Filomat 37:1 (2023), 127–138
https://doi.org/10.2298/FIL2301127B

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Pointwise quasi hemi-slant submanifolds

Selahattin Beyendi[a], Mehmet Akif Akyol[b], Mića S. Stanković[c]

[a] *İnonu University, Faculty of Education, 44000, Malatya, Türkiye*
[b] *Bingol University, Faculty of Arts and Sciences, Department of Mathematics, 12000, Bingöl, Türkiye*
[c] *University of Nis, Faculty of Sciences and Mathematics, Department of Mathematics, 18000, Nis, Serbia*

Abstract. The objective of this paper is to introduce a new class of submanifolds which are called *pointwise quasi hemi-slant submanifolds* in almost Hermitian manifolds which extends quasi hemi-slant, hemi-slant, semi-slant and slant submanifolds in a very natural way. Several basic results in this respect are proved in this paper. Moreover, we obtain some conditions of the distributions which are involved in the definition of the new submanifolds. We also get some results for totally geodesic and mixed totally geodesic conditions for pointwise quasi hemi-slant submanifolds. Finally, we illustrate some examples in order to guaranty the new kind of submanifolds.

1. Introduction

Almost contact geometry and its related topics have been a rich reseach field for geometers due to their applications in wide range of areas of physics as well as in mathematics. One of the interesting and active reseach topic is the theory of submanifolds in differential geometry. The theory has many interesting applications such as economic modeling, mechanics, image processing and computer design. Chen [8] introduced the notion of slant submanifold of an almost Hermitian manifold. It was a naturel generalization of both holomorphic and totally real submanifolds. The theory of submanifolds has been studied by several geometers such as ([3], [4], [12], [13], [15], [33] and [36]).

Later, this interesting notion has been studied broadly by several geometers ([9], [16], [17], [28], [31], [32]). As a generalization of slant submanifolds, there are several kinds of submanifolds: semi-slant submanifolds ([5], [19], [29]), hemi-slant submanifolds ([18], [34]), bi-slant submanifolds ([6], [7], [35]), quasi hemi-slant submanifolds ([23], [24], [25], [26], [27]), pointwise quasi bi-slant submanifolds [2] and quasi bi-slant submanifolds ([1], [22]). In 2012, B. Y. Chen and O. J. Garay [10] studied pointwise slant submanifolds in almost Hermitian manifolds which was first proposed by F. Etayo [14] under the notion of quasi slant submanifold.

In 2013, B. Şahin [30] defined the notion of pointwise semi-slant submanifolds. In 2014, K. S. Park ([20], [21]) defined the notion of pointwise almost h-slant submanifolds and pointwise almost h-semi-slant submanifolds in an almost quaternionic Hermitian manifold. The author obtained some geometrically important properties of these manifolds.

2020 *Mathematics Subject Classification*. Primary 53C15, 53B20
Keywords. Pointwise quasi hemi-slant submanifold; hemi-slant submanifold; pointwise slant submanifold; Kaehler manifold
Received: 11 April 2022; Accepted: 16 May 2022
Communicated by Dragan S. Djordjević
Email addresses: selahattin.beyendi@inonu.edu.tr (Selahattin Beyendi), mehmetakifakyol@bingol.edu.tr (Mehmet Akif Akyol), stmica@mts.rs (Mića S. Stanković)

On the other hand, Akyol and Beyendi [1] initiated the study of quasi bi-slant submanifolds of an almost contact metric manifold by generalizing slant, semi-slant, hemi-slant and bi-slant submanifolds. (See also: [22]).

Taking into account the above studies, we introduce the notion of pointwise quasi-hemi-slant submanifolds, in which includes the classes of anti-invariant, the tangent bundle consists of one invariant and slant distribution which has slant function instead of slant angle, of almost Hermitian manifolds as a generalization of quasi hemi-slant, bi-slant, hemi-slant, semi-slant and slant submanifolds in the present paper.

The paper is organized as follows: In the second section, the basic notions, important definitions and some properties both almost Hermitian manifolds and the geometry of submanifolds are given. In the third section, we define the notion of pointwise quasi-hemi-slant submanifolds and obtain some basic results for the next sections. In the fourth section, we deals with main theorems related to the geometry of distributions. In the last section, we construct some examples of such submanifolds.

2. Preliminaries

In this section, we give the definition of a Kaehler manifold and some background on submanifolds theory.

Let $\widetilde{M}$ be a smooth manifold of dimension $2m$. Then, $\widetilde{M}$ is said to be an almost Hermitian manifold if it admits a tensor field J of type $(1,1)$ and a Riemannian metric g on $\widetilde{M}$ satisfying

$$J^2 = -I, \quad < JX_1, JX_2 >=< X_1, X_2 > \tag{1}$$

for any vector fields X_1, X_2 on $T\widetilde{M}$, where I denotes the identity transformation. The fundamental 2 -form Ω on $\widetilde{M}$ is defined by $\Omega(X_1, X_2) =< X_1, JX_2 >$, $\forall X_1, X_2 \in \Gamma(T\widetilde{M})$, with $\Gamma(T\widetilde{M})$ being the section of tangent bundle $T\widetilde{M}$ of $\widetilde{M}$. An almost Hermitian manifold $\widetilde{M}$ is called a Kaehler manifold [37] if

$$(\widetilde{\nabla}_{X_1} J)X_2 = 0 \tag{2}$$

where $\widetilde{\nabla}$ is the Levi-Civita connection on $\widetilde{M}$ with respect to $<,>$. Let M be a Riemannian manifold isometrically immersed in $\widetilde{M}$ and induced Riemannian metric on M is denoted by the same symbol $<,>$ throughout this paper. Let $\mathcal{A}$ and h denote the shape operator and second fundamental form, respectively, of immersion of M into $\widetilde{M}$. The Gauss and Weingarten formulas of M into $\widetilde{M}$ are given by [9]

$$\widetilde{\nabla}_{X_1} X_2 = \nabla_{X_1} X_2 + h(X_1, X_2) \tag{3}$$

and

$$\widetilde{\nabla}_{X_1} Y_2 = -A_{Y_2} X_1 + \nabla^{\perp}_{X_1} Y_2, \tag{4}$$

for any vector fields $X_1, X_2 \in \Gamma(TM)$ and $Y_2 \in \Gamma(T^{\perp}M)$, where ∇ is the induced connection on M and $\nabla^{\perp}$ represents the connection on the normal bundle $T^{\perp}M$ of M and A_{Y_2} is the shape operator of M with respect to normal vector $Y_2 \in \Gamma(T^{\perp}M)$. Moreover, $\mathcal{A}_{Y_2}$ and h are related by

$$< h(X_1, X_2), Y_2 >=< A_{Y_2} X_1, X_2 > \tag{5}$$

for any vector fields $X_1, X_2 \in \Gamma(TM)$ and $Y_2 \in \Gamma(T^{\perp}M)$.

