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# The Improved Noise Reduction Method for The Vibration Signal Based on Variational Mode Decomposition

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**Abstract.** The variational model decomposition (VMD) has a problem that is difficult to determine the number of intrinsic mode functions (IMF). We use the leaked energy to determine the number of IMFs. And we use the energy concentration rate of the IMF's autocorrelation function and the correlation coefficient between the IMFs and the original signal, define Q as the ratio of the energy concentration and the correlation coefficient, and use Q to determine the noise IMFs in the IMFs. Then, we filter the noise IMFs and use the remaining IMFs to reconstruct signal to achieve noise reduction. Finally, we use the signal-to-noise ratio (SNR) to compare the noise reduction method proposed in this paper and the Empirical Mode Decomposition (EMD) noise reduction method.

## 1. Introduction

Air conditioning manufacturers need to check the quality of air conditioners on production lines according to their vibration signals. The noisy environment interferes with the accuracy of the collected vibration signals. Using effective signal processing methods to filter out noise is an important issue for improving the detection efficiency.

There are many ways to reduce noise on vibration signals. Fourier transform can handle the stationary signal better and obtain the effective frequency information. However, Fourier transform is the average analysis of the data segment, lacking the local information for nonstationary and nonlinear signals. The wavelet transform noise reduction technology improves the ability to deal with nonstationary signal. But, there is a problem that the fundamentalwave of wavelet transform is difficult to choose. The noise reduction effect is not good when the noise is large [1]. Empirical Mode Decomposition (EMD) is a signal decomposition algorithm proposed by Nordeng E. Huang. EMD decomposes and filters complex signals to obtain a series of intrinsic mode function (IMF) with different characteristic scales. This method is suitable for dealing with nonlinear, nonstationary, multicomponent signals [2-5]. However, this method have the problem that mode aliasing [6-7] and produces fault modes at low frequencies [5].

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For the shortcomings of EMD with modal aliasing, end point effects [8] and incomplete mathematical theory support. Therefore, K Dragomiretskiy and D Zosso proposed Variational Mode Decomposition (VMD) . VMD is an entirely nonrecursive model, where the modes are extracted concurrently. The model looks for an ensemble of modes and their respective center frequencies, such that the modes collectively reproduce the input signal, while each being smooth after demodulation into baseband [9]. VMD breaks down complex signals into predefined K IMFs. If the number of decompositions K is appropriate, mode aliasing can be effectively suppressed.

In this paper, in order to reduce the noise of the collected air conditioning vibration signal, we use the VMD to process the signal. We use the leaked energy to determine the number K of decomposed IMFs. And we use the energy concentration ratios of the IMFs's autocorrelation functions and the correlation coefficients between the IMFs and the original signal, define Q as the ratio of energy concentration rate to correlation coefficient. Q is used to determine the noise IMFs. Then we filter out the noise IMFs and reconstruct the signal with remaining IMFs to achieve noise reduction. Last, we compare the noise reduction method proposed in this paper with EMD noise reduction method.

## 2. Variational Mode Decomposition

In VMD, IMF is redefined as an AM-FM signal. VMD introduces the decomposition of the signal into the variational model. The variational problem can be expressed as searching for K IMFs such that the sum of the bandwidths of all the IMFs is the smallest, and the sum of the IMFs is equal to the original input signal $f$. The decomposition principle as follows:

Step 1. According to the preset IMF number K and center angular frequencies $\left\{\omega_{(k)}\right\}$ , the initial IMFs $u_k(t)$ are obtained.

Step 2. The analytical signals their unilateral spectrums of IMFs are obtained through the Hilbert transform:

$$\left(\delta(t) + \frac{j}{\pi t}\right) * u_k(t) \tag{1}$$

Where, $\delta(t)$ is the Dirac distribution and * denote convolution.

Step 3. The exponent term is added to adjust the center frequency of each IMF. The frequency spectrum of each IMF is modulated to the corresponding base band:

$$\left[\left(\delta(t) + \frac{j}{\pi t}\right)^* u_k(t)\right] * e^{-j\omega_k t} \tag{2}$$

Where, $e^{-j\omega_k t}$ is the estimated center frequency.

