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Mappings on Intuitionistic Fuzzy Topology of Soft Sets

Sadi Bayramov[a], Cigdem Gunduz[b]

[a]*Department of Algebra and Geometry, Baku State University, Baku Az1148, Azerbaican*
[b]*Department of Mathematics, Kocaeli University, Kocaeli, 41380-Turkey*

Abstract. The present study is devoted to describe the concepts of continuous mapping, open mapping and closed mapping by using soft points on intuitionistic fuzzy topological spaces. Along, continuous mapping, open mapping and closed mapping on intuitionistic fuzzy topological spaces and their characterizations are also introduced. At the end, some of the crucial properties of the proposed concepts are investigated. Taking advantage of intuitionistic fuzzy topology, we obtain the family of soft topologies and normal topologies. It is clear that the category of intuitionistic fuzzy topological spaces is an extension both the category of soft topological spaces and the category of topological spaces.

1. Introduction

After D. Molodtsov [18] introduced the concept of soft set theory which is completely a new approach for modeling uncertainty, topological structures of soft sets have been studied by some authors in recent years. Since the concept of soft set provides a natural framework for generalizing many concepts of general topology to what may be called soft topological spaces, M. Shabir and M. Naz [21] presented the concept of soft topological spaces which are defined over an initial universe with a fixed set of parameters. It is observed in the last few years that a large number of papers were devoted to the study of soft topological spaces in [1–3, 8, 10, 12, 13, 17, 20, 22].

Since soft set theory has a rich potential, researches on soft set theory and its applications in various fields are progressing rapidly in [4–6, 15, 16]. C. Gunduz(Aras) and S. Bayramov [11] introduced intuitionistic fuzzy soft modules and investigated some important properties. It is known that intuitionistic fuzzy set is a generalization of fuzzy set. After intuitionistic fuzzy set was introduced by K. Atanassov [7], T.K. Mondal and S. K. Samanta [19] initiated a concept of intuitionistic gradation of openness on fuzzy subsets of a nonempty set X. They proved that the category of intuitionistic fuzzy topological spaces and gradation preserving mappings are a topological category. C. Liang and C. Yan [14] defined base and subbase on intuitionistic $I-$fuzzy topological spaces. They studied the base and subbase on the product of intuitionistic $I-$fuzzy topological spaces.

The present study is devoted to describe the concepts of continuous mapping, open mapping and closed mapping by using soft points on intuitionistic fuzzy topological spaces. Along, continuous mapping, open mapping and closed mapping on intuitionistic fuzzy topological spaces and their characterizations are also introduced. At the end, some of the crucial properties of the proposed concepts are investigated.

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Email addresses: baysadi@gmail.com (Sadi Bayramov), carasgunduz@gmail.com (Cigdem Gunduz)

2. Preliminaries

In this section we give some necessary definitions for soft sets. Throughout this paper X and E denote an initial universe set and a set of all parameters, respectively. By A, we will denote a subset of E, i.e $A \subset E$. $SS(X,E)$ denotes the family of all soft sets over X with a fixed set of parameters E.

Definition 2.1. ([18]) A pair (F,A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$.

Definition 2.2. ([16]) For two soft sets (F,A) and (G,B) over X, (F,A) is called a soft subset of (G,B) if

1. $A \subset B$,
2. $F(e)$ and $G(e)$ are identical approximations for each $e \in A$.

This relationship is denoted by $(F,A)\widetilde{\subseteq}(G,B)$.

Definition 2.3. ([16]) The intersection of soft sets (F,A), (G,B) over X is the soft set (H,C) and $H(e) = F(e) \cap G(e)$ for each $e \in C$, where $C = A \cap B$. The soft set is denoted by $(F,A)\widetilde{\cap}(G,B) = (H,C)$.

Definition 2.4. ([16]) The union of soft sets (F,A), (G,B) over X is the soft set (H,C) and

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in \ B - A, \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

for each $e \in C$, where $C = A \cup B$. The soft set is denoted by $(F,A)\widetilde{\cup}(G,B) = (H,C)$.

Definition 2.5. ([21]) The complement of a soft set (F,E), denoted by $(F,E)^c$, is defined $(F,E)^c = (F^c, E)$, where $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X \backslash F(e)$ for all $e \in E$ and F^c is called the soft complement function of F.

Definition 2.6. ([16]) A soft set (F,E) over X is said to be a null soft set if $F(e) = \varnothing$ for all $e \in E$. It is denoted by Φ.

Definition 2.7. ([16]) A soft set (F,E) over X is said to be an absolute soft set if $F(e) = X$ for all $e \in E$. It is denoted by $\widetilde{X}$.

Definition 2.8. ([13]) Let (X,E) and $(Y,E^{'})$ be two soft sets, $f : X \to Y$ and $g : E \to E^{'}$ be two mappings and $(F,A) \subset (X,E)$. Then $(f_g) : (X,E) \to (Y,E^{'})$ is called a soft mapping which is defined as: $(f_g)((F,A)) = f(F)_{g(a)}$ is a soft set in $(Y,E^{'})$ given by

$$f(F)(e^{'}) = \begin{cases} f\left(\underset{e \in g^{-1}(e') \cap A}{\cup} F(e)\right), & \text{if } g^{-1}(e') \cap A \neq \emptyset, \\ \emptyset, & \text{otherwise}, \end{cases}$$

for $e^{'} \in B \subseteq E^{'}$, where $B = g(A) \subseteq E^{'}$. $(f(F), g(A))$ is called soft image of (F,A).

Definition 2.9. ([13]) Let (X,E) and $(Y,E^{'})$ be two soft sets, $(f_g) : (X,E) \to (Y,E^{'})$ be a soft mapping and $(G,C)\widetilde{\subseteq}(Y,E^{'})$. Then $(f_g)^{-1}((G,C)) = f^{-1}(G)_{g^{-1}(C)}$ is a soft set in (X,E) which is defined as:

$$f^{-1}(G)(e) = \begin{cases} f^{-1}(G(g(e))), & \text{if } g(e) \in C, \\ \emptyset, & \text{otherwise}, \end{cases}$$

for $e \in D \subseteq E$, where $D = g^{-1}(C)$. $(f_g)^{-1}((G,C))$ is called soft inverse image of (G,C).