Now, we have the following definition from [10]:

Definition 2.1. *A submanifold M of an almost Hermitian manifold $\widetilde{M}$ is called pointwise slant if, at each point $p \in M$, the Wirtinger angle $\theta(X_1)$ is independent of the choice of nonzero vector $X_1 \in T^*_pM$, where T^*_pM is the tangent space of nonzero vectors. In this case, θ is called slant function of M.*

Definition 2.2. *A submanifold M is called (i) $(\mathfrak{D}_1, \mathfrak{D}_2)$-mixed totally geodesic if $h(Y_3, Y_4) = 0$, for any $Y_3 \in \Gamma(\mathfrak{D}_1)$ and $Y_4 \in \Gamma(\mathfrak{D}_2)$ (ii) $\mathfrak{D}$-totally geodesic if it is $(\mathfrak{D}, \mathfrak{D})$-mixed totally geodesic.*

3. Pointwise quasi hemi-slant submanifolds

In this section, we define a new class of submanifolds which can be considered as a generalization of quasi hemi-slant, hemi-slant, semi-slant, slant etc. submanifolds.

First, we have the following definition.

Definition 3.1. *Let M be an isometrically immersed submanifold in a Kaehler manifold $\widetilde{M}$. Then we say that M is a pointwise quasi hemi-slant submanifold if it is furnished with three orthogonal distributions $(\mathfrak{D}, \mathfrak{D}_\theta, \mathfrak{D}^\perp)$ satisfying the conditions:*

(i) $TM = \mathfrak{D} \oplus \mathfrak{D}_\theta \oplus \mathfrak{D}^\perp$,

(ii) The distribution $\mathfrak{D}$ is invariant, i.e. $J\mathfrak{D} = \mathfrak{D}$,

(iii) For any non-zero vector field $X_1 \in (\mathfrak{D}_\theta)_p$, $p \in M$, the angle θ between JX_1 and $(\mathfrak{D}_\theta)_p$ is slant function and is independent of the choice of the point p and X_1 in $(\mathfrak{D}_\theta)_p$,

(iv) The distribution $\mathfrak{D}^\perp$ is anti- invariant, i.e., $J\mathfrak{D}^\perp \subseteq \mathcal{T}^\perp M$.

We call the angle θ a pointwise quasi hemi-slant angle of M. A pointwise quasi hemi-slant submanifold M is called proper if its pointwise-slant function satisfies $\theta \neq 0, \frac{\pi}{2}$, and θ is not constant on M.

If we represent by d_1, d_2 and d_3 the dimension of $\mathfrak{D}$, $\mathfrak{D}_\theta$ and $\mathfrak{D}^\perp$, respectively, then from our generalized definition of pointwise quasi hemi-slant submanifold M, we can easily see the following particular cases:

(i) If $d_1 = 0$, then M is a pointwise hemi-slant submanifold,

(ii) If $d_2 = 0$, then M is a semi-invariant submanifold,

(iii) If $d_3 = 0$, then M is a pointwise semi-slant submanifold.

Let M be a pointwise quasi hemi-slant submanifold of a Kaehler manifold $\widetilde{M}$. Then, for any $X_1 \in \Gamma(TM)$, we have

$$X_1 = PX_1 + QX_1 + RX_1 \tag{6}$$

where P, Q and R denotes the projections on the distributions $\mathfrak{D}$, $\mathfrak{D}_\theta$ and $\mathfrak{D}^\perp$, respectively.

$$JX_1 = TX_1 + FX_1, \tag{7}$$

where TX_1 and FX_1 are tangential and normal components on M. By using (6) and (7), we get immediately

$$JX_1 = TPX_1 + FPX_1 + TQX_1 + FQX_1 + TRX_1 + FRX_1, \tag{8}$$

here since $J\mathfrak{D} = \mathfrak{D}$, we have $FPX_1 = 0$. Thus we get

$$J(TM) = \mathfrak{D} \oplus T\mathfrak{D}_\theta \oplus F\mathfrak{D}_\theta \oplus J\mathfrak{D}^\perp \tag{9}$$

and

$$T^\perp M = F\mathfrak{D}_\theta \oplus J\mathfrak{D}^\perp \oplus \mu, \tag{10}$$

where μ is the orthogonal complement of $F\mathfrak{D}_\theta \oplus J\mathfrak{D}^\perp$ in $T^\perp M$ and $J\mu = \mu$. Also, for any $Y_3 \in T^\perp M$, we have

$$JY_3 = BY_3 + CY_3, \tag{11}$$

where $BY_3 \in \Gamma(TM)$ and $CY_3 \in \Gamma(T^\perp M)$.

Taking into account of the condition (iii) in Definition (3.1), (7) and (11), we obtain the followings:
$T\mathfrak{D} = D$, $T\mathfrak{D}_\theta = \mathfrak{D}_\theta$, $T\mathfrak{D}^\perp = \{0\}$, $BF\mathfrak{D}_\theta = \mathfrak{D}_\theta$, $BF\mathfrak{D}^\perp = \mathfrak{D}^\perp$.

With the help of (7) and (11), we obtain the following Lemma.

Lemma 3.2. *Let M be a pointwise quasi hemi-slant submanifold of a Kaehler manifold $\widetilde{M}$. Then, we have*

$$\textbf{(a)}\, T^2Y_1 = -(\cos^2\theta)Y_1, \quad \textbf{(b)}\, BFY_1 = -(\sin^2\theta)Y_1,$$

$$\textbf{(c)}\, T^2Y_1 + BFY_1 = -Y_1, \quad \textbf{(d)}\, FTY_1 + CFY_1 = 0,$$

for any $Y_1 \in \Gamma(\mathfrak{D}_\theta)$.

By using (2), Definition (3.1), (7) and (11), we obtain the following Lemma.

Lemma 3.3. *Let M be a pointwise quasi hemi-slant submanifold of a Kaehler manifold $\widetilde{M}$. Then, we have*

(i) $< TY_1, TY_2 >= (\cos^2\theta) < Y_1, Y_2 >,$
(ii) $< FY_1, FY_2 >= (\sin^2\theta) < Y_1, Y_2 >$

for any $Y_1, Y_2 \in \Gamma(\mathfrak{D}_\theta)$.

Proof. The proof follows using similar steps as in Proposition 2.8 of [10]. □

Using the equations (2), (3), (4), (7) and (11) and comparing the tangential and normal components, we have the following:

Lemma 3.4. *Let M be a pointwise quasi hemi-slant submanifold of a Kaehler manifold $\tilde{M}$. Then, we have*

$$\nabla_{X_1}TX_2 - A_{FX_2}X_1 - T\nabla_{X_1}X_2 - Bh(X_1, X_2) = 0$$

and

$$h(X_1, TX_2) + \nabla^{\perp}_{X_1}FX_2 - F(\nabla_{X_1}X_2) - Ch(X_1, X_2) = 0$$

for any $X_1, X_2 \in \Gamma(TM)$.

