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Existence of Solution of Infinite Systems of Singular Integral Equations of Two Variables in $C(I \times I, \ell_p)$ with $I = [0, T], T > 0$ and $1 < p < \infty$ Using Hausdorff Measure of Noncompactness

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Abstract. In this article, we discuss the solvability of infinite systems of singular integral equations of two variables in the Banach sequence spaces $C(I \times I, \ell_p)$ with $I = [0, T], T > 0$ and $1 < p < \infty$ with the help of Meir-Keeler condensing operators and Hausdorff measure of noncompactness. With an example, we illustrate our findings.

1. Introduction

The theory of infinite systems of differential and/or integral equations plays a pivotal role in nonlinear analysis to encountered the real life problems in different fields e.g., the theory of branching processes, the theory of neural nets, the scaling system theory and the theory of algorithms, etc.

Last few years many authors explored the solvability of infinite systems of equations in Banach spaces, we refer to the readers [1, 2, 12–14, 17–19, 28, 30] and reference therein.

Kuratowski [20] was first introduced and analysed the concept of measure of noncompactness in the year 1930. For various forms of noncompactness measures, refer [9] for the viewer. The noncompactness measurements are valuable methods commonly that has been used in theory of fixed points, finite difference, computational equations, abstract spaces and optimization, etc. (see [10, 22]). By using the measure of noncompactness, several authors have already solved the numerous infinite systems of equations (see [3, 5–8, 15, 16, 23–27, 29, 31] for example).

Throughout the article we consider $I = [0, T],\ T > 0$. Suppose E_1 is a real Banach space with the norm $\| \, . \, \|$. Let $B(x_0, d_1)$ be a closed ball in E_1 centered at x_0 and with radius of d_1. If X_1 is a nonempty subset of E_1 then by $\bar{X}_1$ and $\mathrm{Conv}X_1$ we denote the closure and convex closure of X_1. In addition, let $\mathcal{M}_{E_1}$ be the family of all non-empty and bounded subsets of E_1 and $\mathcal{N}_{E_1}$ it consists of all relatively compact sets in its subfamily.

The axiomatic definition of a measure of noncompactness has been formulated by [9].

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Definition 1.1. *A function $\mu_1 : \mathcal{M}_{E_1} \to \mathbb{R}_+$ is said to be a measure of noncompactness if it satisfies the following assertions:*

(i) *the family* $ker\mu_1 = \{X_1 \in \mathcal{M}_{E_1} : \mu_1(X_1) = 0\}$ *is nonempty and* $ker\mu_1 \subset \mathcal{N}_{E_1}$.
(ii) $X_1 \subset Y_1 \implies \mu_1(X_1) \leq \mu_1(Y_1)$.
(iii) $\mu_1(\bar{X_1}) = \mu_1(X_1)$.
(iv) $\mu_1(ConvX_1) = \mu_1(X_1)$.
(v) $\mu_1(\lambda X_1 + (1-\lambda) Y_1) \leq \lambda\mu_1(X_1) + (1-\lambda)\mu_1(Y_1)$ *for* $\lambda \in [0,1]$.
(vi) *if* $X_n^1 \in \mathcal{M}_{E_1}$, $X_n^1 = \bar{X_n^1}$, $X_{n+1}^1 \subset X_n^1$ *for* $n = 1, 2, 3, \ldots$ *and* $\lim_{n\to\infty} \mu_1\left(X_n^1\right) = 0$, *then* $\bigcap_{n=1}^{\infty} X_n^1$ *is nonempty.*

The family kerμ_1 is called a *kernel of measure of noncompactness* μ_1.
A measure μ_1 is said to be sublinear if it satisfies the following assertions:

(1) $\mu_1(\lambda X_1) = |\lambda|\,\mu_1(X_1)$ for $\lambda \in \mathbb{R}$.
(2) $\mu_1(X_1 + Y_1) \leq \mu_1(X_1) + \mu_1(Y_1)$.

A sublinear measure of noncompactness μ_1 satisfying the assertion:

$$\mu_1(X_1 \cup Y_1) = \max\{\mu_1(X_1), \mu_1(Y_1)\}$$

and such that $ker\mu_1 = \mathcal{N}_{E_1}$ is said to be regular.
For a nonempty and bounded subset S of a metric space X_1, the Kuratowski measure of noncompactness is defined as

$$\alpha(S) = \inf\left\{\delta > 0 : S \subset \bigcup_{i=1}^{n} S_i,\ \mathrm{diam}(S_i) \leq \delta \text{ for } 1 \leq i \leq n, n \in \mathbb{N}\right\},$$

where diam(S_i) denotes the diameter of the set S_i, i.e.,

$$\mathrm{diam}(S_i) = \sup\{d(x,y) : x, y \in S_i\}.$$

The Hausdorff measure of noncompactness for a bounded set S is defined by

$$\mathfrak{C}(S) = \inf\{\epsilon > 0 : S \text{ has finite } \epsilon\text{-net in } X_1\}.$$

We again recall the basic properties of the Hausdorff measure od noncompactness.
Let F, F_1 and F_2 be bounded subsets of the metric space (X_1, d). Then

(i) $\mathfrak{C}(F) = 0$ if and only if F is totally bounded;
(ii) $\mathfrak{C}(F) = \mathfrak{C}(\bar{F})$, where $\bar{F}$ denotes the closure of F;
(iii) $F_1 \subset F_2$ implies that $\mathfrak{C}(F_1) \leq \mathfrak{C}(F_2)$;
(iv) $\mathfrak{C}(F_1 \cup F_2) = \max\{\mathfrak{C}(F_1), \mathfrak{C}(F_2)\}$;
(v) $\mathfrak{C}(F_1 \cap F_2) \leq \min\{\mathfrak{C}(F_1), \mathfrak{C}(F_2)\}$.