Definition 2.10. ([21]) Let τ be the collection of soft sets over X. Then τ is said to be a soft topology on X if

1) $\Phi, \widetilde{X}$ belong to τ;

2) the union of any number of soft sets in τ belongs to τ;

3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. Then members of τ are said to be soft open sets in X.

Definition 2.11. ([21]) Let (X, τ, E) be a soft topological space over X. A soft set (F, E) over X is said to be a soft closed in X if its complement $(F, E)^c$ belongs to τ.

Definition 2.12. ([8]) Let (F, E) be a soft set over X. The soft set (F, E) is called a soft point, denoted by (x_e, E), if for the element $e \in E$, $F(e) = \{x\}$ and $F(e') = \emptyset$ for all $e' \in E - \{e\}$ (briefly denoted by x_e).

3. Mappings on Intuitionistic Fuzzy Topology of Soft Sets

Definition 3.1. ([9]) A mapping $(\tau, \tau^*) : SS(X, E) \to [0, 1]$ is called an intuitionistic fuzzy topology on X (briefly IFT) if it satisfies the following conditions:

(i) $\tau(F, E) + \tau^*(F, E) \le 1; \forall (F, E) \in SS(X, E)$,

(ii) $\tau(\Phi) = \tau\left(\widetilde{X}\right) = 1, \tau^*(\Phi) = \tau^*\left(\widetilde{X}\right) = 0$,

(iii) $\tau\left((F, E)\widetilde{\cap}(G, E)\right) \ge \tau(F, E) \wedge \tau(G, E)$, $\tau^*\left((F, E)\widetilde{\cap}(G, E)\right) \le \tau^*(F, E) \vee \tau^*(G, E)$, $\forall (F, E), (G, E) \in SS(X, E)$,

(iv) $\tau\left(\bigcup_{i\in\Delta}(F_i, E)\right) \ge \bigwedge_{i\in\Delta} \tau(F_i, E)$, $\tau^*\left(\bigcup_{i\in\Delta}(F_i, E)\right) \le \bigvee_{i\in\Delta} \tau^*(F_i, E)$, $\forall (F_i, E) \in SS(X, E), i \in \Delta$.

The quadruple (X, E, τ, τ^*) is called an intuitionistic fuzzy topological space of soft sets. Intuitionistic fuzzy topological space (X, E, τ, τ^*) is denoted by $IFTS$.

Example 3.2. Let $X = R, E = \{*\}$ and soft sets $F_k : E \to P(X)$ are defined as follows: for $n \in \mathbb{N}$

$$
\begin{aligned}
F_1(*) &= [0; 1] \\
F_2(*) &= [0; 2] \\
&\cdots \\
F_k(*) &= [0; k] \\
&\cdots
\end{aligned}
$$

Now we consider $(\tau, \tau^*) : SS(X; E) \to [0; 1]$ as follows:

From the definition of (τ, τ^*); (i) and (ii) are clear.

(iii) Let $k \le m$. Then

$$
\begin{aligned}
\tau\left((F_k, E)\widetilde{\cap}(F_m, E)\right) &= \tau(F_k, E) = 1 - \frac{1}{k}, \\
\tau(F_k, E) \wedge \tau(F_m, E) &= \left(1 - \frac{1}{k}\right) \wedge \left(1 - \frac{1}{m}\right) = \left(1 - \frac{1}{k}\right).
\end{aligned}
$$

Thus $\tau\left((F_k, E)\widetilde{\cap}(F_m, E)\right) \ge \tau(F_k, E) \wedge \tau(F_m, E)$ is obtained.

$$
\begin{aligned}
\tau^*\left((F_k, E)\widetilde{\cap}(F_m, E)\right) &= \tau^*(F_k, E) = \frac{1}{k}, \\
\tau^*(F_k, E) \vee \tau^*(F_m, E) &= \frac{1}{k} \vee \frac{1}{m} = \frac{1}{k}.
\end{aligned}
$$

Thus $\tau^*\left((F_k, E)\widetilde{\cap}(F_m, E)\right) \le \tau^*(F_k, E) \vee \tau^*(F_m, E)$ is obtained.

(iv)

$$\tau\left(\underset{k\in\mathbb{N}}{\cup}(F_k,E)\right) = \sup_{k\in\mathbb{N}}\left\{1-\frac{1}{k}\right\},$$

$$\underset{k\in\mathbb{N}}{\wedge}\tau(F_k,E) = \inf_{k\in\mathbb{N}}\left\{1-\frac{1}{k}\right\}.$$

So $\tau\left(\underset{k\in\mathbb{N}}{\cup}(F_k,E)\right) \geq \underset{k\in\mathbb{N}}{\wedge}\tau(F_k,E)$ is obtained.

$$\tau^*\left(\underset{k\in\mathbb{N}}{\cup}(F_k,E)\right) = \inf_{k\in\mathbb{N}}\left\{\frac{1}{k}\right\},$$

$$\underset{k\in\mathbb{N}}{\vee}\tau^*(F_k,E) = \sup_{k\in\mathbb{N}}\left\{\frac{1}{k}\right\}.$$

Hence $\tau^*\left(\underset{k\in\mathbb{N}}{\cup}(F_k,E)\right) \leq \underset{k\in\mathbb{N}}{\vee}\tau^*(F_k,E)$. Then (τ,τ^*) is an intuitionistic fuzzy topology on X.

