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W-Interpolative Ćirić-Reich-Rus Type Contractions on Quasi-Partial B-Metric Space

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Abstract. Karapinar introduced the notion of interpolative Ćirić-Reich-Rus type contractions in the setting of complete metric space. Taking his approach forward, H. Aydi, initiated the concept of w-admissibility and proved some fixed point results on the same. This approach has been applied to partial-metric space as well.

But the question is, can the above result be applied in quasi-partial b-metric space as well ? Our paper deals with the above raised question. The paper discusses how w-admissibility can be used to obtain fixed point results for Ćirić-Reich-Rus type contractions in quasi-partial b-metric space. Few examples are given to justify the result.

1. Introduction and Preliminaries

In 1906, M. Fréchet[1] introduced the term metric space which is considered to be one of the cornerstones in the field of mathematics. Due to its importance and application potential, this notion has been extended and improved by many authors[2],[3],[4],[5]. Later on S.G. Matthews[6] studied partial-metric space and obtained fixed point theorem on it. Then the concept of quasi-partial metric was introduced by Karapinar[7]. Shukla[8] came up with the notion of partial b-metric space as a generalization of partial-metric and b-metric space. Gupta and Gautam[9] generalized quasi-partial metric space and introduced the concept of quasi-partial b-metric space.

The notion of w-orbital admissible maps was introduced by Popescu[10] as a refinement of the concept of α-admissible maps of Samet et al. [11]. Karapinar[12] defined the generalized Kannan-type contraction by adopting the interpolative approach and proved that such an interpolative Kannan-type contraction owns a fixed point in complete metric space. Karapinar[13] also gave the notion of interpolative Ćirić-Reich-Rus type contraction in the framework of partial-metric space. Here in this paper we have stated the fixed point theorem on Ćirić-Reich-Rus type contraction in the framework of complete quasi-partial b-metric space, by taking w-interpolation into account. Some examples are given to verify the effectiveness of the main result. Throughout this paper, R^+ denotes the set of all non-negative real numbers and **N** denotes the set of positive integers.

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Definition 1. A partial metric on a non-empty set X is a function $p : X \times X \to [0, \infty)$ such that for all $u, v, w \in$ X satisfies:

(i) $u = v \longleftrightarrow p(u, u) = p(u, v) = p(v, v)$,

(ii) $p(u, u) \leq p(u, v)$,

(iii) $p(u, v) = p(v, u)$, and

(iv) $p(u, v) \leq p(u, w) + p(w, v) - p(w, w)$.

A partial metric space is a pair (X, p) such that X is a non-empty set and p is a partial metric on X.

Definition 2. A quasi-partial metric on a non-empty set X is a function $q : X \times X \to R^+$ such that for all $u, v, w \in$ X satisfies:

(i) If $q(u, u) = q(u, v) = q(v, v)$, then $u = v$,

(ii) $q(u, u) \leq (u, v)$,

(iii) $q(u, u) \leq (v, u)$, and

(iv) $q(u, v) + q(w, w) \leq q(u, w) + q(w, v)$.

A quasi-partial metric space is a pair (X, q) such that X is a non-empty set and q is a quasi-partial metric on X.

Definition 3. A quasi-partial b-metric on a non-empty set X is a mapping qp_b: $X \times X \to R^+$ such that for some real $s \geq 1$ for all $u, v, w\ \epsilon X$.

(i) $qp_b(u, u) = qp_b(u, v) = qp_b(v, v) \implies u = v$,

(ii) $qp_b(u, u) \leq qp_b(u, v)$,

(iii) $qp_b(u, u) \leq qp_b(v, u)$, and

(iv) $qp_b(u, u) \leq s[qp_b(u, w) + qp_b(w, v)] - qp_b(w, w)$.

A quasi-partial b-metric space is a pair (X, qp_b) such that X is a non-empty set and qp_b is a quasi-partial b-metric on X. The number s is called the coefficient of (X,qp_b).

Definition 4. Let (X, qp_b) be a quasi-partial b-metric space. A self mapping T on X is called an interpolative Ćirić-Reich-Rus type contraction, if there is $\lambda\ \epsilon$ [0,1) and positive reals α, β with $\alpha + \beta < 1$ such that

$$qp_b(T\eta, T\mu) \leq \lambda[qp_b(\eta, \mu)]^{\beta}.[qp_b(\eta, T\eta)]^{\alpha}.[qp_b(\mu, T\mu)]^{(1-\alpha-\beta)},$$

for all η, $\mu\ \epsilon X$.