In case of a Banach space $(X_1, \|.\|)$, the function $\mathfrak{C}$ has some additional properties connected with the linear structure. For example we have

(i) $\mathfrak{C}(F_1 + F_2) \leq \mathfrak{C}(F_1) + \mathfrak{C}(F_2)$,
(ii) $\mathfrak{C}(F + x) = \mathfrak{C}(F)$ for all $x \in X_1$,
(iii) $\mathfrak{C}(\alpha F) = |\alpha|\mathfrak{C}(F)$ for all $\alpha \in \mathbb{R}$.

Definition 1.2. *[4] Let G_1 and G_2 be two Banach spaces and let μ_1 and μ_2 be arbitrary measure of noncompactness on G_1 and G_2, respectively. An operator f from G_1 to G_2 is called a (μ_1, μ_2)-condensing operator if it is continuous and $\mu_2(f(D)) < \mu_1(D)$ for every set $D \subset G_1$ with compact closure.*

Remark 1.3. *If $G_1 = G_2$ and $\mu_1 = \mu_2 = \mu$, then f is called a μ-condensing operator.*

If we consider taking the diameter of a set and the indicator function of a family of non-relatively compact sets (see [4]) as a measure of noncompactness, the contractive maps and the compact maps condense. In 1969, the preceding very fascinating fixed point theorem was proven by Meir and Keeler [21], which would be a generalized form of the notion of Banach contraction.

Definition 1.4. *[21] Let (X, d) be a metric space. Then a mapping O on X is said to be a Meir-Keeler contraction if for any $\epsilon > 0$, there exists $\delta > 0$ such that*

$$\epsilon \leq d(x, y) < \epsilon + \delta \implies d\left(Ox, Oy\right) < \epsilon, \ \forall x, y \in X.$$

Theorem 1.5. *[21] Let (X, d) be a complete metric space. If $O : X \to X$ is a Meir-Keeler contraction, then O has a unique fixed point.*

The preceding results are reported in [3], which are helpful in our assessment.

Definition 1.6. *[3] Let C be a nonempty subset of a Banach space E and let μ be an arbitrary measure of noncompactness on E. We say that an operator $O : C \to C$ is a Meir-Keeler condensing operator if for any $\epsilon > 0$, there exists $\delta > 0$ such that*

$$\epsilon \leq \mu\left(X\right) < \epsilon + \delta \implies \mu\left(O\left(X\right)\right) < \epsilon$$

for any bounded subset X of C.

Theorem 1.7. *[3] Let C be a nonempty, bounded, closed and convex subset of a Banach space E and let μ be an arbitrary measure of noncompactness on E. If $O : C \to C$ is a continuous and Meir-Keeler condensing operator, then O has at least one fixed point and the set of all fixed points of O in C is compact.*

2. Hausdorff measure of noncompactness in sequence spaces

In the Banach space $\left(\ell_p, \| \,.\, \|_{\ell_p}\right)$, the Hausdorff measure of noncompactness $\mathfrak{C}$ is defined as follows (see [9]):

$$\mathfrak{C}_{\ell_p}\left(D\right) = \lim_{n\to\infty}\left[\sup_{u\in D}\left(\sum_{k=n}^{\infty} \mid u_k \mid^p\right)^{\frac{1}{p}}\right], \tag{1}$$

where $u = (u_i)_{i=1}^{\infty} \in \ell_p$ and $D \in \mathcal{M}_{\ell_p}$.

Let us define $C(I \times I, \ell_p)$ denotes the space of all continuous functions defined on $I \times I$ with values in ℓ_p. Then $C(I \times I, \ell_p)$ is also a Banach space with the norm $\| \ x(\varkappa, \wp) \ \|_{C(I\times I,\ell_p)} = \sup\left\{\| \ x(\varkappa, \wp) \ \|_{\ell_p} : \varkappa, \wp \in I\right\}$, where $x(\varkappa, \wp) \in C(I \times I, \ell_p)$. For any non-empty bounded subset $\hat{E}$ of $C(I \times I, \ell_p)$ and $\varkappa, \wp \in I$, let $\hat{E}(\varkappa, \wp) = \{x(\varkappa, \wp) : \varkappa, \wp \in I\}$.

Now, using (1), we conclude that the Housdorff measure of noncompactness for $\hat{E} \subset C(I \times I, \ell_p)$ can be defined by

$$\mathfrak{C}_{C(I\times I,\ell_p)}(\hat{E}) = \sup\left\{\mathfrak{C}_{\ell_p}(\hat{E}(\varkappa, \wp)) : \varkappa, \wp \in I\right\}.$$

In this article, the existence of solution of the following infinite systems of singular integral equations of two variables is discussed

$$x_i(\varkappa, \wp) = h_i\left(\varkappa, \wp, x(\varkappa, \wp)\right) + f_i\left(\varkappa, \wp, x(\varkappa, \wp), \int_0^s \int_0^t \frac{u_i\left(\varkappa, \wp, v, w, x(v, w)\right)}{(\varkappa - v)^{\alpha}(\wp - w)^{\beta}} dv dw\right), \tag{2}$$

where $x(\varkappa, \wp) = (x_i(\varkappa, \wp))_{i=1}^{\infty} \in E$, $(\varkappa, \wp) \in I \times I$ and $x_i(\varkappa, \wp) \in C(I \times I, \mathbb{R})$ for all $i \in \mathbb{N}$ and $\alpha, \beta \in (0, 1)$. $C(I \times I, \mathbb{R})$ denotes the Banach space of all real continuous functions on $I \times I$ with norm $\| \ x \ \| = \sup\left\{\mid x(\varkappa, \wp) \mid : \varkappa, \wp \in I\right\}$ and E is a Banach sequence space $(E, \| \,.\, \|)$.

