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A Soft Fixed Ppoint Theorem of the Weakly Soft susc-Contractions

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Abstract. In this article, we introduce a new notion of weakly soft **susc**-contraction in soft metric spaces and prove a soft fixed point theorem which assure the existence of soft fixed points for the type of weakly soft **susc**-contraction. Our results generalize many recent soft fixed point results in the literature.

1. Introduction and Preliminaries

One of the fundamental research subjects of both the nonlinear functional analysis and topology is the metric fixed point theory. Almost a centcury ago, the fundamentals of theorem of metric fixed point theory was given by Banach. He proved that every contraction possesses a unique fixed point in the setting of complete norm spaces. The analog of his theorem was proved by Caccioppoli in 1930. After then, Banach's fixed point theorem was improved and generalized in the setting of distinct abstract spaces, (see, e.g.[2]-[4],[6]-[8], [13]-[18], [19]-[23], [26],[27]).

Mathematics is established on exact notions where there is no ambiguity. Most of the practical problems in economics, engineering, social science, medical science and so forth cannot be dealt with classical methods because of various types of uncertainties present in these problems. In 1999, Molodtsov [25] introduced a new mathematical tool for dealing with uncertainties, called soft set theory. Soft set is a parameterized general mathematical tool which deal with a collection of approximate descriptions of objects. The approximate description contains two parts : one is a predicate and the other is an approximate value set. In classical mathematics, the mathematical model is constructed and it is complicated, and so the exact solution is not easily obtained. In the soft set theory, we have the opposite approach to solve this problem. The initial description of the object has an approximate nature, and we do not need to introduce the notion of exact solution. In the recent, many papers concerning soft set theory have been published; (see, e.g. [1], [5], [24], [28]). We recall the concepts of soft set theory as follow.

In the sequel, we let $\mathcal{U}$ be an initial universe, let $\mathcal{P}$ be a set of parameters, and let $2^{\mathcal{U}}$ be the collection of all subsets of $\mathcal{U}$.

Definition 1.1. *[25] Let A be a nonempty subset of $\mathcal{P}$. A soft set (T, A) on $\mathcal{U}$ is a set of the form*

$$(T, A) = \{(T(A), A) : \quad A \in \mathcal{P}\},$$

where $T : A \to 2^{\mathcal{U}}$ is a set-valued map such that $T(A) = \phi$ for all $\omega \notin A$, and T is called an approximate function of (T, A). The collection of all soft sets on $\mathcal{U}$ will be denoted by $\mathcal{S}(\mathcal{U})$.

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Definition 1.2. *[24] Let A and B be two nonempty subsets of* $\mathcal{P}$*. The intersection of two soft sets* (T_1, A) *and* (T_2, B) *on* $\mathcal{U}$ *is the soft set* (T_3, C) *where* $C = A \cap B$ *and for each* $p \in C$, $T_3(p) = T_1(p) \cap T_2(p)$*. This is denoted by*

$$(T_1, A)\widetilde{\cap}(T_2, B) = (T_3, C).$$

Definition 1.3. *[24] Let A and B be two nonempty subsets of* $\mathcal{P}$*. The union of two soft sets* (T_1, A) *and* (T_2, B) *on* $\mathcal{U}$ *is the soft set* (T_3, C) *where* $C = A \cup B$ *and for each* $p \in C$,

$$T_3(p) = \begin{cases} T_1(p), & \text{if } p \in A \setminus B; \\ T_2(p), & \text{if } p \in B \setminus A; \\ T_1(p) \cup T_2(p), & \text{if } p \in A \cap B. \end{cases}$$

This relationship is denoted by $(T_1, A)\widetilde{\cup}(T_2, B) = (T_3, C)$

Definition 1.4. *[24] A soft set* (T, A) *on* $\mathcal{U}$ *is said to be a null soft set denoted* $\widetilde{\phi}$ *if for all* $p \in A$, $F(p) = \phi$.

Definition 1.5. *[24] A soft set* (T, A) *on* $\mathcal{U}$ *is said to be an absolute soft set denoted* $\widetilde{A}$ *if for all* $p \in A$, $T(p) = A$.

Definition 1.6. *[24] The complement of soft set* (T, A) *on* $\mathcal{U}$ *is denoted by* (T^c, A) *where* $T^c : X \to 2^{\mathcal{U}}$ *is a mapping given by* $T^c(p) = \mathcal{U} \setminus T(p)$ *for all* $p \in A$.

Applying the concepts of soft set theory, the authors [31] introduced the following notion of soft real numbers.

Definition 1.7. *[31] Let* $B(\mathbb{R})$ *be the collection of all nonempty bounded subsets of* $\mathbb{R}$*, where* $\mathbb{R}$ *denote by the set of all real numbers. Then the mapping* $\varphi : \mathcal{P} \to B(\mathbb{R})$ *is called a soft real mapping. If* $(\varphi, \mathcal{P})$ *is a singleton soft set, then identifying* $(\varphi, \mathcal{P})$ *with the corresponding soft element, it will be called a soft real number and denoted* $\widetilde{a}$, $\widetilde{b}$, $\tilde{c}$ *etc. And,* $\overline{0}$ *and* $\overline{1}$ *are the soft real numbers where* $\overline{0}(\omega) = 0$, $\overline{1}(\omega) = 1$ *for all* $\omega \in \mathcal{P}$*, respectively. Furthermore, we let* $\mathbb{R}^+(\mathcal{P})$ *be denoted by the set of all non-negative soft real numbers.*

