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On Optimality for Mayer Type Problem Governed by a Discrete Inclusion System with Lipschitzian Set-Valued Mappings

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Abstract. Set-valued optimization which is an extension of vector optimization to set-valued problems is a growing branch of applied mathematics. The application of vector optimization technics to set-valued problems and the investigation of optimality conditions has been of enormous interest in the research of optimization problems. In this paper we have considered a Mayer type problem governed by a discrete inclusion system with Lipschitzian set-valued mappings. A necessary condition for K-optimal solutions of the problem is given via local approximations which is considered the lower and upper tangent cones of a set and the lower derivative of the set-valued mappings.

1. Introduction

A Mayer problem, which is a somewhat different classical formulation of a variational problem has a terminal criterion rather than an integral criterion that is, a trajectory is evaluated in terms of where it ultimately terminates. This evaluation does not depend explicitly on the route by which the trajectory reached its terminal point. The form of the problem and the results obtained are particularly suited to trajectory optimization and other modern engineering control problems [1].

The paper [2] is devoted to derive the optimality conditions of Mayer problem for differential inclusions with initial point constraints. By using the discretization method guaranteeing transition to continuous problem, the discrete and discrete-approximation inclusions are investigated. Discrete and continuous time problems with higher order ordinary and partial differential inclusions have wide applications in the field of mathematical economics and in problems of control dynamical system optimization and differential games. In particular, the problems including the higher order discrete and discrete-approximate differential inclusions and the higher order partial differential inclusions are studied by E.N. Mahmudov [3–7].

In [8], Çicek and Mahmudov derived the optimality conditions for second-order discrete Mayer problem with initial boundary constraints using by locally adjoint mapping. The second-order necessary optimality conditions for the Mayer optimal control problem with an arbitrary closed control set is considered in [9].

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Set-valued optimization is an extension of vector optimization to set-valued problems. The application of vector optimization principles to set-valued problems has received increasing attention in recent decades [13, 14, 16–18, 21]. Vector optimization model has found many important applications in decision making problems such as those in economics theory, management science, and engineering design since the introduction of the Pareto optimal solution in 1896 [10].

The functions involved in an optimization problem are often nondifferentiable. This often occurs in many problems encountered in several fields, which can be only described by locally Lipschitz functions. In this regard, and recently, Arana et al. [11] have given new results for K-efficient (optimal) solutions when the involved functions are nondifferentiable.

The investigation of optimality conditions, especially as regards the vector criterion, has received enormous attention in the research of optimization problems and has been studied extensively. Inspired by the above observations, aim of this work is to give a necessary condition for K-optimality of the following Mayer type optimization problem

$$\min x_T \tag{1}$$

$$x_{t+1} \in F_t(x_t), \quad t = 0, ..., T-1 \tag{2}$$

$$x_0 \in M \tag{3}$$

where $T \in \mathbb{N}$, the $F_t : X_t \rightrightarrows X_{t+1}, t = 0, ..., T-1$, are Lipschitzian set-valued mappings in neighborhoods of the points $x_t, t = 0, ..., T-1$ respectively, the $X_t, t = 0, ..., T$, are finite-dimensional Euclidean spaces, and $M \subset X_0$. We also assume that the space X_T is partially ordered by a proper cone K.

A necessary optimality condition for solutions of problem (1)-(3) is obtained via local approximations of sets and set-valued mappings. As such approximations we consider the lower and upper tangent cones of a set and the lower derivative of a set-valued mapping [19, 20].

This work is organized as follows. Section 2 presents the notation and definitions of tanget cones, derivative of a set-valued mapping, locally Lipschitzian set-valued mappings, as well as previous results. In Section 3 we study a Mayer type problem with discrete inclusions involving locally Lipschitzian set-valued mappings, and give a necessary condition for K-optimal solution for the problem.

2. Necessary concepts

In this section we recall some results from the literature that are of interest for our work. Since many definitions and terms in the literature have various interpretations, it is useful to do this to avoid possible misunderstandings. We mostly use the notions introduced in [15, 18–20].