Lemma 3.5. *Let M be a pointwise quasi hemi-slant submanifold of a Kaehler manifold $\tilde{M}$. Then, we have*

$$(\widetilde{\nabla}_{X_1}T)X_2 = A_{FX_2}X_1 + Bh(X_1, X_2),$$

$$(\widetilde{\nabla}_{X_1}F)X_2 = Ch(X_1, X_2) - h(X_1, TX_2)$$

for any $X_1, X_2 \in \Gamma(TM)$.

4. Main Results

Theorem 4.1. *Let M be a pointwise quasi hemi-slant submanifolds of a Kaehler manifold $\widetilde{M}$. Then, the invariant distribution $\mathfrak{D}$ defines a totally geodesic foliation on M if and only if*

$$< T\nabla_{Y_1}TY_2 + Bh(Y_1, TY_2), RY_3 >=< \nabla_{Y_1}TY_2 + h(Y_1, TY_2), JQY_3 >$$

and

$$< \nabla_{Y_1}TY_2, BY_4 >= - < h(Y_1, TY_2), CY_4 >$$

for any $Y_1, Y_2 \in \mathfrak{D}$, $Y_3 = QY_3 + RY_3 \in \Gamma(\mathfrak{D}_\theta \oplus D^{\perp})$ *and* $Y_4 \in \Gamma(TM)^{\perp}$.

Proof. For any $Y_1, Y_2 \in \Gamma(\mathfrak{D}), Y_3 = QY_3 + RY_3 \in \Gamma(\mathfrak{D}_\theta \oplus D^\perp), FY_2 = 0$ and from equations (3) and (7), we have

$$\begin{aligned} <\widetilde{\nabla}_{Y_1} Y_2, Y_3> &=< \widetilde{\nabla}_{Y_1} TY_2, JQY_3 + JRY_3 > \\ &=< \nabla_{Y_1} TY_2 + h(Y_1, TY_2), TQY_3 + FQY_3 > \\ &- < J(\nabla_{Y_1} TY_2 + h(Y_1, TY_2)), RY_3 > . \end{aligned}$$

Taking into acount of (11), the above equation becomes

$$\begin{aligned} <\widetilde{\nabla}_{Y_1} Y_2, Y_3> &=< \nabla_{Y_1} TY_2, TQY_3 > + < h(Y_1, TY_2), FQY_3 > \\ &- < T\nabla_{Y_1} TY_2 + Bh(Y_1, TY_2), RY_3 > . \end{aligned} \tag{12}$$

Now for any $Y_4 \in \Gamma(TM)^\perp$ and $Y_1, Y_2 \in \Gamma(\mathfrak{D})$, we obtain

$$\begin{aligned} <\widetilde{\nabla}_{Y_1} Y_2, Y_4> &=< \widetilde{\nabla}_{Y_1} JY_2, JY_4 > \\ &=< \nabla_{Y_1} TY_2, BY_3 > + < h(Y_1, TY_2), CY_4 > . \end{aligned} \tag{13}$$

The proof comes from (12) and (13). □

Theorem 4.2. *Let M be a pointwise quasi hemi-slant submanifolds of a Kaehler manifold $\widetilde{M}$. Then, the slant distribution $\mathfrak{D}_\theta$ defines a totally geodesic foliation on M if and only if*

$$\sin^2\theta < [Y_1, Y_3], Y_2 > - \sin 2\theta Y_3(\theta) < Y_1, Y_2 >=< B\nabla^\perp_{Y_3} FY_1 - TA_{FY_1} Y_3 - A_{FTY_1} Y_3, Y_2 >$$

and

$$\nabla^\perp_{Y_1} FTY_2 + \nabla^\perp_{Y_1} CFY_2 + h(Y_1, BFY_2) = 0$$

where $Y_1, Y_2 \in \Gamma(\mathfrak{D}_\theta), Y_3 = PY_3 + RY_3 \in \Gamma(\mathfrak{D} \oplus \mathfrak{D}^\perp)$.

Proof. For any $Y_1, Y_2 \in \mathfrak{D}_\theta, Y_3 = PY_3 + RY_3 \in \Gamma(\mathfrak{D} \oplus \mathfrak{D}^\perp)$, by using (1) and (7), we have

$$\begin{aligned} <\widetilde{\nabla}_{Y_1} Y_2, Y_3> &= Y_1 < Y_2, Y_3 > - < Y_2, \widetilde{\nabla}_{Y_1} Y_3 > \\ &= - < [Y_1, Y_3], Y_2 > + < \widetilde{\nabla}_{Y_3} T^2 Y_1, Y_2 > + < \widetilde{\nabla}_{Y_3} FTY_1, Y_2 > \\ &- < \widetilde{\nabla}_{Y_3} FY_1, JY_2 > . \end{aligned}$$

Then from Lemma 3.3 and using the property of slant function, we deduce

$$\begin{aligned} <\widetilde{\nabla}_{Y_1} Y_2, Y_3> &= - < [Y_1, Y_3], Y_2 > + \sin 2\theta Y_3(\theta) < Y_1, Y_2 > - \cos^2\theta < \widetilde{\nabla}_{Y_3} Y_1, Y_2 > \\ &+ < -A_{FTY_1} Y_3, Y_2 > + < J(-A_{FY_1} Y_3 + \nabla^\perp_{Y_3} FY_1), Y_2 > \\ &= - < [Y_1, Y_3], Y_2 > + \sin 2\theta Y_3(\theta) < Y_1, Y_2 > + \cos^2\theta < \widetilde{\nabla}_{Y_1} Y_2, Y_3 > \\ &+ \cos^2\theta < [Y_1, Y_3], Y_2 > - < A_{FTY_1} Y_3, Y_2 > - < TA_{FY_1} Y_3, Y_2 > \\ &+ < B\nabla^\perp_{Y_3} FY_1, Y_2 > . \end{aligned}$$

This implies

$$\begin{aligned} \sin^2\theta < \widetilde{\nabla}_{Y_1} Y_2, Y_3 > &= - \sin^2\theta < [Y_1, Y_3], Y_2 > + \sin 2\theta Y_3(\theta) < Y_1, Y_2 > \\ &- < A_{FTY_1} Y_3, Y_2 > - < TA_{FY_1} Y_3, Y_2 > + < B\nabla^\perp_{Y_3} FY_1, Y_2 > . \end{aligned} \tag{14}$$

