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On Ricci Solitons with a Semi-Symmetric Metric Connection

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Abstract. We find some properties of Ricci solitons with a semi-symmetric metric connection. When the potential vector field is torse-forming, we obtain some characterizations. Applications to submanifolds are also given.

1. Introduction

A linear connection on a Riemannian manifold (M, g) is said to be a *semi-symmetric connection* [13], if its torsion tensor T is of the form

$$T(X, Y) = \varpi(Y)X - \varpi(X)Y,$$

where ϖ is a 1-form defined by

$$\varpi(X) = g(X, P),$$

and P is a vector field on M.

If $\widetilde{\nabla}$ is the Levi-Civita connection of a Riemannian manifold (M, g), then the semi-symmetric metric connection (briefly $SSMC$) $\overset{\circ}{\widetilde{\nabla}}$ is given by

$$\overset{\circ}{\widetilde{\nabla}}_X Y = \widetilde{\nabla}_X Y + \varpi(Y)X - g(X, Y)P, \tag{1}$$

where X, Y, P are vector fields on M [23]. Let $\overset{\circ}{\widetilde{R}}$ and $\widetilde{R}$ denote the curvature tensor fields of $\overset{\circ}{\widetilde{\nabla}}$ and $\widetilde{\nabla}$, respectively. Then from (1), it is easy to see that

$$\overset{\circ}{\widetilde{R}}(X, Y)Z = \widetilde{R}(X, Y)Z - \alpha(Y, Z)X + \alpha(X, Z)Y - g(Y, Z)BX + g(X, Z)BY, \tag{2}$$

where

$$\alpha(X, Y) = g(BX, Y) = (\widetilde{\nabla}_X \varpi)Y - \varpi(X)\varpi(Y) + \frac{1}{2}g(X, Y). \tag{3}$$

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Denote by $\overset{\circ}{\widetilde{Ric}}$ and $\widetilde{Ric}$ the Ricci tensor fields of the connections $\overset{\circ}{\widetilde{\nabla}}$ and $\widetilde{\nabla}$, respectively. Then from (2), it can be easily shown that

$$\overset{\circ}{\widetilde{Ric}} = \widetilde{Ric} - (n-2)\alpha - tr\alpha g, \tag{4}$$

(see [23]).

Let (M, g) be a Riemannian manifold. As an intrinsic geometric flow *Ricci flow*

$$\frac{\partial}{\partial t} g(t) = -2\widetilde{Ric}(g(t))$$

was introduced in [14] by R. S. Hamilton, which is an evolution equation for Riemannian metrics. Ricci solitons correspond to self-similar solutions of Ricci flow. A smooth vector field v on a Riemannian manifold (M, g) is said to define a *Ricci soliton* [15], if there exists a real constant λ such that

$$\frac{1}{2}\pounds_v g + \widetilde{Ric} = \lambda g, \tag{5}$$

where $\pounds_v$ denotes the Lie derivative operator in the direction of the vector field v, Ric denotes the Ricci tensor field of (M, g). We denote it by (v, λ). It can be easily seen that Ricci solitons are natural generalizations of Einstein metrics, any Einstein metric gives a trivial Ricci soliton. A Ricci soliton (v, λ) on a (semi)-Riemannian manifold (M, g) is said to be *shrinking*, *steady* or *expanding* according as λ is positive, zero or negative, respectively [15].

In the recent years, the geometry of Ricci solitons has been studied by many geometers. See, for example, [2], [4], [7], [9], [11], [19], [20], [21] and the references therein. Ricci solitons on submanifolds are also a very popular subject. For these kinds of studies see, for example, [1], [5], [8], [10] and the references therein.

In the present paper, we find some properties of Ricci solitons on Riemannian manifolds endowed with a *SSMC* when the potential vector field is torse-forming with respect to a *SSMC*. As recent studies on torse-forming vector fields see [5], [9] and [16].

The paper is organized as follows. The first section is the Introduction. In Section 2, we consider Riemannian manifolds with a *SSMC*. We find geometric properties of Ricci solitons on these kinds of manifolds when the potential vector field is torse-forming. In Section 3, we obtain some applications to submanifolds.

