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# Two Novel Finite Time Convergent Recurrent Neural Networks for Tackling Complex-Valued Systems of Linear Equation

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**Abstract.** Compared to the linear activation function, a suitable nonlinear activation function can accelerate the convergence speed. Based on this finding, we propose two modified Zhang neural network (ZNN) models using different nonlinear activation functions to tackle the complex-valued systems of linear equation (CVSLE) problems in this paper. To fulfill this goal, we first propose a novel neural network called NRNN-SBP model by introducing the sign-bi-power activation function. Then, we propose another novel neural network called NRNN-IRN model by introducing the tunable activation function. Finally, simulative results demonstrate that the convergence speed of NRNN-SBP and the NRNN-IRN is faster than that of the FTRNN model. On the other hand, these results also reveal that different nonlinear activation function will have a different effect on the convergence rate for different CVSLE problems.

## 1. Introduction

With the development of science and technology, the study for the CVSLE has made great progress, such as a two-step scale-splitting iteration technique for the symmetric CVSLE [1], the blind extraction of communications sources (complex-valued sources) [2], the robot manipulators' control [3], the time-varying linear matrix equation's solution [4], the control of manipulators using image visual servoing scheme [5], the redundant manipulators' control using an adaptive projection neural network [6] and the redundant manipulators' control for tacking the periodic noise [7]. We can describe the CVSLE as

$$HC(t) = a \in \mathbb{C}^n, \tag{1}$$

where $H \in \mathbb{C}^{n\times n}$ and $a \in \mathbb{C}^n$ represent the coefficients in complex field, and $C(t) \in \mathbb{C}^n$ represents a complex-valued vector. Because a complex matrix is composed of the imaginary part and the real part, the complex

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matrix $H$, $C$, and $a$ can be represented as $H = H_{re} + jH_{im}$, $C = C_{re} + jC_{im}$, and $a(t) = a_{re}(t) + ja_{im}(t)$, respectively. In the above equation, the symbol $j = \sqrt{-1}$ represents an imaginary unit. So, we can rewrite the equation (1) as

$$[H_{re} + jH_{im}][C_{re}(t) + jC_{im}] = a_{re} + ja_{im} \in \mathbb{C}^n, \tag{2}$$

where $H_{re}, H_{im} \in \mathbb{R}^{n\times n}$, $C_{re}, C_{im} \in \mathbb{R}^n$, and $a_{re}, a_{im} \in \mathbb{R}^n$. Based on the complex equation's principle, we can rewrite the equation (2) as the following equation [8],

$$\begin{cases} H_{re}C_{re}(t) - H_{im}C_{im}(t) = a_{re} \in \mathbb{R}^n, \\ H_{im}C_{re}(t) + H_{re}C_{im}(t) = a_{im} \in \mathbb{R}^n, \end{cases} \tag{3}$$

Now we can represent the equation (3) as the following compact matrix form

$$\begin{bmatrix} H_{re} & -H_{im} \\ H_{im} & H_{re} \end{bmatrix} \begin{bmatrix} C_{re}(t) \\ C_{im}(t) \end{bmatrix} = \begin{bmatrix} a_{re} \\ a_{im} \end{bmatrix} \in \mathbb{R}^{2n}. \tag{4}$$

Now the equation (4) can be written as

$$Ux(t) = V \in \mathbb{R}^{2n}. \tag{5}$$

where $U = \begin{bmatrix} H_{re} & -H_{im} \\ H_{im} & H_{re} \end{bmatrix}$, $x(t) = \begin{bmatrix} C_{re}(t) \\ C_{im}(t) \end{bmatrix}$, and $V = \begin{bmatrix} a_{re} \\ a_{im} \end{bmatrix}$. Therefore, we can solve the CVSLE in the real domain. Now the methods to tackle the real-valued system of linear equation can be employed to tackle the CVSLE [9–14]. Compared with the Frobenius norm, the lagging error is more suitable to act as the performance criterion of the error matrix. So, the ZNNs using the the lagging error can obtain a desired solution at exponential convergence rate instead of infinitely long time [15–17]. Considering we cannot obtain the original ZNN models' desired solution in finite time, Xiao et al. proposed a novel design formula to improve the ZNN models, which can obtain the desired solution within finite time [18]. Using the above design formula proposed by Xiao, Ding et al. proposed a FTRNN model to solve the CVSLE [19].