Example 1. Let us consider a set X = {2, 4, 8, 10} endowed with $qp_b(u, v) = u + v$.

qp_b(u,v)	2	4	8	10
2	4	6	10	12
4	6	8	12	14
8	10	12	16	18
10	12	14	18	20

Define a self mapping T on X : $\begin{pmatrix} 2 & 4 & 8 & 10 \\ 4 & 2 & 4 & 2 \end{pmatrix}$

qp_b (T2, T4) = qp_b (4, 2) $\leq \lambda$ (qp_b (4,2) + qp_b (T2, 2) + qp_b (4,T4))
= λ (qp_b (4, 2) + qp_b (4, 2) + qp_b (4, 2))
= $\lambda(6 + 6 + 6)$
= 18λ.

On the other hand choose $\beta = 1/2$, $\alpha = 2/5$ and $\lambda = 18/10$. Then, (u, v) ϵ {(8, 10),(10, 8),(8, 8),(10, 10)}. Without loss of generality, we have :

Case 1. u = v = 8 , here,
$qp_b(Tu, Tv) = 16 \leq 40\lambda = \lambda[qp_b(u, v)]^{\beta}.[qp_b(u, Tu)]^{\alpha}.[qp_b(v, Tv)]^{(1-\alpha-\beta)}$

Case 2. u = v = 10 , here,
$qp_b(Tu, Tv) = 4 \leq 44\lambda = \lambda[qp_b(u, v)]^{\beta}.[qp_b(u, Tu)]^{\alpha}.[qp_b(v, Tv)]^{(1-\alpha-\beta)}$

Case 3. u = 8, v = 10, here,
$qp_b(Tu, Tv) = 8 \leq 42\lambda = \lambda[qp_b(u, v)]^{\beta}.[qp_b(u, Tu)]^{\alpha}.[qp_b(v, Tv)]^{(1-\alpha-\beta)}$

Thus, the self mapping T is an interpolative Cirić-Reich-Rus type contraction. Hence, 2 and 4 are the desired fixed points.

Definition 5. Let $w : X \times X \to [0, \infty]$ be a mapping and $X \neq \phi$. A self mapping T : $X \to X$ is said to be a w-orbital admissible if for all $s \in$ X, we have

$$w(s, Ts) \geq 1 \longrightarrow w(Ts, T^2 s) \geq 1$$

H-condition - If $\{\mu_n\}$ is a sequence in X such that $w(\mu_{n+1}, \mu_n) \geq 1$ for each n and $\{\mu_n\} \to \mu \, \epsilon X$ as $n \to \infty$ then there exists $\{\mu_{n(k)}\}$ from $\{\mu_n\}$ such that $w(\mu_{n(k)}, \mu) \geq 1$ for each k.

In this paper we have stated the idea of w-interpolative Ćirić-Reich-Rus type contraction in quasi-partial b-metric space using the notion of w-admissibility.

2. Main Results

Let (X, qp_b) be a quasi-partial b-metric space. The map T : $X \to X$ is said to be a w-interpolative Ćirić-Reich-Rus type contractions if there exists $\lambda \, \epsilon \, \psi$, $w : X \times X \to [0, \infty]$ and positive reals $\alpha, \beta > 0$, verifying $\alpha + \beta < 1$, such that :

$$w(\eta, \mu) qp_b(T\eta, T\mu) \leq \lambda[qp_b(\eta, \mu)]^{\beta}.[qp_b(\eta, T\eta)]^{\alpha}.[qp_b(\mu, T\mu)]^{(1-\alpha-\beta)} \quad (2.1)$$

for all η, $\mu \, \epsilon$ X .

The essential main result is given as follows:

Theorem 1. Let a continuous self mapping T : $X \to X$ is w-orbital admissible and forms a w-interpolative Ćirić-Reich-Rus type contractions on a complete quasi-partial b-metric space (X, qp_b). If there exists $\eta \, \epsilon$ X such that $w(\eta, T\eta) \geq 1$, then T possesses a fixed point in X.