3. Solvability of infinite systems of singular integral equations of two variables in $C(I \times I, \ell_p)$ with $1 < p < \infty$

Assume that

(i) $f_i : I \times I \times C(I \times I, \ell_p) \times \mathbb{R} \to \mathbb{R}\,(i \in \mathbb{N})$ are continuous with

$$\sum_{i\geq 1} \left| f_i\left(\varkappa, \wp, x^0(\varkappa,\wp), 0\right)\right|^p$$

converges to zero for all $\varkappa, \wp \in I$, where $x^0(\varkappa,\wp) = \left(x_i^0(\varkappa,\wp)\right)_{i=1}^{\infty}$ and $x_i^0(\varkappa,\wp) = 0$ for all $i \in \mathbb{N}, (\varkappa,\wp) \in I \times I$. Also there exist $\hat{b}_i, \hat{\psi}_i : I \times I \to \mathbb{R}_+$ $(i \in \mathbb{N})$ which are bounded functions on $I \times I$ such that

$$\begin{aligned}&\left| f_i\left(\varkappa,\wp,x^1(\varkappa,\wp), l_1(\varkappa,\wp)\right) - f_i\left(\varkappa,\wp,x^2(\varkappa,\wp), l_2(\varkappa,\wp)\right)\right|^p \\ &\leq \hat{b}_i(\varkappa,\wp)\left|x_i^1(\varkappa,\wp) - x_i^2(\varkappa,\wp)\right|^p + \hat{\psi}_i(\varkappa,\wp)\left|l_1(\varkappa,\wp) - l_2(\varkappa,\wp)\right|^p,\end{aligned}$$

where $x^1(\varkappa,\wp) = \left(x_i^1(\varkappa,\wp)\right)_{i=1}^{\infty}, x^2(\varkappa,\wp) = \left(x_i^2(\varkappa,\wp)\right)_{i=1}^{\infty} \in C(I \times I, \ell_p); x_i^1(\varkappa,\wp), x_i^2(\varkappa,\wp) \in C(I \times I, \mathbb{R})$ for all $i \in \mathbb{N}$ and $l_1, l_2 : I \times I \to \mathbb{R}$ are bounded.

(ii) $u_i : I \times I \times I \times I \times C(I \times I, \ell_p) \to \mathbb{R}$ $(i \in \mathbb{N})$ are continuous. Moreover

$$\hat{r}_i = \sup\left\{\sum_{k\geq i}\left|\int_0^s\int_0^t \frac{u_k\left(\varkappa,\wp,v,w,x(v,w)\right)}{(\varkappa - v)^{\alpha}(\wp - w)^{\beta}} dvdw\right|^p : \varkappa,\wp,v,w \in I, x(v,w) \in C(I \times I, \ell_p)\right\} < \infty.$$

Also $\sup_i \hat{r}_i = \hat{R}$ and $\lim_{i\to\infty} \hat{r}_i = 0$.

(iii) $h_i : I \times I \times C(I \times I, \ell_p) \to \mathbb{R}$ $(i \in \mathbb{N})$ are continuous and there exist constants $\hat{D}_i \geq 0$ $(i \in \mathbb{N})$ such that

$$\left|h_i(\varkappa,\wp,x(\varkappa,\wp)) - h_i(\varkappa,\wp,y(\varkappa,\wp))\right|^p \leq \hat{D}_i\left|x_i(\varkappa,\wp) - y_i(\varkappa,\wp)\right|^p, \forall i \in \mathbb{N},$$

where $y(\varkappa,\wp) = (y_i(\varkappa,\wp))_{i=1}^{\infty} \in C(I \times I, \ell_p)$ and $y_i(\varkappa,\wp) \in C(I \times I, \mathbb{R})$ for all $i \in \mathbb{N}$ and

$$\sum_{i\geq 1}\left|h_i(\varkappa,\wp,x^0(\varkappa,\wp))\right|^p$$

converges to zero and $\sup_i \hat{D}_i = \hat{D}$.

(iv) Define an operator $\bar{Q}$ on $I \times I \times C(I \times I, \ell_p)$ to $C(I \times I, \ell_p)$ as follows

$$(\varkappa,\wp,x(\varkappa,\wp)) \to (\bar{Q}x)(\varkappa,\wp),$$

where

$$(\bar{Q}x)(\varkappa,\wp) = \left((\bar{Q}_1x)(\varkappa,\wp), (\bar{Q}_2x)(\varkappa,\wp), (\bar{Q}_3x)(\varkappa,\wp), \ldots\right),$$

$$(\bar{Q}_ix)(\varkappa,\wp) = h_i\left(\varkappa,\wp,x(\varkappa,\wp)\right) + f_i\left(\varkappa,\wp,x(\varkappa,\wp), q_i(x)\right)$$

and

$$q_i(x) = \int_0^s\int_0^t \frac{u_i\left(\varkappa,\wp,v,w,x(v,w)\right)}{(\varkappa - v)^{\alpha}(\wp - w)^{\beta}} dvdw, \ i \in \mathbb{N}.$$

(v) Let

$$\sup\left\{\hat{b}_i(\varkappa,\wp) : \varkappa,\wp \in I, i \in \mathbb{N}\right\} = \hat{B} < \infty.$$

(vi) Let $\bar{\psi} = \sup\left\{\hat{\psi}_i(\varkappa,\wp) : \varkappa,\wp \in I, i \in \mathbb{N}\right\}$ and $\gamma : I \times I \to \mathbb{R}_+$ defined by

$$\gamma(\varkappa,\wp) = \left(\varkappa^{1-\alpha}\wp^{1-\beta}\right)^p \sup_i \hat{\psi}_i(\varkappa,\wp).$$

Also let $\hat{\gamma} = \sup\{\gamma(\varkappa,\wp) : \varkappa,\wp \in I\} < \infty$.

