



# Online Optimization Based Adaptive Tracking Control For Redundant Manipulators with Model Uncertainties

Zhihao Xu<sup>a</sup>, Xuefeng Zhou<sup>a</sup>

<sup>a</sup>Guangdong Institute of Intelligent Manufacturing  
Guangdong Key Laboratory of Modern Control Technology

**Abstract.** Tracking control of robot manipulators is always a fundamental problem in robot control, especially for redundant manipulators with higher DOFs. This problem may become more complicated when there exist uncertainties in the robot model. In this paper, we propose an adaptive tracking controller considering the uncertain physical parameters. Based on the coordinate feedback, a Jacobian adaptation strategy is firstly built by updating kinematic parameters online, in which neither cartesian velocity nor joint acceleration is required, making the controller much easier to build. Using the Pseudo-inverse method of Jacobian, the optimal repeatability solution is achieved as the secondary task. Using Lyapunov theory, we have proved that the tracking errors of the end-effector asymptotically converge to zero. Numerical simulations are provided to validate the effectiveness of the proposed tracking method.

## 1. Introduction

Robot manipulators have been already used intensively in the field such as industry, agriculture, space exploration, *etc.* Therefore, the study on robotics, especially robot control, has been a hot topic in recent decades. Aiming at enhancing operating accuracy, tracking control is always a fundamental problem in robot control, and has attracted much attention of researchers.

Among these studies, tracking control in joint space aims at designing controller to drive each joint of the robot to track the predefined trajectories (see, e.g., [1–3] and references therein). Another direction of tracking control is task space tracking, in which the desired trajectory is formulated in cartesian space. The mismatch of control command and object (control command are sent to actuators at every joint while the end-effector is wished to perform in cartesian space) makes it more difficult than joint space tracking. Therefore, inverse kinematics should be solved first, namely, obtaining the required joint-space position or velocity to realize the task-space tracking. This can be done off-line or online. In [4], the desired path in cartesian space is dispersed into a group of key points, and the corresponding joint configuration is

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*Email addresses:* zh.xu@giim.ac.cn (Zhihao Xu), xf.zhou@giim.ac.cn (Xuefeng Zhou)

determined orderly, the desired joint speed and acceleration are derived by interpolation. Similar research can be seen in [5, 6]. This method is now widely used in industrial applications, however, this would cause a certain impact on the real-time performance of the system. For redundant manipulators, there exist infinite joint configurations corresponding to a particular cartesian description. Therefore, a secondary task can be done by adjusting the joints, such as avoiding obstacles, optimizing energy consumption, *etc.*

When physical parameters are perfectly known, a series of studies on real-time controllers are reported in [7, 8]. Actually, robot usually suffers from model uncertainties, including kinematic uncertainties, which may be caused by machining and measurement error. On the other hand, the robot may hold different tools, which will also cause kinematic uncertainties. The parameter drift will result in inaccurate Jacobian, leading to the degraded performance or unpredictable response, and should be compensated. Several calibration methods are proposed to identify the exact parameters before designing controllers [9, 10]. With the development of optical technology, it is possible for researchers to measure the precise position and orientation of the end-effector online. A series of real-time tracking controllers are proposed. Liu *et al.* develop an adaptive tracking scheme, in which the Jacobian is learned online, and a detailed discussion on selecting control gains is taken, stability of the closed-loop system is also proved [12]. In [13], a robust regulation controller is designed, in which actuator saturation is taken into consideration. By Lyapunov theory, semi-global stability is achieved. Another dynamic regulation controller is built in [14], which consists of a transpose Jacobian based item and a gravity compensator. When the desired path is variable, Cheah *et al.* proposes a passive based tracking controller [15], and the global convergence of tracking error is proved. Liu *et al.* uses a fuzzy logic system to learn the uncertain items of the robot's model, and then a tracking control scheme is designed based on sliding mode control. Those studies requires cartesian speed or joint accelerations, which is actually difficult to obtain due to limitations of hardware. Therefore, Wang *et al.* proposed a tracking controller based on a low-pass filter, in which measurement of cartesian speed is omitted [16]. Similar research can be seen in [17]. Similar researches can be also found in [18–20]. The above mentioned studies focus on the general problem of position control on robots with physical uncertainties, and the secondary task is not considered. This restricts the application of redundant robots.

Motivated from the above investigations, in this paper, we focus on the kinematic control problem of redundant manipulators, in which uncertain kinematic parameters are considered. In practice, robots are usually arranged to perform periodic tasks, we select repeatability as the secondary task. To avoid the measurement of both task-space velocity and joint acceleration, a novel adaptive controller is designed, and the secondary task is achieved by optimizing a defined function in the null space of Jacobian matrix. We also offer stability analysis and numerical simulations.

