



Well Posedness Results for Higher-Order Neutral Stochastic Differential Equations Driven by Poisson Jumps and Rosenblatt Process

K. Ramkumar^a, K. Ravikumar^a, A. Anguraj^a, Hamdy M. Ahmed^b

^aDepartment of Mathematics, PSG College of Arts and Science, Coimbatore, 641 046, India.

^bHigher Institute of Engineering, El-Shorouk Academy, El-Shorouk City, Cairo, Egypt.

Abstract. In this article, we investigate the existence, uniqueness and stability of mild solutions for a class of higher-order nonautonomous neutral stochastic differential equations (NSDEs) with infinite delay driven by Poisson jumps and Rosenblatt process in Hilbert space. More precisely, using semigroup theory and successive approximation method, we establish a set of sufficient conditions for obtained the required result. Further, the result is deduced to study the higher-order autonomous system. Finally, examples are provided to demonstrate the obtain results.

1. Introduction

Noise or arbitrary fluctuations are unavoidable and omnipresent in nature as well as in man-made systems, therefore it is better to study the existence, uniqueness of stochastic models rather than deterministic models. The deterministic system often fluctuates due to environmental noise. So, it is important and necessary for us to deal with stochastic differential equations (SDEs). The differential equations which involve randomness in the mathematical description of a given phenomena as known as SDEs. In recent years, SDEs in both finite and infinite dimensions have attracted much attention in many areas due to their applications in describing various phenomenon in population dynamics, electrical engineering, ecology, medicine biology, and other areas of science and engineering. For good introduction to SDEs and their applications see [1–4].

Higher-order differential equations capture the dynamic behavior of many natural phenomena and have found applications in various fields, for example, biology, physics and finance. In many cases, it is advantageous to treat the higher-order SDEs directly rather than converting them to first-order systems. A variety of problems arising in mechanics, molecular dynamics, and quantum mechanics can be described in general by second-order nonlinear differential equations. For instance, it is useful for engineers to model mechanical vibrations or charge on a capacitor or condenser subjected to white noise excitation through a second-order SDEs see [9–12, 25–27]. Due to this reason, researchers interest to focus on second-order

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Email addresses: ramkumarkpsg@gmail.com (K. Ramkumar), ravikumarkpsg@gmail.com (K. Ravikumar), angurajpsg@yahoo.com (A. Anguraj), hamdy_17eg@yahoo.com (Hamdy M. Ahmed)

SDEs. In recent years, existence and stability results for second-order SDEs have been considered by many scholars. Chen [6] discussed the exponential and asymptotical stability for second-order stochastic partial differential equations with infinite delay. Using Banach fixed point theorem, the sufficient conditions for the existenc, exponential stability and as well as almost sure exponential stability of SDEs with Poisson jumps. In the study of Ren and Sakthivel [7], proved the existence and uniqueness of mild solution of second-order neutral stochastic differential equations (NSDEs) with Poisson jumps under Lipschitz and non-Lipschitz conditions. In very recently, Dhanalakshmi and Balasubramaniam [8], investigated the stability of higher-order fractional NSDEs with Poisson jumps and Rosenblatt process via Nussbaum fixed point theorem.

Nowadays various real-life situations can be modeled by using Poisson jumps. For example, if a system jumps from a “normal state” to a “other state”, the strength of systems is random. In order to make realistic model, a jumps term is included in and dynamical systems. Recently, the study of SDEs driven by Poisson jumps has considerable attentions see [8, 9, 13, 14]. On the other hand, the fractional Brownian motion (fBm) are utilized largely due to its self-similarity, stationary of increments and long-range dependence, for more details see [15–17, 24] and the references therein. Firstly, Tudor [18], investigated the Rosenblatt process which is a self-similar process with stationary increments and it appears as limit of long-range dependent stationary series in the Non-Central Limit Theorem. Subsequently, Maejima and Tudor [19] established the new properties of the Rosenblatt distribution. More recently, many researchers, investigated the SDEs with Rosenblatt process, one can refer the articles [8, 20–23].

Moreover, nonautonomous models are important to deal with the changes in the vital rates through time as a result of environmetal variation. Besides, to the best of authors’ knowledge, there is no paper in the literature that is involved with the NSDEs with Poisson jumps and Rosenblatt process. Motivated by this consideration, in this article, we consider the higher-order nonautonomous NSDEs with infinite delay driven by Poisson jumps and Rosenblatt process.

$$\begin{aligned}
 d[u'(t) - \hat{f}_1(t, u_t)] &= \mathfrak{A}(t)[u(t) - \hat{f}_1(t, u_t)]dt + \hat{f}_2(t, u_t)dt + g(t, u_t)dw(t) + \int_{\mathfrak{Z}} h(t, u_t, \eta)\tilde{N}(dt, d\eta) \\
 &+ \sigma(t)dZ_{\mathbb{H}}(t), \quad t \in [0, T], \\
 u_0 &= \varphi \in \mathcal{B}, \quad u'(0) = \zeta,
 \end{aligned}
 \tag{1}$$

where $\mathfrak{A}(t) : \mathcal{D}(\mathfrak{A}(t)) \subset \mathbb{H} \rightarrow \mathbb{H}$, a closed linear operator, which generators an evolution operators $\mathfrak{S}(t, s)$. The functions $\hat{f}_1, \hat{f}_2 : [0, T] \times \mathcal{B} \rightarrow \mathbb{H}$, $g : [0, T] \times \mathcal{B} \rightarrow \mathcal{L}_2^0$, $h : [0, T] \times \mathcal{B} \times \mathfrak{Z} \rightarrow \mathbb{H}$ and $\sigma : [0, T] \rightarrow \mathcal{L}_2^0$ are appropriate mappings specified later. The delay $u_t : (-\infty, 0] \rightarrow \mathbb{H}$ is defined by $u_t(\theta) = u(t + \theta)$ for $t \geq 0$ belongs to the phase space \mathcal{B} , which is defined axiomatically. The initial data $\varphi = \{\varphi(t) : -\infty < t \leq 0\}$ is an \mathfrak{F}_0 -measurable, \mathcal{B} -valued stochastic process independent of t . In $\tilde{N}(dt, d\eta) = N(dt, d\eta) - dt(\lambda d\eta)$ the Poisson measure $\tilde{N}(dt, d\eta)$ denotes the Poisson counting measure.

This manuscript is organized as follows: In section 2, basic notations, preliminaries, and some basic results are recalled to use in the sequel. In section 3, we study the existence and uniqueness of nonautonomous NSDEs with infinite delays driven by Poisson jumps and Rosenblatt process. In section 4, we study the stability through the continuous dependence on initial values. In section 5, we prove the autonomous case of the system. An example are provided to demonstrate the obtain results in section 6 and conclusion is derived in section 7.