(vii) We also assume that $0 < \hat{B} + \hat{D} < 4^{1-p}$.

Theorem 3.1. *Under the hypothesis (i)-(vii), the infinite systems (2) has at least one solution* $x(\varkappa,\wp) = (x_i(\varkappa,\wp))_{i=1}^{\infty} \in C(I \times I, \ell_p)$ *for all* $t,s \in I$ *and* $x_i(\varkappa,\wp) \in C(I \times I, \mathbb{R})$ *for all* $i \in \mathbb{N}$.

Proof. By using (2) and (i)-(vii), for all arbitrarily fixed $t,s \in I$, we have

$$\begin{aligned}
&\| x(\varkappa,\wp) \|_{\ell_p}^p \\
&= \sum_{i\geq 1}\left| h_i(\varkappa,\wp,x(\varkappa,\wp)) + f_i\left(\varkappa,\wp,x(\varkappa,\wp), \int_0^s\int_0^t \frac{u_i(\varkappa,\wp,v,w,x(v,w))}{(\varkappa-v)^{\alpha}(\wp-w)^{\beta}}dvdw\right)\right|^p \\
&\leq 4^{p-1}\sum_{i\geq 1}\left|h_i(\varkappa,\wp,x(\varkappa,\wp)) - h_i\left(\varkappa,\wp,x^0(\varkappa,\wp)\right)\right|^p + 4^{p-1}\sum_{i\geq 1}\left|h_i\left(\varkappa,\wp,x^0(\varkappa,\wp)\right)\right|^p \\
&+ 4^{p-1}\sum_{i\geq 1}\left|f_i\left(\varkappa,\wp,x(\varkappa,\wp), \int_0^s\int_0^t \frac{u_i(\varkappa,\wp,v,w,x(v,w))}{(\varkappa-v)^{\alpha}(\wp-w)^{\beta}}dvdw\right) - f_i\left(\varkappa,\wp,x^0(\varkappa,\wp),0\right)\right|^p \\
&+ 4^{p-1}\sum_{i\geq 1}\left|f_i\left(\varkappa,\wp,x^0(\varkappa,\wp),0\right)\right|^p \\
&\leq 4^{p-1}\sum_{i\geq 1}\left\{\hat{D}_i\,|x_i(\varkappa,\wp)|^p\right\} + 4^{p-1}\sum_{i\geq 1}\left\{\hat{b}_i(\varkappa,\wp)\,|x_i(\varkappa,\wp)|^p + \hat{\psi}_i(\varkappa,\wp)\left|\int_0^s\int_0^t \frac{u_i(\varkappa,\wp,v,w,x(v,w))}{(\varkappa-v)^{\alpha}(\wp-w)^{\beta}}dvdw\right|^p\right\} \\
&\leq 4^{p-1}\left(\hat{B}+\hat{D}\right)\sum_{i\geq 1}|x_i(\varkappa,\wp)|^p + 4^{p-1}\sup_j \hat{\psi}_i(\varkappa,\wp)\sum_{i\geq 1}\left|\int_0^s\int_0^t \frac{u_i(\varkappa,\wp,v,w,x(v,w))}{(\varkappa-v)^{\alpha}(\wp-w)^{\beta}}dvdw\right|^p \\
&\leq 4^{p-1}\left(\hat{B}+\hat{D}\right)\| x(\varkappa,\wp) \|_{\ell_p}^p + 4^{p-1}\bar{\psi}\hat{R}.
\end{aligned}$$

i.e., $\left\{1 - 4^{p-1}\left(\hat{B}+\hat{D}\right)\right\}\| x(\varkappa,\wp) \|_{\ell_p}^p \leq 4^{p-1}\bar{\psi}\hat{R}$ gives $\| x(\varkappa,\wp) \|_{\ell_p}^p \leq \frac{4^{p-1}\bar{\psi}\hat{R}}{1-4^{p-1}(\hat{B}+\hat{D})} = \bar{r}^p$(say).
Thus $\| x(\varkappa,\wp) \|_{\ell_p} \leq \bar{r}$ and hence $\| x(\varkappa,\wp) \|_{C(I\times I,\ell_p)} \leq \bar{r}$.
Therefore $x(\varkappa,\wp) \in C(I \times I, \ell_p)$.
Assume $D = B\left(x^0(\varkappa,\wp),\bar{r}\right)$ be the closed ball with center at $x^0(\varkappa,\wp)$ and radius $\bar{r}$, thus D is an non-empty, bounded, closed and convex subset of $C(I \times I, \ell_p)$. Let us define $\bar{Q} = (\bar{Q}_i)$ be an operator defined as follows. For all arbitrary fixed $\varkappa,\wp \in I$,

$$\left(\bar{Q}x\right)(\varkappa,\wp) = \left\{\left(\bar{Q}_i z\right)(\varkappa,\wp)\right\} = \left\{h_i(\varkappa,\wp,x(\varkappa,\wp)) + f_i(\varkappa,\wp,x(\varkappa,\wp),q_i(x(\varkappa,\wp)))\right\},$$

where $x(\varkappa,\wp) = (x_i(\varkappa,\wp))_{i=1}^{\infty} \in D$ and $x_i(\varkappa,\wp) \in C(I \times I, \mathbb{R})$ for all $i \in \mathbb{N}$.
Since for each $(\varkappa,\wp) \in I \times I$, we have

$$\sum_{i\geq 1}\left|\left(\bar{Q}_i x\right)(\varkappa,\wp)\right|^p = \sum_{i\geq 1}\left|h_i(\varkappa,\wp,x(\varkappa,\wp)) + f_i(\varkappa,\wp,x(\varkappa,\wp),q_i(x(\varkappa,\wp)))\right|^p < \infty.$$