Compared with a linear activation function, a suitable nonlinear activation function can accelerate the convergence rate to some extent. In [20–25] the sign-bi-power activation function is employed to improve the ZNN models, and it can be represented as

$$\Psi(g) = \mathrm{sgn}^b(g) + \mathrm{sgn}^{1/b}(g), \tag{6}$$

where $0 < b < 1$ and

$$\mathrm{sgn}^b(g) = \begin{cases} |g|^b, & \text{if } \ g > 0 \\ 0, & \text{if } \ g = 0 \\ -|g|^b, & \text{if } \ g < 0. \end{cases}$$

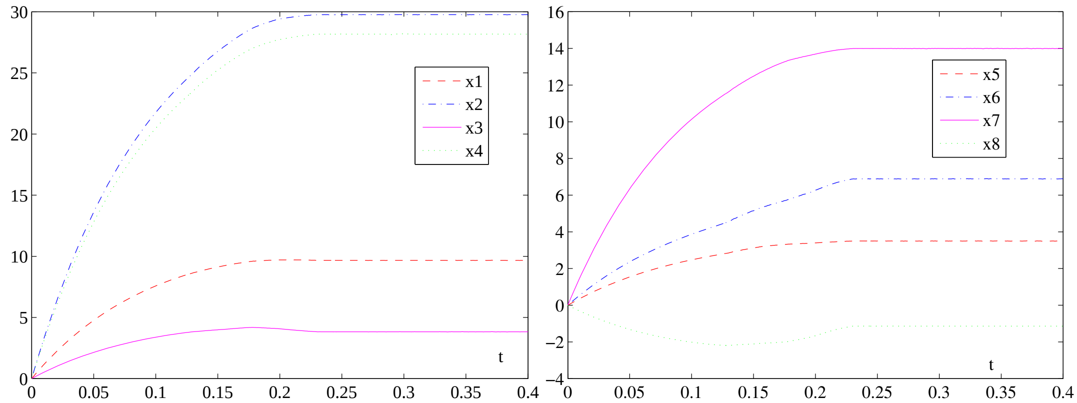
Furthermore Miao et al. proposed a tunable activation function [26], and

$$\Psi(g) = \mathrm{sign}(g)(r_1|g|^b + r_2|g| + r_3|g|^{\frac{1}{b}}), \tag{7}$$

where $0 < p < 1$, $r_1 > 0$, $r_2 > 0$, $r_3 > 0$ and

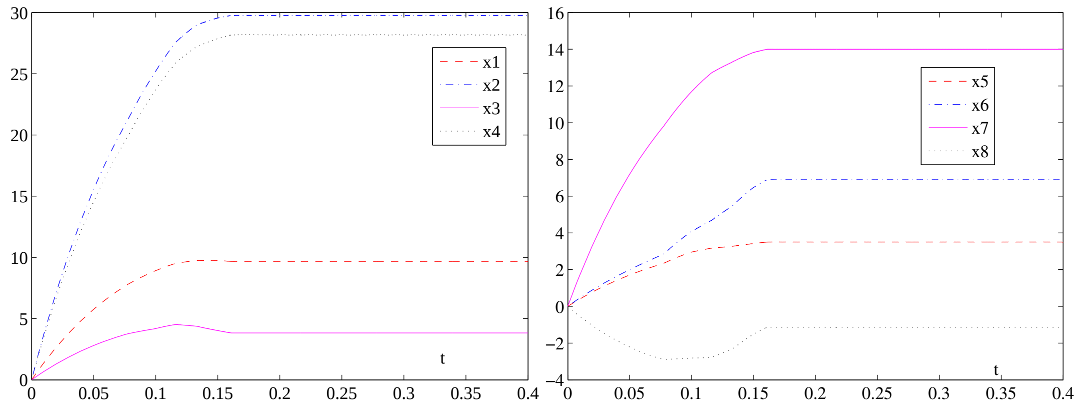
$$\mathrm{sign}(g) = \begin{cases} 1, & \text{if } \ g > 0 \\ 0, & \text{if } \ g = 0 \\ -1, & \text{if } \ g < 0. \end{cases}$$

