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Exponential Behavior of Nonlinear Stochastic Partial Functional Equations Driven by Poisson Jumps and Rosenblatt Process

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Abstract. In this article, we discuss the Asymptotic behaviour of mild solutions of nonlinear stochastic partial functional equations driven by Poisson jumps and Rosenblatt process. The Banach fixed point theorem and the theory of resolvent operator devolped by Grimmer are used. Finally, an illustrative example is given to demonstrate the effectiveness of the obtained results.

1. Introduction

In the past decades, the theory of the non-linear functional differential or integrodifferential equations with resolvent operators has become an active area of investigation due to their applications in many physical phenomena. The resolvent operator is similar to the semigroup operator for abstract differential equations in Banach spaces. However, the resolvent operator does not satisfy semigroup properties. The study of deterministic neutral functional differential equations was initiated by Hale and Mayer [17]. For more details on theory and their applications, we also refer the readers to Hale and Lunel [18], Kolmanovkii and Nosov [19] and so on. In recent years, the quantitative and qualitative properties of solutions to stochastic and stochastic differential equations like existence, uniqueness and stability have been widely examined by many researchers due to various mathematical models in the different areas such as machanics, electronics, control theory, engineering and economics, etc.

Stochastic integrodifferential equations with delay are important for investigating several problems raised from natural phenomena. As far as applications are concerned, stochastic evolution equations have been motivated by such phenomena as wave propagation in random media [15] and turbulence [14]. Important motivations came also from biological sciences, in particular from population biology; see Dawson [20] and Fleming [21]. In addition, the study of neutral stochastic functional differential equations (SFDEs) driven by jumps process also have begin to gain attention and strong growth in recent years (see [15, 16, 22–25] and references therein).

In this work, we shall prove the Asymptotic behaviour of mild solutions of nonlinear stochastic partial

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functional equations driven by Poisson jumps and Rosenblatt process described in the form:

$$\begin{aligned} d\left[x(t)+q(t,x(t-\tau(t)))\right] &= A\left[x(t)+q(t,x(t-\tau(t)))\right]dt \\ &+\Big[\int_0^t \Upsilon(t-s)\Big[x(s)-q(s,x(s-\tau(s)))\Big]+f(t,x(t-\rho))dt\Big]ds \\ &+g(t)dZ_H(t)+\int_{\mathcal{U}} h(t,x(t-\delta,u))\widetilde{N}(dt,du), \quad t\geq 0, \\ x(t) &= \varphi(t), \quad -r\leq t\leq 0, \end{aligned} \tag{1}$$

where A which is the infinitesimal generator of a strongly continuous semigroup $(s(t))_{t>0}$ on a Hilbert space $\mathcal{X}$ with domain $\mathcal{D}(\mathcal{A})$, $\Upsilon(t-s)$ is a closed linear operator on $\mathcal{X}$ with domain $\mathcal{D}(\Upsilon)\supset\mathcal{D}(A)$, $\mathcal{U}\in\mathcal{B}_\sigma(\mathscr{X}-\{0\})$, $\mathcal{B}_\sigma(\mathscr{X}-\{0\})$ is the Borel trace σ-algebra on $\mathscr{X}-\{0\}$ is a Borel set, and $\widetilde{N}$ will be defined later. Z_H is a Rosenblatt process on a real and separable Hilbert space $\mathcal{Y}$, $\tau,\rho,\delta:[0,+\infty)\to[0,r]$ $(r>0)$ are continuous and $f,q:[0,+\infty)\times\mathcal{X}\to\mathcal{X}$, $g:[0,+\infty)\to\mathcal{L}_2^0(\mathcal{Y},\mathcal{X})$, $h:[0,+\infty)\times\mathcal{X}\times\mathcal{U}\to\mathcal{X}$ are appropriate functions. Here $\mathcal{L}_2^0(\mathcal{Y},\mathcal{X})$ denotes the space of all Q -Hilbert-Schmidt operators from $\mathcal{Y}$ into $\mathcal{X}$ (see section 2 below). The initial data $\varphi\in\mathcal{D}:=\mathcal{D}([-r,0],\mathcal{X})$, where $\mathcal{D}$ denote the space of *càdlàg* functions from$[-r,0]$ into $\mathcal{X}$ equipped with supremum norm $|\varsigma|_{\mathcal{D}}=sup_{s\in[-r,0]}\|\varsigma(s)\|_{\mathcal{X}}$, and ς has finite second moment.

The analysis of (1) when $\Upsilon\equiv 0$ was initiated in E.H.Lakhel[22] , where the authors studied the existence and exponential stability of solutions by using a strict contractions principle. The main contribution is towards this direction by presenting conditions to assure existence, uniqueness and exponential stability for such a class of system with the integrodifferential term. Our paper expands the usefulness of stochastic integrodifferential equations since the literature shows results for existence and expoential stability for such equations under semigroup theory.

The remaining of the paper is organized as follows: Section 2 presents notation and preliminary results. Section 3 shows the main result for existence, uniqueness of mild solutions for nonlinear stochastic partial functional equations driven by Poisson jumps and Rosenblatt process and conditions for the exponential stability in mean square. Finally, Section 4 presents an example that illustrates our results.

2. Preliminaries

In this section, we provided some basic results about Poisson process, Resolvent operator and Rosenblatt process.

Poisson Jumps Process: Let $\mathcal{B}_\sigma(\mathcal{H})$ the Borel σ-algebra of $\mathcal{H}$. Let $(p(t))$, $(t\geq 0)$ be an $\mathcal{H}$-valued, σ-finite stationary $\Im_t$-adapted Poisson point process on $(\Omega,\Im,\mathbb{P})$. The counting random measure N defined by

$$N((t_1,t_2]\times\mathcal{U})(w) = \sum_{t_1<s\leq t_2} 1_{\mathcal{U}}(p(s)(w))$$

for any $\mathcal{U}\in\mathcal{B}_\sigma(\mathcal{H}-\{0\})$. where $0\notin\bar{\mathcal{U}}$ is called the Poisson random measure associated to Poisson point process p. The following notation is used

$$N(t,\mathcal{U}) = N((0,t]\times\mathcal{U}).$$

Then it is known that there exists a σ-finite measure $\mathcal{U}$ such that

$$\begin{aligned} \mathbf{E}(N(t,\mathcal{U})) &= \nu(\mathcal{U})t, \\ \mathbf{P}(N(t,\mathcal{U})=k) &= \frac{\exp(-t\nu(\mathcal{U}))(t\nu(\mathcal{U}))^k}{k!}. \end{aligned}$$

This measure ν is said Levy measure. Then the measure $\widetilde{N}$ is defined by

$$\widetilde{N}((0,t]\times\mathcal{U}) = N((0,t]\times\mathcal{U})-t\nu(\mathcal{U}).$$

This measure $\widetilde{N}(dt, du)$ is called the compensated Poisson random measure, and $dt\nu(\mathcal{U})$ is called the compensator (see [18]).