Let X be a finite-dimensional Euclidean space. A set $M \subset X$ is said to be convex if the line segment between any two points in M is contained in M, i.e., if we have

$$x, y \in M, \ \ 0 \leq \lambda \leq 1 \Rightarrow (1-\lambda)x + \lambda y \in M.$$

Define the norm of $x \in X$ by $\|x\| = \sqrt{\langle x, x \rangle}$. For a set $M \subseteq X$, we say that $x \in M$ is an interior point of M if there exists an $\varepsilon > 0$ for which $\{y \in X \ : \ \|y - x\| \leq \varepsilon\} \subseteq M$ holds. The set of all interior points of M is called the interior of M and is denoted by $int(M)$. A set M is said to be open if $int(M) = M$. We say that $M \subseteq X$ is closed if its complement, $X \setminus M$, is open. Let M' denote the set of all accumulation points of M then $cl(M) = M \cup M'$ is the closure of M. The boundary of M is defined by $bd(M) = cl(M) \setminus int(M)$.

A set $K \subseteq X$ is called a cone if it satisfies

$$x \in K, \lambda \geq 0 \Rightarrow \lambda x \in K.$$

We say that K is a convex cone if it is convex and a cone, which means that $\lambda_1 x + \lambda xy \in K$ holds for any $x, y \in K$ and any $\lambda_1, \lambda_2 \geq 0$. A cone K is said to be pointed if $K \cap (-K) = \{0\}$. We say that a cone K is solid if it has a nonempty interior, i.e., if $int(K) \neq \emptyset$. A cone K is called a proper cone if it is closed, convex, solid, and pointed. Let K be a proper cone in X. Then the cone K can induce a partial order $\leq_K$ on X by defining for any $x, y \in X$, $x \leq_K y \Leftrightarrow y - x \in K \setminus \{0\}$, for more see [12].

Let $M \subset X$ be a set and $\mathcal{P}(Y)$ be the family of all subsets of Y. A mapping $F : M \rightrightarrows Y$ is said to be a set-valued mapping defined on M, if for every $x \in M$, $F(x) \subset \mathcal{P}(Y)$. The set of $dom(F) = \{x \ : \ F(x) \neq \emptyset\}$ is called by the domain of F. $F(x)$ is called by the image of x, the set $im(F) := \bigcup_{x \in M} F(x) \subset Y$ is the image of F, and $gph(F) = \{(x, y) \ : \ y \in F(x)\} \subseteq X \times Y$ is called by the graph of F. A set-valued mapping F is said to be convex if its graph $gph(F)$ is convex in $X \times Y$. A set-valued mapping F is said to be convex valued if F(x) is convex in Y. A set-valued mapping F is said to be closed if its graph $gph(F)$ is closed in $X \times Y$.

The closure of a set-valued mapping $F : X \rightrightarrows Y$ is defined as the set-valued mapping $\overline{F} : X \rightrightarrows Y$ whose graph $gph(\overline{F}) := \{(x, y) \ : \ y \in \overline{F}(x)\}$ is the closure of the graph of F, i.e., $gph(\overline{F}) = cl(gph(F))$.

Let $F : X \rightrightarrows Y$ and $G : Y \rightrightarrows Z$ be set-valued mappings. The usual composition product $G \circ F : X \rightrightarrows Z$ of G and F at x is defined by

$$(G \circ F)(x) := \bigcup_{y \in F(x)} G(y). \tag{4}$$

The graph of the composition map $G \circ F$ is defined as $gph(G \circ F) = (\mathbf{I} \times G)(gph(F))$, where $\mathbf{I}$ is the identity map from one set to itself.

Let $F : X \rightrightarrows Y$ be a set-valued mapping and $x_0 \in X$. If there exists a constant $L > 0$ and a neighborhood $U \subset dom(F)$ of x_0 such that

$$\forall x_1, x_2 \in U, \ \ F(x_1) \subset F(x_2) + L\|x_1 - x_2\|B_Y \tag{5}$$

where B_Y is the open unit ball in Y. Then F is said to be a Lipschitzian set-valued mapping around x_0 with the constant L.

The following properties of are obtained directly from the definition of Lipschitzian set-valued mappings and their composition.

Proposition 2.1. *Let X, Y, and Z be finite-dimensional Euclidean spaces, $F : X \rightrightarrows Y$ and $G : Y \rightrightarrows Z$ be set-valued mappings.*
(i) If F is a Lipschitzian set-valued mapping around a point $x \in int(dom\ F)$ and $F(x) \cap int(dom\ G) \neq \emptyset$, then $x \in int(dom(G \circ F))$.
(ii) If G is a Lipschitzian set-valued mapping around a point $x \in int(dom\ F)$ with Lipschitz constant $L_F > 0$ and G is a Lipschitzian set-valued mapping around a point $y \in F(x) \cap int(dom\ G)$ with Lipschitz constant $L_G > 0$, then the composition $G \circ F : X \rightrightarrows Z$ is a Lipschitzian mapping around x with Lipschitz constant $L = L_F \cdot L_G$.