Definition 1.8. *[31] For two soft real numbers, we have*

(1) $\widetilde{a}\widetilde{\leq}\widetilde{b}$ *if* $\widetilde{a}(\omega)\widetilde{\leq}\widetilde{b}(\omega)$*, for all* $\omega \in \mathcal{P}$;
(2) $\widetilde{a}\widetilde{\geq}\widetilde{b}$ *if* $\widetilde{a}(\omega)\widetilde{\geq}\widetilde{b}(\omega)$*, for all* $\omega \in \mathcal{P}$;
(3) $\widetilde{a}\widetilde{<}\widetilde{b}$ *if* $\widetilde{a}(\omega)\widetilde{<}\widetilde{b}(\omega)$*, for all* $\omega \in \mathcal{P}$;
(4) $\widetilde{a}\widetilde{>}\widetilde{b}$ *if* $\widetilde{a}(\omega)\widetilde{>}\widetilde{b}(\omega)$*, for all* $\omega \in \mathcal{P}$.

Definition 1.9. *Let* $\varphi : \mathbb{R}^+(\mathcal{P}) \to \mathbb{R}^+(\mathcal{P})$ *be a soft real mapping. Then*

(1) φ *is said to be soft continuous at* $\widetilde{\tau}\widetilde{\in}\mathbb{R}^+(\mathcal{P})$*, if for every* $\widetilde{\gamma}\widetilde{>}\overline{0}$*, there exists* $\widetilde{\delta}\widetilde{>}\overline{0}$ *such that* $\overline{0}\widetilde{<}\widetilde{a} - \widetilde{\tau}\widetilde{<}\widetilde{\delta}$ *implies*

$$\varphi(\widetilde{a}) - \varphi(\widetilde{\tau})\widetilde{<}\widetilde{\gamma}.$$

Moreover, if $\varphi : \mathbb{R}^+(\mathcal{P}) \to \mathbb{R}^+(\mathcal{P})$ *is soft continuous at every soft real number* $\widetilde{\tau}$ *of* $\mathbb{R}^+(\mathcal{P})$*, then we call* φ *a soft continuous mapping.*

(2) φ *is said to be soft upper semicontinuous, if*

$$\lim_{\widetilde{\gamma}\to\widetilde{\gamma_0}} \sup(\varphi(\widetilde{\gamma}))\widetilde{\leq}\varphi(\widetilde{\gamma_0}), \text{ for all } \widetilde{\gamma_0}\widetilde{\in}\mathbb{R}^+(\mathcal{P}).$$

On the other hand, the authors [29] introduced the following notion of soft points.

Definition 1.10. *[29] A soft set* $(T, \mathcal{P})$ *on* $\mathcal{U}$ *is said to be a soft point if there is exactly one* $p \in \mathcal{P}$ *such that* $T(p) = \{x\}$ *for some* $x \in \mathcal{U}$ *and* $T(\omega) = \phi$ *for all* $\omega \in \mathcal{P} \setminus \{p\}$*. It will be denoted by* $\widetilde{x_p}$.

Let $\widetilde{X}$ be the absolute soft set, where $(T,\mathcal{P}) = \widetilde{X}$, and let $\mathcal{SP}(\widetilde{X})$ be the collection of all soft points of $\widetilde{X}$. In [30], the authors introduced the notion of soft metric on the soft set $\widetilde{X}$.

Definition 1.11. *[30] A mapping $\widetilde{\sigma} : \mathcal{SP}(\widetilde{X}) \times \mathcal{SP}(\widetilde{X}) \to \mathbb{R}^+(\mathcal{P})$ is said to be a soft metric on the soft set $\widetilde{X}$ if $\widetilde{\sigma}$ satisfies the following coditions:*

(i) $\widetilde{\sigma}(\widetilde{x_{p_1}}, \widetilde{y_{p_2}})\widetilde{\geq}\overline{0}$, *for all* $\widetilde{x_{p_1}}, \widetilde{y_{p_2}}\widetilde{\in}\tilde{X}$;
(ii) $\widetilde{\sigma}(\widetilde{x_{p_1}}, \widetilde{y_{p_2}}) = \overline{0}$ *if and only if* $\widetilde{x_{p_1}} = \widetilde{y_{p_2}}$;
(iii) $\widetilde{\sigma}(\widetilde{x_{p_1}}, \widetilde{y_{p_2}}) = \widetilde{\sigma}(\widetilde{y_{p_2}}, \widetilde{x_{p_1}})$, *for all* $\widetilde{x_{p_1}}, \widetilde{y_{p_2}}\widetilde{\in}\widetilde{X}$;
(iv) $\widetilde{\sigma}(\widetilde{x_{p_1}}, \widetilde{z_{p_3}})\widetilde{\leq}\widetilde{\sigma}(\widetilde{x_{p_1}}, \widetilde{y_{p_2}}) + \widetilde{\sigma}(\widetilde{y_{p_2}}, \widetilde{z_{p_3}})$, *for all* $\widetilde{x_{p_1}}, \widetilde{y_{p_2}}, \widetilde{z_{p_3}}\widetilde{\in}\widetilde{X}$.

The soft set $\widetilde{X}$ with a soft metric $\widetilde{\sigma}$ is called a soft metric space and denoted by $(\widetilde{X},\widetilde{\sigma},\mathcal{P})$.