Definition 2.1. *Let $\mathcal{U} \in \mathscr{B}_\sigma(\mathcal{H}-\{0\})$. $P^2([0,T]\times\mathcal{U};\mathcal{X})$ is the space of all predictable mappings $h : [0,T]\times\mathcal{U}\times\Omega \to \mathcal{X}$ for which*

$$\int_0^T \int_{\mathcal{U}} \mathbf{E}\,\|h(t,\nu)\|^2\, dt\lambda(d\nu) < \infty.$$

We may then define the $\mathcal{X}$-valued stochastic integral $\int_0^T \int_{\mathcal{U}} L(t,\nu)\widetilde{N}(dt, d\nu)$, which is a centered square-integral martingale [25].

Rosenblatt Process: In this subsection, we recall some basic concepts on the Rosenblatt process as well as the Wiener integral with respect to it. Consider $(\xi_n)_{n\in Z}$ a stationary Gaussian sequence with mean zero and variance 1 such that its correlation function satisfies that $R(n) := \mathbb{E}(\xi_0\xi_n) = n^{\frac{2H-2}{k}}L(n)$, with $H \in (\frac{1}{2}, 1)$ and L is a slowly varying function at infinity. Let g be a function of Hermite rank k, that is, if g admits the following expansion in Hermite polynomials

$$g(x) = \sum_{j\geq 0} c_j H_j(x), \quad c_j = \frac{1}{j!}\mathbb{E}(g(\xi_0 H_j(\xi_0))),$$

then $k = \min\left\{j|c_j \neq 0\right\} \geq 1$, where $H_j(x)$ is the Hermite polynomial of degree j given by $H_j(x) = (-1)^j e^{\frac{x^2}{2}} \frac{d^j}{dx^j} e^{-\frac{x^2}{2}}$. Then, the Non-Central Limit Theorem (see, for example, Dobrushin & Major [7]) says $\frac{1}{n^H}\sum_{j=1}^{[nt]} g(\xi_j)$ converges as $n \to \infty$, in the sense of finite dimensional distributions, to the process

$$Z_H^k(t) = c(H,k)\int_{\mathbb{R}^k}\int_0^t \left(\prod_{j=1}^k (s-y_j)_+^{\left(-\frac{1}{2}+\frac{1-H}{k}\right)}\right) ds dB(y_1)\cdots dB(y_k), \tag{2}$$

where the above integral is a Wiener-Itô multiple integral of order k with respect to the standard Brownian motion $(B(y))_{y\in\mathbb{R}}$ and $c(H,k)$ is a positive normalization constant depending only on H and k. The process $(Z_H^k(t))_{t\geq 0}$ is called as the Hermite process and it is H self-similar in the sense that for any $c > 0$, $(Z_H^k(ct)) \overset{d}{=} (c^H Z_H^k(t))$ and it has stationary increments.

The fractional Brownian motion (which is obtained from (2) when $k = 1$) is the most used Hermite process to study evolution equations due to its large range of applications. When $k = 2$ in (2), Taqqu [6] named the process as the Rosenblatt process. The stationarity of increments, self-similarity and long range dependence (see Tindel, Tudor and Viens[8]) were made that the Rosenblatt process is very important in practical applications. However, it is noted that Rosenblatt process is not Gaussian. In fact, due their proprieties (long range dependence, self-similarity), the fractional Brownian motion process has large utilization in practical models, for instance in telecommunications and hydrology. So, many researchers prefer to use fractional Brownian motion than other processes because it is Gaussian and it facilitate calculations.

However in concrete situations when the Gaussianity is not plausible for the model, one can use the Rosenblatt process. In recent years, there exists many works that investigated on diverse theoretical aspects of the Rosenblatt process. For example, Leonenko and Ahn[9] gave the rate of convergence to the Rosenblatt process in the Non-Central Limit Theorem and the wavelet-type expansion has been presented by Abry and Pipiras [1]. Tudor [12] established, the representation as a Wiener-Itô multiple integral with respect to the Brownian motion on a finite interval and developed the stochastic calculus with respect to it by using both pathwise type calculus and Malliavin calculus (see also Maejima and Tudor [3]). For more details on Rosenblatt process, we refer the reader to Maejima and Tudor [4, 5]), Pipiras and Taqqu [10] and the references therein.

Consider a time interval $[0, T]$ with arbitrary fixed horizon T and let $\{Z_H(t), t \in [0, T]\}$ be a one-dimensional Rosenblatt process with parameter $H \in (\frac{1}{2}, 1)$. According to the work of Tudor [12], the Rosenblatt process with parameter $H > \frac{1}{2}$ can be written as

$$Z_H(t) = d(H) \int_0^t \int_0^t \left[\int_{y_1 \vee y_2}^t \frac{\partial K^{H'}}{\partial u}(u, y_1) \frac{\partial K^{H'}}{\partial u}(u, y_2) du \right] dB(y_1) dB(y_2), \tag{3}$$

where $K^H(t, s)$ is given by

$$K^H(t, s) = c_H s^{\frac{1}{2}-H} \int_s^t (u - s)^{H-3/2} u^{H-1/2} du \text{ for } t > s,$$

with $c_H = \sqrt{\frac{H(2H-1)}{\beta(2-2H, H-\frac{1}{2})}}$, $\beta(.,.)$ denotes the Beta function, $K^H(t, s) = 0$ when $t \le s$, $(B(t), t \in [0, T])$ is a Brownian motion, $H' = \frac{H+1}{2}$ and $d(H) = \frac{1}{H+1} \sqrt{\frac{H}{2(2H-1)}}$ is a normalizing constant. The covariance of the Rosenblatt process $\{Z_H(t), t \in [0, T]\}$ satisfy

$$\mathbf{E}(Z_H(t) Z_H(s)) = \frac{1}{2} \left(s^{2H} + t^{2H} - |s - t|^{2H} \right).$$