Now, for any $Y_4 \in (TM)^\perp$, we get

$$\begin{aligned} <\widetilde{\nabla}_{Y_1} Y_2, Y_4 > &=-(\sin 2\theta) Y_1(\theta) < Y_2, Y_4 > + \cos^2\theta < \widetilde{\nabla}_{Y_1} Y_2, Y_4 > \\ &- < \nabla^\perp_{Y_1} FTY_2, Y_4 > - < h(Y_1, \mathcal{B}\mathcal{F} Y_2), Y_4 > - < \nabla^\perp_{Y_1} CFY_2, Y_4 > \end{aligned}$$

which gives

$$\sin^2\theta < \widetilde{\nabla}_{Y_1}Y_2, Y_4 >= - < \nabla^{\perp}_{Y_1}FTY_2 + \nabla^{\perp}_{Y_1}\mathcal{CF}Y_2 + h(Y_1, \mathcal{BF}Y_2), Y_4 > . \tag{15}$$

Thus from (14) and (15), which achieves the proof . □

Theorem 4.3. *Let M be a pointwise quasi hemi-slant submanifolds of a Kaehler manifold $\widetilde{M}$. Then, the anti-invariant distribution $\mathfrak{D}^{\perp}$ defines a totally geodesic foliation on M if and only if*

$$< A_{FY_2}Y_1, TPY_3 >=< \nabla_{Y_1}BFY_2 + A_{CFY_2}Y_1, QY_3 >$$

and

$$< A_{FY_2}Y_1, BY_4 >=< \nabla^{\perp}_{Y_1}FY_2, CY_4 >,$$

where $Y_1, Y_2 \in \Gamma(\mathfrak{D}^{\perp}), Y_4 \in (TM)^{\perp}, Y_3 = PY_3 + QY_3 \in \Gamma(\mathfrak{D} \oplus \mathfrak{D}_{\theta})$.

Proof. For $Y_1, Y_2 \in \Gamma(\mathfrak{D}^{\perp}), Y_3 = PY_3 + QY_3 \in \Gamma(\mathfrak{D} \oplus \mathfrak{D}_{\theta})$, by using (1) and (7), we get

$$\begin{aligned} < \widetilde{\nabla}_{Y_1}Y_2, Y_3 > &=< \widetilde{\nabla}_{Y_1}JY_2, JPY_3 + JQY_3 > \\ &=< \widetilde{\nabla}_{Y_1}JY_2, TPY_3 > - < \widetilde{\nabla}_{Y_1}FY_2, QY_3 > . \end{aligned}$$

Taking into account of (4) and (11) in the above equation, we have

$$\begin{aligned} < \widetilde{\nabla}_{Y_1}Y_2, Y_3 > &=< -A_{FY_2}Y_1, TPY_3 > - < \nabla_{Y_1}BFY_2, QY_3 > \\ &+ < A_{CFY_2}Y_1, QY_3 > . \end{aligned} \tag{16}$$

Now for any $Y_1, Y_2 \in \Gamma(\mathfrak{D}^{\perp}), Y_4 \in \Gamma(TM)^{\perp}$, by using (4), (7) and (11), we obtain

$$\begin{aligned} < \widetilde{\nabla}_{Y_1}Y_2, Y_4 > &=< \widetilde{\nabla}_{Y_1}JY_2, JY_4 > \\ &=< -A_{FY_2}Y_1, BY_4 > + < \nabla^{\perp}_{Y_1}FY_2, CY_4 > . \end{aligned} \tag{17}$$

The proof comes from (16) and (17). □

Theorem 4.4. *Let M be a pointwise quasi hemi-slant submanifolds of a Kaehler manifold $\widetilde{M}$. The invariant distribution $\mathfrak{D}$ is integrable if and only if*

$$\begin{aligned} < \nabla_{Y_1}TY_2 - \nabla_{Y_2}JY_1, TQY_3 > &=< h(Y_2, JY_1), FRY_3 > - < h(Y_1, TY_2), FY_3 > \\ &- < Bh(Y_2, JY_1), QY_3 > \end{aligned}$$

where $Y_1, Y_2 \in \Gamma(\mathfrak{D}), Y_3 = QY_3 + RY_3 \in \Gamma(\mathfrak{D}_{\theta} \oplus \mathfrak{D}^{\perp})$.

Proof. For any $Y_1, Y_2 \in \Gamma(\mathfrak{D}), Y_3 = QY_3 + RY_3 \in \Gamma(\mathfrak{D}_{\theta} \oplus \mathfrak{D}^{\perp})$, by using (3) and (7), we obtain

$$\begin{aligned} < [Y_1, Y_2], Y_3 > &=< \widetilde{\nabla}_{Y_1}JY_2, JY_3 > - < \widetilde{\nabla}_{Y_2}JY_1, JY_3 > \\ &=< \widetilde{\nabla}_{Y_1}TY_2, TQY_3 + FQY_3 > + < \widetilde{\nabla}_{Y_1}TY_2, TRY_3 + FRY_3 > \\ &+ < J(\nabla_{Y_2}JY_1 + h(Y_2, JY_1)), QY_3 > - < \widetilde{\nabla}_{Y_2}JY_1, JRY_3 > \end{aligned}$$

by using (11) in the above equation, we have

$$\begin{aligned} < [Y_1, Y_2], Y_3 > &= (\nabla_{Y_1}TY_2, TQY_3) + < h(Y_1, TY_2), FQY_3 + FRY_3 > \\ &+ < T\nabla_{Y_2}JY_1, QY_3 > + < Bh(Y_2, JY_1), QY_3 > \\ &- < h(Y_2, JY_1), FRY_3 > \end{aligned} \tag{18}$$

which proves the assertion. □

Theorem 4.5. *Let M be a pointwise quasi hemi-slant submanifolds of a Kaehler manifold $\widetilde{M}$. The slant distribution $\mathfrak{D}_\theta$ is integrable if and only if*

$$\begin{aligned}&\sin^2\theta < [Y_1, Y_3], Y_2 > -\cos^2\theta < \nabla_{Y_1} Y_2, Y_3 > -\sin 2\theta Y_3(\theta) < Y_1, Y_2 > \\ &=< A_{CFY_1} Y_3 - A_{FTY_1} Y_3 + \nabla_{Y_3} BFY_1, Y_2 > + < A_{FY_1} Y_2 - \nabla_{Y_2} TY_1, TPY_3 >\end{aligned}$$

where $Y_1, Y_2 \in \Gamma(\mathfrak{D}_\theta)$, $Y_3 = PY_3 + RY_3 \in \Gamma(\mathfrak{D} \oplus \mathfrak{D}^\perp)$.