Hence $\left(\bar{Q}x\right)(\varkappa,\wp) \in C(I \times I, \ell_p)$.
Since $\| \left(\bar{Q}x\right)(\varkappa,\wp) - x^0(\varkappa,\wp) \|_{C(I\times I,\ell_p)} \leq \bar{r}$ therefore $\bar{Q}$ is self mapping on D.

We have to show that $\bar{Q}$ is continuous on D.
Let $\epsilon > 0$ and arbitrary $x(\varkappa,\wp) = \left(x_j(\varkappa,\wp)\right)_{j=1}^{\infty}$, $y(\varkappa,\wp) = \left(y_j(\varkappa,\wp)\right)_{j=1}^{\infty} \in D$ such that $\| x(\varkappa,\wp) - y(\varkappa,\wp) \|^p_{C(I\times I,\ell_p)} < \frac{\epsilon^p}{2^p(\hat{B}+\hat{D})}$.
For arbitrarily fixed $\varkappa,\wp \in I$ and $i \in \mathbb{N}$, we have

$$\begin{aligned}
&\left|(\bar{Q}_i x)(\varkappa,\wp) - (\bar{Q}_i y)(\varkappa,\wp)\right|^p \\
&= \left|h_i(\varkappa,\wp,x(\varkappa,\wp)) + f_i(\varkappa,\wp,x(\varkappa,\wp),q_i(x(\varkappa,\wp))) - h_i(\varkappa,\wp,y(\varkappa,\wp)) - f_i(\varkappa,\wp,y(\varkappa,\wp),q_i(y(\varkappa,\wp)))\right|^p \\
&\le 2^{p-1}\left|h_i(\varkappa,\wp,x(\varkappa,\wp)) - h_i(\varkappa,\wp,y(\varkappa,\wp))\right|^p + 2^{p-1}\left|f_i(\varkappa,\wp,x(\varkappa,\wp),q_i(x(\varkappa,\wp))) - f_i(\varkappa,\wp,y(\varkappa,\wp),q_i(y(\varkappa,\wp)))\right|^p \\
&\le 2^{p-1}\left\{\hat{D}_i\left|x_i(\varkappa,\wp) - y_i(\varkappa,\wp)\right|^p + \hat{b}_i(\varkappa,\wp)\left|x_i(\varkappa,\wp) - y_i(\varkappa,\wp)\right|^p + \hat{\psi}_i(\varkappa,\wp)\left|q_i(x(\varkappa,\wp)) - q_i(y(\varkappa,\wp))\right|^p\right\} \\
&\le 2^{p-1}\left(\hat{B}+\hat{D}\right)\left|x_i(\varkappa,\wp) - y_i(\varkappa,\wp)\right|^p \\
&+ 2^{p-1}\hat{\psi}_i(\varkappa,\wp)\left\{\int_0^s\int_0^t \frac{\left|u_i(\varkappa,\wp,v,w,x(v,w)) - u_i(\varkappa,\wp,v,w,y(v,w))\right|}{(\varkappa-v)^\alpha(\wp-w)^\beta}dvdw\right\}^p.
\end{aligned}$$

Let

$$U = \sup\left\{\left|u_i(\varkappa,\wp,v,w,x(v,w)) - u_i(\varkappa,\wp,v,w,y(v,w))\right| : \varkappa,\wp,v,w \in I; x(v,w),y(v,w) \in D, i \in \mathbb{N}\right\}.$$

As $\epsilon \to 0$ we have $U \to 0$ because of assumption (ii) thus we can choose $\frac{2^{p-1}U\hat{\gamma}}{(1-\alpha)^p(1-\beta)^p} < \frac{\epsilon^p}{2^{i+1}}$.
Therefore, we have

$$\begin{aligned}
&\left|(\bar{Q}_i x)(\varkappa,\wp) - (\bar{Q}_i y)(\varkappa,\wp)\right|^p \\
&< 2^{p-1}\left(\hat{B}+\hat{D}\right)\left|x_i(\varkappa,\wp) - y_i(\varkappa,\wp)\right|^p + 2^{p-1}U^p\sup_i\hat{\psi}_i(\varkappa,\wp)\left\{\int_0^s\int_0^t \frac{1}{(\varkappa-v)^\alpha(\wp-w)^\beta}dvdw\right\}^p \\
&= 2^{p-1}\left(\hat{B}+\hat{D}\right)\left|x_i(\varkappa,\wp) - y_i(\varkappa,\wp)\right|^p + 2^{p-1}U^p\sup_i\hat{\psi}_i(\varkappa,\wp)\left\{\frac{\varkappa^{1-\alpha}\wp^{1-\beta}}{(1-\alpha)(1-\beta)}\right\}^p \\
&\le 2^{p-1}\left(\hat{B}+\hat{D}\right)\left|x_i(\varkappa,\wp) - y_i(\varkappa,\wp)\right|^p + \frac{2^{p-1}U^p\hat{\gamma}}{(1-\alpha)^p(1-\beta)^p}.
\end{aligned}$$