The covariance structure of the Rosenblatt process allows to construct Wiener integral with respect to it. We refer to Maejima and Tudor [3] for the definition of Wiener integral with respect to general Hermite processes and to Kruk, Russo, and Tudor [11] for a more general context (see also Tudor [12]). Note that

$$Z_H(t) = \int_0^T \int_0^T I(1_{[0,t]})(y_1, y_2) dB(y_1) dB(y_2),$$

where the operator I is defined on the set of functions $f : [0, T] \to \mathbb{R}$, which takes its values in the set of functions $g : [0, T]^2 \to \mathbb{R}^2$ and is given by

$$I(f)(y_1, y_2) = d(H) \int_{y_1 \vee y_2}^T f(u) \frac{\partial K^{H'}}{\partial u}(u, y_1) \frac{\partial K^{H'}}{\partial u}(u, y_2) du.$$

Let f be an element of the set $\mathcal{E}$ of step functions on $[0, T]$ of the form

$$f = \sum_{i=0}^{n-1} a_i 1_{(t_i, t_{i+1}]}, \; t_i \in [0, T].$$

Then, it is natural to define its Wiener integral with respect to Z_H as

$$\int_0^T f(u) dZ_H(u) := \sum_{i=0}^{n-1} a_i (Z_H(t_{i+1}) - Z_H(t_i)) = \int_0^T \int_0^T I(f)(y_1, y_2) dB(y_1) dB(y_2).$$

Let $\mathcal{H}$ be the set of functions f such that

$$\|f\|_{\mathcal{H}}^2 := 2 \int_0^T \int_0^T (I(f)(y_1, y_2))^2 dy_1 dy_2 < \infty.$$

It follows that (see Tudor[12])

$$\|f\|_{\mathcal{H}}^2 = H(2H - 1) \int_0^T \int_0^T f(u) f(v) |u - v|^{2H-2} du dv.$$

It has been proved in Maejima and Tudor [3] that the mapping

$$f \to \int_0^T f(u) dZ_H(u)$$

defines an isometry from $\mathcal{E}$ to $L^2(\Omega)$ and it can be extended continuously to an isometry from $\mathcal{H}$ to $L^2(\Omega)$ because $\mathcal{E}$ is dense in $\mathcal{H}$. We call this extension as the Wiener integral of $f \in \mathcal{H}$ with respect to Z_H. It is noted that the space $\mathcal{H}$ contains not only functions but its elements could be also distributions. Therefore it is suitable to know subspaces $|\mathcal{H}|$ of $\mathcal{H}$: $|\mathcal{H}| = \left\{ f : [0,T] \to \mathbb{R} | \int_0^T \int_0^T |f(u)||f(v)||u-v|^{2H-2} dudv < \infty \right\}$. The space $|\mathcal{H}|$ is not complete with respect to the norm $\|.\|_{\mathcal{H}}$ but it is a Banach space with respect to the norm

$$\|f\|^2_{|\mathcal{H}|} = H(2H-1) \int_0^T \int_0^T |f(u)||f(v)||u-v|^{2H-2} dudv.$$

As a consequence, we have

$$L^2([0,T]) \subset L^{1/H}([0,T]) \subset |\mathcal{H}| \subset \mathcal{H}.$$

For any $f \in L^2([0,T])$, we have

$$\|f\|^2_{|\mathcal{H}|} \leq 2HT^{2H-1} \int_0^T |f(s)|^2 ds$$

and

$$\|f\|^2_{|\mathcal{H}|} \leq C(H) \|f\|^2_{L^{1/H}([0,T])}, \tag{4}$$

for some constant $C(H) > 0$. Let $C(H) > 0$ stands for a positive constant depending only on H and its value may be different in different appearances.

Define the linear operator K^*_H from $\mathcal{E}$ to $L^2([0,T])$ by

$$(K^*_H f)(y_1, y_2) = \int_{y1 \vee y_2}^T f(t) \frac{\partial \mathcal{K}}{\partial t}(t, y_1, y_2) dt,$$

where $\mathcal{K}$ is the kernel of Rosenblatt process in representation (3)

$$\mathcal{K}(t, y_1, y_2) = 1_{[0,t]}(y_1) 1_{[0,t]}(y_2) \int_{y_1 \vee y_2}^t \frac{\partial K^{H'}}{\partial u}(u, y_1) \frac{\partial K^{H'}}{\partial u}(u, y_2) du.$$

Note that $(K^*_H 1_{[0,t]})(y_1, y_2) = \mathcal{K}(t, y_1, y_2) 1_{[0,t]}(y_1) 1_{[0,t]}(y_2)$. The operator K^*_H is an isometry between $\mathcal{E}$ to $L^2([0,T])$, which can be extended to the Hilbert space $\mathcal{H}$. In fact, for any $s, t \in [0,T]$ we have

$$\begin{aligned} \left\langle K^*_H 1_{[0,t]}, K^*_H 1_{[0,s]} \right\rangle_{L^2([0,T])} &= \langle \mathcal{K}(t,.,.) 1_{[0,t]}, \mathcal{K}(s,.,.) 1_{[0,s]} \rangle_{L^2([0,T])} \\ &= \int_0^{t \wedge s} \int_0^{t \wedge s} \mathcal{K}(t, y_1, y_2) \mathcal{K}(s, y_1, y_2) dy_1 dy_2 \\ &= H(2H-1) \int_0^t \int_0^s |u-v|^{2H-2} dudv \\ &= \langle 1_{[0,t]}, 1_{[0,s]} \rangle_{\mathcal{H}}. \end{aligned}$$

Moreover, for $f \in \mathcal{H}$, we have

$$Z_H(f) = \int_0^T \int_0^T (K^*_H f)(y_1, y_2) dB(y_1) dB(y_2).$$

Let $\{Z_n(t)\}_{n\in\mathbb{N}}$ be a sequence of two-sided one dimensional Rosenblatt process mutually independent on $(\Omega, \Im, \mathbb{P})$. We consider a $\mathcal{Y}$-valued stochastic process $Z_Q(t)$ given by the following series:

$$Z_Q(t) = \sum_{n=1}^{\infty} z_n(t) Q^{1/2} e_n, \quad t \geq 0.$$

Moreover, if Q is a non-negative self-adjoint trace class operator, then this series converges in the space $\mathcal{Y}$, that is, it holds that $Z_Q(t) \in L^2(\Omega, \mathcal{Y})$. Then, we say that the above $Z_Q(t)$ is a $\mathcal{Y}$-valued Q- Rosenblatt process with covariance operator Q. For instance, if $\{\sigma_n\}_{n\in\mathbb{N}}$ is a bounded sequence of non-negative real numbers such that $Qe_n = \sigma_n e_n$, by assuming that Q is a nuclear operator in $\mathcal{Y}$, then the stochastic process

$$Z_Q(t) = \sum_{n=1}^{\infty} z_n(t) Q^{1/2} e_n = \sum_{n=1}^{\infty} \sqrt{\sigma_n} z_n(t) e_n, \quad t \geq 0, \tag{5}$$

is well-defined as a $\mathcal{Y}$-valued Q- Rosenblatt process.