Definition 3.3. ([9]) Let (X,E,τ,τ^*) be an *IFTS*.

a) $(\beta,\beta^*) : SS(X,E) \to [0,1]$ is called a base of (τ,τ^*) if (β,β^*) satisfies the following condition:

$$\tau(F,E) = \underset{\underset{i\in\Delta}{\cup}(G_i,E)=(F,E)}{\vee}\ \underset{i\in\Delta}{\wedge}\beta(G_i,E)$$

and

$$\tau^*(F,E) = \underset{\underset{i\in\Delta}{\cup}(G_i,E)=(F,E)}{\wedge}\ \underset{i\in\Delta}{\vee}\beta^*(G_i,E),\ \forall(F,E)\in SS(X,E).$$

b) $(\varphi,\varphi^*) : SS(X,E) \to [0,1]$ is called a subbase of (τ,τ^*) if $(\widetilde{\varphi},\widetilde{\varphi^*}) : SS(X,E) \to [0,1]$ is a base of (τ,τ^*) if

$$\widetilde{\varphi}(F,E) = \underset{\underset{j\in J}{\cap}(G_j,E)=(F,E)}{\vee}\ \underset{j\in J}{\wedge}\varphi(G_j,E),$$

$$\widetilde{\varphi^*}(F,E) = \underset{\underset{j\in J}{\cap}(G_j,E)=(F,E)}{\wedge}\ \underset{j\in J}{\vee}\varphi^*(G_j,E)$$

are satisfied, where J is a finite set.

Definition 3.4. Let (X,E,τ,τ^*) and (Y,E',γ,γ^*) be two *IFTSs* and $(f,\varphi) : (X,E,\tau,\tau^*) \to (Y,E',\gamma,\gamma^*)$ be a mapping. Then (f,φ) is called a continuous mapping at the soft point $x_e \in (X,E)$ if there exists $x_e \in (F,E) \in SS(X,E)$ such that

$$\tau(F,E) \geq \gamma(G,E'),\ \tau^*(F,E) \leq \gamma^*(G,E') \text{ and } (f,\varphi)(F,E) \subset (G,E')$$

for each arbitrary soft set $(f,\varphi)(x_e) = (f(x))_{\varphi(e)} \in (G,E') \in SS(Y,E')$. If (f,φ) is a continuous mapping for each soft point, then (f,φ) is a continuous mapping.

Theorem 3.5. *Let (X,E,τ,τ^*) and (Y,E',γ,γ^*) be two IFTSs and $(f,\varphi) : (X,E,\tau,\tau^*) \to (Y,E',\gamma,\gamma^*)$ be a mapping. Then (f,φ) is a continuous mapping if and only if*

$$\tau\left((f,\varphi)^{-1}(G,E')\right) \geq \gamma(G,E') \text{ and } \tau^*\left((f,\varphi)^{-1}(G,E')\right) \leq \gamma^*(G,E')$$

are satisfied for each $(G,E') \in SS(Y,E')$.

Proof. Let (f,φ) be a continuous mapping and $(G,E') \in SS(Y,E')$ be an arbitrary soft set. Suppose $x_e \in (f,\varphi)^{-1}(G,E')$ be an arbitrary soft point. Since (f,φ) is a continuous mapping, there exists $x_e \in (F,E) \in SS(X,E)$ such that

$$\tau(F,E) \geq \gamma(G,E'),\ \tau^*(F,E) \leq \gamma^*(G,E') \text{ and } (f,\varphi)(F,E) \subset (G,E').$$

Then

$$(f,\varphi)^{-1}(G,E') = \bigcup_{x_e \in (f,\varphi)^{-1}(G,E')} x_e \subset \bigcup_{x_e \in (f,\varphi)^{-1}(G,E')} (F,E) \subset (f,\varphi)^{-1}(G,E').$$

We have

$$\begin{aligned} \tau\left((f,\varphi)^{-1}(G,E')\right) &= \tau\left(\bigcup_{x_e}(F,E)\right) \geq \wedge\tau(F,E) \geq \gamma(G,E'), \\ \tau^*\left((f,\varphi)^{-1}(G,E')\right) &= \tau^*\left(\bigcup_{x_e}(F,E)\right) \leq \vee\tau^*(F,E) \leq \gamma^*(G,E'). \end{aligned}$$

Conversely, let $x_e \in SS(X,E)$ be an arbitrary soft point and $(f,\varphi)(x_e) \in (G,E')$. From the condition of theorem, $x_e \in (f,\varphi)^{-1}(G,E')$,

$$\begin{aligned} \tau\left((f,\varphi)^{-1}(G,E')\right) &\geq \gamma(G,E'), \\ \tau^*\left((f,\varphi)^{-1}(G,E')\right) &\leq \gamma^*(G,E') \end{aligned}$$

and $(f,\varphi)\left((f,\varphi)^{-1}(G,E')\right) \subset (G,E')$ are satisfied. Thus (f,φ) is a continuous mapping. □

Theorem 3.6. *Let (X,E,τ,τ^*) and (Y,E',γ,γ^*) be two IFTSs and $(f,\varphi) : (X,E,\tau,\tau^*) \to (Y,E',\gamma,\gamma^*)$ be a mapping. Then (f,φ) is a continuous mapping if and only if $(f_r,\varphi_r) : (X,E,\tau_r,\tau_r^*) \to (Y,E',\gamma_r,\gamma_r^*)$ is a continuous mapping on soft bitopological space for each $r \in (0,1]$.*

Proof. Let (f,φ) be a continuous mapping and $(G,E') \in \gamma_r$. Then $\gamma(G,E') \geq r$. Since

$$\tau\left((f,\varphi)^{-1}(G,E')\right) \geq \gamma(G,E') \geq r,$$

$(f,\varphi)^{-1}(G,E') \in \tau_r$. If $(G,E') \in \gamma_r^*$, $\gamma^*(G,E') \leq 1-r$. Since

$$1-r \geq \gamma^*(G,E') \geq \tau^*\left((f,\varphi)^{-1}(G,E')\right),$$

$(f,\varphi)^{-1}(G,E') \in \tau_r^*$ is obtained.