Definition 1.12. *[30] Let $\{\widetilde{x_{p,n}}\}_n$ be a sequence of soft points in a soft metric space $(\widetilde{X},\widetilde{\sigma},\mathcal{P})$. Then the sequence $\{\widetilde{x_{p,n}}\}_n$ is said to be soft convergent in $(\widetilde{X},\widetilde{\sigma},\mathcal{P})$ if there is a soft point $\widetilde{_v}\widetilde{\in}\widetilde{X}$ such that*

$$\lim_{n\to\infty} \widetilde{\sigma}(\widetilde{x_{p,n}}, \widetilde{z_v}) = \overline{0}.$$

Definition 1.13. *[30] Let $\{\widetilde{x_{p,n}}\}_n$ be a sequence of soft points in a soft metric space $(\widetilde{X},\widetilde{\sigma},\mathcal{P})$. Then $\{\widetilde{x_{p,n}}\}_n$ is said to be a soft Cauchy sequence in $(\widetilde{X},\widetilde{\sigma},\mathcal{P})$ if*

$$\lim_{i,j\to\infty} \widetilde{\sigma}(\widetilde{x_{p,i}}, \widetilde{x_{p,j}}) = \overline{0}.$$

Definition 1.14. *[30] A soft metric space $(\widetilde{X},\widetilde{\sigma},\mathcal{P})$ is complete if every Cauchy sequence in $\widetilde{X}$ converges to some soft point of $\widetilde{X}$.*

Later, we introduce the notion of soft continuous mapping on soft metric spaces, as follows:

Definition 1.15. *Let $(\widetilde{X},\widetilde{d},\mathcal{P})$ and $(\tilde{Y},\widetilde{\sigma},\mathcal{P}')$ be two soft metric spaces and $(f,\varphi) : (\widetilde{X},\widetilde{d},\mathcal{P}) \to (\widetilde{Y},\widetilde{\sigma},\mathcal{P}')$. Then we call the soft mapping (f,φ) is soft continuous at the point $\widetilde{x_\lambda}\widetilde{\in}\mathcal{SP}(\widetilde{X})$, if for every $\widetilde{\gamma}\widetilde{>}\overline{0}$, there exists $\widetilde{\delta}\widetilde{>}\overline{0}$ such that $\widetilde{d}(\widetilde{x_\lambda}, \widetilde{y_\mu})\widetilde{<}\widetilde{\delta}$ implies that $\widetilde{\sigma}((f,\varphi)(\widetilde{x_\lambda}), (f,\varphi)(\widetilde{x_\lambda}))\widetilde{<}\widetilde{\gamma}$.*

Fixed point theory plays a fundamental role in mathematics and applied sciences, such as optimization, mathematical models and economic theories. Also, this theory have been applied to show the existence and uniqueness of the solutions of differential equations, integral equations and many other branches of mathematics. Extensions of fixed point theorems to the soft set theory have been studied by some authors.In 2013, Wardowski [32] first established the natural first fixed-point results in the soft set theory, and many authors studied soft metric versions of several important fixed point theorems by using soft set theory, (see, e.g. [9], [10], [12],[15]). In this article, we introduce a new notion of weakly soft **susc**-contraction in soft metric spaces and prove a soft fixed point theorem which assure the existence of soft fixed points for the type of weakly soft **susc**-contractions. Our results generalize many recent soft fixed point results in the literature.

2. Main Results

The following two propositions will play important roles for the soft fixed point theorm 2.5.

Proposition 2.1. *Let* $\varphi : \mathbb{R}^+(\mathcal{P}) \to \mathbb{R}^+(\mathcal{P})$ *be soft upper semicontinuous with* $\varphi(\widetilde{\gamma})\widetilde{<}\widetilde{\gamma}$ *for all* $\widetilde{\gamma}\widetilde{\in}\mathbb{R}^+(\mathcal{P})\backslash\overline{0}$ *and* $\varphi(\overline{0}) = \overline{0}$*. Then there exists a strictly increasing, soft continuous function* $\psi : \mathbb{R}^+(\mathcal{P}) \to \mathbb{R}^+(\mathcal{P})$ *such that*

$$\varphi(\widetilde{\gamma})\widetilde{\leq}\psi(\widetilde{\gamma})\widetilde{<}\widetilde{\gamma}, \textit{ for all } \widetilde{\gamma}\widetilde{\in}\mathbb{R}^+(\mathcal{P})\backslash\overline{0}.$$

Proof. Let $\phi : \mathbb{R}^+(\mathcal{P}) \to \mathbb{R}^+(\mathcal{P})$ be denoted by

$$\phi(\widetilde{\gamma}) = \widetilde{\gamma} - \varphi(\widetilde{\gamma}), \text{ for all } \widetilde{\gamma}\widetilde{\in}\mathbb{R}^+(\mathcal{P}).$$

Since φ is soft upper semicontinuous, we have that ϕ is soft lower semicontinuous, and hence it attains it's minimum in any closed bounded interval of $\mathbb{R}^+(\mathcal{P})$.

For each $\widetilde{n}\widetilde{\in}\mathbb{R}^+(\mathcal{P})$ and $\widetilde{n}$ is soft positive integer, we define four soft sequences $\widetilde{a_{\widetilde{n}}}$, $\widetilde{b_{\widetilde{n}}}$, $\widetilde{c_{\widetilde{n}}}$ and $\widetilde{d_{\widetilde{n}}}$ of soft real numbers by