Proof - Let $\eta \, \epsilon$ X be a point such that $w(\eta, T\eta) \geq 1$ and $\{\eta_n\}$ be a sequence defined by $\eta_n = T^n(\eta) = T(\eta_n)$, where $\eta \geq 0$. If for some n_0, we get $\eta_{n_0} = \eta_{n_0+1}$, then η_{n_0} is a fixed point of T, hence it is proved for this case.

Let $\eta_n \neq \eta_{n+1}$, $n \geq 0$. We have $w(\eta_0, \eta_1) \geq 1$. As T is w-orbital admissible, $w(\eta_1, \eta_2) = w(T\eta_0, T\eta_1) \geq 1$, continuing further $w(\eta_n, \eta_{n+1}) \geq 1$ for all $n \geq 0$.

Let $\eta = \eta_n$, $\mu = \eta_{n-1}$ in (2.1),

$$\begin{aligned} qp_b(\eta_{n+1}, \eta_n) &\le w(\eta_n, \eta_{n-1}) qp_b(T\eta_n, T\eta_{n-1}) \\ &\le \lambda([qp_b(\eta_n, \eta_{n-1}]^\alpha [qp_b(\eta_n, T\eta_n]^\beta [qp_b(\eta_{n-1}, T\eta_{n-1}]^{1-\beta-\alpha} \\ &= \lambda([qp_b(\eta_n, \eta_{n-1}]^\alpha [qp_b(\eta_n, \eta_{n+1}]^\beta [qp_b(\eta_{n-1}, \eta_n]^{1-\beta-\alpha} \\ &= \lambda([qp_b(\eta_{n-1}, \eta_n]^{1-\beta} [qp_b(\eta_n, \eta_{n+1}]^\beta \end{aligned} \tag{2.2}$$

Particularly, as $\lambda(t) < t$ for each $t > 0$.

$$\begin{aligned} qp_b(\eta_{n+1}, \eta_n) &\le \lambda([qp_b(\eta_{n-1}, \eta_n)]^{1-\beta} [qp_b(\eta_n, \eta_{n+1})]^\beta) \\ &< [qp_b(\eta_{n-1}, \eta_n)]^{1-\beta} [qp_b(\eta_n, \eta_{n+1})]^\beta \end{aligned}$$

We derived, $[qp_b(\eta_n, \eta_{n+1})]^{1-\beta} < [qp_b(\eta_{n-1}, \eta_n)]^{1-\beta}$

Therefore, $qp_b(\eta_n, \eta_{n+1}) < qp_b(\eta_{n-1}, \eta_n) \quad$ for all $n \ge 1$ (2.3)

So,$(qp_b(\eta_{n-1}, \eta_n)$ is decreasing .

Let $\lim_{n\to\infty} qp_b(\eta_{n-1}, \eta_n) = L$

Multiplying $qp_b(\eta_{n-1}, \eta_n)^{1-\beta}$ on both sides of (2.3)

$$\begin{aligned} [qp_b(\eta_{n-1}, \eta_n)]^{1-\beta} [qp_b(\eta_n, \eta_{n+1})]^\beta &\le [qp_b(\eta_{n-1}, \eta_n)]^{1-\beta} [qp_b(\eta_{n-1}, \eta_n)^\beta \\ &= qp_b(\eta_{n-1}, \eta_n) \end{aligned} \tag{2.4}$$

From (2.2) and (2.4) ,

$$\begin{aligned} qp_b(\eta_{n+1}, \eta_n) &\le \lambda([qp_b(\eta_{n-1}, \eta_n)]^{1-\beta} [qp_b(\eta_n, \eta_{n+1})]^\beta] \\ &\le \lambda[qp_b(\eta_{n-1}, \eta_n)] \end{aligned}$$

Repeating this argument,we get

$$qp_b(\eta_n, \eta_{n+1}) \le \lambda[qp_b(\eta_{n-1}, \eta_n)] \le \lambda^2[qp_b(\eta_{n-2}, \eta_{n-1})] \le \cdots \lambda^n[qp_b(\eta_0, \eta_1)] \tag{2.5}$$

Taking $n \to \infty$ in (2.5) and using the fact , $\lim_{n\to\infty} \lambda^n(t) = 0$ for all $t > 0$,
We get , L = 0 which implies $\lim_{n\to\infty} qp_b(\eta_n, \eta_{n+1}) = 0$

