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ON A HAUSDORFNESS FUNCTION OF A FUZZY SPACE

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We define the Hausdorfness function $h_X: I^+ \to I$ of a fuzzy topological space (X, τ) as follows

$$h_X(\gamma) := \inf_{x,y \in X} \sup \{ \inf_{z \in X} U^c \vee V^c(z) : U(x) \geq \gamma, V(y) \geq \gamma, U, V \in \tau \}$$

where $U^c = 1-U$, $I=[0,1]$, $I^+ =]0,1]$.

If a fuzzy space (X,τ) is α^*-Hausdorff in the sense of Rodabaugh [1], then $h_X(\gamma)=1$ for all $\gamma \leq \alpha$. The number $h_X(\alpha)$ is interpreted as α^*-Hausdorfness degree of X.

For a topological space X $h_X \equiv 1$ iff X is Hausdorff, otherwise $h_X \equiv 0$. If X is a Hausdorff fuzzy space (in the sense of Pu and Liu [2]), then $h_X \equiv 1$. It is easy to show that $h_X(\gamma) \geq \gamma$ iff $\gamma \in H(X)$, where H (X) is the Hausdorfness spectrum of X [3]. The graph of the function h_X is located on the common border of the spectra $H_i^j(X)$, $i,j \in \{0,1,2\}$.

If $\tau \geq \tau'$ are two fuzzy topologies on X, then $h_{(X,\tau)} \geq h_{(X,\tau')}$. If Y is a subspace of a fuzzy space X, then $h_Y \geq h_X$. If X is a product of fuzzy spaces X_i, $i \in \mathcal{J}$, then $h_X \geq \bigwedge_i h_{X_i}$. In case X is laminated $h_X(\delta) \geq 1-\delta$ for each $\gamma \in I^+$.

Let the closedness function $\xi_M: I^+$ I of a subset M X be defined by the following equality $\xi_M(\gamma) := \inf_{x \in M} \sup \{ \inf U^c(z) : U \in \tau, U(x) \geq \gamma \}$.

Then the Hausdorfness function can be characterized as $h_X = \xi_\Delta$ where Δ is the diagonal in X .

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