The remainder of this paper is organized as follows. In section 2, the basic kinematics of redundant robot is given, we also offer several important properties which will be used in the following sections. In section 3, the detailed discussion of the proposed adaptive controller is illustrated, including adaptive method of model parameters and repeatability optimization. Convergence analysis of the tracking error is also discussed. In section 4, examples and numeral simulations are provided to validate the effectiveness of the proposed tracking method. Finally, conclusions are given in section 5. Before ending the introduction, we highlight the main contributions of this paper as below:

- In this paper, we focus on the situation when there exist unknown physical parameter, which is of great significance in practical engineering.
- In the process of controller design, the measurements of neither task-space velocity nor joint acceleration is not required. Therefore, the proposed controller is easy to realize.
- Repeatability optimization is introduced in the proposed controller, the variable design of weight coefficient ensures the continuous of joint velocities.

## 2. Problem Formulation

The kinematic model of a serial robot manipulator can be described as

$$f(q(t)) = x(t), \quad (1)$$

where  $q(t) \in \mathbb{R}^n$  is the vector of joint angles, and  $x(t) \in \mathbb{R}^m$  is the vector describing the position and orientation of the end-effector in cartesian space.  $f(\bullet) : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is the mapping from joint space to cartesian space,  $f(\bullet)$  is a nonlinear function. Differentiating  $x(t)$  with respect to time  $t$ , the cartesian velocity  $\dot{x}(t)$  is formulated as

$$\dot{x}(t) = J(q(t), a_k)\dot{q}(t), \quad (2)$$

where  $J(q(t), a_k) = \partial f(q(t), a_k) / \partial q(t) \in \mathbb{R}^{m \times n}$  is the Jacobian matrix. For a redundant manipulator,  $n > m$ .  $a_k \in \mathbb{R}^l$  denotes the vector of kinematic parameters, also called physical parameters, while in this paper, mainly refers to length of each joint. Therefore,  $a_k$  is considered constant.

The  $J(q(t), a_k)\dot{q}(t)$  can be decoupled into two parts: physical parameter dependent term and joint angle-speed dependent term, and can be described in the linearization-in-parameter form[15]:

$$J(q(t), a_k)\dot{q}(t) = Y_k(q(t), \dot{q}(t))a_k, \quad (3)$$

where  $Y_k(q(t), \dot{q}(t)) \in \mathbb{R}^{m \times l}$  is called kinematic regressor matrix.

To avoid measuring the task-space velocity, a low-pass filter is used as follows

$$\dot{y} + \lambda_1 y = \lambda_1 \dot{x}, \quad (4)$$

where  $\lambda_1$  is a positive constant and  $y$  is the filtered output of the task-space velocity with initial value  $y(0) = 0$ . Rewriting (4) leads to

$$y = \lambda_1 \dot{x} / (p + \lambda_1), \quad (5)$$

where  $p$  is the Laplace variable.

Combining (3) and (5), we have

$$y = W_k(t)a_k, \quad W_k(t) = \lambda_1 Y_k(q, \dot{q}) / (\lambda_1 + p), \quad (6)$$

where  $W_k(0) = 0$ . For simplicity, we write  $J(q)$ ,  $Y_k(q, \dot{q})$  as  $J$  and  $Y_k$ , respectively.

*Remark 1:* In practical engineering, there are two values of  $a_k$ , namely, actual value  $a_k$  and nominal value  $a_k^n$ .  $a_k^n$  usually refers to parameters provided by the manufacturer or non-calibrated measurement results. However, the actual values of  $a_k$  is usually difficult to obtain.  $a_k$  may differs from its nominal values  $a_k^n$  due to assembly errors and long time operation (such as friction, wearing, etc.), besides, the robot may pick up different tools to perform tasks, which would also lead to kinematic uncertainties. In this case, control methods using  $a_k^n$  directly would lead to large errors, which is unacceptable in accurate tracking control.

### 3. Main Results

In this section, we will show the detailed process of controller design. Firstly, an ideal situation where all parameters are known is firstly considered, and then the basic idea is expanded to the situation of unknown parameters, and the repeatability optimization is done in the null space. Stability of the closed-loop system is also discussed.