2. Preliminaries

In this section, we introduce notations and preliminary results need to establish our results. Let $(\mathbb{H}, \|\cdot\|, \langle \cdot, \cdot \rangle)$ and $(\mathbb{K}, \|\cdot\|, \langle \cdot, \cdot \rangle)$ denote two real separable Hilbert spaces, with their vectors norms and their products, respectively. We denote by $\mathcal{L}(\mathbb{K}; \mathbb{H})$ the set of all linear bounded operators from \mathbb{K} and \mathbb{H} , which is equipped with the usual operator norm $\|\cdot\|$. Let $(\Omega, \mathfrak{F}, \mathbb{P})$ be a complete filtered probability space furnished with complete family of right continuous increasing sub σ -algebras $\{\mathfrak{F}_t, t \in J\}$ satisfying $\mathfrak{F}_t \subset \mathfrak{F}$ an \mathbb{H} -valued random variable is an \mathfrak{F} -measurable function $x(t) : \Omega \rightarrow \mathbb{H}$, and a collection of random variable

$\mathbb{S} = \{x(t, \omega) : \Omega \rightarrow \mathbb{H} : t \in J\}$ is called a stochastic process. Let $\beta_n(t) (n = 1, 2, \dots)$ be a sequence of real valued one-dimensional standard Brownian motions independent of $(\Omega, \mathfrak{F}, \mathbb{P})$. Set $w(t) = \sum_{n=1}^{\infty} \sqrt{\lambda_n} \beta_n(t) \zeta_n(t), t \geq 0$, where, $\lambda_n \geq 0$ are non-negative real numbers and $\{\zeta_n\} (n = 1, 2, \dots)$ is complete orthonormal basis in \mathbb{K} . Let $Q \in \mathcal{L}(\mathbb{K}, \mathbb{H})$ be an operator defined by $Q\zeta_n = \lambda_n \zeta_n$ with finite $Tr(Q) = \sum_{n=1}^{\infty} \lambda_n < \infty$. Then the above \mathbb{K} -valued stochastic process $w(t)$ is called a Q -Wiener process. Let $\Psi \in \mathcal{L}_Q^0(\mathbb{K}, \mathbb{H})$ and define,

$$\|\Psi\|_{\mathcal{L}_Q^0}^2 = Tr(\Psi Q \Psi^*) = \sum_{n=1}^{\infty} \|\sqrt{\lambda_n} \Psi \zeta_n\|^2.$$

If $\|\Psi\|_Q < \infty$, then Ψ is known as Q -Hilbert Schmidt operator. For more details on concepts and theory on SDEs, one can refer the articles [7, 8, 13, 24] and references therein. The axioms of the space \mathcal{B} are established for \mathfrak{F}_0 -measurable functions from $(-\infty, 0]$ into \mathbb{H} endowed with a norm $\|\cdot\|_{\mathcal{B}}$, which satisfies the following axioms:

(a) If $u : (-\infty, T] \rightarrow \mathbb{H}, T > 0$ is such that $u_0 \in \mathcal{B}$, then, for every $t \in [0, T]$, the following conditions hold:

(i) $u_t \in \mathcal{B}$,

(ii) $\|u(t)\| \leq m \|u_t\|_{\mathcal{B}}$,

(iii) $\|u_t\|_{\mathcal{B}} \leq M(t) \sup_{0 \leq s \leq t} \|u(s)\| + N(t) \|u_0\|_{\mathcal{B}}$

where $m > 0$ is a constant, $M, N : [0, +\infty) \rightarrow [1, +\infty)$, M is continuous, N is locally bounded, M, N are independent of $u(\cdot)$.

(b) The space \mathcal{B} is complete.

Let $u : (-\infty, T] \rightarrow \mathbb{H}$ be an \mathfrak{F}_t -adapted measurable process such that \mathfrak{F}_0 -adapted process $u_0 = \varphi \in \mathcal{L}_2^0(\Omega, \mathcal{B})$ then

$$\mathbf{E} \|u_s\|_{\mathcal{B}} \leq N \mathbf{E} \|\varphi\|_{\mathcal{B}} + M \sup_{0 \leq s \leq T} \mathbf{E} \|u_s\|$$

where $N = \sup_{t \in [0, T]} \{N(t)\}$ and $M = \sup_{t \in [0, T]} \{M(t)\}$. Define $\mathfrak{M}^2((-\infty, 0), \mathbb{B})$ be the space of all \mathbb{H} -valued continuous \mathfrak{F}_t -adapted process $u = \{u(t)\}_{-\infty < t \leq T}$ such that

(1) $u_0 = \varphi \in \mathcal{B}$ and $u(t)$ is continuous on $[0, T]$

(2) define the norm $\|\cdot\|_{\mathfrak{M}}$ in $\mathfrak{M}^2((-\infty, T], \mathbb{H})$ by

$$\|x\|_{\mathfrak{M}}^2 = \mathbf{E} \|\varphi\|_{\mathcal{B}}^2 + \mathbf{E} \int_0^T \|u(s)\|^2 dt < \infty. \tag{2}$$

Then, $\mathfrak{M}^2((-\infty, T], \mathbb{H})$ with the norm is a Banach space.

Definition 2.1. [18] The basic concepts of the Rosenblatt process as for as Wiener integral, let $Z_H(t)$ be one-dimensional Rosenblatt process with the Hurst parameter $H \in (\frac{1}{2}, 1)$. Hence the Rosenblatt process with parameter $H > \frac{1}{2}$ representation as

$$Z_H(t) = d(H) \int_0^t \int_0^t \left[\int_0^t \frac{\partial K}{\partial \mathbf{u}}(\mathbf{u}, \zeta_1) \frac{\partial K}{\partial \mathbf{u}}(\mathbf{u}, \zeta_2) d\mathbf{u} \right] dB(\zeta_1) dB(\zeta_2)$$

where $K^H(t, s)$ is defined as

$$K^H(t, s) = C_H s^{\frac{1}{2}-H} \int_s^t (\mathbf{u} - s)^{H-\frac{3}{2}} \mathbf{u}^{H-\frac{3}{2}} d\mathbf{u}, t > s$$

with $C_H = \sqrt{\frac{H(2H-1)}{\gamma(2-2H, H-\frac{1}{2})}}$.

For basic preliminaries and fundamental results on Rosenblatt process can refer the articles therein [18, 19].

Definition 2.2. A map $\mathfrak{S} : [0, T] \times [0, T] \rightarrow \mathcal{L}(\mathbb{H})$ is said to be an evolution operator for equation (1) if the following conditions are fulfilled:

(A1) For each $u \in \mathbb{H}$ the map $(t, s) \rightarrow \Xi(t, s)u$ is continuously differentiable and

(i) For each $t \in [0, T]$, $\Xi(t, t) = 0$.