Definition 2.2. *(Tudor[12]). Let $\varphi : [0, T] \to L_Q^0(\mathcal{Y}, \mathcal{X})$ such that $\sum_{n=1}^{\infty} \|K_H^*(\varphi Q^{1/2} e_n)\|_{L^2([0,T];\mathcal{H})} < \infty$. Then, its stochastic integral with respect to the Rosenblatt process $Z_Q(t)$ is defined, for $t \geq 0$, as follows :*

$$\begin{aligned} \int_0^t \varphi(s) dZ_Q(s) : &= \sum_{n=1}^{\infty} \int_0^t \varphi(s) Q^{1/2} e_n dz_n(s) \\ &= \sum_{n=1}^{\infty} \int_0^t \int_0^t (K_H^*(\varphi Q^{1/2} e_n))(y_1, y_2) dB(y_1) dB(y_2). \end{aligned} \tag{6}$$

Lemma 2.3. *If $\psi : [0, T] \to L_2^0(\mathcal{Y}, \mathcal{X})$ satisfies $\int_0^t \left\|\psi(s)\right\|_{L_2^0}^2 < \infty$, then the above sum in (6) is well defined as an $\mathcal{X}$-valued random variable and we have*

$$\mathbf{E} \left\| \int_0^t \psi(s) dZ_H(s) \right\|^2 \leq 2Ht^{2H-1} \int_0^t \left\|\psi(s)\right\|_{L_2^0}^2 ds.$$

Partial Integrodifferential Equations in Banach Spaces: In this section, we recall some fundamental results needed to establish our main results. For the theory of resolvent operators we refer the reader to [2]. Throughout this paper, $\mathcal{X}$ is a Banach space, A and $\Upsilon(t)$ are closed linear operators on $\mathcal{X}$. Y represents the Banach space $\mathcal{D}(A)$ equipped with the graph norm defined by

$$|y|_Y := |Ay| + |y| \quad \text{for} \;\; y \in Y.$$

The notations $C([0, +\infty); Y), \mathcal{B}(Y, \mathcal{X})$ stand for the space of all continuous functions from $[0, +\infty)$ into Y, the set of all bounded linear operators from Y into $\mathcal{X}$, respectively. In what follows, we suppose the following assumptions:

- **(C1)** A represents the infinitesimal generator of a strongly continuous semigroup on $\mathcal{X}$.
- **(C2)** For all $t \geq 0$, $\Upsilon(s)(t)$ is a closed linear operator from $\mathcal{D}(A)$ to $\mathcal{X}$, and $\Upsilon(t) \in \mathcal{B}(Y, \mathcal{X})$. For any $y \in Y$, the map $t \to \Upsilon(t)y$ is bounded, differentiable and the derivative $t \to \Upsilon'(t)y$ is bounded and uniformly continuous on $\mathbb{R}^+$.

By Grimmer [], under assumptions **(C1)-(C2)**, the Cauchy problem

$$\begin{cases} \rho'(t) = A\rho(t) + \int_0^t \Upsilon(t-s)\rho(s) ds, & \text{for} \quad t \geq 0, \\ \rho(0) = v_0 \in \mathcal{X}. \end{cases} \tag{7}$$

has an associated resolvent operator of bounded linear operator valued function $\mathcal{R}(t) \in L(\mathcal{X})$ for $t \geq 0$.

Definition 2.4. *([2]). A resolvent operator for Eq. (7) is a bounded linear operator valued function $\mathcal{R}(t) \in L(\mathcal{X})$ for $t \geq 0$, satisfying the following properties:*

(i) $\mathcal{R}(0) = I$ and $|\mathcal{R}(t)| \leq Me^{\beta t}$ for some constants M and β.

(ii) For each $x \in \mathrm{H}$, $\mathcal{R}(t)x$ is strongly continuous for $t \geq 0$.

(iii) $\mathcal{R}(t) \in L(Y)$ for $t \geq 0$. For $x \in Y$, $\mathcal{R}(\cdot)x \in C^1([0, +\infty); \mathcal{X}) \cap C([0, +\infty); Y)$ and

$$\begin{aligned} \mathcal{R}'(t)x &= A\mathcal{R}(t)x + \int_0^t \Upsilon(s)(t-s)\mathcal{R}(s)xds \\ &= \mathcal{R}(t)Ax + \int_0^t \mathcal{R}(t-s)\Upsilon(s)xds \quad \text{for } t \geq 0. \end{aligned}$$

Hereafter, the resolvent operator for (7) is assumed to be continuous and exponentially stable.

(C3) The resolvent operator $\mathcal{R}(\cdot)$ is both norm continuous and exponentially stable if $M > 0$ and $\lambda \geq 1$ then

$$\|\mathcal{R}(t)\| \leq Me^{-\lambda t}, \quad t \geq 0.$$

Definition 2.5. *Let the space S_φ denote the set of all càdlàg processes $x(t)$ such that $x(t) = \varphi(t)$, $t \in [-r, 0]$, and there exist some constants $N^* = N^*(\varphi, a) > 0$ and $a > 0$ such that*

$$\mathbf{E}\|x(t)\|^2 \leq N^* e^{-at}, \quad \text{for all } t \geq 0.$$

Definition 2.6. *We denote by $\|\cdot\|_{S_\varphi}$ the norm in S_φ which is defined by*

$$\|x\|_{S_\varphi} = \sup_{t \geq 0} \mathbf{E}\|x(t)\|^2_{\mathcal{X}}, \quad \mathcal{X} \in S_\varphi.$$

Definition 2.7. *An $\mathcal{X}$-valued process $\{x(t) : t \in [-r, T]\}$ is called a mild solution of equation (1) if*
(i) $x(\cdot)$ has càdlàg path, and $\int_0^T \|x(t)\|^2 dt < \infty$ almost surely.
(ii) $x(t) = \varphi(t)$, $-r \leq t \leq 0$.
(iii) For arbitrary $t \in [0, T]$, $x(t)$ satisfies the following integral equation $\mathbb{P}$-a.s.