#### 3.1. Adaptive Tracking Method

Define the tracking error in Cartesian space as

$$e(t) = x(t) - x_d(t), \quad (7)$$

##### 1) Known parameter case

When the kinematic parameters  $a_k$  is perfectly known, the accurate Jacobian matrix  $J$  can be obtained, therefore, the reference trajectory can be designed as

$$\ddot{x}(t) = \ddot{x}_d(t) + k_1 \dot{x}_d(t) - k_2 e(t) - k_1 J \dot{q}(t), \quad (8)$$

where  $k_1$  and  $k_2$  are positive control gains. According to Eq.(2), the Eq.(8) can be reformulated as  $\ddot{x}(t) = \ddot{x}_d(t) + k_1\dot{x}_d(t) - k_2e(t) - k_1\dot{x}(t)$ , by calculating the second derivative of Eq.(7), and substituting Eq.(8), we have

$$\begin{aligned} \ddot{e}(t) &= \ddot{x}(t) - \ddot{x}_d(t) \\ &= k_1\dot{x}_d(t) - k_2e(t) - k_1\dot{x}(t). \end{aligned} \tag{9}$$

Eq.(9) can be rewritten as

$$\ddot{e}(t) + k_1\dot{e}(t) + k_2e(t) = 0, \tag{10}$$

it is obvious that  $e(t)$  will eventually converge to zero, if  $k_1$  and  $k_2$  are Hurwitz. Combining Eq.(8) and Eq.(2), and let initial joint velocity  $\dot{q}(0)$  be 0, one can easily derive the corresponding control signals of joint speed as below

$$\dot{q} = \dot{q}_j + \dot{q}_n \tag{11a}$$

$$\dot{q}_j = \int_0^t J^\dagger [(\ddot{x}_d + k_1\dot{x}_d - k_2e - k_1J\dot{q}) - \dot{J}\dot{q}] dt \tag{11b}$$

$$\dot{q}_n = (I - J^\dagger J)\alpha, \tag{11c}$$

where  $I$  is  $n$ -dimensional identity matrix,  $J^\dagger$  is the Pseudo-Inverse of  $J$ , and  $\dot{q}_n$  is a speed component in the null space of Jacobian,  $\alpha$  can be selected arbitrarily. It is notable that  $J\dot{q}_n = 0$ , indicating that the speed component in the null space has no influence on the movement of end-effector. By getting the time-derivative of Eq.(2) and substituting Eq.(11), Eq.(2), Eq.(7), one can easily verify that the error dynamics under kinematic controller Eq.(11) is the same as Eq.(10), the tracking error will gradually converge to 0.

*Remark 2:* Eq.(8) gives a fundamental description of reference trajectory in the Cartesian space, it is notable that all the required information except  $J(q, a_k)$  on the right side of equation is easy to obtain. This inspires us to design a similar control strategy with the existence of kinematic uncertainties.

2) *Unknown parameter case*

In this situation,  $J$  is unavailable since one can not obtain  $a_k$ , therefore, we use  $\hat{J}$  instead of  $J$  by replacing  $a_k$  with its estimation  $\hat{a}_k$ , and let  $\hat{a}_k(0) = a_k^0$ , then the estimated  $\dot{x}(t)$  is  $\hat{x}(t) = \hat{J}\dot{q}$ . When replacing  $a_k$  by  $\hat{a}_k$ , according to (3), the estimated cartesian speed  $\hat{x}$  satisfies

$$\hat{x} = \hat{J}\dot{q} = Y_k(q, \dot{q})\hat{a}_k, \tag{12}$$

The modified reference trajectory is thus designed as

$$\dot{q}(t) = \int_0^t \{\hat{J}^\dagger[\ddot{x}_d + (k_1 + k_2)\dot{x}_d - k_1k_2e - \dot{\hat{J}}\dot{q} - k_3e] - (k_1 + k_2)\dot{q}\} dt. \tag{13}$$

Since the accurate feedback of cartesian velocity  $\dot{x}$  is unavailable, the derivative of tracking error  $\dot{e} = \dot{x} - \dot{x}_d$  is also unknown, therefore, we define the alternative value of  $\dot{e}$  by using the estimated Cartesian speed  $\hat{x}$ :

$$\Delta\hat{x} = \hat{x} - \dot{x}_d = \hat{J}\dot{q} - \dot{x}_d, \tag{14}$$

then the updating law of kinematic parameters is designed as

$$\hat{a}_k = k_1 Y_k^T(\Delta\hat{x} + k_1e) + k_3 Y_k^T e - W_k^T(t)\Gamma_1(W_k(t)\hat{a}_k - y), \tag{15}$$

where  $\Gamma_1$  is a positive definite diagonal matrix,  $k_1$ ,  $k_2$  and  $k_3$  are positive control gains.

*Remark 3:* Without loss of generality, the initial values of the adaptive kinematic parameters can be selected according to the nominal values, which can be obtained by consulting the instructions or manual measurement. Actually, the adjustment of  $\hat{a}_k(0)$  does affects the tracking process, which can be verified in the following section. The greater the error between  $\hat{a}_k(0)$  and  $a_k$ , the greater the simulation error at the

initial moment. However, the estimated values  $\hat{a}_k$  would finally converge to  $a_k$  according to (15), regardless the exact value of  $\hat{a}_k(0)$ , this can be verified through the stability analysis and numerical experiments.