Proof. For any $Y_1, Y_2 \in \Gamma(\mathfrak{D}_\theta)$, $Y_3 = PY_3 + RY_3 \in \Gamma(\mathfrak{D} \oplus \mathfrak{D}^\perp)$. by using (1) and (7), we have

$$\begin{aligned}< [Y_1, Y_2], Y_3 > &=< \widetilde{\nabla}_{Y_1} Y_2, Y_3 > - < \widetilde{\nabla}_{Y_2} Y_1, Y_3 > \\ &= - < \widetilde{\nabla}_{Y_3} JY_1, JY_2 > - < [Y_1, Y_3], Y_2 > - < \widetilde{\nabla}_{Y_2} JY_1, JY_3 > \\ &=< \widetilde{\nabla}_{Y_3} T^2 Y_1, Y_2 > + < \widetilde{\nabla}_{Y_3} FTY_1, Y_2 > + < \widetilde{\nabla}_{Y_3} JFY_1, Y_2 > \\ &- < [Y_1, Y_3], Y_2 > - < \widetilde{\nabla}_{Y_2} TY_1, JY_3 > - < \widetilde{\nabla}_{Y_2} FY_1, JY_3 > . \end{aligned} \tag{19}$$

On the other hand, taking into account of Lemma 3.3, using the property of slant function, (4), (11), equation (19)

$$\begin{aligned}&< [Y_1, Y_2], Y_3 > = -\sin^2\theta < [Y_1, Y_3], Y_2 > + \cos^2\theta < \widetilde{\nabla}_{Y_1} Y_2, Y_3 > \\ &+ \sin 2(\theta) Y_3(\theta) < Y_1, Y_2 > + < A_{CFY_1} Y_3 - A_{FTY_1} Y_3 + \nabla_{Y_3} BFY_1, Y_2 > \\ &\quad + < A_{FY_1} - \nabla_{Y_2} TY_1, TPY_3 >\end{aligned}$$

which achieves proof. □

Theorem 4.6. *Let M be a pointwise quasi hemi-slant submanifolds of a Kaehler manifold $\widetilde{M}$. The anti-invariant distribution $\mathfrak{D}^\perp$ is integrable if and only if*

$$< A_{JY_1} Y_2 - A_{JY_2} Y_1, TPY_3 >=< T(A_{JY_2} Y_1 - A_{JY_1} Y_2) + B(\nabla^\perp_{Y_2} JY_1 - \nabla^\perp_{Y_1} JY_2), QY_3 >$$

where $Y_1, Y_2 \in \Gamma(\mathfrak{D}^\perp)$, $Y_3 = PY_3 + QY_3 \in \Gamma(\mathfrak{D} \oplus \mathfrak{D}_\theta)$.

Proof. For any $Y_1, Y_2 \in \Gamma(\mathfrak{D}^\perp)$, $Y_3 = PY_3 + QY_3 \in \Gamma(\mathfrak{D} \oplus \mathfrak{D}_\theta)$, by making use of (4) and (7), we have

$$\begin{aligned}< [Y_1, Y_2], Y_3 > &=< \widetilde{\nabla}_{Y_1} JY_2, JY_3 > - < \widetilde{\nabla}_{Y_2} JY_1, JY_3 > \\ &=< -A_{JY_2} Y_1 + \nabla^\perp_{Y_2} JY_1, JPY_3 > + < JA_{JY_2} Y_1, QY_3 > \\ &- < J\nabla^\perp_{Y_1} JY_2, QY_3 > + < A_{JY_1} Y_2 - \nabla^\perp_{Y_2} JY_1, TPY_3 > \\ &- < JA_{JY_1} Y_2, QY_3 > + < J\nabla^\perp_{Y_2} JY_1, QY_3 > .\end{aligned}$$

Then from (11) in the above equation, we have

$$\begin{aligned}< [Y_1, Y_2], Y_3 > &=< TA_{JY_2} Y_1 - TA_{JY_1} Y_2 + B\nabla^\perp_{Y_2} JY_1 - B\nabla^\perp_{Y_1} JY_2, QY_3 > \\ &+ < A_{JY_1} Y_2 - A_{JY_2} Y_1, TPY_3 > .\end{aligned} \tag{20}$$

The proof comes from (20). □

Theorem 4.7. *Let M be a pointwise quasi hemi-slant submanifolds of a Kaehler manifold $\widetilde{M}$. Then, $\mathfrak{D}$ is totally geodesic if and only if*

$$< T\nabla_{Y_1} Y_2 + Bh(Y_1, Y_2), BY_4 >=< A_{CY_4} Y_1, TY_2 > - < \nabla^\perp_{Y_1} CY_4, FY_2 >$$

where $Y_1, Y_2 \in \Gamma(\mathfrak{D})$ and $Y_4 \in \Gamma(TM)^\perp$.

Proof. For any $Y_1, Y_2 \in \Gamma(\mathfrak{D})$ and $Y_4 \in \Gamma(TM)^\perp$, by making use of (1) and (11), we have

$$
\begin{aligned}
< h(Y_1, Y_2), Y_4 > &=< \widetilde{\nabla}_{Y_1} JY_2, JY_4 > \\
&=< J\widetilde{\nabla}_{Y_1} Y_2, BY_4 > + < J\widetilde{\nabla}_{Y_1} Y_2, CY_4 > .
\end{aligned}
$$

Taking into account of (3) and (4) in the above equation, we get

$$
\begin{aligned}
< h(Y_1, Y_2), Y_4 > &=< T\nabla_{Y_1} Y_2, BY_4 > + < Bh(Y_1, Y_2), BY_4 > - < -A_{CY_4} Y_1 + \nabla^\perp_{Y_1} CY_4, JY_2 > \\
&=< T\nabla_{Y_1} Y_2 + Bh(Y_1, Y_2), BY_4 > + < A_{CY_4} Y_1, TY_2 > - < \nabla^\perp_{Y_1} CY_4, FY_2 > . \qquad (21)
\end{aligned}
$$

Hence the proof follows from (21). □

Theorem 4.8. *Let M be a pointwise quasi hemi-slant submanifolds of a Kaehler manifold $\widetilde{M}$. Then, $\mathfrak{D}_\theta$ is totally geodesic if and only if*

$$\cos^2\theta < A_{Y_1} Y_4, Y_2 > + < \nabla^\perp_{Y_1} Y_4, FTY_2 >=< h(Y_1, BY_4) + \nabla^\perp_{Y_1} CY_4, FY_2 >$$

where $Y_1, Y_2 \in \Gamma(\mathfrak{D}_\theta)$ and $Y_4 \in \Gamma(TM)^\perp$.