Conversely, let (f_r,φ_r) be a continuous mapping for each $r \in (0,1]$. If $\gamma(G,E') = r$ for each $(G,E') \in SS(Y,E')$, then $(G,E') \in \gamma_r$. Since (f_r,φ_r) is a continuous mapping, $(f_r,\varphi_r)^{-1}(G,E') \in \tau_r$. Then

$$\tau\left((f,\varphi)^{-1}(G,E')\right) \geq r = \gamma(G,E').$$

If $\gamma^*(G,E') = s$, $\gamma^*(G,E') = s = 1-(1-s)$. This implies that $(G,E') \in \gamma_{1-s}^*$. Hence $(f,\varphi)^{-1}(G,E') \in \tau_{1-s}^*$. So,

$$\tau^*\left((f,\varphi)^{-1}(G,E')\right) \leq 1-(1-s) = s = \gamma^*(G,E').$$

Thus $(f,\varphi) : (X,E,\tau,\tau^*) \to (Y,E',\gamma,\gamma^*)$ is a continuous mapping. □

Theorem 3.7. *Let (X,E,τ,τ^*) and (Y,E',γ,γ^*) be two IFTSs and (β,β^*) be a base of (γ,γ^*) on Y. Then $(f,\varphi) : (X,E,\tau,\tau^*) \to (Y,E',\gamma,\gamma^*)$ is a continuous mapping if and only if $\beta(G,E') \leq \tau\left((f,\varphi)^{-1}(G,E')\right)$ and $\beta^*(G,E') \geq \tau^*\left((f,\varphi)^{-1}(G,E')\right)$ for each $(G,E') \in SS(Y,E')$.*

Proof. Let $(f,\varphi):(X,E,\tau,\tau^*)\to(Y,E',\gamma,\gamma^*)$ be a continuous mapping and $(G,E')\in SS(Y,E')$. Then $\gamma(G,E')\geq\beta(G,E')$ and $\gamma^*(G,E')\leq\beta^*(G,E')$. So

$$\begin{aligned}\tau\left((f,\varphi)^{-1}(G,E')\right) &\geq \gamma(G,E')\geq\beta(G,E'),\\ \tau^*\left((f,\varphi)^{-1}(G,E')\right) &\leq \gamma^*(G,E')\leq\beta^*(G,E')\end{aligned}$$

are holds.

Conversely, let $\beta(G,E')\leq\tau\left((f,\varphi)^{-1}(G,E')\right)$ and $\beta^*(G,E')\geq\tau^*\left((f,\varphi)^{-1}(G,E')\right)$ for each $(G,E')\in SS(Y,E')$. Let $(G,E')=\underset{i\in I}{\cup}(G_i,E')$. We have

$$\begin{aligned}\tau\left((f,\varphi)^{-1}(G,E')\right) &= \tau\left((f,\varphi)^{-1}\left(\underset{i\in I}{\cup}(G_i,E')\right)\right)\\ &= \tau\left(\underset{i\in I}{\cup}(f,\varphi)^{-1}(G_i,E')\right)\\ &\geq \underset{i\in I}{\wedge}\tau\left((f,\varphi)^{-1}(G_i,E')\right)\\ &\geq \underset{i\in I}{\wedge}\beta(G_i,E').\end{aligned}$$

Since this equality is satisfied for arbitrary $(G,E')=\underset{i\in I}{\cup}(G_i,E')$,

$$\tau\left((f,\varphi)^{-1}(G,E')\right)\geq\underset{(G,E')=\underset{i\in I}{\cup}(G_i,E')}{\vee}\ \underset{i\in I}{\wedge}\beta(G_i,E')=\gamma(G,E').$$

Also,

$$\begin{aligned}\tau^*\left((f,\varphi)^{-1}(G,E')\right) &= \tau^*\left((f,\varphi)^{-1}\left(\underset{i\in I}{\cup}(G_i,E')\right)\right)\\ &= \tau^*\left(\underset{i\in I}{\cup}(f,\varphi)^{-1}(G_i,E')\right)\\ &\leq \underset{i\in I}{\vee}\tau^*\left((f,\varphi)^{-1}(G_i,E')\right)\\ &\leq \underset{i\in I}{\vee}\beta^*(G_i,E').\end{aligned}$$

So

$$\tau^*\left((f,\varphi)^{-1}(G,E')\right)\leq\underset{(G,E')=\underset{i\in I}{\cup}(G_i,E')}{\wedge}\ \underset{i\in I}{\vee}\beta^*(G_i,E')=\gamma^*(G,E')$$

are obtained. □

Theorem 3.8. *Let (X,E,τ,τ^*) and (Y,E',γ,γ^*) be two IFTSs and the pair (δ,δ^*) be a subbase of (γ,γ^*). If*

$$\delta(G,E')\leq\tau\left((f,\varphi)^{-1}(G,E')\right),\ \delta^*(G,E')\geq\tau^*\left((f,\varphi)^{-1}(G,E')\right)$$

are satisfied for each $(G,E')\in SS(Y,E')$, then $(f,\varphi):(X,E,\tau,\tau^*)\to(Y,E',\gamma,\gamma^*)$ is a continuous mapping.

Proof. For each $(G,E')\in SS(Y,E')$,

$$\begin{aligned}\delta(G,E') &= \underset{\underset{\lambda\in K}{\cup}(G_\lambda,E')=(G,E')}{\vee}\ \underset{\lambda\in K}{\wedge}\ \underset{\underset{\mu\in K_\lambda}{\cap}(F_\lambda,E')=(G_\lambda,E')}{\vee}\ \underset{\mu\in K_\lambda}{\wedge}\gamma\left((F_\mu,E')\right)\\ &\leq \underset{\underset{\lambda\in K}{\cup}(G_\lambda,E')=(G,E')}{\vee}\ \underset{\lambda\in K}{\wedge}\ \underset{\underset{\mu\in K_\lambda}{\cap}(F_\lambda,E')=(G_\lambda,E')}{\vee}\ \underset{\mu\in K_\lambda}{\wedge}\tau\left((f,\varphi)^{-1}\left(F_\mu,E'\right)\right)\\ &\leq \underset{\underset{\lambda\in K}{\cup}(G_\lambda,E')=(G,E')}{\vee}\ \underset{\lambda\in K}{\wedge}\tau\left((f,\varphi)^{-1}(G_\lambda,E')\right)\end{aligned}$$