We will show that $\{\eta_n\}$ is a Cauchy Sequence ie. $\lim_{n\to\infty} qp_b(\eta_n, \eta_{n+p}) = 0$ for all $p \in \mathbf{N}$.
From (2.5) and using the triangular inequality,

$$\begin{aligned} qp_b(\eta_n, \eta_{n+p}) &\le \lambda^n(qp_b(\eta_0, \eta_1)) + \cdots + \lambda^{n+p-1}(qp_b(\eta_0, \eta_1)) \\ &\le \sum_{i=n}^{\infty} \lambda^i(qp_b(\eta_0, \eta_1)) \end{aligned} \tag{2.6}$$

Taking $n \to \infty$ in (2.6) , we get zero on the right side of the equation. So, $\{\eta_n\}$ is a Cauchy Sequence in (X, qp_b).

Hence, $\{\eta_n\}$ is a Cauchy Sequence in (X, qp_b). As (X, qp_b) is complete, there exists, $\eta \in$ X, such that $\lim_{n\to\infty} qp_b(\eta, \eta_n) = 0$.

As T is continuous, $\eta = \lim_{n\to\infty} \eta_{n+1} = \lim_{n\to\infty} T\eta_n = T(\lim_{n\to\infty} \eta_n) = T\eta$.

Example 2. Let us consider a set X = [0, 3] endowed with $qp_b(u, v) = |\, u - v \,| + u$. Suppose that T is a self mapping defined on X by -

Interval	Values of Tx	Values of w
[0,2]	7/5	1
(2,3]	23/10	0

Let u, v ϵ X such that u ≠ Tu and v ≠ Tv and w(u, v) ≤ 1. Then, u, v ϵ [0, 2] and u, v $\notin \frac{7}{5}$ as we have $Tu = Tv = \frac{7}{5}$.

Hence, (2.1) holds. For $u_0 = 2$,

$$w(2, T2) = w(2, \frac{7}{5}) = 1$$

Now, let u, v ϵ X such that, w(u, v) ≥ 1. It yields that u, v ϵ [0, 2], so Tu = Tv ϵ [0, 2].

Hence, w(Tu, Tv) ≥ 1 , that is w-orbital admissible. Notice, T is not continuous. We shall show that H-condition too holds.

Let there exists a sequence $\{u_n\}$ in X such that $w(u_n, u_{n+1}) \ge 1$ for each n ϵ $\mathbf{N}$. Then , $u_n \subset [0, 2]$.

If $\{u_n\} \to c$ as n $\to \infty$, we have $| u_n - c | \to 0$ as $n \to \infty$.
Hence, c ϵ [0,2] and so, $w(u_n, c) = 1$. All conditions of Theorem 1 holds.
Therefore, $\frac{7}{5}$ and $\frac{23}{10}$ are two fixed points of T.

Theorem 2. Let a self mapping T : $X \to X$ is a w-orbital admissible and forms a w-interpolative Ćirić-Reich-Rus type contraction on a quasi-partial b-metric space(X, qp_b) and suppose the H-condition is fulfilled. If there exists η ϵ X such that $w(\eta_0, T\eta_0) > 1$, then T has a fixed point.

Proof - From Theorem 1, we conclude that the constructed sequence $\{\eta_n\}$ is Cauchy and $\lim_{n\to\infty} qp_b(\eta_n, \eta) = 0$ holds.
Let $\eta \neq T\eta$ (by contradiction)
So, $\eta_{n(k)} \neq T\eta_{n(k)}$ for each $k \geq 0$.
Due to H-condition, there is a partial subsequence $\{\eta_{n(k)}\}$ of $\{\eta_n\}$ such that $w(\eta_{n(k)}, \eta) \geq 1$ for all k.
Since sequences, $qp_b(\eta_{n(k)}, \eta) \to 0$ and $qp_b(\eta_{n(k)}, T\eta_{n(k)}) \to 0$ (as $\eta = T\eta$, shown in Theorem 1) and $qp_b(\eta, T\eta) > 0$, there exists $n \in \mathbf{N}$ such that for all $k \geq n$,
$qp_b(\eta_{n(k)}, \eta) \leq qp_b(\eta, T\eta)$ and $qp_b(\eta_{n(k)}, T\eta_{n(k)} \leq qp_b(\eta, T\eta)$.
Taking $\eta = \eta_{n(k)}$ and $\mu = \eta$ (in equation 1)
$qp_b(\eta_{n(k)+1}, T\eta) \leq w(\eta_{n(k)}, \eta)qp_b(T\eta_{n(k)}, T\eta)$
$\leq \lambda[qp_b(\eta_{n(k)}, \eta)]^{\alpha}[qp_b(\eta_{n(k)}, T\eta_{n(k)}]^{\beta}[qp_b(\eta, T\eta)]^{(1-\alpha-\beta)}$
As λ is non-decreasing ,
$qp_b(\eta_{n(k)+1}, T\eta) \leq \lambda([qp_b(\eta, T\eta)]^{\alpha}[qp_b(\eta, T\eta)]^{\beta}[qp_b(\eta, T\eta)]^{1-\alpha-\beta}]$
$= \lambda(qp_b(\eta, T\eta))$
Let $k \to \infty$,
$0 < (qp_b(\eta, T\eta)) < \lambda(qp_b(\eta, T\eta)) < (qp_b(\eta, T\eta))$, which is a contradiction. Therfore, $\eta = T\eta$.