Therefore

$$\begin{aligned}
&\sum_{i\ge1}\left|(\bar{Q}_i x)(\varkappa,\wp) - (\bar{Q}_i y)(\varkappa,\wp)\right|^p \\
&< 2^{p-1}\left(\hat{B}+\hat{D}\right)\| x(\varkappa,\wp) - y(\varkappa,\wp) \|^p_{C(I\times I,\ell_p)} + \sum_{i\ge1}\frac{\epsilon^p}{2^{i+1}} < \epsilon^p.
\end{aligned}$$

Thus $\| (\bar{Q}x)(\varkappa,\wp) - (\bar{Q}y)(\varkappa,\wp) \|_{C(I\times I,\ell_p)} < \epsilon$. Therefore $\bar{Q}$ is continuous on D.

We have for arbitrary fixed $\varkappa, \wp \in I$,

$$
\begin{aligned}
&\mathfrak{C}_{\ell_p}\left(\hat{Q}(D)\right)\\
&= \lim_{n\to\infty}\left[\sup_{x(\varkappa,\wp)\in D}\left\{\sum_{i\geq n}\left|h_i(\varkappa,\wp,x(\varkappa,\wp)) + f_i(\varkappa,\wp,x(\varkappa,\wp),q_i(x(\varkappa,\wp)))\right|^p\right\}\right]^{\frac{1}{p}}\\
&\leq \lim_{n\to\infty}\sup_{x(\varkappa,\wp)\in D}\left[4^{p-1}\left(\hat{B}+\hat{D}\right)\sum_{i\geq n}|x_i(\varkappa,\wp)|^p + 4^{p-1}\sup_i \hat{\psi}_i(\varkappa,\wp)\sum_{i\geq n}\left|\int_0^s\int_0^t \frac{u_i(\varkappa,\wp,v,w,x(v,w))}{(\varkappa-v)^{\alpha}(\wp-w)^{\beta}}dvdw\right|^p\right]^{\frac{1}{p}}\\
&\leq \lim_{n\to\infty}\sup_{x(\varkappa,\wp)\in D}\left[4^{p-1}\left(\hat{B}+\hat{D}\right)\sum_{i\geq n}|x_i(\varkappa,\wp)|^p + 4^{p-1}\hat{r}_n\sup_i\hat{\psi}_i(\varkappa,\wp)\right]^{\frac{1}{p}}.
\end{aligned}
$$

i.e.,

$$\mathfrak{C}_{\ell_p}\left(\hat{Q}(D)\right) \leq 4^{1-\frac{1}{p}}\left(\hat{B}+\hat{D}\right)^{\frac{1}{p}}\mathfrak{C}_{\ell_p}(D)$$

gives

$$\mathfrak{C}_{C(I\times I,\ell_p)}\left(\hat{Q}(D)\right) \leq 4^{1-\frac{1}{p}}\left(\hat{B}+\hat{D}\right)^{\frac{1}{p}}\mathfrak{C}_{C(I\times I,\ell_p)}(D).$$

We observe that $\mathfrak{C}_{C(I\times I,\ell_p)}(\bar{Q}(D)) \leq 4^{1-\frac{1}{p}}\left(\hat{B}+\hat{D}\right)^{\frac{1}{p}}\mathfrak{C}_{C(I\times I,\ell_p)}(D) < \epsilon \Rightarrow \mathfrak{C}_{C(I\times I,\ell_p)}(D) < \frac{\epsilon}{4^{1-\frac{1}{p}}(\hat{B}+\hat{D})^{\frac{1}{p}}}$.

Taking $\delta = \frac{\epsilon\left(1-4^{1-\frac{1}{p}}(\hat{B}+\hat{D})^{\frac{1}{p}}\right)}{4^{1-\frac{1}{p}}(\hat{B}+\hat{D})^{\frac{1}{p}}}$, we get $\epsilon \leq \mathfrak{C}_{C(I\times I,\ell_p)}(D) < \epsilon+\delta$. Therefore $\bar{Q}$ is a Meir-Keeler condensing operator defined on the set $D \subset C(I\times I,\ell_p)$. So $\bar{Q}$ satisfies all the assertions of Theorem 1.7 which implies $\bar{Q}$ has a fixed point in D. Hence the systems (2) has a solution in $C(I\times I,\ell_p)$. □

4. Example

Consider the following infinite systems of singular integral equations

$$x_i(\varkappa,\wp) = \frac{1}{8+\varkappa^2\wp^2}\sum_{j=i}^{i+1}\left(\frac{x_j(\varkappa,\wp)}{j^2}\right) + \sum_{j=i}^{i+1}\left(\frac{\left|x_j(\varkappa,\wp)\right|}{4^2j^2}\right) + \frac{1}{e^{\varkappa\wp}}\int_0^s\int_0^t \frac{\sin^2\left(1+\sum_{j=1}^{2i}x_j(v,w)\right)}{(\varkappa\wp+i^2)(\varkappa-v)^{\frac{1}{2}}(\wp-w)^{\frac{1}{2}}}dvdw, \tag{3}$$

where $i \in \mathbb{N}$ and $\varkappa, \wp \in I = [0,1]$. Here

$$h_i(\varkappa,\wp,x(\varkappa,\wp)) = \frac{1}{8+\varkappa^2\wp^2}\sum_{j=i}^{i+1}\left(\frac{x_j(\varkappa,\wp)}{j^2}\right),$$