Now, we are ready to offer a theorem about the task-space tracking problem for robots with uncertain physical parameters using the proposed adaptive controller as below

*Theorem 1:* The control error  $e(t)$  for a redundant manipulator described by (7) globally converges to 0, provided the joint speed controller described as (13), along with the kinematic adaptation law (15).

*Proof:* Differentiating (7) and substituting (3) and (12), we have

$$\begin{aligned} \dot{e} &= \dot{x} - \dot{\hat{x}} + \dot{\hat{x}} - \dot{x}_d \\ &= Y_k a_k - Y_k \hat{a}_k + \dot{\hat{x}} - \dot{x}_d \\ &= -Y_k \tilde{a}_k + \Delta \dot{\hat{x}}. \end{aligned} \tag{16}$$

Calculating derivative of function  $\Delta \hat{x}$  with respect to time and substituting Eq.(14), (16) yields

$$\begin{aligned} \frac{d}{dt}(\Delta \hat{x}) &= \hat{J} \dot{q} + \hat{J} \ddot{q} - \ddot{x}_d \\ &= (k_1 + k_2) \dot{x}_d - k_1 k_2 e - k_3 e - k_2 \hat{J} \dot{q} - k_1 \hat{J} \ddot{q} \\ &= k_2 \dot{x}_d - k_2 \hat{J} \dot{q} - k_1 k_2 e - k_3 e + k_1 \dot{x}_d - k_1 (\dot{x}_d + \dot{e} + Y_k \tilde{a}_k) \\ &= -k_2 \Delta \dot{\hat{x}} - k_2 k_1 e - k_3 e - k_1 Y_k \tilde{a}_k - k_1 \dot{e}, \end{aligned} \tag{17}$$

where  $\tilde{a}_k = a_k - \hat{a}_k$  represents the difference between the real value of physical parameters  $a_k$  and the estimated one  $\hat{a}_k$ . Eq.(17) can be reformulated as

$$\frac{d}{dt}(\Delta \hat{x} - k_1 e) = -k_2(\Delta \dot{\hat{x}} + k_1 e) - k_3 e - k_1 Y_k \tilde{a}_k. \tag{18}$$

Define the Lyapunov function candidate as

$$V = (\Delta \hat{x} + k_1 e)^T (\Delta \hat{x} + k_1 e) / 2 + k_3 e^T e / 2 + \tilde{a}_k^T \tilde{a}_k / 2. \tag{19}$$

Differentiating (19) and substituting (15),(16) and (17), we have

$$\begin{aligned} \dot{V} &= (\Delta \hat{x} + k_1 e)^T d(\Delta \hat{x} + k_1 e) / dt + k_3 e^T \dot{e} + \tilde{a}_k^T \dot{\tilde{a}}_k \\ &= (\Delta \hat{x} + k_1 e)^T (-k_2(\Delta \dot{\hat{x}} + k_1 e) - k_3 e - k_1 Y_k \tilde{a}_k) + \tilde{a}_k^T (k_1 Y_k^T (\Delta \hat{x} + k_1 e) + k_3 Y_k^T e - W_k^T(t) \Gamma_1 (W_k(t) \hat{a}_k - y) \\ &\quad + k_3 e^T (-Y_k \tilde{a}_k + \Delta \dot{\hat{x}}) \\ &= -k_2 (\Delta \hat{x} + k_1 e)^T (\Delta \hat{x} + k_1 e) - k_1 k_3 e^T e - \tilde{a}_k^T W_k^T(t) \Gamma_1 W_k(t) \tilde{a}_k \\ &\leq 0. \end{aligned} \tag{20}$$

Then we arrive at the conclusion that  $\Delta \hat{x}$ ,  $e$  and  $\tilde{a}_k$  are all bounded. Based on Eq.(14) and (3),  $\hat{J} \dot{q}$ ,  $\hat{a}_k$  and  $Y_k \tilde{a}_k$  are bounded.  $W_k(t) \tilde{a}_k$  is the output of a stable system with bounded input  $Y_k(t) \tilde{a}_k$ ,  $W_k(t) \tilde{a}_k$  is also bounded. Based on Eq.(15),  $\hat{a}_k$  is bounded. Differentiating  $W_k(t) \tilde{a}_k$  with respect to time, we have

$$\frac{d}{dt}(W_k(t) \tilde{a}_k) = \lambda_1 (Y_k - W_k(t)) \tilde{a}_k + W_k(t) \dot{\tilde{a}}_k. \tag{21}$$

$d(W_k(t) \tilde{a}_k) / dt$  is also bounded. Then we have  $\dot{e}$ ,  $d(\Delta \hat{x}) / dt$  and  $d(W_k(t) \tilde{a}_k) / dt$  are all bounded, which means the time derivative of (20),  $\dot{V}$  is bounded. Using Barbalat's Lemma, we have  $\Delta \hat{x} + k_1 e \rightarrow 0$ ,  $e \rightarrow 0$ , as  $t \rightarrow \infty$ .