Example 3. Let us consider a set X = [2, 5] endowed with $qp_b(u, v) = | \ln (\frac{u}{v}) |$. Suppose that T is a self mapping defined on X by -

Interval	Values of Tx	Values of w
[2, 3]	9/100	1
(3, 5]	5/100	0

Let u, v ϵ X such that u ≠ Tu and v ≠ Tv and w(u,v) ≤ 1 . Then, u, v ϵ [2, 3] and u, v $\notin \frac{9}{100}$ as we have $Tu = Tv = \frac{9}{100}$.
Hence, (2.1) holds. For $u_0 = 2$,

$$w(2, T2) = w(2, \frac{9}{100}) = 1.$$

.
Now, let u, v ϵ X such that, w(u, v) ≥ 1. It yields that u, v ϵ [2, 3], so Tu = Tv ϵ [2, 3].
Hence, w(Tu, Tv) ≥ 1, that is w-orbital admissible. Notice, T is not continuous. We shall show that H-condition too holds.
Let there exist a sequence $\{u_n\}$ in X such that $w(u_n, u_{n+1}) \geq 1$ for each $n \in \mathbf{N}$. The, $u_n \subset$ [2,3] .
If $\{u_n\} \to c$ as $n \to \infty$, we have $| u_n - c | \to 0$ as $n \to \infty$.
Hence, c ϵ [2, 3] and so, $w(u_n, c) = 1$. All conditions of Theorem 1 holds.
Therefore, $\frac{9}{100}$ and $\frac{5}{100}$ are two fixed points of T.

Corollary 1. Let A_1 and A_2 be two non-empty subsets of a complete quasi-partial b-metric space (X, qp_b) are closed. Consider that $T : A_1 \cup A_2 \to A_1 \cup A_2$ satisfies,

$$w(\eta, \mu)qp_b(T\eta, T\mu) \leq \lambda([qp_b(\eta, \mu)]^{\beta}.[qp_b(\eta, T\eta)]^{\alpha}.[qp_b(\mu, T\mu)]^{(1-\alpha-\beta)}$$

for all $\eta \epsilon A_1$ and $\mu \epsilon A_2$ such that η, $\mu \notin$ Fix(T), where $\lambda \epsilon \psi$ and $\alpha, \beta > 0$ are positive reals such that $\alpha + \beta < 1$. If T(A_1) $\subseteq A_2$ and T(A_2) $\subseteq A_1$, then there exists a fixed point of T in $A_1 \cap A_2$.

Corollary 2. Let A_1 and A_2 be two non-empty subsets of a complete quasi-partial b-metric space (X, qp_b) are closed. Consider that $T : A_1 \cup A_2 \to A_1 \cup A_2$ satisfies,

$$w(\eta, \mu)qp_b(T\eta, T\mu) \leq \lambda([qp_b(\eta, T\eta)]^{\beta}.[qp_b(\mu, T\mu)]^{(1-\alpha-\beta)}$$

for all $\eta \epsilon A_1$ and $\mu \epsilon A_2$, such that η , $\mu \notin$ Fix(T) , where $\lambda \epsilon \phi$ and $0 < \beta < 1$. If T(A_1) $\subseteq A_2$ and T(A_2) $\subseteq A_1$, then there exists a fixed point of T in $A_1 \cap A_2$.