(ii) For all $t, s \in [0, T]$ and each $u \in \mathbb{H}$, $\frac{\partial}{\partial t} \Xi(t, s)u|_{t=s} = u$ and $\frac{\partial}{\partial t} \Xi(t, s)u|_{t=s} = -u$.

(A2) For all $s, t \in [0, T]$ if $u \in \mathcal{D}$, then $\Xi(t, s)u \in \mathcal{D}$, the map $(t, s) \rightarrow \Xi(t, s)u$ is of class \mathcal{C}^2 and (i) $\frac{\partial^2}{\partial t^2} \Xi(t, s)u = \mathfrak{A}(t)\Xi(t, s)x$.

(ii) $\frac{\partial^2}{\partial s^2} \Xi(t, s)u = \Xi(t, s)\mathfrak{A}(s)x$.

(iii) $\frac{\partial^2}{\partial s \partial t} \Xi(t, s)u|_{t=s} = 0$.

(A3) For all $s, t \in [0, T]$, if $x \in \mathcal{D}$, then $\frac{\partial}{\partial s} \Xi(t, s)x \in \mathcal{D}$, there exist $\frac{\partial^3}{\partial t^2 \partial s} \Xi(t, s)x$, $\frac{\partial^3}{\partial s^2 \partial t} \Xi(t, s)x$ and

(i) $\frac{\partial^3}{\partial t^2 \partial s} \Xi(t, s)x = \mathfrak{A}(t)\frac{\partial}{\partial s} \Xi(t, s)x$. Moreover, the map $(t, s) \rightarrow \mathfrak{A}(t)\frac{\partial}{\partial s} \Xi(t, s)x$ is continuous.

(ii) $\frac{\partial^3}{\partial t^2 \partial s} \Xi(t, s)x = \frac{\partial}{\partial t} \Xi(t, s)\mathfrak{A}(s)x$.

Moreover, we assume that there exists an evolution operator $\Xi(t, s)$ associated to the operators $\mathfrak{A}(t)$. Also, we define the operator $\mathfrak{C}(t, s) = -\frac{\partial \Xi(t, s)}{\partial s}$. Furthermore, $L > 0$ such that

$$\sup_{0 \leq s, t \leq T} \|\Xi(t, s)\|^2 \leq L, \text{ and } \sup_{0 \leq s, t \leq T} \|\mathfrak{C}(t, s)\|^2 \leq L.$$

Definition 2.3. A continuous stochastic process $u : (-\infty, T] \rightarrow \mathbb{H}$ is said to be a mild solution of (1) if

(i) $u(t)$ is \mathfrak{F} -adapted and $\{u_t : t \in [0, T]\}$ is \mathcal{B} -valued,

(ii) $\int_0^T \|u(t)\|^2 < \infty$, \mathbb{P} -a.s.,

(iii) For each $t \in [0, T]$, $u(t)$ satisfies the following integral equation:

$$\begin{aligned} u(t) &= \mathfrak{C}(t, 0)\varphi(0) + \Xi(t, 0) \left[\zeta - \mathfrak{f}_g(0, \varphi) \right] + \mathfrak{f}_g(s, u_s) + \int_0^t \Xi(t, s)\mathfrak{f}(s, u_s)ds \\ &+ \int_0^t \Xi(t, s)g(s, u_s)dw(s) + \int_0^t \int_{\mathfrak{J}} \Xi(t, s)h(s, u_s, \eta)\tilde{N}(ds, d\eta) \\ &+ \int_0^t \Xi(t, s)\sigma(s)dZ_H(s). \end{aligned} \tag{3}$$

(iv) $u_0 = \varphi \in \mathcal{B}$.

3. Existence and Uniqueness Results

In this section, we prove the existence, uniqueness and stability of mild solutions for nonautonomous NSDEs with infinite delay driven by Poisson jumps and Rosenblatt process. In order to prove the result, we impose the conditions as follows:

(H1) The functions $\mathfrak{f}_g, \mathfrak{f} : [0, T] \times \mathcal{B} \rightarrow \mathbb{H}$, $g : [0, T] \times \mathcal{B} \rightarrow \mathcal{L}_2^0$, $h : [0, T] \times \mathcal{B} \times \mathfrak{J} \rightarrow \mathbb{H}$ and $\sigma : [0, T] \rightarrow \mathcal{L}_2^0$ satisfy $\forall t \in [0, T], x, y \in \mathcal{B}$

$$(i) \|\mathfrak{f}(t, x) - \mathfrak{f}(t, y)\|^2 \vee \|g(t, x) - g(t, y)\|^2 \leq \kappa(\|x - y\|_{\mathcal{B}}^2),$$

$$(ii) \int_{\mathfrak{J}} \|\mathfrak{h}(s, x, \eta) - \mathfrak{h}(s, y, \eta)\|^2 \lambda(d\eta)ds \vee$$

$$\left(\int_{\mathfrak{J}} \|\mathfrak{h}(s, x, \eta) - \mathfrak{h}(s, y, \eta)\|^4 \lambda(d\eta)ds \right)^{1/2} \leq \kappa(\|x - y\|_{\mathcal{B}}^2),$$

$$(iii) \left(\int_{\mathfrak{J}} \|\mathfrak{h}(s, x, \eta) - \mathfrak{h}(s, y, \eta)\|^4 \lambda(d\eta)ds \right)^{1/2} \leq \kappa(\|x - y\|_{\mathcal{B}}^2),$$

where $\kappa(\cdot)$ is a concave, nondecreasing, continuous function from \mathbb{R}_+ to \mathbb{R}_+ such that $\kappa(0) = 0$, $\kappa(\vartheta) > 0$ for $\vartheta > 0$ and $\int_{0^+} \frac{d\vartheta}{\kappa(\vartheta)} = \infty$.

(H2) There exists $M_{\mathfrak{f}_g} > 0$ such that $M_{\mathfrak{f}_g} < \frac{1}{14}$ for any $x, y \in \mathcal{D}$

$$\|\mathfrak{f}_g(t, x) - \mathfrak{f}_g(t, y)\|^2 \leq M_{\mathfrak{f}_g} \|x - y\|^2, \quad \mathfrak{f}_g(t, 0) = 0, \quad t \geq 0.$$

(H3) The function $\sigma : [0, T] \rightarrow \mathcal{L}_Q^2$ satisfies

$$\left\| \int_a^b \sigma(s) dZ_H(s) \right\|^2 \leq \mathbf{C}(H) t^{2H} \left(\sup_{t \in [0, T]} \|\sigma(t)\|_{\mathcal{L}_Q^0(\mathbb{K}, \mathbb{H})}^2 \right).$$

(H4) For all $t \in [0, T]$ there exists $\kappa_0 > 0$,

$$\|\mathfrak{f}(t, 0)\|^2 \vee \|\mathfrak{g}(t, 0)\|^2 \vee \int_3 \|b(t, 0, \eta)\|^2 \leq \kappa_0.$$