Proof. For any $Y_1, Y_2 \in \Gamma(\mathfrak{D}_\theta)$ and $Y_4 \in \Gamma(TM)^\perp$, using (1) and (7), we obtain

$$
\begin{aligned}
< h(Y_1, Y_2), Y_4 > &= - < \widetilde{\nabla}_{Y_1} JY_4, JY_2 > \\
&=< \widetilde{\nabla}_{Y_1} Y_4, JTY_2 + JFY_2 > \\
&=< \widetilde{\nabla}_{Y_1} Y_4, T^2 Y_2 + FTY_2 > + < \widetilde{\nabla}_{Y_1} Y_4, JFY_2 > .
\end{aligned}
$$

Then from (3), (4), (11) and Lemma 3.3, we have

$$
\begin{aligned}
< h(Y_1, Y_2), Y_4 > &=< -A_{Y_1} Y_4 + \nabla^\perp_{Y_1} Y_4, -\cos^2\theta Y_2 + FTY_2 > - < \widetilde{\nabla}_{Y_1} BY_4 + CY_4, FY_4 > \\
&= \cos^2\theta < A_{Y_1} Y_4, Y_2 > + < \nabla^\perp_{Y_1} Y_4, FTY_2 > - < h(Y_1, BY_4), FY_2 > - < \nabla^\perp_{Y_1} CY_4, FY_2 > . \qquad (22)
\end{aligned}
$$

The proof comes from (22). □

Theorem 4.9. *Let M be a pointwise quasi hemi-slant submanifolds of a Kaehler manifold $\widetilde{M}$. Then, $\mathfrak{D}^\perp$ is totally geodesic if and only if*

$$< A_{Y_4} Y_1, BFY_2 >=< \nabla^\perp_{Y_1} Y_4, CFY_2 >$$

where $Y_1, Y_2 \in \Gamma(\mathfrak{D}^\perp)$ and $Y_4 \in \Gamma(TM)^\perp$.

Proof. For any $Y_1, Y_2 \in \Gamma(\mathfrak{D}^\perp)$ and $Y_4 \in \Gamma(TM)^\perp$, by using (1), (7) and the fact that $TY_2 = 0$, we have

$$< h(Y_1, Y_2), Y_4 >=< \widetilde{\nabla}_{Y_1} Y_2 >= - < \widetilde{\nabla}_{Y_1} JY_4, JY_2 >= - < \widetilde{\nabla}_{Y_1} JY_4, FY_2) > .$$

On the other hand, using (4) and (11), we get

$$< h(Y_1, Y_2), Y_4 >= - < A_{Y_4} Y_1, BFY_2 > + < \nabla^\perp_{Y_1} Y_4, CFY_2 > \qquad (23)$$

which gives the proof. □

Theorem 4.10. *Let M be a pointwise quasi hemi-slant submanifolds of a Kaehler manifold $\widetilde{M}$. Then, $\mathfrak{D} - \mathfrak{D}_\theta$ mixed totally geodesic if and only if*

$$< h(Y_1, TY_2) + \nabla^\perp_{Y_1} FY_2, CY_4 >=< \nabla_{Y_1} Y_2, TBY_2 > + < h(Y_1, Y_2), FBY_4 >$$

where $Y_1 \in \Gamma(\mathfrak{D})$, $Y_2 \in \Gamma(\mathfrak{D}_\theta)$ and $Y_4 \in \Gamma(TM)^\perp$.

Proof. For any $Y_1 \in \Gamma(\mathfrak{D})$, $Y_2 \in \Gamma(\mathfrak{D}_\theta)$ and $Y_4 \in \Gamma(TM)^\perp$, from (1) and (11), we obtain

$$< h(Y_1, Y_2), Y_4 >=< \widetilde{\nabla}_{Y_1} JY_2, JY_4 >= - < \widetilde{\nabla}_{Y_1} Y_2, JBY_4 + JCY_4 > .$$

Taking into account of (3), (4) and (7), we have

$$\begin{aligned} < h(Y_1, Y_2), Y_4 > &=< h(Y_1, TY_2), CY_4 > + < \nabla^\perp_{Y_1} FY_2, CY_4 > \\ &- < \nabla_{Y_1} Y_2, TBY_4 > - < h(Y_1, Y_2), FBY_4 > . \end{aligned} \quad (24)$$

The proof comes from (24). □

Theorem 4.11. *Let M be a pointwise quasi hemi-slant submanifolds of a Kaehler manifold $\widetilde{M}$. Then, $\mathfrak{D} - \mathfrak{D}^\perp$ mixed totally geodesic if and only if*

$$< F\nabla_{Y_1} Y_2 + Ch(Y_1, Y_2), CY_4 >= - < \nabla_{Y_1} TY_2, BY_4 >$$

where $Y_1 \in \Gamma(\mathfrak{D})$, $Y_2 \in \Gamma(\mathfrak{D}^\perp)$ and $Y_4 \in \Gamma(TM)^\perp$.

Proof. For any $Y_1 \in \Gamma(\mathfrak{D})$, $Y_2 \in \Gamma(\mathfrak{D}^\perp)$ and $Y_4 \in \Gamma(TM)^\perp$, by using (1) and (11), we have

$$< h(Y_1, Y_2), Y_4 >=< \widetilde{\nabla}_{Y_1} Y_2, Y_4 >=< \widetilde{\nabla}_{Y_1} JY_2, BY_4 > + < J(\widetilde{\nabla}_{Y_1} Y_2, CY_4) > .$$

By virtue of (3) and (7), we get

$$< h(Y_1, Y_2), Y_4 >=< \nabla_{Y_1} TY_2, BY_4 > + < F\nabla_{Y_1} Y_2 + Ch(Y_1, Y_2), CY_4 > . \quad (25)$$

The proof comes from (25). □

Theorem 4.12. *Let M be a pointwise quasi hemi-slant submanifolds of a Kaehler manifold $\widetilde{M}$. Then, $\mathfrak{D}_\theta - \mathfrak{D}^\perp$ mixed totally geodesic if and only if*

$$FA_{FY_2} Y_1 = C\nabla^\perp_{Y_1} FY_2$$

where $Y_1 \in \Gamma(\mathfrak{D}_\theta)$, $Y_2 \in \Gamma(\mathfrak{D}^\perp)$ and $Y_4 \in \Gamma(TM)^\perp$.

Proof. For any $Y_1 \in \Gamma(\mathfrak{D}_\theta)$, $Y_2 \in \Gamma(\mathfrak{D}^\perp)$ and $Y_4 \in \Gamma(TM)^\perp$, by making use of (1) and (7), we have

$$< h(Y_1, Y_2), Y_4 >=< \widetilde{\nabla}_{Y_1} JY_2, JY_4 >=< \widetilde{\nabla}_{Y_1} FY_2, JY_4 > .$$

Taking into account of (4) and (11), we get

$$\begin{aligned} < h(Y_1, Y_2), Y_4 > &= - < J\widetilde{\nabla}_{Y_1} FY_2, Y_4 > \\ &=< JA_{FY_2} Y_1, Y_4 > - < \nabla^\perp_{Y_1} FY_2, Y_4 > \\ &=< FA_{FY_2} Y_1 - C\nabla^\perp_{Y_1} FY_2, Y_4 > . \end{aligned} \quad (26)$$

The proof comes from (26). □

Finally, we mention the following examples.