*Remark 4:* We have proved the convergence of the tracking error under the condition of kinematic uncertainties. In fact, when  $a_k$  is perfectly known, Eq.(13) will be degenerated as

$$\dot{q}(t) = \int_0^t [J^\dagger (\ddot{x}_d + (k_1 + k_2) \dot{x}_d - k_1 k_2 e - \dot{J} \dot{q} - k_3 e) - (k_1 + k_2) \dot{q}] dt, \tag{22}$$

**Algorithm 1** The proposed tracking method

**Input:** Parameters  $k_1, k_2, k_3, K, \Gamma_1, \epsilon$ , initial states  $\dot{q}(0) = 0, q(0)$ , nominal kinematic parameter  $\hat{a}_k(0)$ , desired path  $x_d(t), \dot{x}_d(t)$  and  $\ddot{x}_d(t)$ , task duration  $T_e$ , feedback of end effector  $x(t)$ , analytical expressions of estimated Jacobian matrix  $\hat{J}$  and kinematic regressor matrix  $Y_k$ .

**Output:** To achieve task-space tracking of the redundant manipulator

1. Initialize  $a_k(0) \leftarrow a_k^n$ .
2.  $x, q, \dot{q} \leftarrow$  Sensor readings
3. Calculate  $e, y$ , and  $W_k(t)$  by Equation (7), (5) and (6)
4. Update  $K$  by Equation (27)
5. Update  $q_j$  by  $\dot{q}_j \leftarrow$  Equation (26b)
6. Update  $q_n$  by  $\dot{q}_n \leftarrow$  Equation (26c)
7. calculate the output  $\dot{q}$  by Equation (26a)
8. Update  $\hat{a}_k$  by  $\hat{a}_k \leftarrow$  by Equation (26d)

**Until** ( $t > T_e$ )

which has the similar form compared with Eq.(11). Therefore, known parameter case described in Eq.(11) can be considered as a special form of Eq.(13).

*Remark 5:* The control velocity  $\dot{q}$  in Eq.(13) is not the final result of this paper. The velocity component in null space is ignored, although it has no effect on the movement of end-effector as well as the stability proof, this part can not be neglected, because the redundancy mechanism is of great engineering significance to the manipulator.

3) *Repeatability optimization*

In this subsection, a repeatability optimization scheme is developed in the null space of Jacobian matrix, this will help to improve the stability and reliability of robots in periodic tasks.

The function describing robot’s repeatability is selected as

$$F(q) = -K(q - q_{ini})^T(q - q_{ini})/2, \tag{23}$$

where  $K$  is a positive parameter scaling the weight of repeatability optimization,  $q_{ini}$  is the initial value of the joint angles. By using gradient projection method, velocity component in null space can be calculated as

$$\alpha = [\partial(F(q))/\partial(q_1), \dots, \partial(F(q))/\partial(q_n)]. \tag{24}$$

Combining Eq.(24) and Eq.(23), we have

$$\alpha = [q_{int}(1) - q(1), \dots, q_{int}(i) - q(i), \dots, q_{int}(n) - q(n)]^T. \tag{25}$$

where  $q_{int}(i)$  and  $q(i)$  represent the  $i$ th element of  $q_{int}$  and  $q$ , respectively,  $i = 1, \dots, n$ .

Then the complete form of the proposed adaptive controller is

$$\dot{q} = \dot{q}_j + \dot{q}_n \tag{26a}$$

$$\dot{q}_j = \int_0^t [J^+(\ddot{x}_d + (k_1 + k_2)\dot{x}_d - k_1k_2e - \hat{J}\dot{q} - k_3e) - (k_1 + k_2)\dot{q}]dt \tag{26b}$$

$$\dot{q}_n = (I - J^+J)[q_{int}(1) - q(1), \dots, q_{int}(i) - q(i), \dots, q_{int}(n) - q(n)]^T \tag{26c}$$

$$\hat{a}_k = k_1 Y_k^T(\Delta \hat{x} + k_1 e) + k_3 Y_k^T e - W_k^T(t)\Gamma_1(W_k(t)\hat{a}_k - y) \tag{26d}$$

Remarkable that at the beginning stage of the tracking cycle, the repeatability is less important, and then it rises as the task continues. To this end, we set  $K$  as a variable:

$$K = \begin{cases} 0 & NT \leq t < NT + T/2, \\ K^* & \frac{2NT + T}{2} < t < (N + 1)T \end{cases} \tag{27}$$