2. Ricci solitons on Riemannian manifolds with a *SSMC*

In this section, we consider Ricci solitons on Riemannian manifolds with a *SSMC*. Using (1), we have

$$(\widetilde{\pounds}_v g)(X, Y) = g(\overset{\circ}{\widetilde{\nabla}}_X v, Y) + g(X, \overset{\circ}{\widetilde{\nabla}}_Y v) - 2\varpi(v)g(X, Y) + g(X, v)\varpi(Y) + g(Y, v)\varpi(X). \tag{6}$$

Hence using equation (6), the soliton equation (5) with respect to a *SSMC* can be written as

$$\frac{1}{2}\left(g(\overset{\circ}{\widetilde{\nabla}}_X v, Y) + g(X, \overset{\circ}{\widetilde{\nabla}}_Y v)\right) + \frac{1}{2}\left(g(X, v)\varpi(Y) + g(Y, v)\varpi(X)\right) + \widetilde{Ric}(X, Y) = (\lambda + \varpi(v))\, g(X, Y). \tag{7}$$

A vector field v on a Riemannian manifold (M, g) is called *torse-forming* [22], if

$$\widetilde{\nabla}_X v = cX + \varphi(X)v,$$

where c is a smooth function, φ is a 1-form and $\widetilde{\nabla}$ is the Levi-Civita connection of g.

In particular, if $\varphi = 0$, then v is called a *concircular vector field* [12] and if $c = 0$, then v is called a *recurrent vector field* [19].

Assume that P is a parallel unit vector field with respect to the Levi-Civita connection $\widetilde{\nabla}$. Using (1), we have

$$\overset{\circ}{\widetilde{\nabla}}_X P = X - \varpi(X)P.$$

So we get:

Proposition 2.1. *Let (M, g) be a Riemannian manifold endowed with a SSMC. If P is a parallel unit vector field with respect to the Levi-Civita connection $\widetilde{\nabla}$ then, P is a torse-forming vector field with respect to a SSMC of the form $\overset{\circ}{\widetilde{\nabla}}_X P = X - \varpi(X)P$.*

A non-flat Riemannian manifold (M, g) $(n \geq 3)$ is called *a hyper-generalized quasi-Einstein manifold* [18], if its Ricci tensor field is not identically zero and satisfies

$$\widetilde{Ric} = a_1 g + a_2 A \otimes A + a_3 (A \otimes B + B \otimes A) + a_4 (A \otimes D + D \otimes A),$$

where a_1, a_2, a_3 and a_4 are functions on M and A, B and D are non-zero 1-forms. If $a_3 = a_4 = 0$, then M is called a *quasi-Einstein manifold* [6]. If $a_2 = a_3 = a_4 = 0$, then (M, g) is *an Einstein manifold* [3]. *The functions* a_1, a_2, a_3 and a_4 are called *associated functions*.

Now let (M, g) be a Riemannian manifold endowed with a *SSMC* and υ a torse-forming potential vector field with respect to a *SSMC* on M. Then from (1), we have

$$\overset{\circ}{\widetilde{\nabla}}_X \upsilon = cX + \varphi(X)\upsilon.$$

So by (7), we can write

$$\widetilde{Ric}(X, Y) = (\lambda - c + \varpi(\upsilon))\, g(X, Y) - \frac{1}{2}\{g(X, \upsilon)\varphi(Y) + g(Y, \upsilon)\varphi(X)\} - \frac{1}{2}\{g(X, \upsilon)\varpi(Y) + g(Y, \upsilon)\varpi(X)\}.$$

Hence we can state the following Theorem:

Theorem 2.2. *Let (M, g) be a Riemannian manifold admitting a SSMC and υ a torse-forming potential vector field with respect a SSMC on M. Then (M, g) is a Ricci soliton (υ, λ) if and only if there exists a constant λ such that*

$$\widetilde{Ric}(X, Y) = (\lambda - c + \varpi(\upsilon))\, g(X, Y) - \frac{1}{2}\{g(X, \upsilon)\varpi(Y) + g(Y, \upsilon)\varpi(X)\} - \frac{1}{2}\{g(X, \upsilon)\varphi(Y) + g(Y, \upsilon)\varphi(X)\}. \tag{8}$$