Following the Pareto concept of optimality, a point $\tilde{x} \in M$ is called a nondominated minimal point of M if

$$M \cap (\tilde{x} - K(x)) = \{\tilde{x}\}, \ \ \forall x \in M.$$

If $K(x) = K$ for all $x \in X$ with K some nontrivial pointed convex cone then the definitions of a nondominated element of a set M with respect to K-optimal and of a minimal element of a set M with respect to K-optimal coincide with the concepts of an optimal element of M in the space X partially ordered by the convex cone K.

The inclusion relation of the system (2) determines a finite point sequence. That sequence $\{x_t\}_{t=0}^T$ is called a trajectory of the system (2) − (3). We call that $\{x_t\}_{t=0}^T$ is the zero trajectory if $x_t = 0$ for $t = 0, 1, ..., T$. If the terminal point $\tilde{x}_T$ of the trajectory is a nondominated minimal in the attainability set of system (2) − (3) with respect to the cone K, i.e., if there is no trajectory $\{\tilde{y}_t\}_{t=0}^T$ of system (2) − (3) such that $\tilde{x}_T - \tilde{y}_T \in K \setminus \{0\}$, than the trajectory $\{\tilde{x}_t\}_{t=0}^T$ is said to be K-optimal solution [11].

Obviously, a K-optimal trajectory $\{x_t\}_{t=0}^T$ for the problem (1)-(3) is necessarily K-optimal; in general, the converse fails.

A necessary optimality condition for solutions of problem (1)-(3) is obtained via local approximations of sets and set-valued mappings. As such approximations we consider the lower and upper tangent cones of a set and the lower derivative of a set-valued mapping. In this connection, we use the notions introduced in [19, 20].

The lower tangent cone of a subset $M \subset X$ at a point $x \in cl(M)$ is defined as the set

$$T_l(x; M) = \liminf_{t \to 0^+} \frac{M - x}{t}.$$

The upper tangent cone of a subset $M \subset X$ at a point $x \in cl(M)$ is defined as the set

$$T_u(x; M) = \limsup_{t \to 0^+} \frac{M - x}{t}.$$

It is very convenient to use the following characterization of the lower and upper tangent cones.

$$h \in T_l(x; M) \text{ if and only if } \forall \{t_k\}_{k=1}^{\infty} \to 0^+, \exists \{h^k\}_{k=1}^{\infty} \to h \text{ such that } x + t_k h^k \in M, \forall k \in \mathbb{N}$$

and

$$h \in T_u(x; M) \text{ if and only if } \exists \{t_k\}_{k=1}^{\infty} \to 0^+, \exists \{h^k\}_{k=1}^{\infty} \to h \text{ such that } x + t_k h^k \in M, \forall k \in \mathbb{N},$$

or equivalently

$$h \in T_u(x; M) \text{ if and only if } \exists \{r_k\}_{k=1}^{\infty} \to +\infty, \exists \{h^k\}_{k=1}^{\infty} \to h \text{ such that } r_k(h^k - x) \to h,$$

where $0 < t_k \in \mathbb{R}, h^k \in M, \forall k \in \mathbb{N}$, lim inf and lim sup stand for the Painlevé-Kuratowski upper and lower limits. It follows from properties of lower and upper limits that the sets $T_l(x; M)$ and $T_u(x; M)$ are closed cones and $T_l(x; M) \subset T_u(x; M)$. We also know that if M is a convex set, then $T_l(x; M\}) = T_u(x; M)$ is also convex. It is obvious that the lower and upper tangent cones to a singleton is obviously reduced to $\{0\}$, i.e.,$T_l(x_0; \{x_0\}) = T_u(x_0; \{x_0\} = \{0\}$.