(1) $\widetilde{a_{\widetilde{n}}} = \min_{\widetilde{\gamma}\widetilde{\in}[\widetilde{n},\widetilde{n+1}]} \phi(\widetilde{\gamma})$;
(2) $\widetilde{b_{\widetilde{n}}} = \min_{\widetilde{\gamma}\widetilde{\in}[\widetilde{\frac{1}{n+1}},\widetilde{\frac{1}{n}}]} \phi(\widetilde{\gamma})$;
(3) $\widetilde{c_{\widetilde{1}}}, \widetilde{d_{\widetilde{1}}} = \min\{\widetilde{a_{\widetilde{1}}}, \widetilde{b_{\widetilde{1}}}\}$;
(4) $\widetilde{c_{\widetilde{n}}} = \min\{\widetilde{c_{\widetilde{1}}}, \widetilde{a_{\widetilde{1}}}, \widetilde{a_{\widetilde{2}}}, \cdots, \widetilde{a_{\widetilde{n}}}\}$, for all $\widetilde{n}\widetilde{\geq}\widetilde{2}$;
(5) $\widetilde{d_{\widetilde{n}}} = \min\{\widetilde{c_{\widetilde{1}}}, \widetilde{b_{\widetilde{1}}}, \widetilde{b_{\widetilde{2}}}, \cdots, \widetilde{b_{\widetilde{n}}}, \widetilde{\frac{1}{n(n+1)}}\}$, for all $\widetilde{n}\widetilde{\geq}\widetilde{2}$,

and let $\psi : \mathbb{R}^+(\mathcal{P}) \to \mathbb{R}^+(\mathcal{P})$ satisfy the following:

(i) $\psi(\widetilde{0}) = \widetilde{0}$, $\psi(\widetilde{n})\widetilde{=}\widetilde{n} - \widetilde{c_{\widetilde{n}}}$, $\psi(\widetilde{\frac{1}{n}}) = \widetilde{\frac{1}{n}} - \widetilde{d_{\widetilde{n}}}$;
(ii) $\psi(\widetilde{\gamma}) = (\widetilde{\gamma} - \widetilde{n})\psi(\widetilde{n+1}) + (\widetilde{n+1-\gamma})\psi(\widetilde{n})$, if $\widetilde{n}\widetilde{\leq}\widetilde{\gamma}\widetilde{\leq}\widetilde{n+1}$;
(iii) $\psi(\widetilde{\gamma}) = \psi(\frac{1}{n+1}) + \widetilde{n}(\widetilde{n+1})[\psi(\widetilde{\frac{1}{n}}) - \psi(\frac{1}{n+1})(\widetilde{\gamma} - \frac{1}{n+1})$.

Then, we are easily to conclude that $\psi : \mathbb{R}^+(\mathcal{P}) \to \mathbb{R}^+(\mathcal{P})$ is strictly increasing, soft continuous and

$$\varphi(\widetilde{\gamma})\widetilde{\leq}\psi(\widetilde{\gamma})\widetilde{<}\widetilde{\gamma}, \text{ for all } \widetilde{\gamma}\widetilde{\in}\mathbb{R}^+(\mathcal{P})\backslash\overline{0}.$$

□

Proposition 2.2. *Let* $\varphi : \mathbb{R}^+(\mathcal{P}) \to \mathbb{R}^+(\mathcal{P})$ *be soft upper semicontinuous with* $\varphi(\widetilde{\gamma})\widetilde{<}\widetilde{\gamma}$ *for all* $\widetilde{\gamma}\widetilde{\in}\mathbb{R}^+(\mathcal{P})\backslash\overline{0}$ *and* $\varphi(\overline{0}) = \overline{0}$*. Then* $\lim_{n\to\infty} \varphi^n(\widetilde{\gamma}) = \overline{0}$ *for all* $\widetilde{\gamma}\widetilde{>}\overline{0}$*, where* φ^n *denotes the n-th iteration of* φ*.*

Proof. Applying Proposition 2.1, there exists a strictly increasing, soft continuous function $\psi : \mathbb{R}^+(\mathcal{P}) \to \mathbb{R}^+(\mathcal{P})$ such that

$$\varphi(\widetilde{\gamma})\widetilde{\leq}\psi(\widetilde{\gamma})\widetilde{<}\widetilde{\gamma}, \text{ for all } \widetilde{\gamma}\widetilde{\in}\mathbb{R}^+(\mathcal{P})\backslash\overline{0},$$

and the function ψ is invertible.

Let $\widetilde{\gamma}\widetilde{>}\overline{0}$ be fixed. We claim that $\lim_{n\to\infty} \psi^n(\widetilde{\gamma}) = \overline{0}$. Suppose, on the the contrary, that $\lim_{n\to\infty} \psi^{-n}(\widetilde{\gamma}) = \widetilde{\eta}$ for some positive soft real number $\widetilde{\eta}$. Then we can conclude

$$\begin{aligned}\widetilde{\eta} &= \lim_{n\to\infty} \psi^{-n}(\widetilde{\gamma}) \\ &= \psi^{-1}(\lim_{n\to\infty} \psi^{-n+1}(\widetilde{\gamma})) \\ &= \psi^{-1}(\widetilde{\gamma}) \\ &\widetilde{>}\widetilde{\eta}.\end{aligned}$$

This implies a contradiction. So $\lim_{n\to\infty} \psi^n(\widetilde{\gamma}) = \overline{0}$, and hence we have that $\lim_{n\to\infty} \varphi^n(\widetilde{\gamma}) = \overline{0}$ for all $\widetilde{\gamma}\widetilde{>}\overline{0}$. □

We introduce the following new notion of weakly soft **susc**-contractions.