If υ is a concircular vector field with respect a *SSMC*, then we can state the following corollary:

Corollary 2.3. *Let (M, g) be a Riemannian manifold admitting a SSMC and υ a concircular potential vector field with respect to a SSMC on M. Assume that ϖ is the g dual of υ. Then (M, g) is a Ricci soliton (υ, λ) if and only if M is a quasi-Einstein manifold with associated functions $\lambda - c + \|\upsilon\|^2$, -1.*

Now assume that a 1-form η is the g-dual of υ. Then from (8), we have

$$\widetilde{Ric}(X, Y) = (\lambda - c + \varpi(\upsilon))\, g(X, Y) - \frac{1}{2}\{\eta(X)\varpi(Y) + \eta(Y)\varpi(X)\} - \frac{1}{2}\{\eta(X)\varphi(Y) + \eta(Y)\varphi(X)\}.$$

Then we can state the following theorem:

Theorem 2.4. *Let (M, g) be a Riemannian manifold admitting a SSMC and υ a torse-forming potential vector field with respect to a SSMC on M. Assume that a 1-form η is the g dual of υ. Then (M, g) is a Ricci soliton (υ, λ) if and only if M is a hyper-generalized quasi-Einstein manifold with associated functions $\lambda - c + \varpi(\upsilon), 0, -\frac{1}{2}$ and $-\frac{1}{2}$.*

Using (4), the equation (8) can be written as

$$\widetilde{\overset{\circ}{Ric}}(X,Y) = (\lambda - c + \varpi(v) - tr\alpha)\, g(X,Y) - (n-2)\alpha(X,Y)$$
$$-\frac{1}{2}\{g(X,v)\varpi(Y) + g(Y,v)\varpi(X)\} - \frac{1}{2}\{g(X,v)\varphi(Y) + g(Y,v)\varphi(X)\}.$$

So we can state the following corollary:

Corollary 2.5. *Let (M, g) be a Riemannian manifold admitting a SSMC and v a torse-forming potential vector field with respect to a SSMC on M. Then (M, g) is a Ricci soliton (v, λ) if and only if*

$$\widetilde{\overset{\circ}{Ric}}(X,Y) = (\lambda - c + \varpi(v) - tr\alpha)\, g(X,Y) - (n-2)\alpha(X,Y)$$
$$-\frac{1}{2}\{g(X,v)\varpi(Y) + g(Y,v)\varpi(X)\} - \frac{1}{2}\{g(X,v)\varphi(Y) + g(Y,v)\varphi(X)\}. \tag{9}$$

Now assume that P is a parallel unit vector field with respect to the Levi-Civita connection, i.e., $\widetilde{\nabla}P = 0$ and $\|P\| = 1$. Then

$$(\widetilde{\nabla}_X\varpi)Y = \widetilde{\nabla}_X\varpi(Y) - \varpi(\widetilde{\nabla}_XY) = 0.$$

So from (3), $\alpha(X,Y) = -\varpi(X)\varpi(Y) + \frac{1}{2}g(X,Y)$ and $tr\alpha = \frac{n}{2} - 1$. Thus by (9), we have

$$\widetilde{\overset{\circ}{Ric}}(X,Y) = (\lambda - c + \varpi(v) - n + 2)g(X,Y) + (n-2)\,\varpi(X)\varpi(Y)$$
$$-\frac{1}{2}\{g(X,v)\varpi(Y) + g(Y,v)\varpi(X)\} - \frac{1}{2}\{g(X,v)\varphi(Y) + g(Y,v)\varphi(X)\}.$$

Hence we can state the following theorem:

Theorem 2.6. *Let (M, g) be a Riemannian manifold admitting a SSMC, P a parallel unit vector field with respect to the Levi-Civita connection $\widetilde{\nabla}$ and v a torse-forming potential vector field with respect a SSMC on M. Assume that a 1-form η is the g-dual of v. Then (M, g) is a Ricci soliton (v, λ) if and only if M is a hyper-generalized quasi-Einstein manifold with respect to a SSMC with corresponding functions $\lambda - c + \varpi(v) - n + 2, n - 2, -\frac{1}{2}, -\frac{1}{2}$.*

3. Submanifolds

Let $\left(\widetilde{M}, \widetilde{g}\right)$ be an $(n + d)$-dimensional Riemannian manifold endowed with a *SSMC* $\widetilde{\overset{\circ}{\nabla}}$ and the Levi-Civita connection $\widetilde{\nabla}$. Let M be an n-dimensional submanifold of $\left(\widetilde{M}, \widetilde{g}\right)$. On the submanifold M, let us denote the induced connection by $\overset{\circ}{\nabla}$ and the induced Levi-Civita connection by ∇.

The Gauss formulas and Weingarten formulas with respect to $\widetilde{\nabla}$ and $\widetilde{\overset{\circ}{\nabla}}$ can be written as:

$$\widetilde{\nabla}_XY = \nabla_XY + \sigma(X,Y),$$

$$\widetilde{\overset{\circ}{\nabla}}_XY = \overset{\circ}{\nabla}_XY + \overset{\circ}{\sigma}(X,Y), \quad X, Y \in \chi(M),$$

and

$$\widetilde{\nabla}_XN = -A_NX + \nabla^{\perp}_XN,$$

$$\overset{\circ}{\widetilde{\nabla}}_X N = -\overset{\circ}{A}_N X + \nabla^{\perp}_X N,$$

respectively, where $X, Y \in \chi(M)$, σ is the second fundamental form, N is a unit normal vector field and A_N is the shape operator of M in $\left(\widetilde{M}, \widetilde{g}\right)$ and $\overset{\circ}{\sigma}$ is a normal valued $(0,2)$-tensor field and $\overset{\circ}{A}$ is a $(1,1)$-tensor field on M [17]. Denote by the tangential and normal parts of P by P^T and $P^{\perp}$, respectively. Then from the formula (7) in [17], we have

$$\overset{\circ}{\sigma}(X, Y) = \sigma(X, Y) - g(X, Y)P^{\perp} \tag{10}$$

and

$$\overset{\circ}{A}_N X = A_N X - \varpi(N)X. \tag{11}$$

It is known from [17] that the induced connection $\overset{\circ}{\nabla}$ on a submanifold of a Riemannian manifold endowed with a *SSMC* is also a *SSMC*.

Now assume that $(\widetilde{M}, \widetilde{g})$ is a Riemannian manifold admitting a *SSMC* and υ is a torse-forming vector field with respect to a *SSMC* on $\widetilde{M}$. Let (M, g) be a submanifold of $(\widetilde{M}, \widetilde{g})$. Denote by υ^T and $\upsilon^{\perp}$, the tangential and normal parts of υ, respectively. Then using (1), we have

$$\begin{aligned}\overset{\circ}{\widetilde{\nabla}}_X \upsilon &= \overset{\circ}{\widetilde{\nabla}}_X \left(\upsilon^T + \upsilon^{\perp}\right) = \overset{\circ}{\widetilde{\nabla}}_X \upsilon^T + \overset{\circ}{\widetilde{\nabla}}_X \upsilon^{\perp} \\ &= \widetilde{\nabla}_X \upsilon^T + \varpi(\upsilon^T)X - g(X, \upsilon^T)P^T - g(X, \upsilon^T)P^{\perp} + \widetilde{\nabla}_X \upsilon^{\perp} + \varpi(\upsilon^{\perp})X \\ &= cX + \varphi(X)\upsilon^T + \varphi(X)\upsilon^{\perp}.\end{aligned}$$

So by using of Gauss and Weingarten formulas and by the equality of the tangential and normal parts, we find

$$\nabla_X \upsilon^T = (c - \varpi(\upsilon))\, X + g(X, \upsilon^T)P^T + A_{\upsilon^{\perp}} X + \varphi(X)\upsilon^T \tag{12}$$

and

$$\sigma\left(X, \upsilon^T\right) + \nabla^{\perp}_X \upsilon^{\perp} - g(X, \upsilon^T)P^{\perp} = \varphi(X)\upsilon^{\perp}.$$