Further, in order to prove the result, let us now introduce the successive approximations as follows:

$$\begin{aligned} u^0(t) &= \mathfrak{C}(t, 0)\varphi(0) + \mathfrak{E}(t, 0) [\zeta - \mathfrak{f}_g(0, \varphi)], \quad t \in [0, T], \\ u^n(t) &= \mathfrak{C}(t, 0)\varphi(0) + \mathfrak{E}(t, 0) [\zeta - \mathfrak{f}_g(0, \varphi)] + \mathfrak{f}_g(s, u_s^n) + \int_0^t \mathfrak{C}(t, s)\mathfrak{f}(s, u_s^{n-1})ds \\ &+ \int_0^t \mathfrak{E}(t, s)\mathfrak{g}(s, u_s^{n-1})dw(s) + \int_0^t \int_3 \mathfrak{E}(t, s)h(s, u_s^{n-1}, \eta)\tilde{N}(ds, d\eta) \\ &+ \int_0^t \mathfrak{E}(t, s)\sigma(s)dZ_H(s). \tag{4} \\ u^n(t) &= \varphi(t), \quad -\infty < t \leq 0, \quad n \geq 1. \end{aligned}$$

Lemma 3.1. Assume that the conditions (H1) – (H4) hold. Then, for all $t \in [-\infty, T], n \geq 0$ there exists a constant C_1 such that $\mathbf{E} \|u^n\|^2 \leq C_1$.

Proof: For all $t \in (-\infty, T]$, the sequence $u^n(t), n \geq 1$ is bounded. It is obvious that $u^0(t) \in \mathfrak{M}^2((-\infty, T], \mathbb{H})$. By induction, we show that $u^n(t) \in \mathfrak{M}^2((-\infty, T], \mathbb{H})$. Now, we have

$$\begin{aligned} \mathbf{E} \left(\sup_{0 \leq s \leq t} \|u^n(s)\|^2 \right) &\leq 7\mathbf{L}\mathbf{E} \|\varphi\|^2 + 14\mathbf{L}\mathbf{E} \|\zeta\|^2 + 14\mathbf{L}M_{\mathfrak{f}_g} \mathbf{E} \|\varphi\|^2 \\ &+ 7M_{\mathfrak{f}_g} \mathbf{L}\mathbf{E} \|u^n(s)\|^2 + 14\mathbf{L}T\mathbf{E} \int_0^t \kappa \left(\|u_s^{n-1}\|^2 \right) ds + 14\mathbf{L}T^2\kappa_0 \\ &+ 14\mathbf{L}\mathbf{E} \int_0^t \kappa \left(\|u_s^{n-1}\|^2 \right) ds + 14\mathbf{L}T\kappa_0 \\ &+ 21\mathbf{L}\mathbf{E} \int_0^t \kappa \left(\|u_s^{n-1}\|^2 \right) ds + 14\mathbf{L}T\kappa_0 + 7\mathbf{L}\mathbf{C}(H)T^{2H} \\ &\leq \mathfrak{Q}_1 + 7M_{\mathfrak{f}_g} \mathbf{E} \|u_s^n\|^2 + 7\mathbf{L}(2T + 5)\mathbf{E} \int_0^t \kappa \left(\|u_s^{n-1}\|^2 \right) ds \end{aligned}$$

where $\mathfrak{Q}_1 = 7\mathbf{L}\mathbf{E} \|\varphi\|^2 + 14\mathbf{L}\mathbf{E} \|\zeta\|^2 + 14\mathbf{L}M_{\mathfrak{f}_g} \mathbf{E} \|\varphi\|^2 + 14\mathbf{L}T^2\kappa_0 + 28\mathbf{L}T\kappa_0 + 14\mathbf{L}T(T + 2)\kappa_0 + 7\mathbf{L}\mathbf{C}(H)T^{2H}$. Given that $\kappa(\cdot)$ is a concave and $\kappa(0) = 0$, we can find positive constants a and b such that

$$\kappa(v) \leq a + bv, \quad \text{for all } v \geq 0.$$

So, let us derive that

$$\begin{aligned} \mathbf{E} \left(\sup_{0 \leq s \leq t} \|u^n(s)\|^2 \right) &\leq \mathfrak{Q}_1 + 7La(2T + 5)T + 7M_{f_g} \mathbf{E} \|u_s^n\|^2 + 7L(2T + 5)b \int_0^t \mathbf{E} \left(\|u^{n-1}(s)\|^2 \right) ds \\ &\leq \mathfrak{Q}_2 + 7M_{f_g} \mathbf{E} \|u_s^n\|^2 + 7L(2T + 5)b \int_0^t \mathbf{E} \left(\sup_{0 \leq s \leq t} \|u^{n-1}(s)\|^2 \right) ds \end{aligned}$$

where $\mathfrak{Q}_2 = \mathfrak{Q}_1 + 7La(2T + 5)T$ and noting that

$$\begin{aligned} \mathbf{E} \left(\sup_{0 \leq s \leq t} \|u^n(s)\|^2 \right) &\leq \mathfrak{Q}_2 + 14M_{f_g} \mathbf{E} \|\varphi\|^2 + 14M_{f_g} \mathbf{E} \sup_{0 \leq s \leq t} \|u_s^n\|^2 \\ &+ 14Lb(2T + 5)T \mathbf{E} \|\varphi\|^2 + 14Lb(2T + 5) \int_0^t \mathbf{E} \left(\sup_{0 \leq s \leq t} \|u_s^{n-1}\|^2 \right) ds \\ &\leq \frac{1}{1 - 14M_{f_g}} \left\{ \mathfrak{Q}_3 + 14Lb(2T + 5) \int_0^t \mathbf{E} \left(\sup_{0 \leq s \leq t} \|u_s^{n-1}\|^2 \right) ds \right\} \end{aligned}$$

where $\mathfrak{Q}_3 = \mathfrak{Q}_2 14M_{f_g} \mathbf{E} \|\varphi\|^2 + 14Lb(2T + 5)T \mathbf{E} \|\varphi\|^2$. On the other hand, for any $k \geq 1$,