Compared with the ZNN models using the linear activation function, the ZNN models using the suitable nonlinear activation function will have better convergent effect. As we know, different nonlinear activation function will have different convergence performance, so two different kinds of neural network models for




(a) Element of real part of $C(t)$ (b) Element of imaginary part of $C(t)$

Figure 1: Output trajectories of neural states $C(t)$ synthesized by FTRNN model (12) of example 1




(a) Element of real part of $C(t)$ (b) Element of imaginary part of $C(t)$

Figure 2: Output trajectories of neural states $C(t)$ synthesized by IRNN-SBP model (13) of example 1

tackling the CVSLE can be derived by using a sign-bi-power activation function and a tunable activation function [27–31].

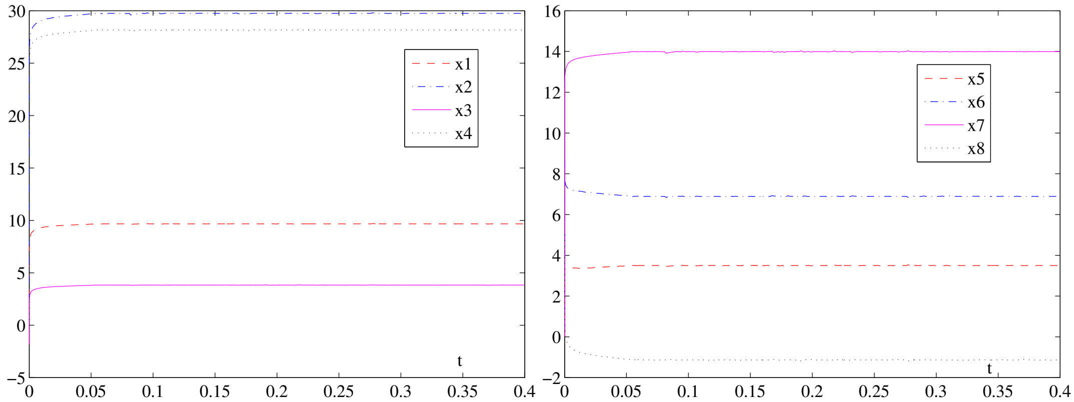
The remaining parts are divided into three sections. Section 2, two different kinds of finite time convergent neural network models for tackling CVSLE are presented. Section 3, the corresponding simulation results are given to verify the performance of the two improved neural network models. Finally, the corresponding conclusions are given in section 4.

Now we can summarize the main contributions of this paper as follows:

1) Two modified ZNN models using different nonlinear activation functions to tackle the CVSLE problems are proposed in this paper.

2) As compared to the ZNN using linear activation function, our proposed models using different nonlinear activation functions have better convergence performance. Furthermore, the corresponding theoretical proof is given to guarantee the convergence property of our proposed models.

3) The results reveal that different nonlinear activation function will have a different effect on the convergence rate for different CVSLE problems.




(a) Element of real part of $C(t)$ (b) Element of imaginary part of $C(t)$

Figure 3: Output trajectories of neural states $X(t)$ synthesized by IRNN-IRN model (14) of example 1

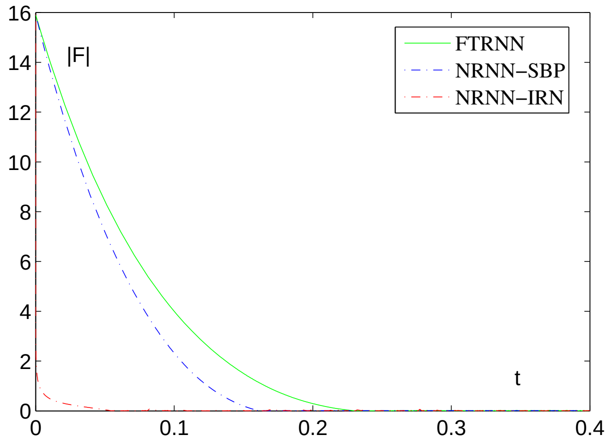



Figure 4: The evolution of the corresponding residual errors of example 1.