The lower derivative of a set-valued mapping $F : X \rightrightarrows Y$ at a point $(x, y) \in gph(F)$ is defined as the set-valued mapping $D_l F(x, y) : X \rightrightarrows Y$, whose graph is the lower tangent cone of the set $gph(F)$ at the point (x, y), i.e., $gph(D_l F(x, y)) = T_l((x, y); gph(F))$. The upper derivative of $F : X \rightrightarrows Y$ at a point $(x, y) \in gph(F)$ is defined as the set-valued mapping $D_u F(x, y) : X \rightrightarrows Y$, whose graph is the upper tangent cone of the set $gph(F)$ at the point (x, y), i.e., $gph(D_u F(x, y)) = T_u((x, y); gph(F))$. The circatangent derivative of $F : X \rightrightarrows Y$ at a point $(x, y) \in gph(F)$ is defined as the set-valued mapping $D_C F(x, y) : X \rightrightarrows Y$, whose graph is the Clarke tangent cone of the set $gph(F)$ at the point (x, y), i.e., $gph(D_C F(x, y)) = T_C((x, y); gph(F))$. We have the following inclusions

$$D_C F(x, y)(u) \subset D_l F(x, y)(u) \subset D_u F(x, y)(u), \quad \forall u \in X.$$

Let's consider the composition $F_t \circ F_{t-1} : X_{t-1} \rightrightarrows X_t, t = 1, ..., T$ defined analogous to (4)

$$(F_t \circ F_{t-1})(x_{t-1}) = \bigcup_{x_t \in F_{t-1}(x_{t-1})} F_t(x_t).$$

Theorem 2.2. *[20] Consider set-valued maps $F : X \rightrightarrows Y$ and $G : Y \rightrightarrows Z$. Fix $x_0 \in dom(F)$, $y_0 \in F(x_0) \cap dom(G)$ and $z_0 \in G(y_0)$. If F and G are closed and satisfy the transversality condition $lm(D_C F(x_0, y_0)) - dom(D_C G(y_0, z_0)) = Y$ then $D_l G(y_0, z_0) \circ D_l F(x_0, y_0) \subset D_l(G \circ F)(x_0, z_0)$.*

The following property of the lower derivative of the composition of set-valued mappings can be derived from Theorem 2.2.

Lemma 2.3. *Let $F : X \rightrightarrows Y$ and $G : Y \rightrightarrows Z$ be set-valued mappings, and let G be a Lipschitzian set-valued mapping in a neighborhood of a point $y \in F(x) \cap dom(G)$. Then for any points $(x, y) \in gph(F)$ and $(y, z) \in gph(G)$ we have*

$$D_l G(y, z) \circ D_l F(x, y) \subset D_l(G \circ F)(x, z).$$

The following result is crucial in this work, see [21].

Theorem 2.4. *Let K be a convex subset of X with $0 \in bd(K)$, M be a subset of X and $\tilde{x} \in M$ a nondominated minimal point of M. Then*

$$T_u(\tilde{x}; M) \cap (-int(K)) = \emptyset.$$

Proof. Suppose that $x \in T_u(\tilde{x}; M) \cap (-int(K))$. Since $0 \notin int(K)$, $x \neq 0$. There exist some sequences $\{t_k\}_{k=1}^{\infty} \to +\infty$ and $\{x^k\}_{k=1}^{\infty} \to x$ such that $t_k(x^k - \tilde{x}) \to x$. Since x is an interior point of $-K$, there exist an open ball $N(x) \subset -int(K)$ and a positive integer number $\bar{k}$, such that if $k \geq \bar{k}$ we have $t_k(x^k - \tilde{x}) \in N(x)$. Choose $k_0 \geq \bar{k}$, such that $t_{k_0} \geq 1$ and $x^{k_0} \neq x$ (since $y \neq 0$ and $t_k \to +\infty$, such k_0 can be chosen). Thus there exists $x_0 \in int(K)$ such that $t_{k_0}(x^{k_0} - \tilde{x}) = -x_0, \tilde{x} - x^{k_0} = \frac{x_0}{t_{k_0}}$. Since $0 \in bd(K)$ and K is a convex set, we have

$$(0, x_0] = \{\lambda x_0 \ : \ 0 < \lambda \leq 1\} \subset int(K).$$

Thus $\frac{x_0}{t_{k_0}} \in int(K)$. This contradicts the fact that $\tilde{x}$ is a nondominated minimal point of M. □

3. Main results

To give some optimality conditions for solutions of problem (1)-(3) we will need the following propositions and lemma.