$$\begin{aligned} \max_{1 \leq n \leq k} \mathbf{E} \sup_{0 \leq s \leq t} \|u^{n-1}(s)\|^2 &\leq \mathbf{E} \|u^0(s)\|^2 + \max_{1 \leq n \leq k} \mathbf{E} \sup_{0 \leq s \leq t} \|u^{n-1}(s)\|^2, \\ \max_{1 \leq n \leq k} \mathbf{E} \sup_{0 \leq s \leq t} \|u^n(s)\|^2 &\leq \frac{1}{1 - 14M_{f_g}} \left\{ \mathfrak{Q}_3 + 14Lb(2T + 5) \int_0^t \mathbf{E} \left(\|u_s^{n-1}\|^2 \right) ds \right. \\ &+ \left. 14Lb(2T + 5) \int_0^t \max_{1 \leq n \leq k} \mathbf{E} \|u^n(r)\|^2 ds \right\} \\ &\leq \frac{1}{1 - 14M_{f_g}} \left\{ \mathfrak{Q}_3 + 14Lb(2T + 5) [2\mathbf{E} \|\varphi\|^2 + 4LE \|\zeta\|^2 + 4L f_g \mathbf{E} \|\varphi\|^2] T \right. \\ &+ \left. 14Lb(2T + 5) \int_0^t \max_{1 \leq n \leq k} \mathbf{E} \left(\sup_{0 \leq s \leq t} \|u^n(r)\| \right)^2 ds \right\} \\ &\leq \mathfrak{Q}_4 + \mathfrak{Q}_5 \int_0^t \max_{1 \leq n \leq k} \mathbf{E} \left(\sup_{0 \leq s \leq t} \|u^n(s)\| \right)^2 ds. \end{aligned}$$

where

$$\begin{aligned} \mathfrak{Q}_4 &= \frac{1}{1 - 14M_{f_g}} \left\{ \mathfrak{Q}_3 + 14Lb(2T + 5) [2\mathbf{E} \|\varphi\|^2 + 4LE \|\zeta\|^2 + 4L f_g \mathbf{E} \|\varphi\|^2] T \right\} \\ \mathfrak{Q}_5 &= \frac{14Lb(2T + 5)}{1 - 14M_{f_g}} \end{aligned}$$

$$\max_{1 \leq n \leq k} \mathbf{E} \left(\sup_{0 \leq s \leq t} \|u^n(s)\|^2 \right) \leq \mathfrak{Q}_4 + \mathfrak{Q}_5 \int_0^t \max_{1 \leq n \leq k} \mathbf{E} \left(\sup_{0 \leq s \leq t} \|u^n(s)\| \right)^2 ds.$$

Using the Gronwall inequality in the above inequality, we get

$$\max_{1 \leq n \leq k} \mathbf{E} \left(\sup_{0 \leq s \leq t} \|u^n(s)\|^2 \right) \leq \mathfrak{Q}_4 e^{\mathfrak{Q}_5 T}.$$

Since, k is arbitrary, we have

$$\mathbf{E} \left(\sup_{0 \leq s \leq t} \|u^n(s)\|^2 \right) \leq \mathfrak{Q}_4 e^{\mathfrak{Q}_5 T}, \quad \forall 0 \leq t \leq T, n \geq 1.$$

Hence by the result, we have

$$\begin{aligned} \mathbf{E} \|u^n\|^2 &= \mathbf{E} \|u_0^2\|^2 + \int_0^t \mathbf{E} \|u^n(s)\|^2 ds \\ &\leq \mathbf{E} \|\varphi\|^2 + T\mathfrak{D}_4 e^{\mathfrak{D}_5 T} \leq C_1 < \infty, \end{aligned}$$

where $C_1 = \mathbf{E} \|\varphi\|^2 + T\mathfrak{D}_4 e^{\mathfrak{D}_5 T}$. Hence the proof.

Lemma 3.2. *If the assumptions of Lemma 3.1 are satisfied, then there exist positive constants C_2, C_3 such that*

$$\begin{aligned} \mathbf{E} \left(\sup_{0 \leq s \leq t} \|u^{n+m}(s) - u^n(s)\|^2 \right) &\leq C_2 \int_0^t \kappa \left(\mathbf{E} \left(\|u^{n+m-1}(r) - u^{n-1}(r)\|^2 \right) \right) ds \\ \mathbf{E} \left(\sup_{0 \leq s \leq t} \|u^{n+m}(s) - u^n(s)\|^2 \right) &\leq C_3 t. \end{aligned}$$

for all $0 \leq t \leq T, n, m \geq 1$.

Proof: By the definition of u^n , we obtain

$$\begin{aligned} &\mathbf{E} \left(\sup_{0 \leq s \leq t} \|u^{n+m}(s) - u^n(s)\|^2 \right) \\ &\leq 4\mathbf{E} \left(\sup_{0 \leq s \leq t} \|\tilde{f}_g(s, u^{n+m}) - \tilde{f}_g(s, u^n)\|^2 \right) \\ &+ 4\mathbf{E} \left(\sup_{0 \leq s \leq t} \left\| \int_0^s \Xi(t, s) [\tilde{f}(s, u^{n+m-1}) - \tilde{f}(s, u^{n-1})] ds \right\|^2 \right) \\ &+ 4\mathbf{E} \left(\sup_{0 \leq s \leq t} \left\| \int_0^s \Xi(t, s) [g(s, u^{n+m-1}) - g(s, u^{n-1})] d\omega(s) \right\|^2 \right) \\ &+ 4\mathbf{E} \left(\sup_{0 \leq s \leq t} \left\| \int_0^s \int_3 \Xi(t, s) [\tilde{h}(s, u^{n+m-1}, \eta) - \tilde{h}(s, u^{n-1}, \eta)] \tilde{N}(ds, d\eta) \right\|^2 \right) \\ &\leq \frac{\mathfrak{D}_6}{1 - 4M_{\tilde{f}_g}} \int_0^t \tau \left(\sup_{0 \leq s \leq t} \|u^{n+m-1}(r) - u^{n-1}(r)\|^2 \right) ds \\ &\leq C_2 \int_0^t \kappa(2C_1) ds = C_3 t. \end{aligned}$$

Theorem 3.3. *Assume that the assumptions of Lemma 3.1 and Lemma 3.2 are hold. Then, the system (1) has a unique mild solution of $\mathfrak{M}^2((-\infty, T], \mathbb{H})$.*

Proof: Step 1: Let us show that $u^n(t), t \in [0, T]$ is a Cauchy sequence.