5. Examples

Example 5.1. *For $\theta \in (0, \frac{\pi}{2})$, consider a submanifold M of a Kaehler manifold $\widetilde{M}$ defined by immersion ψ as follows:*

$$\psi(r, s, t, \theta, u, v) = (\frac{t}{\sqrt{3}}, \frac{r}{\sqrt{2}} + \frac{1}{\sqrt{5}}, \frac{u}{\sqrt{2}} + \frac{1}{\sqrt{5}}, \sin\theta, \frac{v}{\sqrt{3}}, 0, \cos\theta, 0, \frac{s}{\sqrt{3}}, 0).$$

We can easily to see that the tangent bundle of M is spanned by the tangent vectors

$$E_1 = \frac{1}{\sqrt{2}}\frac{\partial}{\partial y_1}, \quad E_2 = \frac{1}{\sqrt{3}}\frac{\partial}{\partial x_5}, \quad E_3 = \frac{1}{\sqrt{3}}\frac{\partial}{\partial x_1},$$
$$E_4 = \cos\theta\frac{\partial}{\partial y_2} - \sin\theta\frac{\partial}{\partial x_4}, \quad E_5 = \frac{1}{\sqrt{2}}\frac{\partial}{\partial x_2}, \quad E_6 = \frac{1}{\sqrt{3}}\frac{\partial}{\partial x_3}.$$

We define Kaehler structure J of $\mathbb{R}^{10}$ by,

$$J(\frac{\partial}{\partial x_i}) = -\frac{\partial}{\partial y_i}, \ J(\frac{\partial}{\partial y_j}) = \frac{\partial}{\partial x_j}, \ 1 \le i, j \le 5.$$

We get

$$JE_1 = \frac{1}{\sqrt{2}}\frac{\partial}{\partial x_1}, \quad JE_2 = -\frac{1}{\sqrt{3}}\frac{\partial}{\partial y_5}, \quad JE_3 = -\frac{1}{\sqrt{3}}\frac{\partial}{\partial y_1},$$
$$JE_4 = \cos\theta\frac{\partial}{\partial x_2} + \sin\theta\frac{\partial}{\partial y_4}, \quad JE_5 = -\frac{1}{\sqrt{2}}\frac{\partial}{\partial y_2}, \quad JE_6 = -\frac{1}{\sqrt{3}}\frac{\partial}{\partial y_3}.$$

Then $\mathfrak{D} = span\{E_1, E_3\}$ is holomorphic distribution, $\mathfrak{D}_\theta = span\{E_4, E_5\}$ is pointwise slant with slant function $\cos^{-1}(\frac{\cos\theta}{\sqrt{2}})$ and $\mathfrak{D}^\perp = span\{E_2, E_6\}$ is anti-invariant distribution. Thus ψ defines a proper 6-dimensional pointwise quasi hemi-slant submanifold M in $\widetilde{M}$.

Example 5.2. *For $v \neq 0, 1$ and $\theta \in (0, \frac{\pi}{2})$, consider a submanifold M of a Kaehler manifold $\widetilde{M}$ defined by immersion ψ as follows:*

$$\psi(v, u, \alpha, r, s, t, w, \theta) = (u, \alpha, v\cos(u+\alpha), \pi, v\sin(u+\alpha), \sqrt{3}, r, e, \frac{s}{\sqrt{2}}, \frac{t}{\sqrt{2}}, \frac{w}{\sqrt{3}}, \frac{\theta}{\sqrt{3}}).$$

We can easily see that the tangent bundle of M is spanned by the tangent vectors

$$E_1 = \cos(u+\alpha)\frac{\partial}{\partial x_3} + \sin(u+\alpha)\frac{\partial}{\partial x_5},$$
$$E_2 = \frac{\partial}{\partial x_1} - v\sin(u+\alpha)\frac{\partial}{\partial x_3} + v\cos(u+\alpha)\frac{\partial}{\partial x_5},$$
$$E_3 = \frac{\partial}{\partial x_2} - v\sin(u+\alpha)\frac{\partial}{\partial x_3} + v\cos(u+\alpha)\frac{\partial}{\partial x_5},$$
$$E_4 = \frac{\partial}{\partial x_7}, \quad E_5 = \frac{1}{\sqrt{2}}\frac{\partial}{\partial x_9}, \quad E_6 = \frac{1}{\sqrt{2}}\frac{\partial}{\partial x_{10}},$$
$$E_7 = \frac{1}{\sqrt{3}}\frac{\partial}{\partial x_{11}}, \quad E_8 = \frac{1}{\sqrt{3}}\frac{\partial}{\partial x_{12}}.$$

We define Kaehler structure J of $\mathbb{R}^{12}$ by

$$J(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}) = (-x_2, x_1, -x_4, x_3, -x_6, x_5, -x_8, x_7, -x_{10}, x_9, -x_{12}, x_{11}).$$

We obtain

$$
\begin{aligned}
JE_1 &= -\cos(u+\alpha)\frac{\partial}{\partial x_4} - \sin(u+\alpha)\frac{\partial}{\partial x_6},\\
JE_2 &= -\frac{\partial}{\partial x_2} + v\sin(u+\alpha)\frac{\partial}{\partial x_4} - v\cos(u+\alpha)\frac{\partial}{\partial x_6},\\
JE_3 &= \frac{\partial}{\partial x_1} + v\sin(u+\alpha)\frac{\partial}{\partial x_4} - v\cos(u+\alpha)\frac{\partial}{\partial x_6},\\
JE_4 &= -\frac{\partial}{\partial x_8}, \qquad JE_5 = -\frac{1}{\sqrt{2}}\frac{\partial}{\partial x_{10}}, \qquad JE_6 = \frac{1}{\sqrt{2}}\frac{\partial}{\partial x_9},\\
JE_7 &= -\frac{1}{\sqrt{3}}\frac{\partial}{\partial x_{12}}, \qquad JE_8 = \frac{1}{\sqrt{3}}\frac{\partial}{\partial x_{11}}.
\end{aligned}
$$

Then $\mathfrak{D} = span\{E_5, E_6, E_7, E_8\}$ *is holomorphic distribution,* $\mathfrak{D}_\theta = span\{E_2, E_3\}$ *is pointwise slant with slant function* $\cos^{-1}(\frac{1}{1+v^2})$ *and* $\mathfrak{D}^\perp = span\{E_1, E_4\}$ *is anti-invariant distribution. Thus* ψ *defines a proper* 8*-dimensional pointwise quasi hemi-slant submanifold* M *in* $\widetilde{M}$*.*