Definition 2.3. *Let $(\widetilde{X},\tilde{\sigma},\mathcal{P})$ be a soft metric space and let $\varphi:\mathbb{R}^+(\mathcal{P})\to\mathbb{R}^+(\mathcal{P})$ be soft upper semicontinuous with $\varphi(\widetilde{\gamma})\widetilde{<}\widetilde{\gamma}$ for all $\widetilde{\gamma}\widetilde{\in}\mathbb{R}^+(\mathcal{P})\backslash\overline{0}$ and $\varphi(\overline{0})=\overline{0}$. A mapping $(T,\chi):(\widetilde{X},\widetilde{\sigma},\mathcal{P})\to(\widetilde{X},\widetilde{\sigma},\mathcal{P})$ is called a weakly soft φ-**susc**-contraction if for each soft points $\widetilde{x_\mu},\widetilde{y_\tau}\widetilde{\in}\mathcal{SP}(\widetilde{X})$,*

$$\begin{aligned}&\widetilde{\sigma}((T,\chi)(\widetilde{x_\mu}),(T,\chi)(\widetilde{y_\tau}))\\ \widetilde{\leq}\,&\varphi\left(\max\left\{\widetilde{\sigma}(\widetilde{x_\mu},\widetilde{y_\tau}),\widetilde{\sigma}(\widetilde{x_\mu},(T,\chi)(\widetilde{x_\mu})),\widetilde{\sigma}(\widetilde{y_\tau},(T,\chi)(\widetilde{y_\tau}))\right\}\right).\end{aligned}$$

Example 2.4. *Let $(\widetilde{\mathbb{R}},\widetilde{\sigma},\mathcal{P})$ be a soft metric space with the following metrics*

$$\sigma_\chi(p,\tau)=\begin{cases}\max\{|p|,|\tau|\}, & \text{if } p\neq\tau\\ 0, & \text{if } p=\tau\end{cases};$$

$$\sigma(x,y)=|x-y|,\quad \text{and}$$

$$\widetilde{\sigma}(\widetilde{x_p},\widetilde{y_\tau})=\frac{4}{5}\sigma_\chi(p,\tau)+\sigma(x,y),$$

where $\mathcal{P}=[0,\infty)$, $\chi(t)=\frac{3}{4}t$ for $t\in[0,\infty)$.

Let $\varphi,\psi:\mathbb{R}^+(\mathcal{P})\to\mathbb{R}^+(\mathcal{P})$ denote by

$$\varphi(\widetilde{\gamma})=\begin{cases}\frac{5}{6}\widetilde{\gamma} & \text{if } \widetilde{\gamma}\widetilde{\geq}\widetilde{1}\\ \frac{2}{3}\widetilde{\gamma} & \text{if } \widetilde{0}\widetilde{\leq}\widetilde{\gamma}\widetilde{<}\widetilde{1}\end{cases},$$

and

$$\psi(\widetilde{\gamma})=\frac{5}{6}\widetilde{\gamma}.$$

Then φ is soft upper semicontinuous, and ψ is strictly increasing and soft continuous with

$$\varphi(\widetilde{\gamma})\widetilde{\leq}\psi(\widetilde{\gamma})\widetilde{<}\widetilde{\gamma},\ \text{for all } \widetilde{\gamma}\widetilde{\in}\mathbb{R}^+(\mathcal{P})\backslash\overline{0},\ \text{and } \varphi(\overline{0})=\psi(\overline{0})=\overline{0}$$

Let $(T,\chi)(x_p)=\widetilde{\frac{2}{5}x_{\chi(p)}}$. Then

$$\begin{aligned}&\tilde{\sigma}((T,\chi)(\widetilde{x_p}),(T,\chi)(\widetilde{y_\tau}))\\ =&\widetilde{\sigma}(\widetilde{\frac{2}{5}x_{\frac{3}{4}p}},\widetilde{\frac{2}{5}y_{\frac{3}{4}\tau}})\\ =&\frac{3}{5}\max\{|p|,|\tau|\}+\frac{2}{5}|x-y|,\end{aligned}$$

and

(1) $\widetilde{\sigma}(\widetilde{x_p},\widetilde{y_\tau})=\frac{4}{5}\max\{|p|,|\tau|\}+|x-y|,$
(2) $\widetilde{\sigma}(\widetilde{x_p},\widetilde{\frac{2}{5}x_{\frac{3}{4}p}})=\frac{4}{5}\max\{|p|,|\frac{3}{4}p|\}+|x-\frac{2}{5}x|=\frac{4}{5}|p|+\frac{3}{5}|x|,$
(3) $\widetilde{\sigma}(\widetilde{y_\tau},\widetilde{\frac{2}{5}y_{\frac{3}{4}\tau}})=\frac{4}{5}\max\{|\tau|,|\frac{3}{4}\tau|\}+|y-\frac{2}{5}y|=\frac{4}{5}|\tau|+\frac{3}{5}|y|.$

*Thus, (T,χ) is a weakly soft φ-**susc**-contraction on the soft metric space $(\widetilde{\mathbb{R}},\widetilde{\sigma},\mathcal{P})$.*

Applying Proposition 2.1 and Proposition 2.2, we prove the following soft fixed point theorem for the type of weakly soft **susc**-contractions.

Theorem 2.5. *Let $(\widetilde{X},\widetilde{\sigma},\mathcal{P})$ be a complete soft metric space, and let $\varphi:\mathbb{R}^+(\mathcal{P})\to\mathbb{R}^+(\mathcal{P})$ be soft upper semicontinuous with $\varphi(\widetilde{\gamma})\widetilde{<}\widetilde{\gamma}$ for all $\widetilde{\gamma}\widetilde{\in}\mathbb{R}^+(\mathcal{P})\backslash\overline{0}$ and $\varphi(\overline{0})=\overline{0}$. Let $(T,\chi):(\widetilde{X},\tilde{\sigma},\mathcal{P})\to(\widetilde{X},\widetilde{\sigma},\mathcal{P})$ be a weakly soft φ-**susc**-contraction on $(\widetilde{X},\widetilde{\sigma},\mathcal{P})$. Then (T,χ) has a soft fixed point, that is, there exists a soft point $\widetilde{x_\tau}\widetilde{\in}\mathcal{SP}(\widetilde{X})$ such that $(T,\chi)(\widetilde{x_\tau})=\widetilde{x_\tau}$.*