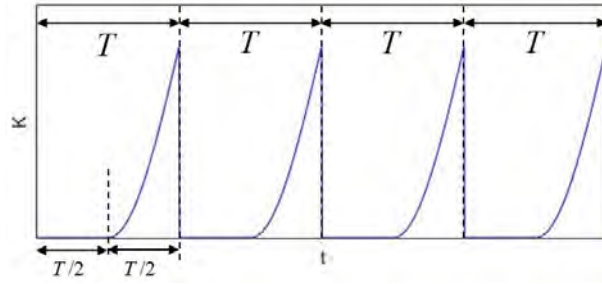
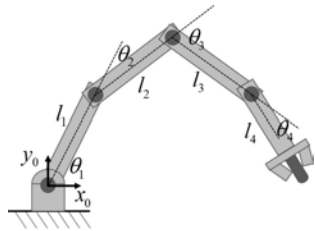


Figure 1: change curve of  $K$  with time  $t$ .



(a)

Joint	$a_i$ (m)	$\alpha_i$ (rad)	$d_i$ (m)
1	0.3	0	0
2	0.3	0	0
3	0.1	0	0
4	0.2	0	0

(b)

Figure 2: The 4-DOF redundant manipulator to be simulated in this paper. (a) Physical structure of the 4-link robot manipulator. (b) D-H parameters.

where  $K^* = K_{max}(1 - \cos(\pi(t - NT - T/2)/T))$ ,  $N = 0, 1, 2, \dots$  are natural numbers,  $T$  is the period of cyclic motion. If  $t < NT + T/2$ , the robot has just left the initial state to perform a task, thus we let  $K = 0$ , this will cause  $\alpha = 0$ , the joint control velocity is the same as (13). When  $t > NT + T/2$ ,  $K$  increases from 0 to maximum value  $K_{max}$  with time, forcing the robot to repeat the initial state. The change curve of  $K$  with time is shown in Fig.(1).

*Remark 6:* The main reason for this selection of  $K$  is to ensure the continuity of joint speed signals during a motion cycle. Notable that the discontinuities of  $K$  still appear at the moment  $T = NT$ . If the robot can repeat the initial joint state,  $q - q_{ini}$  would converge to 0, so  $\alpha$  can be also regarded as continuous. Therefore, the definition of  $K$  in (27) is acceptable.

#### 4. Numeral Simulations

In this section, simulations on a planar 4 link redundant manipulator are carried out to show the effectiveness of the proposed control scheme. The physical structure and D-H parameters are given in Figure 2. Firstly, we will verify the effectiveness of the proposed controller Eq.(26) in the presence of physical uncertainties. Secondly, the repeatability optimization performance is checked, finally, more discussions when the robot is required to track a cardioid curve(there exists non-conductive point) are carried out to show the robustness of the proposed tracking strategy.

##### 4.1. Simulation Settings

The vector of initial joint angles is selected as  $q_{ini} = [0, 0, \pi/2, 0]^T$  rad, and the corresponding cartesian position is  $x_{ini} = [0.6, 0.3]^T$ . Since the exact value of kinematic parameters(see  $d_i$  in Table. I), we assume the nominal values to be  $a_k^n = [0.25, 0.25, 0.12, 0.18]^T$  m, and let  $\hat{a}_k(0) = a_k^n$ . The control gains  $k_1, k_2$  and  $k_3$  are set to be  $k_1 = 50, k_2 = 10, k_3 = 50$ . The positive constant scaling the updating speed of  $\hat{a}_k$  is selected as  $\Gamma = 10$ , and  $K_{max}$  is set as 10. The time constant of low-pass filter is  $\lambda = 40$ . It is notable that matrix  $\hat{J}$  is essential in the proposed tracking controller, which is used to estimate the actual Jacobian matrix  $J(q, a_k)$ . To further show the detail of the proposed controller, analytical expression of  $\hat{J}$  is given in appendix I

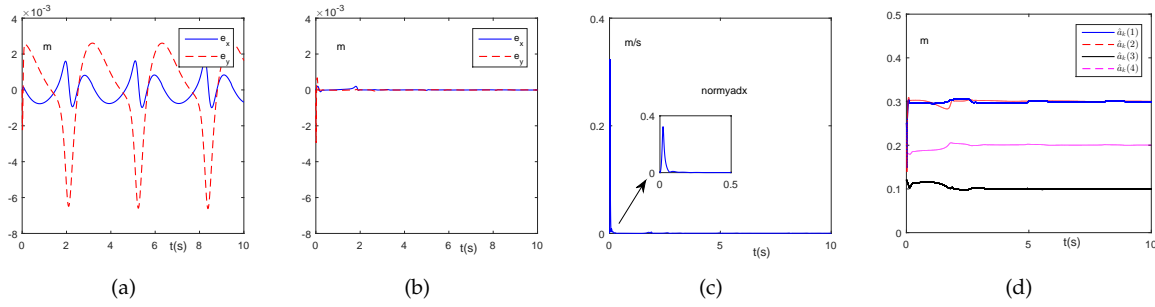


Figure 3: Simulation results with and without parameter estimation when tracking a circle. (a) Tracking errors without parameter estimation. (b) Tracking errors with parameter estimation. (c) Error norm of the estimated cartesian speed. (d) Estimated parameter  $\hat{a}_k$ .