$$\begin{aligned} x(t) &= \mathcal{R}(t)\left[\varphi(0) + q(0, \varphi(-\tau(0)))\right] - q(t, x(\tau(t))) - \int_0^t \mathcal{R}(t-s)f(s, x(\rho))ds + \int_0^t \mathcal{R}(t-s)g(s)dZ_H(s) \\ &+ \int_0^t \int_{\mathcal{U}} \mathcal{R}(t-s)h(s, x(\delta), u)\widetilde{N}(ds, du) \end{aligned}$$

3. Main results

In this section, we consider existence, uniqueness and exponential stability of mild solution to equation (1). In order to prove the main result in this section, we require the following assumptions:

(C4) The functions $f(t, \cdot), q(t, \cdot)$ and $h(t, \cdot)$ satisfy global Lipschitz conditions: there exists $L, \bar{L} > 0$ such that for any $x, y \in \mathcal{X}$

$$\begin{aligned} \left\|f(t, x) - f(t, y)\right\|^2_{\mathcal{X}} &\leq L\left\|x - y\right\|^2_{\mathcal{X}}, \\ \left\|q(t, x) - q(t, y)\right\|^2_{\mathcal{X}} &\leq \bar{L}\left\|x - y\right\|^2_{\mathcal{X}}, \\ \int_{\mathcal{U}} \left\|h(t, x, u) - h(t, y, u)\right\|^2_{\mathcal{X}} \nu(du) &\leq L\left\|x - y\right\|^2_{\mathcal{X}}, \end{aligned}$$

and $f(t, 0) = q(t, 0) = h(t, 0, u) = 0$, $t \geq 0$.

(C5) The function q is continuous in the quadratic mean sense: for all $x, y \in \mathcal{D}([0,T]; L^2(\Omega,\mathcal{X}))$

$$\lim_{t\to s}\left\|q(t,x)-q(t,y)\right\|^2 = 0.$$

(C6) There exists a real number $\lambda > 0$ such that the function $q : [0,+\infty) \to \mathcal{L}_2^0(\mathcal{Y},\mathcal{X})$ satisfies

$$\int_0^{+\infty} e^{2\lambda s}\left\|q(s)\right\|_{L_2^0}^2 ds < \infty.$$

Theorem 3.1. *Suppose that* **(C1)-(C6)** *hold and that*

$$3\Big[\bar{L} + M^2L\lambda^{-2} + M^2L(2\lambda)^{-1}\Big] < 1.$$

If the initial value $\varphi(t)$ satisfies

$$\mathbf{E}\left\|\varphi(t)\right\|^2 \leq N_0\mathbf{E}\left|\varphi\right|_{\mathcal{D}}^2 e^{-at}, \quad t\in[-r,0],$$

for $N_0 > 0$ and $a > 0$, then for all $T > 0$, equation (1) has a unique mild solution on $[-r,T]$ and exponentially decays to zero mean square, i.e., there exists a pair of positive constants $a > 0$ and $N^* = N^*(\varphi,a)$ such that

$$\mathbf{E}\left\|x(t)\right\|^2 \leq N^*e^{-at}, \quad t \geq 0.$$

Proof. Define the mapping $\mathcal{G}$ on S_φ by

$$\mathcal{G}(x)(t) = \begin{cases} \varphi(t), \quad t\in[-r,0], \\ \mathcal{R}(t)\left[\varphi(0)+q(0,\varphi(-\tau(0)))\right] - q(t,x(\tau(t))) - \int_0^t \mathcal{R}(t-s)f(s,x(\rho))ds + \int_0^t \mathcal{R}(t-s)g(s)dZ_H(s) \\ + \int_0^t\int_{\mathcal{U}} \mathcal{R}(t-s)h(s,x(\delta),u)\widetilde{N}(ds,du) \end{cases}$$

Now, we prove that $\mathcal{G}$ has a fixed point in S_φ. The remaining argument for proving the above theorem are divided into the following three main steps:
Step 1: We show that $\mathcal{G}(S_\varphi) \subset S_\varphi$. Let $x \in S_\varphi$, then we have

$$\begin{aligned} \|\mathbf{E}\mathcal{G}(x)(t)\|_{\mathcal{X}}^2 &\leq 5\Big[\mathbf{E}\left\|\mathcal{R}(t)\left[\varphi(0)+q(0,\varphi(-\tau(0)))\right]\right\|_{\mathcal{X}}^2 + \mathbf{E}\left\|q(t,x(t-\tau(t)))\right\|_{\mathcal{X}}^2 \\ &+ \mathbf{E}\left\|\int_0^t \mathcal{R}(t-s)f(s,x(s-\rho(s)))ds\right\|_{\mathcal{X}}^2 + \mathbf{E}\left\|\int_0^t \mathcal{R}(t-s)g(s)dZ_H(s)\right\|_{\mathcal{X}}^2 \\ &+ \mathbf{E}\left\|\int_0^t \mathcal{R}(t-s)h(s,x(s-\delta(s)),u)\widetilde{N}(ds,du)\right\|_{\mathcal{X}}^2\Big] \\ &= 5\sum_{k=1}^{5}\Gamma_k. \end{aligned} \tag{8}$$

Without loss of generally, we may assume that $0 < a < \lambda$. Now, let us estimate the terms on right-hand side of inequality (8).

Let $N^* = N^*(\varphi,a) > 0$ and $a > 0$ such that

$$\begin{aligned} \Gamma_1 &\leq M^2\mathbf{E}\left\|\varphi(0)+q(0,\varphi(-\tau(0)))\right\|^2 \\ &\leq K_1e^{-\lambda t}, \end{aligned} \tag{9}$$

where $K_1 = M^2\mathbf{E}\left\|\varphi(0) + q(0, \varphi(-\tau(0)))\right\|^2 < +\infty$.
By using assumption **(C4)** we obtain

$$\begin{aligned}\Gamma_2 &\leq \mathbf{E}\left\|q(t, x(t-\tau(t))) - q(t,0)\right\|_{\mathcal{X}}^2\\ &\leq \bar{L}\mathbf{E}\left\|x(t-\tau(t))\right\|_{\mathcal{X}}^2\\ &\leq \bar{L}\left[N^* e^{-(t-\tau(t))} + \mathbf{E}\left\|\varphi(t-\tau(t))\right\|^2\right]\\ &\leq \bar{L}\left[N^* + N_0\mathbf{E}\left\|\varphi\right\|_{\mathcal{D}}^2\right]e^{-at}e^{ar}\\ &\leq K_2 e^{-at}. \end{aligned} \tag{10}$$