## 2. Two Different Finite Time Zhang Neural Network

Now according to the equation (5), we can write the error function $F(t)$ of ZNN as the following formula

$$F(t) = Ux(t) - V \in \mathbb{R}^{2n}. \tag{8}$$

So from the ZNN's design formula $\dot{F}(t) = -q\Psi(F(t))$, we can derive the following formula

$$U\dot{x}(t) = -q\Psi(Ux(t) - V), \tag{9}$$

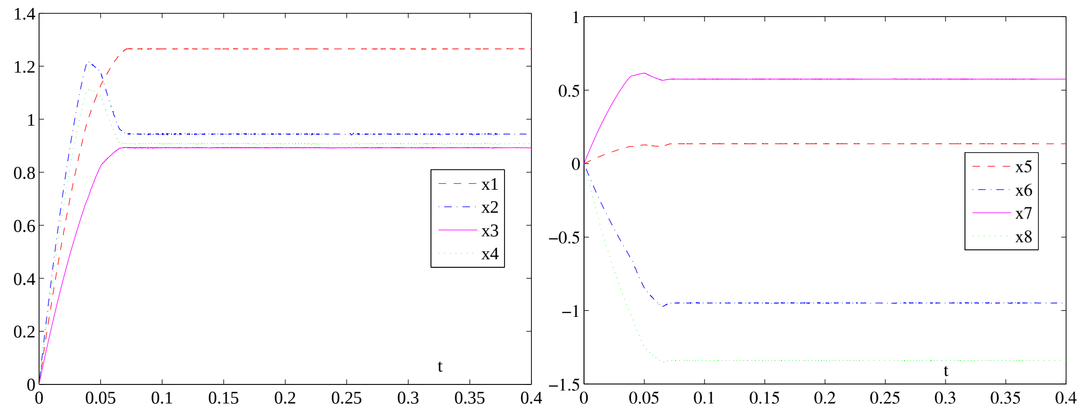
where $q > 0$ is a coefficient for adjusting the convergence rate, and $\Psi(\cdot)$ represents an activation function array. This is an original ZNN model for online tackling the CVSLE .

If we use the new design formula proposed by Xiao [18], we will have

$$\frac{\mathrm{d}F(t)}{\mathrm{d}t} = -q\Psi(s_1F(t) + s_2F^{w/z}(t)), \tag{10}$$

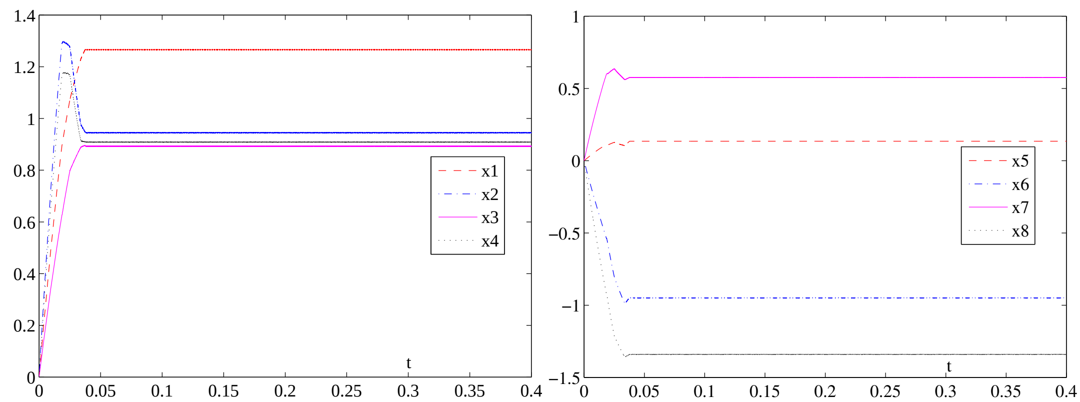
where $s_1$ and $s_2$ meet $s_1 > 0$, $s_2 > 0$, and $z$ and $w$ are the odd integer and meet $z > w > 0$. Then we have