References

[1] M. A. Akyol and S. Beyendi, A note on quasi bi-slant submanifolds of cosymplectic manifolds, Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat. 69(2), (2020), 1508-1521.
[2] M. A. Akyol, S. Beyendi, T. Fatima and A. Ali, Pointwise quasi bi-slant submanifolds, Filomat 36(19), (2022), 6687-6697.
[3] A. Benjancu and N. Papaghuic, Semi-invariant submanifolds of a Sasakian manifold, An. St. Univ. AI. I. Cuza. Iasi. Math.(N.S.), 27, (1981), 163-170.
[4] J. L. Cabrerizo, A. Carriazo and L. M. Fernandez, Slant submanifolds in Sasakian manifolds, Glasg. Math. J., 42, (2000), 125-138.
[5] J. L. Cabrerizo, A. Carriazo, L. M. Fernandez and M. Fernandez, Semi-slant submanifolds of a Sasakian manifold, Geom. Dedic. 78(2), (1999), 183-199.
[6] A. Carriazo, New developments in slant submanifolds theory, Narasa Publishing Hause New Delhi, India, 2002.
[7] A. Carriazo, Bi-slant immersions. In: Proceeding of the ICRAMS 2000, Kharagpur, pp. (2000), 88-97.
[8] B. Y. Chen, Geometry of slant submanifolds, Katholieke Universiteit Leuven, Leuven, Belgium, View at Zentralblatt Math., 1990.
[9] B. Y. Chen, Slant immersions, Bull. Austral. Math. Soc., 41 (1990), 135-147.
[10] B. Y. Chen and O. J. Garay, Pointwise slant submanifolds in almost Hermitian manifolds. Turk. J. Math. 36, (2012), 630-640.
[11] B. Y. Chen and S. Uddin, Warped product pointwise bi-slant submanifolds of Kaehler Manifolds. Publ. Math. Debrecen, 92(1-2): (2018), 183-199.
[12] U. C. De and P. Majhi, On invariant submanifolds of Kenmotsu manifolds, Journal of Geometry, 106, (2015), 109-122.
[13] U. C. De, Y. Matsuyama and A. K. Sengupta, Generalized CR-submanifolds of a T-manifold, J. Korea Soc. Math. Educ. Ser. B Pure Appl. Math., 11 (2004), 175-187.
[14] F. Etayo, On quasi-slant submanifolds of an almost Hermitian manifold. Publ. Math. Debrecen. 53, (1998), 217-223.
[15] M. Kon, Remarks on anti-invariant submanifolds of a Sasakian manifold, Tensor (N.S.), 30 (1976), 239-245.
[16] A. Lotta, Slant submanifolds in contact geometry, Bulletin Mathematical Society Roumanie, 39 (1996), 183-198.
[17] K. Matsumoto, I. Mihai Y. Tazawa, Ricci tensor of slant submanifolds in complex space forms, Kodai Math.J. 26, (2003), 85-94.
[18] M. A. Lone, M. S. Lone and M. H. Shahid, Hemi-slant submanifolds of cosymplectic manifolds, Cogent Mathematics 3(1) (2016); 1204143.
[19] N. Papaghuic, Semi-slant submanifold of Kaehlerian manifold, An. St. Univ. AI. I. Cuza. Iasi. Math.(N.S.), 9, (1994), 55-61.
[20] K. S. Park, Pointwise slant and pointwise semi-slant submanifolds in almost contact metric manifolds. Mathematics (2020), 8(6), 985; https://doi.org/10.3390/math8060985.
[21] K. S. Park, On the pointwise slant submanifolds. In Hermitian-Grassmannian Submanifolds; Suh, Y., Ohnita, Y., Zhou, J., Kim, B., Lee, H., Eds.; Springer Proceedings in Mathematics and Statistics, 203; Springer: Singapore, 2017.
[22] S. Y. Perktaş, A. M. Blaga, and E. Kılıç, Almost bi-slant submanifolds of an almost contact metric manifold. J. Geom. 112(2), (2021), 1-23.
[23] R. Prasad, S. K. Verma and S. Kumar, Quasi hemi-slant submanifolds of Sasakian manifolds, Journal of Mathematical and Computational Science, 10(2), (2020), 418-435.
[24] R. Prasad, P. K. Singh, and A. K. Rai, On quasi hemi-slant submanifolds of nearly Kaehler manifolds, Diff. Geom. Dyn. Systems., 23, (2021), pp., 188-202.
[25] R. Prasad, S. K. Verma, S. Kumar and S. K. Chaubey, Quasi hemi-slant submanifolds of cosymplectic manifolds, Korean J. Math. 28(2), (2020), 257-273.
[26] R. Prasad, S. S. Shukla, A. Haseeb and S. Kumar, Quasi hemi-slant submanifolds of Kaehler manifolds, Honam Math. J. 42, 4, pp. (2020), 795-809.
[27] M. S. Siddesha, M. M. Praveena and C. S Bagewadi, On quasi hemi-slant submanifolds of LP-cosymplectic manifolds, Math., Anal., Appl., 3(3), (2021), 39-49.

[28] B. Şahin, Slant submanifolds of quaternion Kaehler manifolds, Commum. Korean Math. Soc. 22, (2007), 123-135.
[29] B. Şahin, Non-existence of warped product semi-slant submanifolds of Kaehler manifolds, Geom. Dedicata, 117, (2006), 195-202.
[30] B. Şahin, Warped product pointwise semi-slant submanifolds of Kaehler manifolds, Port. Math. 70, (2013), 252-268.
[31] B. Şahin, Slant submanifolds of an almost product Riemannian manifold, Journal of the Korean Mathematical Society, 43(4), (2006), 717-732.
[32] B. Şahin and S. Keleş, Slant submanifolds of Kaehler product manifolds, Turkish Journal of Mathematics, 31, (2007), 65-77.
[33] M. H. Shahid, On semi-invariant submanifolds of a nearly Sasakian manifold, Indian J. Pure and Appl. Math., 94, (10), (1993), 571-580.
[34] H. M. Taştan and F. Özdemir, The geometry of hemi-slant submanifolds of a locally product Riemannian manifold, Turkish Journal of Mathematics, 39, (2015), 268-284.
[35] S. Uddin, B. Y. Chen and F. R. Al-Solamy, Warped product bi-slant immersions in Kaehler manifolds. Mediterr. J. Math. 14(2), (2017), 14–95.
[36] Y. Tashiro, On contact structures of hypersurfaces in almost complex manifolds, I, Tohoku Math. J., 15 (1963), 62-78.
[37] K. Yano and M. Kon, Structures on manifolds, Series in Pure Mathematics, World Scientific Publishing Co., Singapore, 1984.