where $K_1 = \bar{L}\left[N^* + N_0\mathbf{E}\left\|\varphi\right\|_{\mathcal{D}}^2\right]e^{a\tau}$.
Applying Holder's inequality and **(C4)**, we have

$$\begin{aligned}\Gamma_3 &\leq \mathbf{E}\left\|\int_0^t \mathcal{R}(t-s)f(s, x(s-\rho(s)))ds\right\|^2\\ &\leq M^2L\lambda^{-1}\int_0^t e^{-\lambda(t-s)}\left[N^* + N_0\mathbf{E}\left|\varphi\right|_{\mathcal{D}}^2\right]e^{-as}e^{ar}ds\\ &\leq M^2L\lambda^{-1}(\lambda-a)^{-1}\left[N^* + N_0\mathbf{E}\left|\varphi\right|_{\mathcal{D}}^2\right]e^{-at}e^{ar}\\ &\leq K_3e^{-at}. \end{aligned} \tag{11}$$

where $K_3 = M^2L\lambda^{-1}(\lambda-a)^{-1}\left[N^* + N_0\mathbf{E}\left|\varphi\right|_{\mathcal{D}}^2\right]e^{ar}$
By using Lemma 2.1, we get

$$\begin{aligned}\Gamma_4 &\leq \mathbf{E}\left\|\int_0^t \mathcal{R}(t-s)g(s)dZ_H(s)\right\|^2\\ &\leq 2M^2Ht^{2H-1}\int_0^t e^{-2\lambda(t-s)}\left\|\sigma(s)\right\|_{L_2^0}^2 ds \end{aligned} \tag{12}$$

If $\gamma < \lambda$, then the following estimate holds

$$\begin{aligned}\Gamma_4 &\leq 2M^2Ht^{2H-1}\int_0^t e^{-2\lambda(t-s)}e^{-2\gamma(t-s)}e^{2\gamma(t-s)}\left\|g(s)\right\|_{L_2^0}^2 ds\\ &\leq 2M^2Ht^{2H-1}e^{-2\gamma t}\int_0^t e^{-2(\lambda-\gamma)(t-s)}e^{2\gamma s}\left\|g(s)\right\|_{L_2^0}^2 ds\\ &\leq 2M^2Ht^{2H-1}e^{-2\gamma t}\int_0^t e^{2\gamma s}\left\|g(s)\right\|_{L_2^0}^2 ds \end{aligned} \tag{13}$$

If $\gamma > \lambda$, then the following estimate holds

$$\Gamma_4 \leq 2M^2Ht^{2H-1}e^{-2\lambda t}\int_0^t e^{2\gamma s}\left\|g(s)\right\|_{L_2^0}^2 ds \tag{14}$$

In virtue of (12), (13) and (14), we obtain

$$\Gamma_4 \leq K_4e^{-min(\lambda,\gamma)t}, \tag{15}$$

where $K_4 = 2M^2Ht^{2H-1}\int_0^t e^{2\gamma s}\left\|g(s)\right\|^2_{L_2^0} ds < +\infty$.
Applying **(C3)** and **(C4)**, we get

$$\begin{aligned}\Gamma_5 &\leq \mathbf{E}\left\|\int_0^t\int_{\mathcal{U}}\mathcal{R}(t-s)h(s,x(s-\delta(t)),u)\widetilde{N}(ds,du)\right\|^2\\ &\leq M^2\mathbf{E}\int_0^t\int_{\mathcal{U}}e^{-2\lambda(t-s)}\left\|h(s,x(s-\delta(t)),u)\right\|^2 v(du)ds\\ &\leq M^2L\left[N^*+N_0\mathbf{E}\left|\varphi\right|^2_{\mathcal{D}}\right]e^{-at}e^{a\tau}\int_0^t e^{(-2\lambda+a)(t-s)}ds\\ &\leq M^2L\left[N^*+N_0\mathbf{E}\left|\varphi\right|^2_{\mathcal{D}}\right]e^{a\tau}(2\lambda-a)^{-1}e^{-at}\\ &\leq K_5e^{-at},\end{aligned} \tag{16}$$

where $K_5 = M^2L\left[N^*+N_0\mathbf{E}\left|\varphi\right|^2_{\mathcal{D}}\right]e^{a\tau}(2\lambda-a)^{-1} < +\infty$.
From the above inequalities (9),(10),(15),(16) and (8) together imply that

$$\mathbf{E}\left\|\mathcal{G}(x)(t)\right\|^2 \leq \bar{M}e^{-\bar{a}t},\ \ t\geq 0,\ \ \bar{M}>0,\ \ \bar{a}>0.$$

Step 2: Next, we show that $\mathcal{G}(x)(t)$ is *càdlàg* processes on S_φ.
Let $0<t<T$ and let $\gamma>0$ be sufficiently small. Then for $x(t)\in S_\varphi$, we have

$$\begin{aligned}&\mathbf{E}\left\|\mathcal{G}(x)(t+\gamma)-\mathcal{G}(x)(t)\right\|^2\\ &\leq 5\mathbf{E}\left\|\mathcal{R}(t+\gamma)-\mathcal{R}(t)\left[\varphi(0)-q(0,\varphi(-\tau(0)))\right]\right\|^2\\ &\quad+5\mathbf{E}\left\|q(t+\gamma,x(t+\gamma-\tau(t+\gamma)))-q(t,x(t-\tau(t)))\right\|^2\\ &\quad+5\mathbf{E}\left\|\int_0^{t+\gamma}\mathcal{R}(t+\gamma-s)f(s,x(s-\rho(s)))ds-\int_0^t\mathcal{R}(t-s)f(s,x(s-\rho(s)))ds\right\|^2\\ &\quad+5\mathbf{E}\left\|\int_0^{t+\gamma}\mathcal{R}(t+\gamma-s)g(s)dZ_H(s)-\int_0^t\mathcal{R}(t-s)g(s)dZ_H(s)\right\|^2\\ &\quad+5\mathbf{E}\left\|\int_0^{t+\gamma}\int_{\mathcal{U}}\mathcal{R}(t+\gamma-s)h(s,x(s-\delta(s)),u)\widetilde{N}(ds,du)-\int_0^t\int_{\mathcal{U}}\mathcal{R}(t-s)h(s,x(s-\delta(s)),u)\widetilde{N}(ds,du)\right\|^2\\ &=5\sum_{k=1}^{5}\mathbf{E}\left\|I_i(t+\gamma)-I_i(t)\right\|^2.\end{aligned}$$