$$U\dot{x}(t) = -q\Psi(s_1(Ux(t) - V) + s_2(Ux(t) - V)^{w/z}). \tag{11}$$




(a) Element of real part of $C(t)$ (b) Element of imaginary part of $C(t)$

Figure 5: Output trajectories of neural states $C(t)$ synthesized by FTRNN model (12) of example 2




(a) Element of real part of $C(t)$ (b) Element of imaginary part of $C(t)$

Figure 6: Output trajectories of neural states $C(t)$ synthesized by IRNN-SBP model (13) of example 2.

If we choose a linear activation function, we can have the following formula

$$U\dot{x}(t) = -q(s_1(Ux(t) - V) + s_2(Ux(t) - V)^{w/z}), \tag{12}$$

which is called the FTRNN model for online dealing with the CVSLE [19].

### 2.1. NRNN-SBP Model

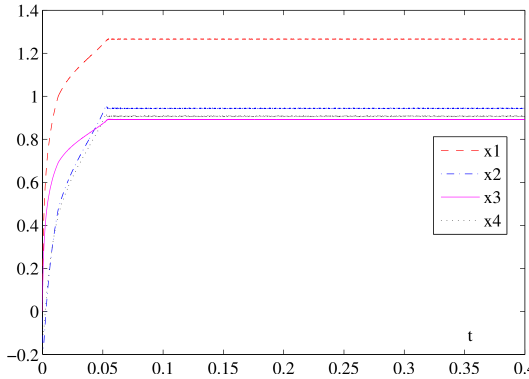
If we introduce a sign-bi-power activation function (6), we can rewrite the equation (12) as

$$\begin{aligned} U\dot{x}(t) = &-q(\mathrm{sgn}^b(s_1(Ux(t) - V) + s_2(Ux(t) - V)^{w/z}) \\ &+\mathrm{sgn}^{1/b}(s_1(Ux(t) - V) + s_2(Ux(t) - V)^{w/z})), \end{aligned} \tag{13}$$

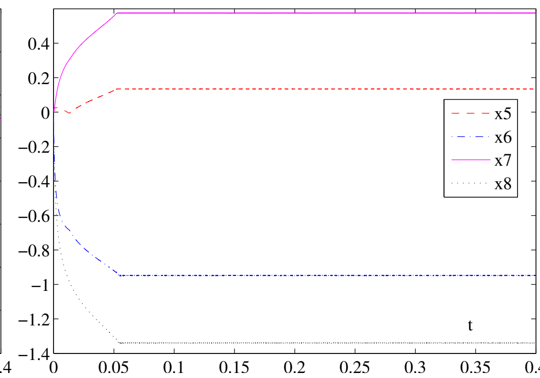
here $q > 0$, $1 > b > 0$, $s_1 > 0$, $s_2 > 0$, and

$$\mathrm{sgn}^b(g) = \begin{cases} |g|^b, & \text{if} \quad g > 0 \\ 0, & \text{if} \quad g = 0 \\ -|g|^b, & \text{if} \quad g < 0. \end{cases}$$

This model is called the NRNN-SBP model for online tackling the CVSLE.