Let $v_1(\vartheta) = C_2 \kappa(\vartheta)$. Choose $T_1 \in [0, T]$ such that $v_1(C_3 \vartheta) \leq C_3$ for $\vartheta \in [0, T_1]$. We first introduce two sequences of functions $\phi_{n,m}(t)_{m,n \in \mathbb{N}_+}$ and $\phi_n(t)_{n \in \mathbb{N}_+}$ by

$$\begin{aligned} \phi_1(t) &= C_3 t, \\ \phi_{n+1}(t) &= \int_0^t v_1(\phi_n(\vartheta)) d\vartheta, \\ \phi_{m,n}(t) &= \mathbf{E} \sup_{0 \leq \vartheta \leq t} \|u^{m+n}(\vartheta) - u^n(\vartheta)\|^2. \end{aligned}$$

Then $\phi_n(t)_{n \in \mathbb{N}_+}$ is monotonically decreasing when $n \rightarrow \infty$ and $0 \leq \phi_{m,n}(t) \leq \phi_n(t)$ for all $m, n \geq 1, t \in [0, T_1]$. In fact, it is obvious that $\phi_{1,m}(t) \leq \phi_1(t)$ and

$$\begin{aligned} \phi_{2,m}(t) &= \mathbf{E} \sup_{0 \leq \vartheta \leq t} \|x^{m+2}(\vartheta) - x^2(\vartheta)\|^2 \\ &\leq \int_0^t v_1 E \left(\sup_{0 \leq \vartheta \leq s} \|x^{m+1}(\vartheta) - x^1(\vartheta)\|^2 \right) ds \\ &\leq \int_0^t v_1(\phi_1(s)) ds \\ &= \phi_2(t) \leq C_3 t = \phi_1(t), \end{aligned}$$

which implies that $\phi_2(t) \leq \phi_1(t)$. Now assume the results holds for n , then

$$\begin{aligned} \phi^{n+1,m}(t) &= \mathbf{E} \sup_{0 \leq \vartheta \leq t} \|u^{m+n+1}(\vartheta) - u^{n+1}(\vartheta)\|^2 \\ &\leq \int_0^t v_1(\phi_{m,n}(s)) ds \\ &\leq \int_0^t v_1(\phi_n(s)) ds \\ &= \phi_{n+1}(t) \leq \int_0^t v_1(\phi_{n-1}(s)) ds \\ &= \phi_n(t). \end{aligned}$$

This shows that $\phi_n(t)$ is a nonnegative and decreasing continuous function on $[0, T_1]$ by induction on n , so we can define a function $\phi_n(t)$ by $\phi_k(t) \downarrow \phi(t)$, and it is easy to verify that $\phi(0) = 0$ and $\phi(t)$ is a continuous function on $[0, T_1]$. Consequently, $\phi(t) = \lim_{n \rightarrow \infty} \phi_n(t) = \lim_{n \rightarrow \infty} \int_0^t v_1(\phi_{n-1}(s)) ds = \int_0^t v_1(\phi(s)) ds$. From $\phi(0) = 0, \int_{0^+} \frac{d\vartheta}{v_1(\vartheta)} = +\infty$ together with Bihari inequality, we obtain $\phi(t) \equiv 0$. Thus $0 \leq \phi_{n,n}(t) \leq \phi_n(b_1) \rightarrow 0$ as $n \rightarrow \infty$. This shows that $x^n(t), t \in [0, T_1]$ is a Cauchy sequence in $\mathfrak{M}^2((-\infty, T], \mathbb{H})$. The Borel-Cantelli lemma shows that as $n \rightarrow \infty, u^n(t) \rightarrow u(t)$ holds uniformly for $0 \leq t \leq T$. So, taking limits on both sides of (5), for all $-\infty \leq t \leq T$, we obtain that $u(t)$ is a solution of (1).

Step 2: Uniqueness Let $u(t), v(t)$ be two solutions of (1). Then the uniqueness is obvious on the interval $[-\infty, 0]$, and for $0 \leq t \leq T$, it is easy to show that by using Lemma 3.2, we have

$$\mathbf{E} \sup_{0 \leq s \leq t} \|u(s) - v(s)\|^2 \leq C_2 \int_0^t \kappa \left(\sup_{0 \leq r \leq s} \|u(r) - v(r)\|^2 \right) ds.$$

The Bihari inequality yields that

$$\mathbf{E}(\sup_{0 \leq s \leq t} \|u(s) - v(s)\|^2) = 0, \quad 0 \leq t \leq T.$$

Therefore, $u(t) = v(t)$ for all $0 \leq t \leq T$.

4. Stability Results

In this section, we will give the continuous dependence of solutions on the initial value by means of a corollary of Bihari’s inequality.

Definition 4.1. A mild solution $u^{\zeta,u}(t)$ with initial value (ζ, u) is said to be stable in mean square if for all $\epsilon > 0$ there exists $\delta > 0$ such that

$$\mathbf{E} \left(\sup_{0 \leq s \leq T} \|u^{\zeta,u}(s) - u^{\xi,u}(s)\|^2 \right) \leq \epsilon, \quad \text{when } \mathbf{E} \|\zeta - \xi\|^2 + \mathbf{E} \|u - v\|^2 \leq \delta,$$

where $u^{\xi,u}(t)$ is another solution of (1) with initial value (ξ, v) .

Lemma 4.2. (Bihari inequality)[5]: Let $T > 0$ and $u_0 \geq 0$, $u(t), v(t)$ be continuous functions on $[0, T]$. Let $K : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ be a concave continuous and nondecreasing function such that $K(r) > 0$ for $r > 0$. If

$$u(t) \leq u_0 + \int_0^t v(s)K(u(s))ds \text{ for } 0 \leq t \leq T,$$

then

$$u(t) \leq G^{-1}\left(G(u_0) + \int_0^t v(s)ds\right)$$

for all $t \in [0, T]$ such that

$$G(u_0) + \int_0^t v(s)ds \in \text{Dom}(G^{-1}),$$

where $G(r) = \int_1^r \frac{ds}{K(s)}$, for $r \geq 0$ and G^{-1} is the inverse function of the G . In particular, if, moreover, $u_0 = 0$ and $\int_0^+ \frac{ds}{K(s)} = +\infty$, then $u(t) = 0$ for all $t \in [0, T]$.

Lemma 4.3. [7] Let the assumption of Lemma 4.2 hold. If

$$u(t) \leq u_0 + \int_0^t v(s)K(u(s))ds \text{ for } 0 \leq t \leq T,$$

then

$$u(t) \leq G^{-1}\left(G(u_0) + \int_0^t v(s)ds\right), \quad t \in [0, T]$$

such that

$$G(u_0) + \int_0^t v(s)ds \in \text{Dom}(G^{-1}),$$

where $G(r) = \int_1^r \frac{ds}{K(s)}$, for $r \geq 0$ and G^{-1} is the inverse function of the G .

Corollary 4.4. [7] Let the assumptions of Lemma 4.2 hold and $v(t) \geq 0$ for $t \in [0, T]$. If for all $\epsilon > 0$, there exists $t_1 \geq 0$ such that for $0 \leq u_0 < \epsilon$, $\int_{t_1}^T v(s)ds \leq \int_{u_0}^\epsilon \frac{ds}{K(s)}$ holds. Then for every $t \in [t_1, T]$, the estimate $u(t) \leq \epsilon$ holds.

Theorem 4.5. Assume that the conditions of Theorem 3.3 are satisfied, then the solution of (1) is stable in mean square.