Proof. Let $\widetilde{x^0_{\tau_0}}\widetilde{\in}\mathcal{SP}(\widetilde{X})$ be given. For each $n \in \mathbb{N} \cup \{0\}$, we put

$$\widetilde{x^{n+1}_{\tau_{n+1}}} = ((T,\chi)(\widetilde{x^n_{\tau_n}})) = (T^{n+1}(\widetilde{x^0_{\tau_0}}))_{\chi^{n+1}(\tau_0)}.$$

Then for each $n \in \mathbb{N} \cup \{0\}$, we have

$$\begin{aligned}&\widetilde{\sigma}(\widetilde{x^n_{\tau_n}}, \widetilde{x^{n+1}_{\tau_{n+1}}}) = \widetilde{\sigma}((T,\chi)(\widetilde{x^{n-1}_{\tau_{n-1}}}), (T,\chi)(\widetilde{x^n_{\tau_n}}))\\ &\widetilde{\leq}\varphi\left(\max\left\{\widetilde{\sigma}(\widetilde{x^{n-1}_{\tau_{n-1}}}, \widetilde{x^n_{\tau_n}}), \widetilde{\sigma}(\widetilde{x^{n-1}_{\tau_{n-1}}}, (T,\chi)(\widetilde{x^{n-1}_{\tau_{n-1}}})), \widetilde{\sigma}(\widetilde{x^n_{\tau_n}}, (T,\chi)(\widetilde{x^n_{\tau_n}}))\right\}\right)\\ &= \varphi\left(\max\left\{\widetilde{\sigma}(\widetilde{x^{n-1}_{\tau_{n-1}}}, \widetilde{x^n_{\tau_n}}), \widetilde{\sigma}(\widetilde{x^{n-1}_{\tau_{n-1}}}, \widetilde{x^n_{\tau_n}}), \widetilde{\sigma}(\widetilde{x^n_{\tau_n}}, \widetilde{x^{n+1}_{\tau_{n+1}}})\right\}\right).\end{aligned}$$

If $\widetilde{\sigma}(\widetilde{x^{n-1}_{\tau_{n-1}}}, \widetilde{x^n_{\tau_n}})\widetilde{\leq}\widetilde{\sigma}(\widetilde{x^n_{\tau_n}}, \widetilde{x^{n+1}_{\tau_{n+1}}})$, then by the conditions of the function φ, we have

$$\widetilde{\sigma}(\widetilde{x^n_{\tau_n}}, \widetilde{x^{n+1}_{\tau_{n+1}}})\widetilde{\leq}\varphi(\widetilde{\sigma}(\widetilde{x^n_{\tau_n}}, \widetilde{x^{n+1}_{\tau_{n+1}}}))\widetilde{<}\widetilde{\sigma}(\widetilde{x^n_{\tau_n}}, \widetilde{x^{n+1}_{\tau_{n+1}}}),$$

which implies a contradiction. Thus, for each $n \in \mathbb{N} \cup \{0\}$,

$$\widetilde{\sigma}(\widetilde{x^n_{\tau_n}}, \widetilde{x^{n+1}_{\tau_{n+1}}})\widetilde{\leq}\varphi(\widetilde{\sigma}(\widetilde{x^{n-1}_{\tau_{n-1}}}, \widetilde{x^n_{\tau_n}})).$$

By induction, we can conclude that

$$\begin{aligned}&\widetilde{\sigma}(\widetilde{x^n_{\tau_n}}, \widetilde{x^{n+1}_{\tau_{n+1}}})\\ &\widetilde{\leq}\varphi(\widetilde{\sigma}(\widetilde{x^{n-1}_{\tau_{n-1}}}, \widetilde{x^n_{\tau_n}}))\\ &\widetilde{\leq}\varphi^2(\widetilde{\sigma}(\widetilde{x^{n-2}_{\tau_{n-2}}}, \widetilde{x^{n-1}_{\tau_{n-1}}}))\\ &\widetilde{\leq}\cdots\\ &\widetilde{\leq}\varphi^n(\widetilde{\sigma}(\widetilde{x^0_{\tau_0}}, \widetilde{x^1_{\tau_1}})).\end{aligned}$$

Since φ is soft upper semicontinuous and by Proposition 2.2, we can get

$$\lim_{n\to\infty}\widetilde{\sigma}(\widetilde{x^n_{\tau_n}}, \widetilde{x^{n+1}_{\tau_{n+1}}}) = \overline{0}.$$

We claim that the sequence $\{\widetilde{x^n_{\tau_n}}\}$ is soft Cauchy, that is, the following result (∗) holds:
For every $\widetilde{\varepsilon}$, there exists $n_0 \in \mathbb{N}$ such that if $n, k \geq n_0$, then

$$\widetilde{\sigma}(\widetilde{x^{k_r}_{\tau_{k_r}}}, \widetilde{x^{n_r}_{\tau_{n_r}}})\widetilde{<}\widetilde{\varepsilon}. \tag{*}$$

Suppose that the above statesment (∗) is false. Then there exists $\widetilde{\epsilon}\widetilde{>}\overline{0}$ such that, for any $r \in \mathbb{N}$, there are $n_r, k_r \in \mathbb{N}$ with $n_r > k_r \geq r$ satisfying that

(i) n_r is even and k_r is odd;
(ii) $\widetilde{\sigma}(\widetilde{x^{k_r}_{\tau_{k_r}}}, \widetilde{x^{n_r}_{\tau_{n_r}}})\widetilde{\geq}\widetilde{\epsilon}$
(iii) n_r is the smallest even number such that the condition (ii) holds.