(a) Element of real part of $C(t)$




(b) Element of imaginary part of $C(t)$

Figure 7: Output trajectories of neural states $C(t)$ synthesized by IRNN-IRN model (14) of example 2

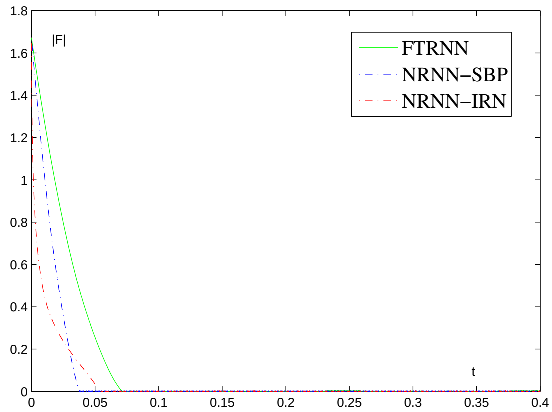



Figure 8: The evolution of the corresponding residual errors of example 2.

### 2.2. *NRNN-IRN Model*

If we introduce a tunable activation function (7), we will have the following formula

$$\begin{aligned} U\dot{x}(t) =& -q\text{sign}(s_1(Ux(t)-V)+s_2(Ux(t)-V)^{w/z})(r_1|s_1(Ux(t)-V) \\ &+s_2(Ux(t)-V)^{w/z}|^b + r_2|s_1(Ux(t)-V)+s_2(Ux(t)-V)^{w/z}| \\ &+r_3|s_1(Ux(t)-V)+s_2(Ux(t)-V)^{w/z}|^{\frac{1}{b}}), \end{aligned} \tag{14}$$

where $q>0$, $1>b>0$, $r_1>0$, $r_2>0$, $r_3>0$, $s_1>0$, $s_2>0$, $z$ and $w$ are the odd integer and satisfy $z>w>0$, and

$$\text{sign}(g)=\begin{cases} 1, & \text{if } \ g>0 \\ 0, & \text{if } \ g=0 \\ -1, & \text{if } \ g<0. \end{cases}$$

Then the equation (14) is called the NRNN-IRN model for online tackling the CVSLE.

### 2.3. *Theorem Analysis*

*Theorem:* The state matrix $x(t)$ of the model (13) (or the model (14)) will converge to its theoretical solution, whatever its initial random value $x(0)$ is.

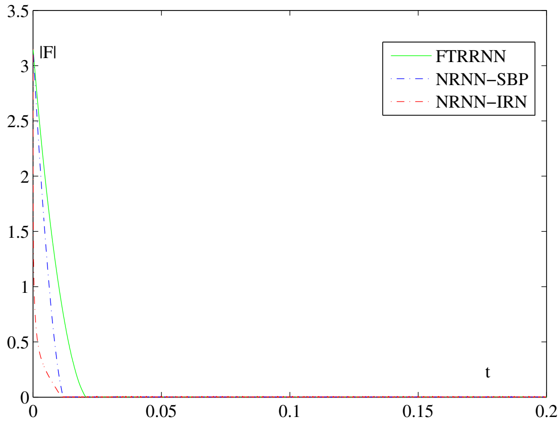



Figure 9: The evolution of the corresponding residual errors of example 3.

*Proof:* According the definition $F(t) = Ux(t) - V$, the model (13) can be written as

$$\dot{F}(t) = -q\Psi_1(s_1F(t) + s_2F(t)^{w/z}), \tag{15}$$

where $\Psi_1(\cdot) = \text{sgn}^b(\cdot)$. Now suppose $R(F(t)) = s_1F(t) + s_2F(t)^{w/z}$, we have

$$\dot{F}(t) = -q\Psi_1(R(F(t))). \tag{16}$$

As we know, the matrix $F(t)$'s each element shows the identical dynamic, then we have

$$\dot{f}_{ij}(t) = -q\Psi_1(R(f_{ij}(t))), \tag{17}$$

where $f_{ij}(t)$ represents the matrix $F(t)$'s $ij$th element. Now we define the following Lyapunov function $V_f = f_{ij}^2(t)$. Then we have

$$\dot{V}_f = -2qf_{ij}(t)\Psi_1(R(f_{ij}(t))). \tag{18}$$

As $\Psi_1(\cdot)$ and $R(\cdot)$ are all odd and monotonic increasing functions, $\Psi_1(R(f_{ij}(t)))$ must be also monotonic increasing. So we can find that $\dot{V}_f$ is negative definite and $f_{ij}(t)$ will globally converge to 0. This means that each element of matrix $x(t)$ will obtain its theoretical solution with time.

