE OPERATORS AND FIXED POINTS OF MAPPINGS

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Well-known fixed point theorems of L.P.Belluce and W.A.Kirk [1] are extended similarly as in [2].

A mapping $S: PX \mapsto PX$, where X is a set and by PX the set of all subsets of X is denoted, satisfying for all $A, B \in PX$ the following conditions: 1) $A \subset B \Rightarrow S(A) \subset S(B)$; 2) $A \subset S(A)$; 3) $S(S(A)) = S(A)$; is called a closure operator on X. Closure operator S on X is called algebraic if for every $A \in PX$ and $x \in S(A)$ there exists a finite $B \subset A$ satisfying: $x \in S(B)$.

<u>Theorem</u>. Let X be a metric space with a distance d and let S be an algebraic closure operator on X. For every $x \in X$ and $r \in R_+$ define $B(x,r) := \{ y \in X \mid d(y,x) \leq r \}$ and suppose that $S(B(x,r)) = B(x,r)$. For every $A \in PX$ suppose that $S(\overline{S(A)}) = \overline{S(A)} =: T(A)$. Let X be T-compact, where T-compactness is defined in the same manner as for the case of a topological closure operator. Let $q \in]0,1[$ and let f be a continuous selfmap of X satisfying $d(f(x), f(y)) \leq \max\{d(x,y); q\max\{d(x,f(x)); d(y,f(y)); d(x,f(y)); d(y,f(x))\}\}$ for every $x, y \in X$. Suppose that for every $x \in X$ $(x \neq f(x))$ there exists $y \in S(\{ f^n(x) \mid n \in N \}) =: A(x)$ satisfying:
$\inf\{ \sup\{ d(y, f^m(x)) \mid m \geq n\} \; n \in N\} < \operatorname{diam} A(x)$.

Then f has a fixed point.

REFERENCES

[1] L.P.BELLUCE, W.A.KIRK, Fixed point theorems for certain classes of nonexpansive mappings, Proc.Amer.Math.Soc, 20(1969), 141-146.

[2] А.ЛИЕПИНЬШ, Колыбельная для маленького тигрёнка о неподвижных точках, Топологические пространства и их отображения, Рига, 1983, 61-69.