Then in view of (12), we get

$$\begin{aligned}(\pounds_{\upsilon^T} g)(X, Y) &= g(\nabla_X \upsilon^T, Y) + g(X, \nabla_Y \upsilon^T) \\ &= 2\,(c - \varpi(\upsilon))\, g(X, Y) + g(X, \upsilon^T)\varpi(Y) + g(Y, \upsilon^T)\varpi(X) + 2\widetilde{g}(\sigma(X, Y), \upsilon^{\perp}) + \varphi(X)g(Y, \upsilon^T) + \varphi(Y)g(X, \upsilon^T).\end{aligned}$$

Hence equation (5) gives us

$$\begin{aligned}Ric(X, Y) &= (\lambda - c + \varpi(\upsilon))\, g(X, Y) - \widetilde{g}(\sigma(X, Y), \upsilon^{\perp}) \\ &- \frac{1}{2}\left\{g(X, \upsilon^T)\varpi(Y) + g(Y, \upsilon^T)\varpi(X)\right\} - \frac{1}{2}\left\{\varphi(X)g(Y, \upsilon^T) + \varphi(Y)g(X, \upsilon^T)\right\}.\end{aligned}$$

So we can state the following theorem:

Theorem 3.1. *Let M be an n-dimensional submanifold isometrically immersed into a Riemannian manifold $\left(\widetilde{M}, \widetilde{g}\right)$ endowed with a SSMC and υ a torse-forming vector field with respect to a SSMC on $\widetilde{M}$. Then (M, g) is a Ricci soliton (υ^T, λ) if and only if the Ricci tensor field Ric of M satisfies:*

$$\begin{aligned}Ric(X, Y) &= (\lambda - c + \varpi(\upsilon))\, g(X, Y) - \widetilde{g}(\sigma(X, Y), \upsilon^{\perp}) \\ &- \frac{1}{2}\left\{g(X, \upsilon^T)\varpi(Y) + g(Y, \upsilon^T)\varpi(X)\right\} - \frac{1}{2}\left\{\varphi(X)g(Y, \upsilon^T) + \varphi(Y)g(X, \upsilon^T)\right\}\end{aligned} \tag{13}$$

for every $X, Y \in \chi(M)$.

If M is $v^\perp$-umbilical, then $A_{v^\perp} = kI$, where k is a function on M and I is the identity map [8]. So we have $\widetilde{g}(\sigma(X,Y), v^\perp) = g(A_{v^\perp}X, Y) = kg(X,Y)$. Then from (13), we have

$$Ric(X,Y) = (\lambda - c + \varpi(v) - k)\, g(X,Y)$$
$$-\frac{1}{2}\left\{g(X,v^T)\varpi(Y) + g(Y,v^T)\varpi(X)\right\} - \frac{1}{2}\left\{\varphi(X)g(Y,v^T) + \varphi(Y)g(X,v^T)\right\}.$$

So we can state the following theorem:

Theorem 3.2. *Let M be an n-dimensional $v^\perp$-umbilical submanifold isometrically immersed into a Riemannian manifold $\left(\widetilde{M}, \widetilde{g}\right)$ endowed with a SSMC and v a torse-forming vector field with respect to a SSMC on $\widetilde{M}$. Assume that a 1-form η is the g dual of v^T. Then (M, g) is a Ricci soliton (v^T, λ) if and only if it is a hyper-generalized quasi-Einstein manifold with associated functions $(\lambda - c + \varpi(v) - k)$, $0, -\frac{1}{2}$ and $-\frac{1}{2}$.*

Since the induced connection $\overset{\circ}{\nabla}$ on a submanifold of a Riemannian manifold endowed with a *SSMC* is also a *SSMC*, from (4), (10) and (13), we also have

$$\overset{\circ}{Ric}(X,Y) = \left(\lambda - c + \varpi(v^T) - tr\alpha\right) g(X,Y) - (n-2)\alpha(X,Y)$$
$$-\widetilde{g}(\overset{\circ}{\sigma}(X,Y), v^\perp) - \frac{1}{2}\left\{g(X,v^T)\varpi(Y) + g(Y,v^T)\varpi(X)\right\} - \frac{1}{2}\left\{\varphi(X)g(Y,v^T) + \varphi(Y)g(X,v^T)\right\},$$

where $\overset{\circ}{Ric}$ denotes the Ricci tensor field of the induced *SSMC*.