Similarly, we can use the above method to prove the convergence property of the model (14).

This proof is successful. ■

## 3. Computer Simulation

Now the following three illustrative examples will be carried to compare the NRNN-SBP model (13) and the NRNN-IRN model (14) with the FTRNN model (12). To display the different convergent rate, the trail of each neural-state and the residual error norm $\|F(t)\|_2$ are shown in the corresponding figures. For comparison, we choose a same group of parameters $q = 10$, $w = 1$, $b = \frac{1}{8}$, $z = 5$, $w = 1$, $r_1 = r_2 = r_3 = 1$, $s_1 = s_2 = 1$ in the example 1, the example 2 and the example 3.

Example 1:

$$H_1C_1(t) = a_1 \in \mathbb{C}^n,$$

where

$$H_1 = \begin{bmatrix} -0.5444+0.6754j & -0.8967-0.5434j & 0.2352-0.7875j & 0.7854+0.9544j \\ 0.7724+0.8944j & -0.8244+0.8478j & -0.8525-0.5486j & 0.8651-0.3564j \\ 0.8544-0.3521j & -0.8215-0.6234j & -0.7445+0.3647j & 0.7443+0.9634j \\ 0.7455-0.3647j & -0.7542+0.3645j & 0.7411+0.3644j & 0.8564-0.7565j \end{bmatrix}$$

and

$$a_1 = \begin{bmatrix} 4.5634 + 8.5445j \\ 2.3432 + 5.8452j \\ 3.4533 - 7.3564j \\ 4.5323 - 5.7842j \end{bmatrix}.$$

According to equation (5) we have

$$U_1 = \begin{bmatrix} -0.5444 & -0.8967 & 0.2352 & 0.7854 & -0.6754 & 0.5434 & 0.7875 & -0.9544 \\ 0.7724 & -0.8244 & -0.8525 & 0.8651 & -0.8944 & -0.8478 & 0.5486 & 0.3564 \\ 0.8544 & -0.8215 & -0.7445 & 0.7443 & 0.3521 & 0.6234 & -0.3647 & -0.9634 \\ 0.7455 & -0.7542 & 0.7411 & 0.8564 & 0.3647 & -0.3645 & -0.3644 & 0.7565 \\ 0.6754 & -0.5434 & -0.7875 & 0.9544 & -0.5444 & -0.8967 & 0.2352 & 0.7854 \\ 0.8944 & 0.8478 & -0.5486 & -0.3564 & 0.7724 & -0.8244 & -0.8525 & 0.8651 \\ -0.3521 & -0.6234 & 0.3647 & 0.9634 & 0.8544 & -0.8215 & -0.7445 & 0.7443 \\ -0.3647 & 0.3645 & 0.3644 & -0.7565 & 0.7455 & -0.7542 & 0.7411 & 0.8564 \end{bmatrix}$$

and $V_1 = [4.5634 \quad 2.3432 \quad 3.4533 \quad 4.5323 \quad 8.5445 \quad 5.8452 \quad -7.3564 \quad -5.7842]^{\mathrm{T}}$, where T represents the matrix $V_1$'s transpose.

Example 2:

$$H_2C_2(t) = a_2 \in \mathbb{C}^n.$$

Suppose $H_2 = H_1$, and

$$a_2 = \begin{bmatrix} 0.5121 + 0.3654j \\ 0.7451 + 0.3564j \\ 0.8542 - 0.3647j \\ 0.8422 - 0.3843j \end{bmatrix}.$$