Proof: Let $u^{\zeta, u}(s), u^{\xi, u}(s)$ be solutions of (1) with initial value (ζ, u) and (ξ, u) respectively. This implies that $u(t)$ and $v(t)$ be two solutions of (1) with initial value (ζ, u) and (ξ, u) . Then we have

$$\mathbf{E}\left(\sup_{0 \leq s \leq t} \|u(s) - v(s)\|^2\right) \leq \frac{6L(3+2)M_{f_g}}{1-6M_{f_g}} \mathbf{E}\|\zeta - \xi\|^2 + \frac{6L(T+3)}{1-6M_{f_g}} \int_0^t \kappa(\|u(r) - v(r)\|^2) ds$$

Let $\kappa_1(\vartheta) = \frac{6L(3+2)M_{f_g}}{1-6M_{f_g}} \kappa(\vartheta)$, where κ is a concave increasing function from \mathbb{R}^+ to \mathbb{R}^+ such that $\kappa(0) = 0$, $\kappa(\vartheta) > 0$ for $\vartheta > 0$ and $\int_{0^+} \frac{d\vartheta}{\kappa(\vartheta)} = +\infty$. Then, $\kappa_1(\vartheta)$ is concave from \mathbb{R}^+ to \mathbb{R}^+ such that $\kappa(0) = 0$, $\kappa_1(\vartheta) \geq \kappa(\vartheta)$ for $0 \leq \vartheta \leq 1$ and $\int_{0^+} \frac{d\vartheta}{\kappa(\vartheta)} = +\infty$. Now for any $\epsilon > 0$, $\epsilon_1 = \frac{1}{2}\epsilon$, we have $\lim_{s \rightarrow 0} \int_s^{\epsilon_1} \frac{d\vartheta}{\kappa_1(\vartheta)} = \infty$. Then, there is a positive constant $\delta < \epsilon_1$, such that $\int_\delta^{\epsilon_1} \frac{d\vartheta}{\kappa_1(\vartheta)} \geq T$.

Let

$$\vartheta_0 = \frac{6L(3+2)M_{f_g}}{1-6M_{f_g}} \mathbf{E}\|\varphi_1 - \varphi_2\|^2,$$

$$\vartheta(t) = \mathbf{E}\|x - y\|_{\mathcal{B}}^2, \quad \varsigma(t) = 1,$$

when $\vartheta_0 \leq \delta \leq \epsilon_1$. Corollary 4.4, shows that

$$\int_{\vartheta_0}^{\epsilon_1} \frac{du}{\kappa_1(\vartheta)} \geq \int_{\delta}^{\epsilon_1} \frac{du}{\kappa_1(\vartheta)} \geq T = \int_0^T c(s)ds.$$

So, for any $t \in [0, T]$, the estimate $\vartheta(t) \leq \epsilon_1$ hold. This completes the proof of the theorem.

5. Autonomous Case

Next, we consider the autonomous case of Equations (1). Put $\mathfrak{A}(t) = \mathfrak{A}$ the higher-order nonautonomous NSDEs with infinite delay of (1) becomes

$$\begin{aligned} d[u'(t) - \hat{f}_1(t, u_t)] &= \mathfrak{A}[u(t) - \hat{f}_1(t, u_t)]dt + \hat{f}_2(t, u_t)dt + g(t, u_t)dw(t) + \int_3^{\infty} h(t, u_t, \eta)\tilde{N}(dt, d\eta) \\ &+ \sigma(t)dZ_H(t), \quad t \in [0, T], \\ u_0 &= \varphi \in \mathcal{B}, \quad u'(0) = \zeta, \end{aligned} \tag{5}$$

where \mathfrak{A} is the infinitesimal generator of a strongly continuous cosine family $\mathfrak{C}(t)$ on \mathbb{H} . Let the functions $\hat{f}_1, \hat{f}_2, g, h$ and σ are defined as in equation (1). Now, we will present some facts about cosine families of operators.

Definition 5.1. The one parameter family $\{\mathfrak{C}(t) : t \in \mathbb{R}\} \in \mathcal{L}(\mathbb{H}, \mathbb{H})$ satisfying that

- (i) $\mathfrak{C}(0) = I$
- (ii) $\mathfrak{C}(t)x$ is continuous in t on \mathbb{R} , for all $x \in \mathbb{H}$
- (iii) $\mathfrak{C}(t + s) + \mathfrak{C}(t - s) = 2\mathfrak{C}(t)\mathfrak{C}(s)$ for all $t, s \in \mathbb{R}$ is called a strongly continuous cosine family.

Definition 5.2. A continuous stochastic process $u : (-\infty, T] \rightarrow \mathbb{H}$ is said to be a mild solution of (5) if

- (i) $u(t)$ is \mathfrak{F} -adapted and $\{u_t : t \in [0, T]\}$ is \mathcal{B} -valued,
- (ii) $\int_0^T \|u(t)\|^2 < \infty$, \mathbb{P} -a.s.,
- (iii) For each $t \in [0, T]$, $u(t)$ satisfies the following integral equation:

$$\begin{aligned} u(t) &= \mathfrak{C}(t)\varphi(0) + \mathfrak{S}(t) [\zeta - \hat{f}_g(0, \varphi)] + \hat{f}_g(s, u_s) + \int_0^t \mathfrak{S}(t - s)\hat{f}(s, u_s)ds \\ &+ \int_0^t \mathfrak{S}(t - s)g(s, u_s)dw(s) + \int_0^t \int_3^{\infty} \mathfrak{S}(t - s)h(s, u_s, \eta)\tilde{N}(ds, d\eta) \\ &+ \int_0^t \mathfrak{S}(t - s)\sigma(s)dZ_H(s). \end{aligned} \tag{6}$$

- (iv) $u_0 = \varphi \in \mathcal{B}$.

Next, we provide the existence, uniqueness and stability of mild solution to the autonomous NSDEs with infinite delays of (5).

Theorem 5.3. Assume that the cosine family of operators $\{\mathfrak{C}(t) : t \in [0, T]\}$ on \mathbb{H} and the corresponding sine family $\{\mathfrak{S}(t) : t \in [0, T]\}$ satisfy the conditions

$$\|\mathfrak{C}(t)\|^2 \leq L, \quad \|\mathfrak{S}(t)\|^2 \leq L, \quad t \geq 0$$

for a positive constant L . Further, assume that the conditions **(H1)** – **(H4)** hold. Then, there exists a unique mild solution of (5) in $\mathfrak{M}^2((-\infty, T], \mathbb{H})$.