Proposition 3.1. *Let $F : X \rightrightarrows Y$ be a set-valued mapping and $(x, y) \in gph(F)$, then for any $v \in X$ we have $D_l F(x, y)(v) \subset D_u F(x, y)(v)$.*

Proof. Let's take any $w \in D_l F(x; y)(v))$, then from the definition of the lower derivative we have $(v, w) \in gph(D_l F(x, y))$, namely $(v, w) \in T_l((x, y); gph(F))$. Therefore from the definition we have that for any sequence $\{t_k\}_{k=1}^{\infty} \to 0^+$ there exists sequences $\{v^k\}_{k=1}^{\infty} \to v$ and $\{w^k\}_{k=1}^{\infty} \to w$ such that $y + t_k w^k \in F(x + t_k v^k)$ for all $k \in \mathbb{N}$. Thus using the upper tangent cone of $gph(F)$ at (x, y) we have $(v, w) \in T_u((x, y); gph(F))$, namely $(v, w) \in gph(D_u F(x, y))$. Thus we have get $w \in D_u F(x, y)(v)$. □

Lemma 3.2. *Let the set-valued mapping $F : X \rightrightarrows Y$ be convex-valued and Lipschitzian around a point $x \in X$. Then the lower derivative $D_l F(x, y) : X \rightrightarrows Y$ at a point $(x, y) \in gpg(F)$ is convex-valued for any $v \in X$.*

Proof. Let $p, r \in D_l F(x, y)(v), v \in X$. Then, for any sequence $\{t_k\}_{k=1}^{\infty} \to 0^+$, there exist sequences $\{v^{1k}\}_{k=1}^{\infty} \to v$ and $\{v^{2k}\}_{k=1}^{\infty} \to v$ and sequences $\{p^k\}_{k=1}^{\infty} \to p$ and $\{r^k\}_{k=1}^{\infty} \to r$ such that $y + t_k p^k \in F(x + t_k v^{1k})$ and $y + t_k r^k \in F(x + t_k v^{2k})$, for all $k \in \mathbb{N}$. Since F is Lipschitzian around the point x, there exists $L > 0$ such that for all k large enough,

$$y + t_k r^k \in F(x + t_k v^{1k}) + L t_k \|v^{2k} - v^{1k}\|$$

so that we can find another sequence $w^k \to r$ such that

$$y + t_k w^k \subset F(x + t_k v^{1k}).$$

Now, $F(x + t_k v^{1k})$ being convex, we deduce that for all $\lambda \in [0, 1]$,

$$y + t_k((1 - \lambda)p^k + \lambda w^k) \in F(x + t_k v^{1k}).$$

Since $(1 - \lambda)p^k + \lambda w^k$ converges to $(1 - \lambda)v + \lambda w$ we have $(1 - \lambda)v + \lambda w \in D_l F(x, y)(u)$. □

Proposition 3.3. *Let $F : X \rightrightarrows Y$ be a Lipschitzian set-valued mapping around a point $x \in M \subset X$. Then for any $y \in F(x)$ we have the following inclusion*

$$D_l F(x, y)(T_u(x; M)) \subset T_u(y; F(M)).$$

Proof. Let's take any $u \in T_u(x; M)$ and $v \in D_l F(x; y)(u))$. Then there exists sequences $\{t_k\}_{k=1}^{\infty} \to 0^+$, $\{h^k\}_{k=1}^{\infty} \to u$, $\{u^k\}_{k=1}^{\infty} \to u$ and $\{v^k\}_{k=1}^{\infty} \to v$ such that $x + t_k h^k \in M$ and $y + t_k v^k \in F(x + t_k u^k)$ for all $k \in \mathbb{N}$. Since F is Lipschitzian set-valued mapping around the point $x \in M \subset X$ with the constant $L > 0$, we can infer that

$$y + t_k v^k \in F(x + t_k h^k) + L t_k \left\| u^k - h^k \right\|$$

so that there exists another sequence $\{w^k\}_{k=1}^{\infty} \to v$ such that

$$y + t_k w^k \in F(x + t_k h^k) \subset F(M).$$

This last inclusion implies that $v \in T_u(y; F(M))$. □

Note that in the case of a singleton $M \subset X$ the inclusion converse to the above-proved one is valid for any set-valued mapping F.