$$f_i(\varkappa,\wp,x(\varkappa,\wp),q_i(x(\varkappa,\wp))) = \sum_{j=i}^{i+1}\left(\frac{\left|x_j(\varkappa,\wp)\right|}{4^2j^2}\right) + \frac{1}{e^{\varkappa\wp}}q_i(x(\varkappa,\wp)),$$

$$q_i(x(\varkappa,\wp)) = \int_0^s\int_0^t \frac{\sin^2\left(1+\sum_{j=1}^{2i}x_j(v,w)\right)}{(\varkappa\wp+i^2)(\varkappa-v)^{\frac{1}{2}}(\wp-w)^{\frac{1}{2}}}dvdw,$$

$$u_i(\varkappa,\wp,v,w,x(v,w)) = \sin^2\left(1+\sum_{j=1}^{2i}x_j(v,w)\right)$$

and $\alpha = \beta = \frac{1}{2}$.
Now if $x(\varkappa,\wp) = (x_i(\varkappa,\wp)), y(\varkappa,\wp) = (y_i(\varkappa,\wp)) \in C(I\times I,\ell_p)$ then $(f_i(\varkappa,\wp,x(\varkappa,\wp),q_i(x(\varkappa,\wp)))) \in C(I\times I,\ell_p)$ and $(h_i(\varkappa,\wp,x(\varkappa,\wp))) \in C(I\times I,\ell_p)$, since for arbitrary fixed $\varkappa,\wp\in I$,

$$\begin{aligned}
&\sum_{i\geq 1}|h_i(\varkappa,\wp,x(\varkappa,\wp))|^p\\
&\leq \sum_{i\geq 1}\left|\frac{1}{8+\varkappa^2\wp^2}\sum_{j=i}^{i+1}\left(\frac{x_j(\varkappa,\wp)}{j^2}\right)\right|^p\\
&= \frac{1}{(8+\varkappa^2\wp^2)^p}\sum_{i\geq 1}\left|\sum_{j=i}^{i+1}\left(\frac{x_j(\varkappa,\wp)}{j^2}\right)\right|^p\\
&\leq \frac{1}{8^p}\sum_{i\geq 1}\left\{2^{p-1}\sum_{j=i}^{i+1}\frac{\left|x_j(\varkappa,\wp)\right|^p}{j^{2p}}\right\}\\
&\leq \frac{2^{p-1}}{8^p}\sum_{i\geq 1}\left\{\sum_{j=i}^{i+1}\left|x_j(\varkappa,\wp)\right|^p\right\}\\
&\leq \frac{1}{4^p}\parallel x(\varkappa,\wp)\parallel^p_{C(I\times I,\ell_p)}<\infty
\end{aligned}$$

and

$$\begin{aligned}
&\sum_{i\geq 1}\left|f_i(\varkappa,\wp,x(\varkappa,\wp),q_i(x(\varkappa,\wp)))\right|^p\\
&= \sum_{i\geq 1}\left|\sum_{j=i}^{i+1}\left(\frac{\left|x_j(\varkappa,\wp)\right|}{4^2j^2}\right)+\frac{1}{e^{\varkappa\wp}}q_i(x(\varkappa,\wp))\right|^p\\
&\leq 2^{p-1}\sum_{i\geq 1}\left\{\left|\sum_{j=i}^{i+1}\left(\frac{\left|x_j(\varkappa,\wp)\right|}{4^2j^2}\right)\right|^p+\left|\frac{1}{e^{\varkappa\wp}}q_i(x(\varkappa,\wp))\right|^p\right\}\\
&\leq 2^{p-1}\sum_{i\geq 1}\left\{2^{p-1}\sum_{j=i}^{i+1}\left(\frac{\left|x_j(\varkappa,\wp)\right|^p}{4^{2p}j^{2p}}\right)+\left|\frac{1}{e^{\varkappa\wp}}q_i(x(\varkappa,\wp))\right|^p\right\}\\
&\leq 2^{p-1}\sum_{i\geq 1}\left\{\frac{2^p}{4^{2p}}\left|x_i(\varkappa,\wp)\right|^p+\frac{\left|q_i(x(\varkappa,\wp))\right|^p}{e^{p\varkappa\wp}}\right\}\\
&= \frac{1}{2.4^p}\parallel x(\varkappa,\wp)\parallel^p_{\ell_p}+\frac{2^{p-1}}{e^{p\varkappa\wp}}\sum_{i\geq 1}\left|q_i(x(\varkappa,\wp))\right|^p.
\end{aligned}$$

Let $\sum\limits_{i\geq 1}\frac{1}{i^{2p}} = B$. Since $p>1$, we have $B<\infty$.
Again

$$\begin{aligned}
&\left|q_i(x(\varkappa,\wp))\right|\\
&\leq \frac{1}{i^2}\int_0^s\int_0^t\frac{1}{(\varkappa-v)^{\frac{1}{2}}(\wp-w)^{\frac{1}{2}}}dvdw\\
&= \frac{4\sqrt{\varkappa\wp}}{i^2}.
\end{aligned}$$