Proof: First, we consider the sequence of successive approximate as follows:

$$\begin{aligned}
 u^0(t) &= \mathfrak{C}(t)\varphi(0) + \mathfrak{S}(t) [\zeta - \hat{f}_g(0, \varphi)], \quad t \in [0, T], \\
 u^n(t) &= \mathfrak{C}(t)\varphi(0) + \mathfrak{S}(t) [\zeta - \hat{f}_g(0, \varphi)] + \hat{f}_g(s, u_s^n) + \int_0^t \mathfrak{C}(t-s)\hat{f}(s, u_s^{n-1})ds \\
 &+ \int_0^t \mathfrak{S}(t-s)g(s, u_s^{n-1})dw(s) + \int_0^t \int_3 \mathfrak{S}(t-s)h(s, u_s^{n-1}, \eta)\tilde{N}(ds, d\eta) \\
 &+ \int_0^t \mathfrak{S}(t-s)\sigma(s)dZ_H(s). \\
 u^n(t) &= \varphi(t), \quad \infty < t \leq 0, \quad n \geq 1.
 \end{aligned} \tag{7}$$

The proof of this theorem is similar to that of Theorem 3.3, and one can easily prove that solution of system (5) and hence, it is omitted.

Theorem 5.4. Let $u^{\zeta, \mu}(t)$ and $u^{\xi, \nu}(t)$ be solutions of (5) with initial value (ζ, u) and (ξ, v) respectively. Assume the assumptions **(H1)** – **(H4)** hold. Then, the solution of (5) is stable in mean square.

The proof of this theorem is similar to that of Theorem 4.3, and one can easily prove that solution of system (5) and hence, it is omitted.

6. Example

In this section, we consider the stochastic wave equation driven by Poisson jumps and Rosenblatt process of the form

$$\begin{aligned}
 \partial \left[\frac{\partial x(t, \zeta)}{\partial t} - \hat{f}_1(t, x(t-\tau, \zeta)) \right] &= \frac{\partial^2}{\partial \zeta^2} [x(t, \zeta) - \hat{f}_1(t, x(t-\tau, \zeta))] \partial t \\
 &+ \hat{f}_2(t, x(t-\tau), \zeta) \partial t + \hat{g}(t, x(t-\tau), \zeta) dw(t) \\
 &+ \int_3 \hat{h}(t, x(t-\tau)\eta, \zeta) \tilde{N}(dt, d\eta) + \hat{\sigma}(t) dZ_H(t), \quad \tau > 0, \quad t \in [0, 1], \\
 x(\theta, \zeta) &= \varphi(\theta, \zeta), \quad \theta \in (-\infty, 0], \quad 0 \leq \zeta \leq 2\pi, \\
 x(t, 0) &= x(t, 2\pi) = 0, \quad t \in [0, 1], \\
 \frac{\partial x(0, \zeta)}{\partial t} &= \psi(\zeta), \quad 0 \leq \zeta \leq 2\pi,
 \end{aligned} \tag{8}$$

where $dZ_H(s)$ is the Rosenblatt process, $\hat{f}_1, \hat{f}_2, g, h$ and σ are appropriate functions. Let $\mathbb{H} = \mathbb{K} = \mathcal{L}^2(\mathfrak{I}, \mathbb{C})$, where \mathfrak{I} is defined as the quotient $\mathbb{R}/2\pi\mathbb{Z}$ and $\mathcal{L}^2(\mathfrak{I}, \mathbb{C})$ denotes the Sobolev space of 2π -periodic functions $x : \mathbb{R} \rightarrow \mathbb{C}$ such that $x'' \in \mathbb{H}$. Define the operator $\mathfrak{A}x(\zeta) = \frac{\partial^2}{\partial \zeta^2} x(\zeta)$ with domain $\mathcal{D}(\mathfrak{A}) = \mathcal{L}^2(\mathfrak{I}, \mathbb{C})$. It is known that \mathfrak{A}_0 is the infinitesimal generator of a strongly continuous cosine function $\mathfrak{C}_0(t)$ and is given by

$$\mathfrak{C}_0(t)u = \sum_{n \in \mathbb{Z}} \cos(nt) \langle u, x_n \rangle x_n, \quad t \in \mathbb{R},$$

with corresponding sine function

$$\mathfrak{S}_0(t)u = t \langle u, x_0 \rangle x_0 + \sum_{n \in \mathbb{Z}, n \neq 0} \frac{\sin(nt)}{n} \langle u, x_n \rangle x_n, \quad t \in \mathbb{R}.$$

Also, \mathfrak{A}_0 has discrete spectrum and the spectrum of \mathfrak{A}_0 consists of eigenvalues $-n^2$ for $n \in \mathbb{Z}$, with associated eigenvectors $x_n(\zeta) = \frac{1}{2\pi} e^{in\zeta}$, $n \in \mathbb{Z}$. Furthermore, the set $\{x_n : n \in \mathbb{Z}\}$ is an orthonormal basis of \mathbb{H} . In particular, $\mathfrak{A}_0 = \sum_{n \in \mathbb{Z}} -n^2 \langle u, x_n \rangle x_n$ for $u \in \mathcal{D}(\mathfrak{A}_0)$. Also, it is clear that $\|\mathfrak{C}_0(t)\| \leq 1$ for all $t \in \mathbb{R}$ and

hence $\mathbb{C}_0(\cdot)$ is uniformly bounded on \mathbb{R} and $\mathfrak{Z}(t, s) : \mathbb{H} \rightarrow \mathbb{H}$ is well defined and satisfies the conditions of Definition 2.2. Let $\varphi(\theta)\zeta = \varphi(\theta, \zeta)$, $(\theta, \zeta) \in (-\infty, 0] \times [0, 2\pi]$, $x(t)\zeta = x(t, \zeta)$.

Define $\hat{f}_1, \hat{f}_2 : [0, T] \times \mathcal{B} \rightarrow \mathbb{H}$, $g : [0, T] \times \mathcal{B} \rightarrow \mathcal{L}_2^0$, $h : [0, T] \times \mathcal{B} \times \mathcal{Z} \rightarrow \mathbb{H}$ and $\sigma : [0, T] \rightarrow \mathcal{L}_2^0$ by $\hat{f}_1(t, x)(\cdot) = \hat{f}_1(t, x)(\cdot)$, $\hat{f}_2(t, x)(\cdot) = \hat{f}_2(t, x)(\cdot)$, $g(t, x)(\cdot) = \hat{g}(t, x)(\cdot)$, $h(t, x)(\cdot) = \hat{h}(t, x)(\cdot)$, and $\sigma(t, x)(\cdot) = \hat{\sigma}(t, x)(\cdot)$. Then, the system (8) can be rewritten as the abstract form from the system (1). Further, all the assumptions of Theorem 4.2, have been satisfied, so we can conclude that the mild solution of the system (8) is stable.

7. Conclusion

This manuscript addresses, we investigate the existence, uniqueness and stability of mild solutions for a class of higher-order nonautonomous NSDEs with infinite delay driven by Poisson jumps and Rosenblatt process in Hilbert space. More precisely, using semigroup theory and successive approximation method, we establish a set of sufficient conditions for obtained the required result. Further, the result is deduced to study the higher-order autonomous system. Finally, examples are provided to demonstrate the obtain results. Further, this result could be extended to investigate the controllability of higher-order nonautonomous NSDEs with infinite delay in future.

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