$$\leq \bigvee_{\underset{\lambda\in K}{\cup}(G_\lambda,E')=(G,E')} \tau\left((f,\varphi)^{-1}\left(\underset{\lambda\in K}{\cup}(G_\lambda,E')\right)\right)$$
$$= \tau\left((f,\varphi)^{-1}(G,E')\right),$$
$$\delta^*(G,E') = \bigwedge_{\underset{\lambda\in K}{\cup}(G_\lambda,E')=(G,E')} \bigvee_{\lambda\in K} \bigwedge_{\underset{\mu\in K_\lambda}{\cap}(F_\lambda,E')=(G_\lambda,E')} \bigvee_{\mu\in K_\lambda} \gamma^*\left(\left(F_\mu,E'\right)\right)$$
$$\geq \bigwedge_{\underset{\lambda\in K}{\cup}(G_\lambda,E')=(G,E')} \bigvee_{\lambda\in K} \bigwedge_{\underset{\mu\in K_\lambda}{\cap}(F_\lambda,E')=(G_\lambda,E')} \bigvee_{\mu\in K_\lambda} \tau^*\left((f,\varphi)^{-1}\left(F_\mu,E'\right)\right)$$
$$\geq \bigwedge_{\underset{\lambda\in K}{\cup}(G_\lambda,E')=(G,E')} \bigvee_{\lambda\in K} \tau^*\left((f,\varphi)^{-1}(G_\lambda,E')\right)$$
$$\geq \bigwedge_{\underset{\lambda\in K}{\cup}(G_\lambda,E')=(G,E')} \tau^*\left((f,\varphi)^{-1}\left(\underset{\lambda\in K}{\cup}(G_\lambda,E')\right)\right)$$
$$= \tau^*\left((f,\varphi)^{-1}(G,E')\right)$$

are hold. □

Definition 3.9. Let (X,E,τ,τ^*) and (Y,E',γ,γ^*) be two *IFTSs* and (f,φ) be a mapping from (X,E,τ,τ^*) to (Y,E',γ,γ^*). The mapping (f,φ) is called an open mapping if it satisfies the following condition:

$$\tau(F,E) \leq \gamma((f,\varphi)(F,E)) \text{ and } \tau^*(F,E) \geq \gamma^*((f,\varphi)(F,E))$$

for each $(F,E) \in SS(X,E)$.

Theorem 3.10. *Let (X,E,τ,τ^*) and (Y,E',γ,γ^*) be two IFTSs and $(f,\varphi):(X,E,\tau,\tau^*)\to(Y,E',\gamma,\gamma^*)$ be a mapping and (β,β^*) be a base of (τ,τ^*). If*

$$\beta(F,E) \leq \gamma((f,\varphi)(F,E)) \text{ and } \beta^*(F,E) \geq \gamma^*((f,\varphi)(F,E))$$

are satisfied for each $(F,E) \in SS(X,E)$, then (f,φ) is an open mapping.

Proof. For each $(F,E) \in SS(X,E)$,

$$\begin{aligned}
\tau(F,E) &= \bigvee_{\underset{i\in I}{\cup}(F_i,E)=(F,E)} \bigwedge_{i\in I} \beta((F_i,E)) \\
&\leq \bigvee_{\underset{i\in I}{\cup}(F_i,E)=(F,E)} \bigwedge_{i\in I} \gamma((f,\varphi)(F_i,E)) \\
&\leq \bigvee_{\underset{i\in I}{\cup}(F_i,E)=(F,E)} \gamma\left((f,\varphi)\left(\underset{i\in I}{\cup}(F_i,E)\right)\right) \\
&= \gamma((f,\varphi)(F,E))
\end{aligned}$$

and

$$\begin{aligned}
\tau^*(F,E) &= \bigwedge_{\underset{i\in I}{\cup}(F_i,E)=(F,E)} \bigvee_{i\in I} \beta^*((F_i,E)) \\
&\geq \bigwedge_{\underset{i\in I}{\cup}(F_i,E)=(F,E)} \bigvee_{i\in I} \gamma^*((f,\varphi)(F_i,E)) \\
&\geq \bigwedge_{\underset{i\in I}{\cup}(F_i,E)=(F,E)} \gamma^*\left((f,\varphi)\left(\underset{i\in I}{\cup}(F_i,E)\right)\right) \\
&= \gamma^*((f,\varphi)(F,E))
\end{aligned}$$

are holds. □

Now by using the mapping $(f,\varphi): SS(X,E) \to (Y,E',\gamma,\gamma^*)$ and (γ,γ^*), we define intuitionistic fuzzy topology on $SS(X,E)$ such that (f,φ) is a continuous mapping.

Theorem 3.11. *Let $(Y, E', \gamma, \gamma^*)$ be an IFTS and $(f, \varphi) : SS(X, E) \to (Y, E', \gamma, \gamma^*)$ be a mapping of soft sets. Then define $(\tau, \tau^*) : SS(X, E) \to [0, 1]$ by:*

$$\begin{aligned}\tau(F,E) &= \bigvee_{f^{-1}(G,E')=(F,E)} \gamma(G,E'),\\ \tau^*(F,E) &= \bigwedge_{f^{-1}(G,E')=(F,E)} \gamma^*(G,E')\end{aligned}$$

for each $(F, E) \in SS(X, E)$.Then (τ, τ^*) is an IFT on X and (f, φ) is a continuous mapping.