By (i) and (ii), we conclude that

$$\begin{aligned}&\widetilde{\epsilon}\widetilde{\leq}\widetilde{\sigma}(\widetilde{x^{k_r}_{\tau_{k_r}}}, \widetilde{x^{n_r}_{\tau_{n_r}}})\\ &\widetilde{\leq}\widetilde{\sigma}(\widetilde{x^{k_r}_{\tau_{k_r}}}, \widetilde{x^{n_r-2}_{\tau_{n_r-2}}}) + \widetilde{\sigma}(\widetilde{x^{n_r-2}_{\tau_{n_r-2}}}, \widetilde{x^{n_r-1}_{\tau_{n_r-1}}}) + \widetilde{\sigma}(\widetilde{x^{n_r-1}_{\tau_{n_r-1}}}, \widetilde{x^{n_r}_{\tau_{n_r}}})\\ &\widetilde{\leq}\widetilde{\epsilon} + \widetilde{\sigma}(\widetilde{x^{n_r-2}_{\tau_{n_r-2}}}, \widetilde{x^{n_r-1}_{\tau_{n_r-1}}}) + \widetilde{\sigma}(\widetilde{x^{n_r-1}_{\tau_{n_r-1}}}, \widetilde{x^{n_r}_{\tau_{n_r}}}).\end{aligned}$$

Letting $r \to \infty$, we obtain that

$$\lim_{r\to\infty} \widetilde{\sigma}(\widetilde{x^{k_r}_{\tau_{k_r}}}, \widetilde{x^{n_r}_{\tau_{n_r}}}) = \widetilde{\epsilon}.$$

On the other hand, we have

$$\begin{aligned}
&\widetilde{\epsilon}\widetilde{\leq}\widetilde{\sigma}(\widetilde{x^{k_r-1}_{\tau_{k_r-1}}}, \widetilde{x^{n_r-1}_{\tau_{n_r-1}}})\\
&\widetilde{\leq}\widetilde{\sigma}(\widetilde{x^{k_r-1}_{\tau_{k_r-1}}}, \widetilde{x^{n_r-3}_{\tau_{n_r-3}}}) + \widetilde{\sigma}(\widetilde{x^{n_r-3}_{\tau_{n_r-3}}}, \widetilde{x^{n_r-2}_{\tau_{n_r-2}}}) + \widetilde{\sigma}(\widetilde{x^{n_r-2}_{\tau_{n_r-2}}}, \widetilde{x^{n_r-1}_{\tau_{n_r-1}}})\\
&\widetilde{\leq}\widetilde{\epsilon} + \widetilde{\sigma}(\widetilde{x^{n_r-3}_{\tau_{n_r-3}}}, \widetilde{x^{n_r-2}_{\tau_{n_r-2}}}) + \widetilde{\sigma}(\widetilde{x^{n_r-2}_{\tau_{n_r-2}}}, \widetilde{x^{n_r-1}_{\tau_{n_r-1}}}).
\end{aligned}$$

Letting $r \to \infty$, we obtain that

$$\lim_{r\to\infty} \widetilde{\sigma}(\widetilde{x^{k_r-1}_{\tau_{k_r-1}}}, \widetilde{x^{n_r-1}_{\tau_{n_r-1}}}) = \widetilde{\epsilon}.$$

By above arguement, we obtain that

$$\begin{aligned}
&\widetilde{\sigma}(\widetilde{x^{k_r}_{\tau_{k_r}}}, \widetilde{x^{n_r}_{\tau_{n_r}}})\\
&=\widetilde{\sigma}((T,\chi)(\widetilde{x^{k_r-1}_{\tau_{k_r-1}}}), (T,\chi)(\widetilde{x^{n_r-1}_{\tau_{n_r-1}}}))\\
&\widetilde{\leq}\varphi\left(\max\left\{\widetilde{\sigma}(\widetilde{x^{k_r-1}_{\tau_{k_r-1}}}, \widetilde{x^{n_r-1}_{\tau_{n_r-1}}}), \widetilde{\sigma}(\widetilde{x^{k_r-1}_{\tau_{k_r-1}}}, (T,\chi)(\widetilde{x^{k_r-1}_{\tau_{kr-1}}})), \widetilde{\sigma}(\widetilde{x^{n_r-1}_{\tau_{n_r-1}}}, (T,\chi)(\widetilde{x^{n_r-1}_{\tau_{n_r-1}}}))\right\}\right)\\
&\widetilde{\leq}\varphi\left(\max\left\{\widetilde{\sigma}(\widetilde{x^{k_r-1}_{\tau_{k_r-1}}}, \widetilde{x^{n_r-1}_{\tau_{n_r-1}}}), \widetilde{\sigma}(\widetilde{x^{k_r-1}_{\tau_{k_r-1}}}, \widetilde{\xi^{k_r}_{\tau_{kr}}}), \widetilde{\sigma}(\widetilde{x^{n_r-1}_{\tau_{n_r-1}}}, \widetilde{x^{n_r}_{\tau_{n_r}}})\right\}\right).
\end{aligned}$$