Therefore

$$\begin{aligned}
&\sum_{i\geq 1}\left|f_i(\varkappa,\wp,x(\varkappa,\wp),q_i(x(\varkappa,\wp)))\right|^p\\
&\leq \frac{1}{2.4^p}\parallel x(\varkappa,\wp)\parallel_{\ell_p}^p+\frac{2^{p-1}}{e^{p\varkappa\wp}}\sum_{i\geq 1}\left(\frac{4\sqrt{\varkappa\wp}}{i^2}\right)^p\\
&=\frac{1}{2.4^p}\parallel x(\varkappa,\wp)\parallel_{\ell_p}^p+\frac{2^{p-1}4^p(\sqrt{\varkappa\wp})^p}{e^{p\varkappa\wp}}\sum_{i\geq 1}\frac{1}{i^{2p}}\\
&\leq \frac{1}{2.4^p}\parallel x(\varkappa,\wp)\parallel_{C(I\times I,\ell_p)}^p+\frac{2^{3p-1}B}{(\sqrt{2e})^p}<\infty.
\end{aligned}$$

Now

$$\begin{aligned}
&\left|f_i(\varkappa,\wp,x(\varkappa,\wp),q_i(x(\varkappa,\wp)))-f_i(\varkappa,\wp,y(\varkappa,\wp),q_i(y(\varkappa,\wp)))\right|^p\\
&\leq 2^{p-1}\left|\sum_{j=i}^{i+1}\frac{\left|x_j(\varkappa,\wp)\right|-\left|y_j(\varkappa,\wp)\right|}{4^2j^2}\right|^p+2^{p-1}\left|\frac{q_i(x(\varkappa,\wp))-q_j(y(\varkappa,\wp))}{e^{\varkappa\wp}}\right|^p\\
&\leq 2^{p-1}\left|\sum_{j=i}^{i+1}\frac{\left|x_j(\varkappa,\wp)-y_j(\varkappa,\wp)\right|}{4^2j^2}\right|^p+2^{p-1}\left|\frac{q_i(x(\varkappa,\wp))-q_j(y(\varkappa,\wp))}{e^{\varkappa\wp}}\right|^p\\
&\leq 4^{p-1}\sum_{j=i}^{i+1}\frac{\left|x_j(\varkappa,\wp)-y_j(\varkappa,\wp)\right|^p}{4^{2p}j^{2p}}+\frac{2^{p-1}}{e^{p\varkappa\wp}}\left|q_i(x(\varkappa,\wp))-q_j(y(\varkappa,\wp))\right|^p\\
&\leq \frac{2}{4^{p+1}}\left|x_i(\varkappa,\wp)-y_i(\varkappa,\wp)\right|^p+\frac{2^{p-1}}{e^{p\varkappa\wp}}\left|q_i(x(\varkappa,\wp))-q_j(y(\varkappa,\wp))\right|^p.
\end{aligned}$$

Here $\hat{b}_i(\varkappa,\wp)=\frac{2}{4^{p+1}},\ \hat{\psi}_i(\varkappa,\wp)=\frac{2^{p-1}}{e^{p\varkappa\wp}}$ are both bounded functions for all $\varkappa,\wp\in I,\ i\in\mathbb{N}$ and $\sum_{i\geq 1}\left|f_i\left(\varkappa,\wp,x^0(\varkappa,\wp),0\right)\right|$ converges to zero. Also we have $\hat{B}=\frac{2}{4^{p+1}}$ and $\bar{\psi}=2^{p-1}$.
Again

$$\begin{aligned}
&\left|h_i(\varkappa,\wp,x(\varkappa,\wp))-h_i(\varkappa,\wp,y(\varkappa,\wp))\right|^p\\
&=\left|\frac{1}{8+\varkappa^2\wp^2}\sum_{j=i}^{i+1}\left(\frac{x_j(\varkappa,\wp)-y_j(\varkappa,\wp)}{j^2}\right)\right|^p\\
&\leq \frac{2^{p-1}}{8^p}\sum_{j=i}^{i+1}\frac{\left|x_j(\varkappa,\wp)-y_j(\varkappa,\wp)\right|^p}{j^{2p}}\\
&\leq \frac{1}{4^p}\left|x_i(\varkappa,\wp)-y_i(\varkappa,\wp)\right|^p.
\end{aligned}$$

Here $\hat{D}_i = \frac{1}{4^p}$ so we have $\hat{D} = \frac{1}{4^p}$. Therefore $0 < \hat{B} + \hat{D} < 4^{1-p}$.
Again we have $\gamma(\varkappa, \wp) = 2^{p-1}.\left(\frac{\sqrt{\varkappa\wp}}{e^{\varkappa\wp}}\right)^p$ is bounded and $\hat{\gamma} = 2^{p-1}.\left(\frac{1}{\sqrt{2}e}\right)^p$. Since

$$\left|\int_0^s\int_0^t \frac{\sin^2\left(1+\sum_{j=1}^{2i} x_j(v,w)\right)}{(\varkappa\wp+i^2)(\varkappa-v)^{\frac{1}{2}}(\wp-w)^{\frac{1}{2}}}dvdw\right|$$

$$\leq \frac{4\sqrt{\varkappa\wp}}{\varkappa\wp+i^2} \leq \frac{4}{\sqrt{\varkappa\wp+i^2}} \leq \frac{4}{i}.$$

Therefore $\hat{r}_i \leq 4^p \sum_{k\geq i} \frac{1}{k^p} = 4^p B_1 < \infty$ for all $i \in \mathbb{N}$, where $B_1 = \sum_{k\geq 1} \frac{1}{k^p} < \infty$ as $p > 1$.
Thus $\lim_{i\to\infty} \hat{r}_i = 0$ and $\hat{R} = 4^p B_1$.
It is obvious that h_i, f_i and u_i are continuous functions. So all the assumptions from (i)-(vii) are satisfied. Hence by Theorem 3.1, we conclude that the systems (3) has a solution in $C(I \times I, \ell_p)$.

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