Step 4. The L2 norm of the gradient of demodulated signal in equation (2) are calculated, the bandwidth of each IMF is estimated, and the constraint that the sum of IMFs is equal to the original input signal $f$ need be satisfied .

$$\min_{\{\mathrm{u_u}\}\{\omega_\mathrm{k}\}} \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t}\right)^* u_k(t)\right] e^{-j\omega_\mathrm{e} t} \right\|^2 \quad \text{s.t.} \sum_k u_k(t) = f \tag{3}$$

Where,$\{u_k(t)\} = \{u_1(t) \cdots u_k(t)\}$ are IMFs, $\{\omega_k\} = \{\omega_1 \cdots \omega_k\}$ are the angular frequency center of IMFs.

Step 5. The augmented Lagrangian function is introduced to turn constrained variational problem to unconstrained variational problem.

$$\zeta\left(u_k, \omega_k, \lambda\right) = \alpha \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi}\right)^* u_k(t)\right] e^{-j\omega_k t} \right\|^2 + \left\| f(t) - \sum_k u_k(t) \right\|^2 + < \lambda(t), f(t) - \sum_k u_k(t) > \tag{4}$$

Where,$\alpha$is the balance factor, $\lambda$ is Lagrange operator, ¡ ¿ is the inner product operation.

Step 6. The alternating direction method of multiplication (ADMM) [10] is used to solve the above variational problem. $u_k^{n+1}, \omega_k^{n+1}, \lambda_k^{n+1}$ are updated by seeking the saddle point of the Lagrange expression alternately.

$$u_k^{n+1} \arg\min_{u_k \in x} \left\{ \alpha \left\| \partial_t \left[ \left( \delta(t) + \frac{j}{\pi t} \right)^* u_k(t) \right] e^{-j\omega_k t} \right\|^2 + \left\| f(t) - \sum_i u_i \left( t + \frac{\lambda(t)}{2} \right) \right\|^2 \right\} \tag{5}$$

Where, $\omega_k$ equivalent to $\omega_k^{n+1}$, $\sum_i u_i(t)$ equivalent to $\sum_i u_i(t)^{n+1}$.

Step 7. Parseval/Plancherel Fourier isometric transformation is used to convert equation (5) to the frequency domain. Then, $\omega - \omega_k$ is used to replace the variable $\omega$ in the first term. Last, convert it to a non-negative frequency region integral form.

$$\hat{u}_k^{n+1} = \arg\min_{u_k, \hat{u} \in x} \left\{ \alpha \left\| j(\omega - \omega_k) \left[ (\operatorname{sgn}(\omega) + 1) * \hat{u}_k(\omega) \right] \right\|^2 + \left\| \hat{f}(\omega) - \sum_i \hat{u}_i(\omega) + \frac{\hat{\lambda}(\omega)}{2} \right\| \right\} \tag{6}$$

Step 8. Formula (6) is conversioned to nonnegative frequency interval integral form to obtain the update method of $\hat{u}_k^{n+1}(\omega)$:

$$\hat{u}_k^{n+1}(\omega) = \frac{\hat{f}(\omega) - \sum_{i \neq k} \hat{u}_i(\omega) + \hat{\lambda}(\omega)/2}{1 + 2\alpha (\omega - \omega_k)^2} \tag{7}$$

Step 9. Similarly, the update method of $\omega_k^{n+1}$ is obtained:

$$\omega_k^{n+1} = \frac{\int_0^\infty \omega \left| \hat{u}_k(\omega) \right| d\omega}{\int_0^\infty \left| \hat{u}_k(\omega) \right|^2 d\omega} \tag{8}$$

Step 10. The accuracy factor $\varepsilon > 0$, if the iteration stop condition $\frac{\sum \|u_k^{n+1} - u_k^n\|^2}{\sum \|u_k^n\|^2} < \varepsilon$ is satisfied, K IMFs are obtained.

## 3. Determination of K

Before the signal is decomposed by VMD, the number K of IMF must be determined. If K is too small, multiple signal components may appear in the same IMF, or part of the signal components cannot be calculated. If K is too large, the same signal component will appear in multiple IMFs, and the center frequencies of the IMFs will overlap [11].