By assumptions **(C4)**, we have

$$\begin{aligned}&\mathbf{E}\left\|I_5(t+\gamma)-I_5(t)\right\|^2\\ &\leq 2\mathbf{E}\left\|\int_0^t\int_{\mathcal{U}}\left[\mathcal{R}(t+\gamma-s)-\mathcal{R}(t-s)\right]h(s,x(\delta(s)),u)\widetilde{N}(ds,du)\right\|^2\\ &+2\mathbf{E}\left\|\int_0^t\int_{\mathcal{U}}\left[\mathcal{R}(t+\gamma-s)\right]h(s,x(\delta(s)),u)\widetilde{N}(ds,du)\right\|^2\\ &\leq 2M^2\left\|\mathcal{R}(\gamma)-I\right\|\mathbf{E}\int_0^t\int_{\mathcal{U}}e^{-2\lambda(t-s)}\left\|h(s,x(s-\delta(s)),u)\right\|^2 v(du)ds\\ &+2M^2\mathbf{E}\int_0^{t+\gamma}\int_{\mathcal{U}}e^{-2\lambda(t-s)}\left\|h(s,x(s-\delta(s)),u)\right\|^2 v(du)ds\end{aligned} \tag{17}$$

and

$$\begin{aligned}
&\mathbf{E}\int_0^t\int_{\mathcal{U}} e^{-2\lambda(t-s)}\,\|h(s,x(s-\delta(s)),u)\|^2\,v(du)ds\\
&\leq L\int_0^t e^{-2\lambda(t-s)}\left[N^*+N_0\mathbf{E}\left|\varphi\right|_{\mathcal{D}}^2\right]e^{-as}e^{a\tau}ds\\
&\leq L\int_0^t e^{-2\lambda(t-s)}\left[N^*+N_0\mathbf{E}\left|\varphi\right|_{\mathcal{D}}^2\right]e^{a\tau}(2\lambda-a)^{-1}e^{-at}. \qquad (18)
\end{aligned}$$

Inequality (18) implies that there exists a constant $B > 0$ such that

$$\mathbf{E}\int_0^t\int_{\mathcal{U}} e^{-2\lambda(t-s)}\,\|h(s,x(s-\delta(s)),u)\|^2\,v(du)ds \leq B. \qquad (19)$$

Using the norm continuity of the resolvent operator, inequalities (17) and (19) and Lebesgue's dominated convergence theorem, it follows that $\mathbf{E}\left\|I_5(t+\gamma)-I_5(t)\right\|_{\mathcal{X}}^2 \to 0$ as $\gamma \to 0$.

Similarly, we can verify that $\mathbf{E}\left\|I_k(t+\gamma)-I_k(t)\right\|_{\mathcal{X}}^2 \to 0$ as $\gamma \to 0$, $k = 1,2,3,4$. Th above arguments show that $t \to \mathcal{G}(x)(t)$ is *càdlàg* process. Thus $\mathcal{G}(S_\varphi) \subset S_\varphi$.

Step 3: Next, we show that $\mathcal{G} : S_\varphi \to S_\varphi$ is a contraction mapping. Now, fix $x, y \in S_\varphi$, we have

$$\begin{aligned}
&\mathbf{E}\left\|\mathcal{G}(x)(t)-\mathcal{G}(y)(t)\right\|_{\mathcal{X}}^2\\
&\leq 3\Bigg[\mathbf{E}\left\|q(t,x(t-\tau(t)))-q(t,y(t-\tau(t)))\right\|_{\mathcal{X}}^2\\
&\quad+\mathbf{E}\left\|\int_0^t \mathcal{R}(t-s)\left[f(s,x(s-\rho(s)))-f(s,y(s-\rho(s)))\right]ds\right\|_{\mathcal{X}}^2\\
&\quad+\mathbf{E}\left\|\int_0^t \mathcal{R}(t-s)\left[h(s,x(s-\delta(s)),u)-h(s,y(s-\delta(s)),u)\right]\widetilde{N}(ds,du)\right\|_{\mathcal{X}}^2\Bigg]\\
&=3\sum_{k=1}^{3}\Delta_k. \qquad (20)
\end{aligned}$$

Applying bf(C4), we have

$$\begin{aligned}
\Delta_1 &\leq \mathbf{E}\left\|q(t,x(t-\tau(t)))-q(t,y(t-\tau(t)))\right\|_{\mathcal{X}}^2\\
&\leq \bar{L}\sup_{t\geq 0}\mathbf{E}\left\|x(t)-y(t)\right\|_{\mathcal{X}}^2. \qquad (21)
\end{aligned}$$

Using Holder's inequality and bf(C4), we have

$$\begin{aligned}
\Delta_2 &\leq \mathbf{E}\left\|\int_0^t \mathcal{R}(t-s)\left[f(s,x(s-\rho(s)))-f(s,y(s-\rho(s)))\right]ds\right\|_{\mathcal{X}}^2\\
&\leq M^2L\left[\int_0^t e^{-\lambda(t-s)}ds\right]^2\sup_{t\geq 0}\mathbf{E}\left\|x(t)-y(t)\right\|_{\mathcal{X}}^2\\
&\leq M^2L\lambda^{-2}\sup_{t\geq 0}\mathbf{E}\left\|x(t)-y(t)\right\|_{\mathcal{X}}^2. \qquad (22)
\end{aligned}$$

Similarly

$$\begin{aligned}\Delta_3 &\leq \mathbf{E}\left\|\int_0^t \mathcal{R}(t-s)\left[h(s,x(s-\delta(s)),u)-h(s,y(s-\delta(s)),u)\right]\widetilde{N}(ds,du)\right\|_{\mathcal{X}}^2 \\ &\leq M^2L\left[\int_0^t e^{-\lambda(t-s)}ds\right]^2 \sup_{t\geq 0}\mathbf{E}\left\|x(t)-y(t)\right\|_{\mathcal{X}}^2 \\ &\leq M^2L(2\lambda)^{-1}\sup_{t\geq 0}\mathbf{E}\left\|x(t)-y(t)\right\|_{\mathcal{X}}^2. \end{aligned} \tag{23}$$