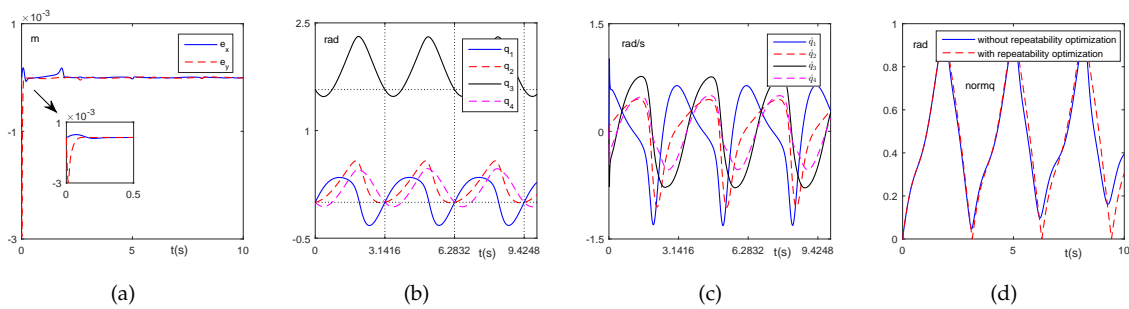


Figure 4: Simulation results when tracking a circle using repeatability optimization. (a) Tracking error. (b) Joint angles. (c) Joint speed. (d) Comparison of  $\|q - q_{ini}\|_2$  with and without repeatability optimization.

#### 4.2. Verification of Parameter Estimation

Comparative simulations is firstly carried out to show the effectiveness the proposed updating law (15). The desired path to be tracked is defined as  $x_d(t) = 0.4 + 0.2\cos(2t)$ ,  $y_d(t) = 0.3 + 0.2\sin(2t)$ . In the first simulation, the nomina values are used directly in the tracking control according to Eq. (13). By contrast,  $\hat{a}_k$  is updated using (15) in the comparable simulation, and  $\alpha$  is set to be zero (*i.e.*, we didn't use repeatability in this part). Simulation results are shown in Fig. (3). Both controllers ensure the boundedness of the tracking error. When  $a_k$  is known, the tracking errors along two axes are about 6mm and 2mm, this is mainly benefit from the closed-loop control mechanism. The tracking errors using parameter updating are less than 1mm, showing the effectiveness of the proposed controller under the condition of unknown models. The estimated parameter are shown in Fig.3(d),  $\hat{a}_k$  slowly converge to  $a_k$  with time. The error norm of the estimated cartesian speed reduce to zero rapidly (Fig. 3(c)).

#### 4.3. Verification of Repeatability Optimization

Then we check the effectiveness of repeatability optimization. Based on the simulation of previous part, we introduce the proposed repeatability optimization scheme (*i.e.*, the controller is the same as the adaptive tracking controller in the previous part except  $\alpha \neq 0$ .) Simulation results are shown in Fig. 3. The curve of tracking error  $e$  is the same as the one when  $\alpha = 0$ , showing the property that the velocity component in null space have no influence on the cartesian movement(Fig. 3(a)). Joint angles and speed are given in Fig. 3(b) and Fig. 3(c), when  $t = T, 2T, 3T$ , the robot is guaranteed to return to its initial configuration, and the joint speed keeps smooth at all time, showing the effectiveness of the proposed repeatability optimization method. The curve of repeatability function is is shown Fig. 3(d), when  $t = T, 2T, 3T \dots$ ,  $\|q - q_{ini}\|_2$  increases if repeatability optimization is not used, on the contrary,  $\|q - q_{ini}\|_2$  changes periodically.



