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Corrigendum to "On the Mazur-Ulam Theorem in Non-Archimedean Fuzzy Anti-2-Normed Spaces"

Javier Cabello Sánchez[a], José Navarro Garmendia[a]

[a]*Departamento de Matemáticas, Universidad de Extremadura, Avenida de Elvas s/n, 06006 Badajoz, Spain*

Abstract. In this note we correct a paper by D. Kang ("On the Mazur-Ulam theorem in non-Archimedean fuzzy anti-2-normed spaces", Filomat, 2017). The research in that paper applies to what the author calls strictly convex spaces. Nevertheless, we prove that this notion is void: there is no single space that satisfies the definition.

1. Introduction

In [2, Theorem 3.5], we can see the following non-Archimedean fuzzy version of the classical Mazur-Ulam theorem:

Theorem 1.1. *Let X, Y be non-Archimedean fuzzy anti-2-normed spaces over a certain type of non-Archimedean field $\mathbb{K}$. If both X and Y are strictly convex, then any centred fuzzy 2-isometry $f : X \to Y$ is an additive map.*

The class of strictly convex fuzzy anti-2-normed spaces was introduced in that paper (cf. [2, Definition 2.5]), although there appeared no examples there. In this note, we prove:

Proposition 1.2. *There are no such strictly convex spaces at all.*

As a consequence, the statement of the above theorem is void.

Let us also point out that the situation is similar in the different non-Archimedean versions of the Mazur-Ulam theorem that have appeared in recent years (cf. [1] and references in [2]).

2. Voidness of the Notion of Strictly Convex Fuzzy Space

We reflect the following definition just for the sake of completeness.

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Email addresses: coco@unex.es (Javier Cabello Sánchez), navarrogarmendia@unex.es (José Navarro Garmendia)

Definition 2.1. A *non-Archimedean fuzzy anti-2-normed space* is a linear space X over a non-Archimedean field $(\mathbb{K}, |\cdot|)$ together with a fuzzy anti-2-norm; that is to say, with a function $N : X^2 \times \mathbb{R} \to [0,1]$ such that, for all $x, y \in X$ and all $s, t \in \mathbb{R}$:

(A2N-1) if $t \leq 0$, then $N(x, y, t) = 1$,

(A2N-2)] if $t > 0$, then $N(x, y, t) = 0$ if and only if x and y are linearly dependent,

(A2N-3) $N(x, y, t) = N(y, x, t)$,

(A2N-4) $N(x, cy, t) = N(x, y, t/|c|)$ for any non-zero $c \in \mathbb{K}$

(A2N-5) $N(x, y+z, \max\{s,t\}) \leq \max\{N(x, y, s), N(x, z, t)\}$,

(A2N-6) $N(x, y, *)$ is a non-increasing function of $\mathbb{R}$ and $\lim_{t\to\infty} N(x, y, t) = 0$.

Definition 2.2. ([2, Definition 2.5]) A non-Archimedean fuzzy anti-2-normed space (X, N) is *strictly convex* if

$$N(x, y, s) = N(x, z, t) = N(x, y+z, \max\{s,t\}) \quad \Rightarrow \quad y = z \text{ and } s = t\,. \tag{1}$$

Proposition 2.3. *There are no strictly convex spaces at all –in the sense of the above definition.*

Proof. Any non-Archimedean fuzzy anti-2-norm N satisfies, by (A2N-2), that, for any $x \in X$, and any $s, t \in (0, \infty)$,

$$N(x, -x, s) = N(x, 2x, t) = N(x, x, \max\{s,t\}) = 0.$$

As these equalities are valid *for any* $s, t \in (0, \infty)$, it is clear that no fuzzy anti-2-normed space (X, N) may fulfil condition (1) –not even the zero linear space. □

References

[1] J. Cabello Sánchez, J. Navarro Garmendia, A precision on the concept of strict convexity in non-Archimedean analysis, Nonlinear Anal. 207 (2021) 112304.

[2] D. Kang, On the Mazur-Ulam theorem in non-Archimedean fuzzy anti-2-normed spaces, Filomat 31 (2017) 4915–4923.