Similarly, according to equation (5) we have

$$U_2 = \begin{bmatrix} -0.5444 & -0.8967 & 0.2352 & 0.7854 & -0.6754 & 0.5434 & 0.7875 & -0.9544 \\ 0.7724 & -0.8244 & -0.8525 & 0.8651 & -0.8944 & -0.8478 & 0.5486 & 0.3564 \\ 0.8544 & -0.8215 & -0.7445 & 0.7443 & 0.3521 & 0.6234 & -0.3647 & -0.9634 \\ 0.7455 & -0.7542 & 0.7411 & 0.8564 & 0.3647 & -0.3645 & -0.3644 & 0.7565 \\ 0.6754 & -0.5434 & -0.7875 & 0.9544 & -0.5444 & -0.8967 & 0.2352 & 0.7854 \\ 0.8944 & 0.8478 & -0.5486 & -0.3564 & 0.7724 & -0.8244 & -0.8525 & 0.8651 \\ -0.3521 & -0.6234 & 0.3647 & 0.9634 & 0.8544 & -0.8215 & -0.7445 & 0.7443 \\ -0.3647 & 0.3645 & 0.3644 & -0.7565 & 0.7455 & -0.7542 & 0.7411 & 0.8564 \end{bmatrix}$$

and $V_2 = [0.5121 \quad 0.7451 \quad 0.8542 \quad 0.8422 \quad 0.3654 \quad 0.3564 \quad -0.3647 \quad -0.3843]^{\mathrm{T}}$, where T represents the matrix $V_2$'s transpose.

Example 3:

$$H_3C_3(t) = a_3 \in \mathbb{C}^n.$$

Suppose $H_3 = H_2$, and

$$a_3 = \begin{bmatrix} 1.5321 + 0.2444j \\ 1.2845 + 0.9589j \\ 1.2845 - 0.9589j \\ 1.5436 \end{bmatrix}$$

Then ccording to equation (5) we have

$$U_3 = \begin{bmatrix} -0.5444 & -0.8967 & 0.2352 & 0.7854 & -0.6754 & 0.5434 & 0.7875 & -0.9544 \\ 0.7724 & -0.8244 & -0.8525 & 0.8651 & -0.8944 & -0.8478 & 0.5486 & 0.3564 \\ 0.8544 & -0.8215 & -0.7445 & 0.7443 & 0.3521 & 0.6234 & -0.3647 & -0.9634 \\ 0.7455 & -0.7542 & 0.7411 & 0.8564 & 0.3647 & -0.3645 & -0.3644 & 0.7565 \\ 0.6754 & -0.5434 & -0.7875 & 0.9544 & -0.5444 & -0.8967 & 0.2352 & 0.7854 \\ 0.8944 & 0.8478 & -0.5486 & -0.3564 & 0.7724 & -0.8244 & -0.8525 & 0.8651 \\ -0.3521 & -0.6234 & 0.3647 & 0.9634 & 0.8544 & -0.8215 & -0.7445 & 0.7443 \\ -0.3647 & 0.3645 & 0.3644 & -0.7565 & 0.7455 & -0.7542 & 0.7411 & 0.8564 \end{bmatrix}$$

and $V_3 = [1.5321 \quad 1.2845 \quad 1.2845 \quad 1.5436 \quad 0.2444 \quad 0.9589 \quad -0.9589 \quad 0]^{\mathrm{T}}$, where T represents the matrix $V_3$'s transpose.

From the Figs.1-9, we can find the NRNN-SBP model (13) and the NRNN-IRN model (14) have higher convergence rate than the FTRNN model (12). Furthermore from the example 1 and the example 3, we can find that the NRNN-IRN model has the fastest convergence rate. However, from the example 2, we can find that the NRNN-SBP model has the fastest convergence rate. This phenomenon reveals that different nonlinear activation function will have a different effect on the convergence rate for different CVSLE problems.

## 4. Conclusions

It is the first time to build two different finite-time convergent ZNN models to tackle the CVSLE in this paper. The results show that the convergence speed of NRNN-SBP and the NRNN-IRN is faster than that of the FTRNN model, and also reveal that different nonlinear activation function will have a different effect to accelerate the convergence rate for different CVSLE problems. So how to choose a suitable nonlinear activation function for a different CVSLE problem will be our next research.

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