Proof. It is clear that $\tau(\Phi) = \tau\left(\widetilde{X}\right) = 1$, $\tau^*(\Phi) = \tau^*\left(\widetilde{X}\right) = 0$. Now,

$$\begin{aligned}\tau\left((F_1,E)\widetilde{\cap}(F_2,E)\right) &= \vee\left\{\gamma(G,E') : (f,\varphi)^{-1}(G,E') = (F_1,E)\widetilde{\cap}(F_2,E)\right\}\\ &\geq \vee\left\{\gamma\left((G_1,E')\widetilde{\cap}(G_2,E')\right) : (f,\varphi)^{-1}\left((G_1,E')\widetilde{\cap}(G_2,E')\right) = (F_1,E)\widetilde{\cap}(F_2,E)\right\}\\ &\geq \left(\vee\left\{\gamma(G_1,E') : (f,\varphi)^{-1}(G_1,E') = (F_1,E)\right\}\right) \wedge \left(\vee\left\{\gamma(G_2,E') : (f,\varphi)^{-1}(G_2,E') = (F_2,E)\right\}\right)\\ &= \tau(F_1,E) \wedge \tau(F_2,E),\end{aligned}$$

$$\begin{aligned}\tau^*\left((F_1,E)\widetilde{\cap}(F_2,E)\right) &= \wedge\left\{\gamma^*(G,E') : (f,\varphi)^{-1}(G,E') = (F_1,E)\widetilde{\cap}(F_2,E)\right\}\\ &\leq \wedge\left\{\gamma^*\left((G_1,E')\widetilde{\cap}(G_2,E')\right) : (f,\varphi)^{-1}\left((G_1,E')\widetilde{\cap}(G_2,E')\right) = (F_1,E)\widetilde{\cap}(F_2,E)\right\}\\ &\leq \left(\wedge\left\{\gamma^*(G_1,E') : (f,\varphi)^{-1}(G_1,E') = (F_1,E)\right\}\right) \wedge \left(\vee\left\{\gamma^*(G_2,E') : (f,\varphi)^{-1}(G_2,E') = (F_2,E)\right\}\right)\\ &= \tau^*(F_1,E) \vee \tau^*(F_2,E)\end{aligned}$$

are holds. Furthermore,

$$\begin{aligned}\tau\left(\bigcup_{i\in\Delta}(F_i,E)\right) &= \vee\left\{\gamma(G,E') : (f,\varphi)^{-1}(G,E') = \bigcup_{i\in\Delta}(F_i,E)\right\}\\ &\geq \vee\left\{\gamma\left(\bigcup_{i\in\Delta}(G_i,E')\right) : (f,\varphi)^{-1}\left(\bigcup_{i\in\Delta}(G_i,E')\right) = \bigcup_{i\in\Delta}(F_i,E)\right\}\\ &\geq \vee\left\{\bigwedge_{i\in\Delta}\gamma(G_i,E') : (f,\varphi)^{-1}(G_i,E') = (F_i,E)\right\}\\ &= \bigwedge_{i\in\Delta}\left(\vee\left\{\gamma(G_i,E') : (f,\varphi)^{-1}(G_i,E') = (F_i,E)\right\}\right)\\ &= \bigwedge_{i\in\Delta}\tau(F_i,E).\end{aligned}$$

So we have

$$\begin{aligned}\tau^*\left(\bigcup_{i\in\Delta}(F_i,E)\right) &= \wedge\left\{\gamma^*(G,E') : (f,\varphi)^{-1}(G,E') = \bigcup_{i\in\Delta}(F_i,E)\right\}\\ &\leq \wedge\left\{\gamma^*\left(\bigcup_{i\in\Delta}(G_i,E')\right) : (f,\varphi)^{-1}\left(\bigcup_{i\in\Delta}(G_i,E')\right) = \bigcup_{i\in\Delta}(F_i,E)\right\}\\ &\leq \wedge\left\{\bigvee_{i\in\Delta}\gamma^*(G_i,E') : (f,\varphi)^{-1}(G_i,E') = (F_i,E)\right\}\\ &= \bigvee_{i\in\Delta}\left(\wedge\left\{\gamma^*(G_i,E') : (f,\varphi)^{-1}(G_i,E') = (F_i,E)\right\}\right)\\ &= \bigvee_{i\in\Delta}\tau^*(F_i,E).\end{aligned}$$

It implies that (τ, τ^*) is an *IFT* on *X*. It is clear that (f, φ) is a continuous mapping. □

Theorem 3.12. *Let (X, E, τ, τ^*) be an IFTS and $(f, \varphi) : (X, E, \tau, \tau^*) \to SS(Y, E')$ be a mapping of soft sets. Then define $(\gamma, \gamma^*) : SS(Y, E') \to [0, 1]$ by:*

$$\begin{aligned}\gamma(G,E') &= \tau\left((f,\varphi)^{-1}(G,E')\right),\\ \gamma^*(G,E') &= \tau^*\left((f,\varphi)^{-1}(G,E')\right),\ \forall (G,E') \in SS(Y,E').\end{aligned}$$

Then (γ, γ^*) is an IFT on Y and (f, φ) is a continuous mapping.

Proof. It is clear that $\gamma(\Phi) = \gamma\left(\widetilde{Y}\right) = 1$, $\gamma^*(\Phi) = \gamma^*\left(\widetilde{Y}\right) = 0$ and $\gamma(G, E') + \gamma^*(G, E') \leq 1$. Now,

$$\begin{aligned}\gamma\left((G_1,E')\widetilde{\cap}(G_2,E')\right) &= \tau\left((f,\varphi)^{-1}(G_1,E')\widetilde{\cap}(G_2,E')\right)\\ &= \tau\left((f,\varphi)^{-1}(G_1,E')\widetilde{\cap}(f,\varphi)^{-1}(G_2,E')\right)\end{aligned}$$

$$\geq \tau\left((f,\varphi)^{-1}(G_1,E')\right)\wedge\tau\left((f,\varphi)^{-1}(G_2,E')\right)$$
$$=\gamma(G_1,E')\wedge\gamma(G_2,E'),$$
$$\gamma^*\left((G_1,E')\widetilde{\cap}(G_2,E')\right)=\tau^*\left((f,\varphi)^{-1}(G_1,E')\widetilde{\cap}(G_2,E')\right)$$
$$=\tau^*\left((f,\varphi)^{-1}(G_1,E')\widetilde{\cap}(f,\varphi)^{-1}(G_2,E')\right)$$
$$\leq\tau^*\left((f,\varphi)^{-1}(G_1,E')\right)\vee\tau^*\left((f,\varphi)^{-1}(G_2,E')\right)$$
$$=\gamma^*(G_1,E')\vee\gamma^*(G_2,E')$$