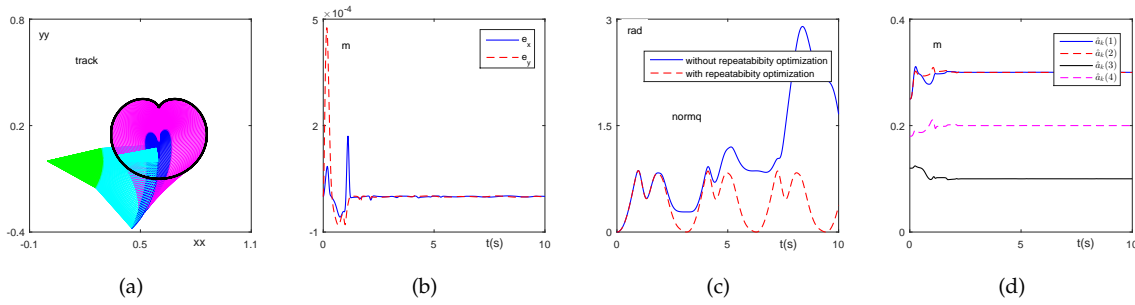


Figure 5: Simulation results when tracking a cardioid curve. (a) Motion trajectory of the manipulator. (b) Tracking error. (c) Comparison of  $\|q - q_{ini}\|_2$  with and without repeatability optimization. (d) Estimated parameter  $\hat{a}_k$ .

#### 4.4. Cardioid Tracking

To further verify the proposed control scheme, the robot is controlled to track a cardioid curve on the plane. The desired path is defined as  $x_d(t) = 0.1(2\sin(2t) - \sin(4t)) + 0.6m$ ,  $y_d(t) = 0.1(2\cos(2t) - \cos(4t)) + 0.2m$ . Simulation results are shown in Fig. (5). The trajectory of the end-effector and the corresponding configurations of the robot is shown in Fig. 5(a). The corresponding tracking errors are given in Fig. 5(b), maximum error is about 0.5mm, showing that the robot successfully track the given trajectory.  $\|q - q_{ini}\|_2$  is guaranteed to 0 when  $t = T, 2T, 3T$  (Fig. 5(c)), and the estimated kinematic parameters are shown in Fig. 5(d). All in all, the proposed controller ensures stable tracking under the condition of model uncertainties, and the repeatability is also achieved.

## 5. Conclusions

In this paper, an adaptive tracking controller is designed for redundant manipulators. Model uncertainties and repeatability are considered. The control scheme requires neither joint accelerations nor cartesian velocity, which is more suitable in practical engineering. By using pseudo-inverse method, repeatability is optimized in the null space of Jacobian, the continuous of joint speed is also guaranteed. Future studies will concentrate on the experimental validation of the proposed controller.

## Appendix I

Given the joint angle  $q = [q_1, q_2, q_3, q_4]^T$  and the estimated  $\hat{a}_k = [\hat{a}_k(1), \hat{a}_k(2), \hat{a}_k(3), \hat{a}_k(4)]^T$ . By simplifying  $\cos(q_i) = c_i$ ,  $\sin(q_i) = s_i$ ,  $\hat{a}_k(i) = a_i$ , the analytical expression of  $\hat{J}$  is given as below.

$$\hat{J}(1, 1) = -a_1s_1 - a_2s_{12} - a_3s_{123} - a_4s_{1234}, \hat{J}(1, 2) = -a_2s_{12} - a_3s_{123} - a_4s_{1234}, \hat{J}(1, 3) = -a_3s_{123} - a_4s_{1234}, \hat{J}(1, 4) = -a_4s_{1234}, \hat{J}(2, 1) = a_1c_1 + a_2c_{12} + a_3c_{123} + a_4c_{1234}, \hat{J}(2, 2) = a_2c_{12} + a_3c_{123} + a_4c_{1234}, \hat{J}(2, 3) = a_3c_{123} + a_4c_{1234}, \hat{J}(2, 4) = a_4c_{1234}.$$

Based on the analytical expression of  $\hat{J}$  given above,  $\hat{J}$  can be formulated as follows.

$$\begin{aligned} \hat{J}(1, 1) &= -a_1c_1\dot{q}_1 - a_2c_{12}(\dot{q}_1 + \dot{q}_2) - a_3c_{123}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3) - a_4c_{1234}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4), \hat{J}(1, 2) = -a_2c_{12}(\dot{q}_1 + \dot{q}_2) - a_3c_{123}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3) - a_4c_{1234}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4), \\ \hat{J}(1, 3) &= -a_3c_{123}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3) - a_4c_{1234}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4), \hat{J}(1, 4) = -a_4c_{1234}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4), \hat{J}(2, 1) = -a_1s_1\dot{q}_1 - a_2s_{12}(\dot{q}_1 + \dot{q}_2) - a_3s_{123}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3) - a_4s_{1234}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4), \\ \hat{J}(2, 2) &= -a_2s_{12}(\dot{q}_1 + \dot{q}_2) - a_3s_{123}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3) - a_4s_{1234}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4), \hat{J}(2, 3) = -a_3s_{123}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3) - a_4s_{1234}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4), \\ \hat{J}(2, 4) &= -a_4s_{1234}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4). \end{aligned}$$

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