Example 4. Let us consider a set X = $A_1 \cup A_2$ endowed with $qp_b(u, v) = \mid u - v \mid$. Let $A_1 = [1, 6]$ and $A_2 = [0, 5]$.

Interval	Values of Tx
A_1	2
A_2	3

Interval	Values of w
$(A_1 \times A_2) \cup (A_2 \times A_1)$	1
Otherwise	0

Let u, v ϵ X such that u $\neq$ Tu and v $\neq$ Tv and w(u, v) $\leq$ 1. Then, u, v ϵ $(A_1 \times A_2) \cup (A_2 \times A_1)$ and u, v $\notin$ 2 as we have $Tu = Tv = 2$. Hence, equation 1 holds. For $u_0 = 3$,

$$w(3, T3) = w(3, 2) = 1.$$

Now, let u, v ϵ X such that, w(u, v) $\geq$ 1. It yields that for u, v ϵ $(A_1 \times A_2) \cup (A_2 \times A_1)$, T($A_1$) $\subseteq A_2$ and T(A_2) $\subseteq A_1$. Hence, w(Tu, Tv) $\geq$ 1 , that is w-orbital admissible. Notice, T is not continuous. We shall show that H-condition too holds.

Let sequence $\{u_n\}$ in X such that $w(u_n, u_{n+1}) \geq 1$ for each $n \in \mathbf{N}$. Then , $u_n \subset [1, 5]$.

If $\{u_n\} \to c$ as $n \to \infty$, we have $\mid u_n - c \mid \to 0$ as $n \to \infty$.

Hence, c ϵ [1, 5] and so, $w(u_n, c) = 1$. All conditions of Theorem 1 holds. $A_1 \cap A_2 = [1, 5]$, which satisfies the corollary. Therefore, 2 and 3 are two fixed points of T and they belong to $A_1 \cap A_2$.

Corollary 3. Let T be a self mapping on a complete quasi-partial b-metric space, (X, qp_b) such that

$$w(\eta, \mu)qp_b(T\eta, T\mu) \leq \lambda([qp_b(\eta, T\eta)]^{\beta}.[qp_b(\mu, T\mu)]^{(1-\beta)}$$

for all η, $\mu \epsilon$ Fix(T), where $0 < \beta < 1$. Then T admits a fixed point in X.

Example 5. Suppose a set X = $[0, \frac{\Pi}{2}]$ is endowed with $qp_b(u, v) = \sin u + \sin v$. Suppose that T is a self mapping defined on X by -

Interval	Values of Tx	Values of w
$[0, \frac{\Pi}{4}]$	$\frac{1}{2}$	1
$(\frac{\Pi}{4}, \frac{\Pi}{2}]$	$1 + \frac{\sqrt{3}}{2}$	0

Let u, v ϵ X such that u $\neq$ Tu and v $\neq$ Tv and w(u, v) $\leq$ 1 . Then, u, v ϵ $[0, \frac{\Pi}{4}]$ and u, v $\notin \frac{1}{2}$ as we have $Tu = Tv = \frac{1}{2}$.

Hence, (2.1) holds. For $u_0 = \frac{\Pi}{6}$,

$$w(\frac{\Pi}{6}, T\frac{\Pi}{6}) = w(\frac{\Pi}{6}, \frac{1}{2}) = 1.$$

Now, let u, v ϵ X such that, w(u, v) ≥ 1. It yields that u, v ϵ [1, 2], so Tu = Tv ϵ $[0, \frac{\Pi}{4}]$.

Hence, w(Tu, Tv) ≥ 1, that is w-orbital admissible. Notice, T is not continuous. We shall show that H-condition too holds.

Let there exists a sequence $\{u_n\}$ in X such that $w(u_n, u_{n+1}) \geq 1$ for each $n \in \mathbf{N}$. Then, $u_n \subset [0, \frac{\Pi}{4}]$.

If $\{u_n\} \to c$ as $n \to \infty$, we have $| u_n - c | \to 0$ as $n \to \infty$.

Hence, c ϵ $[0, \frac{\Pi}{4}]$ and so, $w(u_n, c) = 1$. All conditions of Theorem 1 holds.

Therefore, $\frac{1}{2}$ and $1 + \frac{\sqrt{3}}{2}$ are two fixed points of T.

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