are obtained. Moreover,

$$\gamma\left(\underset{i\in\Delta}{\cup}(G_i,E')\right)=\tau\left((f,\varphi)^{-1}\left(\underset{i\in\Delta}{\cup}(G_i,E')\right)\right)$$
$$=\tau\left(\underset{i\in\Delta}{\cup}(f,\varphi)^{-1}(G_i,E')\right)$$
$$\geq\underset{i\in\Delta}{\wedge}\tau\left((f,\varphi)^{-1}(G_i,E')\right)=\underset{i\in\Delta}{\wedge}\gamma(G_i,E'),$$
$$\gamma^*\left(\underset{i\in\Delta}{\cup}(G_i,E')\right)=\tau^*\left((f,\varphi)^{-1}\left(\underset{i\in\Delta}{\cup}(G_i,E')\right)\right)$$
$$=\tau^*\left(\underset{i\in\Delta}{\cup}(f,\varphi)^{-1}(G_i,E')\right)$$
$$\leq\underset{i\in\Delta}{\vee}\tau^*\left((f,\varphi)^{-1}(G_i,E')\right)=\underset{i\in\Delta}{\vee}\gamma^*(G_i,E').$$

Thus (γ,γ^*) is an *IFT* on Y and (f,φ) is a continuous mapping. □

Theorem 3.13. *Let $\left\{\left(X_\lambda,E_\lambda,\tau_\lambda,\tau^*_\lambda\right)\right\}_{\lambda\in\wedge}$ be a family of IFTSs, $X=\prod_{\lambda\in\wedge}X_\lambda$ be a set, $E=\prod_{\lambda\in\wedge}E_\lambda$ be a parameter set and for each $\lambda\in\Lambda$, $p_\lambda:X\to X_\lambda$ and $q_\lambda:E\to E_\lambda$ be two projections maps. Define $(\beta,\beta^*):SS(Y,E')\to[0,1]$ by:*

$$\beta(G,E')=\vee\left\{\overset{n}{\underset{j=1}{\wedge}}\tau_{\alpha_j}\left(F_{\alpha_j},E_{\alpha_j}\right):(F,E)=\overset{n}{\underset{j=1}{\cap}}\left(p_{\alpha_j},q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j},E_{\alpha_j}\right)\right\},$$
$$\beta^*(G,E')=\wedge\left\{\overset{n}{\underset{j=1}{\vee}}\tau^*_{\alpha_j}\left(F_{\alpha_j},E_{\alpha_j}\right):(F,E)=\overset{n}{\underset{j=1}{\cap}}\left(p_{\alpha_j},q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j},E_{\alpha_j}\right)\right\}.$$

Then (β,β^*) is a base on IFTS and $(p_\lambda,q_\lambda):\left(X,E,\tau_\beta,\tau^*_\beta\right)\to\left(X_\lambda,E_\lambda,\tau_\lambda,\tau^*_\lambda\right)$ are continuous maps for each $\lambda\in\Lambda$.

Proof. Now we check conditions of base for (β,β^*).

$$\beta\left(\widetilde{X}\right)=\vee\left\{\overset{n}{\underset{j=1}{\wedge}}\tau_{\alpha_j}\left(F_{\alpha_j},E_{\alpha_j}\right):\widetilde{X}=\overset{n}{\underset{j=1}{\cap}}\left(p_{\alpha_j},q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j},E_{\alpha_j}\right)\right\}$$
$$=\vee\left\{\overset{n}{\underset{j=1}{\wedge}}\tau_{\alpha_j}\left(X_{\alpha_j},E_{\alpha_j}\right)\right\}=1$$

is holds. Similarly, $\beta(\Phi)=1$ and $\beta^*(\Phi)=\beta^*\left(\widetilde{X}\right)=0$ are obtained. Now,

$$
\begin{aligned}
\beta(F,E)\wedge\beta(G,E) &= \left(\bigvee_{\bigcap_{j=1}^{n}\left(p_{\alpha_j},q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j},E_{\alpha_j}\right)=(F,E)} \bigwedge_{j=1}^{n}\tau_{\alpha_j}\left(F_{\alpha_j},E_{\alpha_j}\right)\right)\wedge \\
&\quad \left(\bigvee_{\bigcap_{i=1}^{k}\left(p_{\gamma_i},q_{\gamma_i}\right)^{-1}\left(G_{\gamma_i},E_{\gamma_i}\right)=(G,E)} \bigwedge_{i=1}^{k}\tau_{\gamma_i}\left(G_{\gamma_i},E_{\gamma_i}\right)\right) \\
&= \bigvee_{\bigcap_{j=1}^{n}\left(p_{\alpha_j},q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j},E_{\alpha_j}\right)=(F,E)}\ \bigvee_{\bigcap_{i=1}^{k}\left(p_{\gamma_i},q_{\gamma_i}\right)^{-1}\left(G_{\gamma_i},E_{\gamma_i}\right)=(G,E)} \\
&\quad \left(\left(\bigwedge_{j=1}^{n}\tau_{\alpha_j}\left(F_{\alpha_j},E_{\alpha_j}\right)\right)\wedge\left(\bigwedge_{i=1}^{k}\tau_{\gamma_i}\left(G_{\gamma_i},E_{\gamma_i}\right)\right)\right) \\
&= \bigvee_{\cap\left(\left(\left(p_{\alpha_j},q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j},E_{\alpha_j}\right)\right)\cap\left(\left(p_{\gamma_i},q_{\gamma_i}\right)^{-1}\left(G_{\gamma_i},E_{\gamma_i}\right)\right)\right)=(F,E)\cap(G,E)} \\
&\quad \left(\left(\bigwedge_{j=1}^{n}\tau_{\alpha_j}\left(F_{\alpha_j},E_{\alpha_j}\right)\right)\wedge\left(\bigwedge_{i=1}^{k}\tau_{\gamma_i}\left(G_{\gamma_i},E_{\gamma_i}\right)\right)\right) \\
&\leq \bigvee_{\bigcap_{\lambda}\left(p_{\theta_\lambda},q_{\theta_\lambda}\right)^{-1}\left(H_{\theta_\lambda},E_{\theta_\lambda}\right)=(F,E)\cap(G,E)} \tau_{\theta_\lambda}\left(H_{\theta_\lambda},E_{\theta_\lambda}\right) \\
&= \beta\left((F,E)\,\widetilde{\cap}\,(G,E)\right)
\end{aligned}
$$