Applying Proposition 2.1, there exists a strictly increasing, soft continuous function $\psi : \mathbb{R}^+(\mathcal{P}) \to \mathbb{R}^+(\mathcal{P})$ such that

$$\begin{aligned}
&\widetilde{\sigma}(\widetilde{x^{k_r}_{\tau_{k_r}}}, \widetilde{x^{n_r}_{\tau_{n_r}}})\\
&\widetilde{\leq}\varphi\left(\max\left\{\widetilde{\sigma}(\widetilde{x^{k_r-1}_{\tau_{k_r-1}}}, \widetilde{x^{n_r-1}_{\tau_{n_r-1}}}), \widetilde{\sigma}(\widetilde{x^{k_r-1}_{\tau_{k_r-1}}}, \widetilde{x^{k_r}_{\tau_{kr}}}), \widetilde{\sigma}(\widetilde{x^{n_r-1}_{\tau_{n_r-1}}}, \widetilde{x^{n_r}_{\tau_{n_r}}})\right\}\right)\\
&\widetilde{\leq}\psi\left(\max\left\{\widetilde{\sigma}(\widetilde{x^{k_r-1}_{\tau_{k_r-1}}}, \widetilde{x^{n_r-1}_{\tau_{n_r-1}}}), \widetilde{\sigma}(\widetilde{x^{k_r-1}_{\tau_{k_r-1}}}, \widetilde{x^{k_r}_{\tau_{kr}}}), \widetilde{\sigma}(\widetilde{x^{n_r-1}_{\tau_{n_r-1}}}, \widetilde{x^{n_r}_{\tau_{n_r}}})\right\}\right).
\end{aligned}$$

Letting $r \to \infty$, we get $\widetilde{\epsilon}\widetilde{\leq}\psi(\widetilde{\epsilon})\widetilde{<}\widetilde{\epsilon}$. This implies a contradiction. So the sequence $\{\widetilde{x^{n}_{\tau_n}}\}$ is soft Cauchy. Since $(\widetilde{X}, \widetilde{\sigma}, \mathcal{P})$ is complete, there exists $\widetilde{x^*_\tau}\widetilde{\in}\widetilde{X}$ such that

$$\widetilde{x^{n}_{\tau_n}} \to \widetilde{x^*_\tau} \text{ as } n \to \infty,$$

that is,

$$\widetilde{\sigma}(\widetilde{x^{n}_{\tau_n}}, \widetilde{x^*_\tau}) \to \overline{0} \text{ as } n \to \infty.$$

And, we also have that

$$\begin{aligned}
&\widetilde{\sigma}((T,\chi)(\widetilde{x^*_\tau}), \widetilde{x^*_\tau})\\
&\widetilde{\leq}\widetilde{\sigma}((T,\chi)(\widetilde{x^{n}_{\tau_n}}), (T,\chi)(\widetilde{x^*_\tau})) + \widetilde{\sigma}((T,\chi)(\widetilde{x^{n}_{\tau_n}}), \widetilde{x^*_\tau})\\
&\widetilde{<}\varphi\left(\max\left\{\widetilde{\sigma}(\widetilde{x^{n}_{\tau_n}}, \widetilde{x^*_\tau}), \widetilde{\sigma}(\widetilde{x^{n}_{\tau_n}}, (T,\chi)(\widetilde{x^{n}_{\tau_n}})), \widetilde{\sigma}(\widetilde{x^*_\tau}, (T,\chi)(\widetilde{x^*_\tau}))\right\}\right) + \widetilde{d}(\widetilde{x^{n+1}_{\tau_{n+1}}}, \widetilde{x^*_\tau})\\
&\widetilde{<}\varphi\left(max\left\{\widetilde{\sigma}(\widetilde{\xi^{n}_{\tau_n}}, \widetilde{\xi^*_\tau}), \widetilde{\sigma}(\widetilde{\xi^{n}_{\tau_n}}, \widetilde{\xi^{n+1}_{\tau_{n+1}}}), \widetilde{\sigma}(\widetilde{x^*_\tau}, (T,\chi)(\widetilde{x^*_\tau}))\right\}\right) + \widetilde{\sigma}(\widetilde{x^{n+1}_{\tau_{n+1}}}, \widetilde{x^*_\tau})\\
&\widetilde{<}\psi\left(\max\left\{\widetilde{\sigma}(\widetilde{x^{n}_{\tau_n}}, \widetilde{x^*_\tau}), \widetilde{\sigma}(\widetilde{x^{n}_{\tau_n}}, \widetilde{x^{n+1}_{\tau_{n+1}}}), \widetilde{\sigma}(\widetilde{x^*_\tau}, (T,\chi)(\widetilde{x^*_\tau}))\right\}\right) + \widetilde{\sigma}(\widetilde{x^{n+1}_{\tau_{n+1}}}, \widetilde{x^*_\tau}).
\end{aligned}$$

Taking $n \to \infty$, we get that

$$\begin{aligned}
&\widetilde{\sigma}((T,\chi)(\widetilde{x^*_\tau}), \widetilde{\xi^*_\tau}) \\
&\widetilde{\leq}\psi\left(\max\left\{\overline{0}, \overline{0}, , \widetilde{\sigma}(\widetilde{x^*_\tau}, (T,\chi)(\widetilde{x^*_\tau}))\right\}\right) + \overline{0} \\
&\widetilde{<}\widetilde{\sigma}((T,\chi)(\widetilde{x^*_\tau}), \widetilde{x^*_\tau}),
\end{aligned}$$

and this is a contradiction unless $\widetilde{\sigma}((T,\chi)(\widetilde{x^*_\tau}), \widetilde{x^*_\tau}) = \overline{0}$. Thus, $(T,\chi)(\tilde{x}^*_\tau) = \tilde{x}^*_\tau$, and hence, $\tilde{x}^*_\tau$ is a soft fixed point of the mapping (T,χ). □

Example 2.6. *Applying Example 2.4, we can conclude that $\overline{0_0}$ is a soft fixed point of the weakly soft φ-**susc**-contraction (T,χ).*

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