Proposition 3.4. *Let $F : X \rightrightarrows Y$ be a Lipschitzian set-valued mapping around a point $x \in M \subset X$. Then for any $y \in F(x)$ we have the following inclusion $T_l(y; F(x)) \subset D_l F(x, y)(0)$ are valid.*

One can derive the following result from Proposition 3.3.

Proposition 3.5. *Let $F_t : X_t \rightrightarrows X_{t+1}$ be a Lipschitzian mappings in neighborhoods of the points $x_t, t = 0, ..., T - 1$ respectively, and $x_0 \in M \subset X_0$. Then for any $x_{t+1} \in F_t(x_t)$ we have the following inclusion*

$$D_l F_{t+1}(x_{t+1}, x_{t+2})(T_u(x_{t+1}; F_t(M))) \subset T_u(x_{t+2}; F_{t+1} \circ \cdots \circ F_0(M)), \quad t = 0, ..., T - 2. \tag{6}$$

Let us now return to optimality conditions for solutions of problem (1)-(3). Consider the composition $F_T := F_{T-1} \circ F_{T-2} \circ \cdots \circ F_0$ of the set-valued mapping $F_t : X_t \rightrightarrows X_{t+1}, t = 0, ..., T - 1$, where X_t are finite-dimensional Euclidean spaces.

Theorem 3.6. *Let the trajectory $\{x_k\}_{k=0}^{T}$ is a K-optimal solution of problem (1)-(3) and let F_t be Lipschitzian set-valued mappings in neighborhoods of the points $x_t, t = 0, ..., T - 1$, respectively. Then the zero trajectory is a K-optimal solution of problem of the associated problem below*

$$\min x'_T \tag{7}$$

$$x'_{t+1} \in D_l F_t(x_t, x_{t+1})(x'_t), \quad t = 0, ..., T - 1 \tag{8}$$

$$x'_0 \in T_u(x_0; M) \tag{9}$$

Proof. Let $\{x_k\}_{k=0}^{T}$ be a K-optimal solution trajectory of problem (1)-(3). Since K is a proper cone, K is convex and $0 \in bd(K)$. Let's define the composition $F_T := F_{T-1} \circ F_{T-2} \circ \cdots \circ F_0$, so $F_T : X_0 \rightrightarrows X_T$. On the other hand, due to $F_T(M) = F_{T-1} \circ F_{T-2} \circ \cdots \circ F_0(M)$ and the inclusions (2), $x_T \in F_T(M) \subset X_T$ is satisfied. Hence, thanks to Theorem 2.4, we have

$$T_u(x_T; F_{T-1} \circ F_{T-2} \circ \cdots \circ F_0(M)) \cap (-int(K)) = T_u(x_T; F_T(M)) \cap (-int(K)) = \emptyset. \tag{10}$$

If we consider Proposition 2.1, 3.3, 3.5 and Lemma 2.3 together the following inclusion is obtained

$$D_l F_{T-1}(x_{T-1}, x_T)(T_u(x_{T-1}; F_{T-2} \circ F_{T-3} \circ \cdots \circ F_0(M))) \subset (T_u(x_T; F_{T-1} \circ F_{T-2} \circ \cdots \circ F_0(M))) = T_u(x_T; F_T(M)). \quad (11)$$

From (10) and (11) we have

$$D_l F_{T-1}(x_{T-1}, x_T) \circ D_l F_{T-2}(x_{T-2}, x_{T-1}) \circ \cdots \circ D_l F_0(x_0, x_1)(T_u(x_0; M)) \cap (-int(K)) = \emptyset. \quad (12)$$

With Proposition 3.4, the relation (12) is equivalent to the fact that the zero trajectory is a K-optimal solution of problem (7)-(9). $\square$

4. Discussion and Conclusions

In this study, we focused on K-optimality for a Mayer type problem governed by a discrete inclusion system with Lipschitzian set-valued mappings, that is given with (1)-(3). The main result we have obtained is a necessary K-optimality condition for solutions of the problem which is given Theorem 3.6. To obtain the necessary condition we have used the lower and upper tangent cones of a set and the lower derivative of a set-valued mapping. Unlike the paper [8], we have considered a first order discrete Mayer type problem, used set-valued mappings more than one and the terminal point not in a specific set. One can consider the problem of finding trajectories of the system (1)-(3), whose terminal points provide the minimum of some real-valued function on the attainability set of this system, and it can be obtained necessary optimality conditions for this problem.

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