and

$$
\begin{aligned}
\beta^*(F,E)\vee\beta^*(G,E) &= \left(\bigwedge_{\bigcap_{j=1}^{n}\left(p_{\alpha_j},q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j},E_{\alpha_j}\right)=(F,E)} \bigvee_{j=1}^{n}\tau^*_{\alpha_j}\left(F_{\alpha_j},E_{\alpha_j}\right)\right)\vee \\
&\quad \left(\bigwedge_{\bigcap_{i=1}^{k}\left(p_{\gamma_i},q_{\gamma_i}\right)^{-1}\left(G_{\gamma_i},E_{\gamma_i}\right)=(G,E)} \bigvee_{i=1}^{k}\tau^*_{\gamma_i}\left(G_{\gamma_i},E_{\gamma_i}\right)\right) \\
&= \bigwedge_{\bigcap_{j=1}^{n}\left(p_{\alpha_j},q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j},E_{\alpha_j}\right)=(F,E)}\ \bigwedge_{\bigcap_{i=1}^{k}\left(p_{\gamma_i},q_{\gamma_i}\right)^{-1}\left(G_{\gamma_i},E_{\gamma_i}\right)=(G,E)} \\
&\quad \left(\left(\bigvee_{j=1}^{n}\tau^*_{\alpha_j}\left(F_{\alpha_j},E_{\alpha_j}\right)\right)\vee\left(\bigvee_{i=1}^{k}\tau^*_{\gamma_i}\left(G_{\gamma_i},E_{\gamma_i}\right)\right)\right) \\
&= \bigwedge_{\cap\left(\left(\left(p_{\alpha_j},q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j},E_{\alpha_j}\right)\right)\cap\left(\left(p_{\gamma_i},q_{\gamma_i}\right)^{-1}\left(G_{\gamma_i},E_{\gamma_i}\right)\right)\right)=(F,E)\cap(G,E)} \\
&\quad \left(\left(\bigvee_{j=1}^{n}\tau^*_{\alpha_j}\left(F_{\alpha_j},E_{\alpha_j}\right)\right)\vee\left(\bigvee_{i=1}^{k}\tau^*_{\gamma_i}\left(G_{\gamma_i},E_{\gamma_i}\right)\right)\right) \\
&\geq \bigwedge_{\bigcap_{\lambda}\left(p_{\theta_\lambda},q_{\theta_\lambda}\right)^{-1}\left(H_{\theta_\lambda},E_{\theta_\lambda}\right)=(F,E)\cap(G,E)} \tau^*_{\theta_\lambda}\left(H_{\theta_\lambda},E_{\theta_\lambda}\right) \\
&= \beta^*\left((F,E)\,\widetilde{\cup}\,(G,E)\right).
\end{aligned}
$$

Thus (β,β^*) is satisfied conditions of base. Now we show that the projection mapping (p_λ,q_λ) : $\left(X,E,\tau_\beta,\tau^*_\beta\right)\to\left(X_\lambda,E_\lambda,\tau_\lambda,\tau^*_\lambda\right)$ is continuous maps for each $\lambda\in\Lambda$. Indeed for each $(F_\lambda,E_\lambda)\in SS(X_\lambda,E_\lambda)$,
$\tau\left((p_\lambda,q_\lambda)^{-1}(F_\lambda,E_\lambda)\right)\geq\beta\left((p_\lambda,q_\lambda)^{-1}(F_\lambda,E_\lambda)\right)$

$$
\begin{aligned}
&= \vee \left\{ \bigwedge_{j=1}^{n} \tau_{\alpha_j}\left(F_{\alpha_j}, E_{\alpha_j}\right) : \left(p_{\alpha_j}, q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j}, E_{\alpha_j}\right) = \left(p_\lambda, q_\lambda\right)^{-1}\left(F_\lambda, E_\lambda\right) \right\} \\
&\geq \tau_\lambda\left(F_\lambda, E_\lambda\right),
\end{aligned}
$$

$$
\begin{aligned}
\tau^*\left(\left(p_\lambda, q_\lambda\right)^{-1}\left(F_\lambda, E_\lambda\right)\right) &\leq \beta^*\left(\left(p_\lambda, q_\lambda\right)^{-1}\left(F_\lambda, E_\lambda\right)\right) \\
&= \wedge \left\{ \bigvee_{j=1}^{n} \tau^*_{\alpha_j}\left(F_{\alpha_j}, E_{\alpha_j}\right) : \left(p_{\alpha_j}, q_{\alpha_j}\right)^{-1}\left(F_{\alpha_j}, E_{\alpha_j}\right) = \left(p_\lambda, q_\lambda\right)^{-1}\left(F_\lambda, E_\lambda\right) \right\} \\
&\leq \tau^*_\lambda\left(F_\lambda, E_\lambda\right)
\end{aligned}
$$

are obtained. Thus the proof is completed. □

4. Conclusion

Intuitionistic fuzzy topological spaces are an important generalization of topological spaces. In this paper, we introduce the concepts of continuous mapping, open mapping and closed mapping by using soft points on intuitionistic fuzzy topological spaces. Along, continuous mapping, open mapping and closed mapping on intuitionistic fuzzy topological spaces and their characterizations are also introduced. At the end, some of the crucial properties of the proposed concepts are investigated. On the other hand, we can give topological structures in these algebraic structures by applying gradation functions to Boolean algebras and distributive Lattice with 0,1 elements. This can open new horizons in the study of these structures.We hope that the results of this study may help in the investigation of intuitionistic fuzzy topological spaces on soft sets.

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