This inequalities (21),(22),(23) and (20) together imply

$$\mathbf{E}\left\|\mathcal{G}(x)(t)-\mathcal{G}(y)(t)\right\|_{\mathcal{X}}^2 \leq 3\Big[\bar{L}+M^2L\lambda^{-2}+M^2L(2\lambda)^{-1}\Big]\times\sup_{t\geq 0}\mathbf{E}\left\|x(t)-y(t)\right\|_{\mathcal{X}}^2. \tag{24}$$

Therefore, by the condition of the theorem it follows that $\mathcal{G}$ is a contractive mapping. Then the fixed point theorem implies that system (1) prossesses a unique mild solution and this solution is exponential stable in mean square. □

4. Example

Consider the neutral stochastic integrodifferential equation with Poisson jumps:

$$\begin{aligned}\frac{\partial}{\partial t}\left[x(t,\zeta)+\bar{q}(t,x(t-\tau),\zeta)\right] &= \frac{\partial^2}{\partial\zeta^2}\left[x(t,\zeta)+\bar{q}(t,x(t-\tau),\zeta)\right]+\int_0^t \bar{b}(t-s)\frac{\partial^2}{\partial\zeta^2}\left[x(t,\zeta)+\bar{q}(t,x(t-\tau),\zeta)\right]ds \\ &+ \bar{f}(t,x(t-\rho(t),\zeta))dt+e^{-t}dZ_H(t)+\int_{\mathcal{U}}\bar{h}(t,x(t-\delta(t)),u)\widetilde{N}(ds,du),\ t\geq 0, \\ x(t,0) &= x(t,\pi)=0, \quad t\geq 0. \\ x(s,\zeta) &= \varphi(s,\zeta), \quad \varphi(s,\cdot)\in L^2([0,\pi]), \quad -r\leq s\leq 0. \end{aligned} \tag{25}$$

where $r>0$. Let $\mathcal{X}=L^2(0,\pi)$ with the norm $\|\cdot\|$, $e_n=\sqrt{\frac{2}{n}}\sin nx$, $n=1,2,...$ denote the complete orthonormal basis in $\mathcal{X}$ and Z_H is a Rosenblatt process.

Define $A:\mathcal{D}(A)\subset\mathcal{X}\to\mathcal{X}$ by $A=\frac{\partial^2}{\partial z^2}$ with $\mathcal{D}(A)=\mathcal{X}^2(0,\pi)\bigcap\mathcal{X}_0^2(0,\pi)$. Then

$$Ax = -\sum_{n=1}^{\infty} n^2 < x, e_n > e_n, \quad x\in\mathcal{D}(A).$$

where $(e_n)_{n\in\mathbb{N}}$ is the orthonormal set of eigenvectors A. It is easy to prove that A is the infinitesimal generator of a strongly continuous semigroup $(S(t))_{t\geq 0}$ thus, **(C1)** is true and $\|S(t)\|\leq\frac{1}{(e^t)}\leq 1$. We denote by $\Theta(t):\mathcal{D}(A)\subset\mathcal{X}\to\mathcal{X}$ the operator defined by $\Theta(t)z=\bar{b}(t)Az$ for $t\geq 0$ and $z\in\mathcal{D}(A)$.

Let $\mathcal{H}=\mathbb{R}$ and let $\mathcal{U}=\{z\in\mathcal{H}:0<|z|\leq c, c>0\}$. We suppose that

(1) For $t\geq 0$, $\bar{q}(t,0)=\bar{f}(t,0)=\bar{h}(t,0,u)=0$.

(2) There exists a positive constant $\bar{k}$ such that

$$\left\|\bar{q}(t,x)-\bar{q}(t,y)\right\|_{\mathcal{X}}^2 \leq \bar{k}\left\|x-y\right\|_{\mathcal{X}}^2.$$

(3) There exist a positive constant $\bar{k}_1$ such that

$$\left\|\bar{f}(t,x)-\bar{f}(t,y)\right\|_{\mathcal{X}}^2 \vee \int_{\mathcal{U}}\left\|h(t,x,u)-h(t,y,u)\right\|_{\mathcal{X}}^2 v(du) \leq \bar{k}_1\left\|x-y\right\|_{\mathcal{X}}^2.$$

Define the functions $q, f : \mathbb{R}^+ \times \mathcal{D} \to \mathcal{X}$, $h : \mathbb{R}^+ \times \mathcal{H} \times \mathcal{D} \to \mathcal{X}$ by

$$\begin{aligned} q(t,x)(\zeta) &= \bar{q}(t, x(t-\tau(t),\zeta)), \\ f(t,x)(\zeta) &= \bar{f}(t, x(t-\tau(t),\zeta)), \\ g(s)(\zeta) &= e^{-t}, \\ h(t,x)(\zeta) &= \bar{h}(t, x(t-\tau(t),\zeta),u), \end{aligned}$$

for $\zeta \in [0,\pi]$. then equation (1) takes the following form:

$$\begin{aligned} d\left[x(t) + q(t, x(t-\tau(t)))\right] &= A\left[x(t) + q(t, x(t-\tau(t)))\right]dt \\ &\quad + \Big[\int_0^t \Upsilon(t-s)\Big[x(s) - q(s, x(s-\tau(s)))\Big] + f(t, x(t-\rho))dt\Big]ds \\ &\quad + g(t)dZ_H(t) + \int_{\mathcal{U}} h(t, x(t-\delta,u))\widetilde{N}(dt,du), \quad t \geq 0, \\ x(t) &= \varphi(t), \quad -r \leq t \leq 0, \end{aligned} \tag{26}$$

Moreover, if $\bar{b}$ is bounded and a C^1 function such that its derivative $\bar{b}'$ is bounded and uniformly continuous, then **(C1)**-**(C2)** are satisfied, and hence, by Theorem 2.1 in [24], equation (7) has a resolvent operator $(\mathcal{R}(t))_{t\geq 0}$ on $\mathcal{X}$. We assume moreover that there exists $x > a_1 > 1$ and $\bar{b}(t) < e^{-xt/a_1}$ for all $t \geq 0$. Thanks to Lemma 5.2 in [24], we have the following estimates $\|\mathcal{R}(t)\| \leq e^{-\lambda t}$, where $\lambda = 1 - 1/a$. Consequently, all hypothese of theorem 3. 1 are fulfilled. Therefore, equation (1) prossesses a unique mild solution, which is exponentially stable, provided that

$$\left[\bar{L} + M^2 L\lambda^{-2} + M^2 L(2\lambda)^{-1}\right] < \frac{1}{3}.$$

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