So we can state the following corollary:

Corollary 3.3. *Let M be an n-dimensional submanifold isometrically immersed into a Riemannian manifold $\left(\widetilde{M}, \widetilde{g}\right)$ endowed with a SSMC and v a torse-forming vector field with respect to a SSMC on $\widetilde{M}$. Then (M, g) is a Ricci soliton (v^T, λ) with respect to the SSMC if and only if the Ricci tensor field $\overset{\circ}{Ric}$ of the induced SSMC of M satisfies:*

$$\overset{\circ}{Ric}(X,Y) = \left(\lambda - c + \varpi(v^T) - tr\alpha\right) g(X,Y) - (n-2)\alpha(X,Y)$$
$$-\widetilde{g}(\overset{\circ}{\sigma}(X,Y), v^\perp) - \frac{1}{2}\left\{g(X,v^T)\varpi(Y) + g(Y,v^T)\varpi(X)\right\} - \frac{1}{2}\left\{\varphi(X)g(Y,v^T) + \varphi(Y)g(X,v^T)\right\}$$

for every $X, Y \in \chi(M)$.

If P is a parallel unit vector field with respect to the Levi-Civita connection $\widetilde{\nabla}$, then we have

$$\overset{\circ}{Ric}(X,Y) = \left(\lambda - c + \varpi(v^T) - n + 2\right) g(X,Y) + (n-2)\varpi(X)\varpi(Y)$$
$$-\widetilde{g}(\overset{\circ}{\sigma}(X,Y), v^\perp) - \frac{1}{2}\left\{g(X,v^T)\varpi(Y) + g(Y,v^T)\varpi(X)\right\} - \frac{1}{2}\left\{\varphi(X)g(Y,v^T) + \varphi(Y)g(X,v^T)\right\}. \quad (14)$$

If M is $v^\perp$-umbilical, then by (11), we have

$$\overset{\circ}{A}_{v^\perp}X = (k - \varpi(v^\perp))\, X,$$

which gives us

$$(k - \varpi(v^\perp))\, g\,(X,Y) = g\left(\overset{\circ}{A}_{v^\perp}X, Y\right) = \widetilde{g}(\overset{\circ}{\sigma}(X,Y), v^\perp).$$

Hence from (14), we find

$$\overset{\circ}{Ric}(X,Y) = (\lambda - c - k + \varpi(v) - n + 2)\; g(X,Y) + (n-2)\varpi(X)\varpi(Y)$$
$$-\frac{1}{2}\left\{\varphi(X)g(Y,v^T) + \varphi(Y)g(X,v^T)\right\} - \frac{1}{2}\left\{g(X,v^T)\varpi(Y) + g(Y,v^T)\varpi(X)\right\}.$$

So we can state the following theorem:

Theorem 3.4. *Let M be a $v^{\perp}$-umbilical submanifold isometrically immersed into a Riemannian manifold $\left(\widetilde{M}, \widetilde{g}\right)$ endowed with a SSMC and v a torse-forming vector field with respect to a SSMC on $\widetilde{M}$. Assume that a 1-form η is the g dual of v^T and P is a parallel unit vector field with respect to the Levi-Civita connection $\widetilde{\nabla}$. Then (M, g) is a Ricci soliton (v^T, λ) if and only if it is a hypergeneralized quasi-Einstein manifold with respect to a SSMC with associate functions $\lambda - c - k + \varpi(v) - n + 2, n - 2, \frac{1}{2}$ and $-\frac{1}{2}$.*

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