The limited duration signal is $x(i), i = 0, 1, 2, 3 \ldots N$, The MIFs decomposed by VMD are $c_j(i), j = 1, 2, 3 \ldots K, i = 0, 1, 2, 3 \ldots N$. The energy of $x(i)$ is

$$E = \sum_{i=0}^{N} x^2(i) \tag{9}$$

The energy of the $j$th MIF is

$$E_j = \sum_{i=0}^{N} c_j^2(i) \tag{10}$$

If IMFs are orthogonal, the leaked energy $E_L$ is:

$$E_L = E - \sum_{j=1}^{k} E_j = 0 \tag{11}$$

IMFs has an approximate orthogonality if K is appropriate [11], and the leaked energy $E_L$ will be small.

## 4. Determination of Noise IMF

### 4.1. Characteristic of the autocorrelation function of white noise and energy concentration ratio

The autocorrelation function of the signal containing the periodic component still has obvious periodicity when t change. The autocorrelation function of white noise have the max value when $t = 0$, but decays rapidly to 0 with the change of $t$ [10]. The energy concentration ratio is defined as the ratio of the energy of the signal over the time period $[-t, t]$ to the energy of the signal over the entire time period [11].The energy concentration ratio of the $j$th IMF is $\eta_j$, and its discrete expression is:

$$\eta_j[-n, n] = \frac{E_{jn}}{E_j} = \frac{\sum_{i=n}^{n} xcorr(i)}{\sum_{i=N}^{N} xcorr(i)} \tag{12}$$

Where, $xcorr_j(i)$ is the The autocorrelation function of the jth IMF, Ejn is the energy of $xcorr_j(i)$ between the time period $[-n, n]$ , Ej is the total energy of $xcorr_j(i)$.

If the $j$1th IMF is noise IMF, the $\eta_j$ will be large , Otherwise, $\eta_j$ will be small.

### 4.2. correlation coefficient

The correlation coefficient shows the strength and direction of the linear relationship between two random variables, the correlation coefficient between the signal $x(i)$ and the $j$th IMF is $\rho_j$:

IF the $j$th IMF is noise IMF, The $\rho_j$ will be small. Otherwise, the $\rho_j$ will be large.

$$\rho_j = \frac{\sum_{i=1}^{N} (x_i - \bar{x}) \left(c_{ji} - \bar{c}_j\right)}{\sqrt{\sum_{i=1}^{N} (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^{N} \left(c_{ji} - \bar{c}_j\right)^2}} \tag{13}$$

IF the $j$th IMF is noise IMF, The $\rho_j$ will be small. Otherwise, the $\rho_j$ will be large.

### 4.3. eew noise IMF judgment factor Q

Defined $Q$ as:

$$Q_j = \frac{\eta_j}{\rho_j} \tag{14}$$

$Q_j$ is defined as the ratio of the $\eta_j$ and the $\rho_j$, which can amplify the difference between the noise IMF and the IMF containing the signal components.

## 5. Simulation Signal Analysis

The simulation signal 1 $x_1(t)$ and signal 2 $x_2(t)$ are:

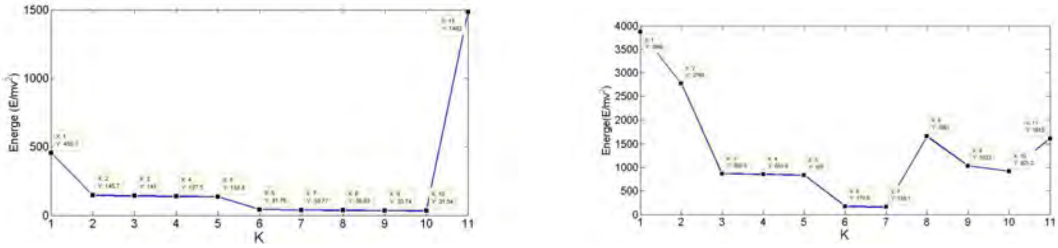
$$x_1(t) = \cos(2\pi \cdot 30t) + 0.2\sin(2\pi \cdot 50t) + 0.33\sin(2\pi \cdot 80t) + \eta_1 \tag{15}$$

$$x_2(t) = \cos(2\pi \cdot 50t) + 0.8\sin(2\pi \cdot 90t) + 0.5\sin(2\pi \cdot 120t) + 0.6\sin(2\pi \cdot 220t) + \eta_2 \tag{16}$$

Where, $\eta_1$ is the white noise with intensity 0.1. $\eta_2$ is the white noise with intensity 0.2. Unit of each component is MV. Signal sampling rate is 2000.

### 5.1. determination of K

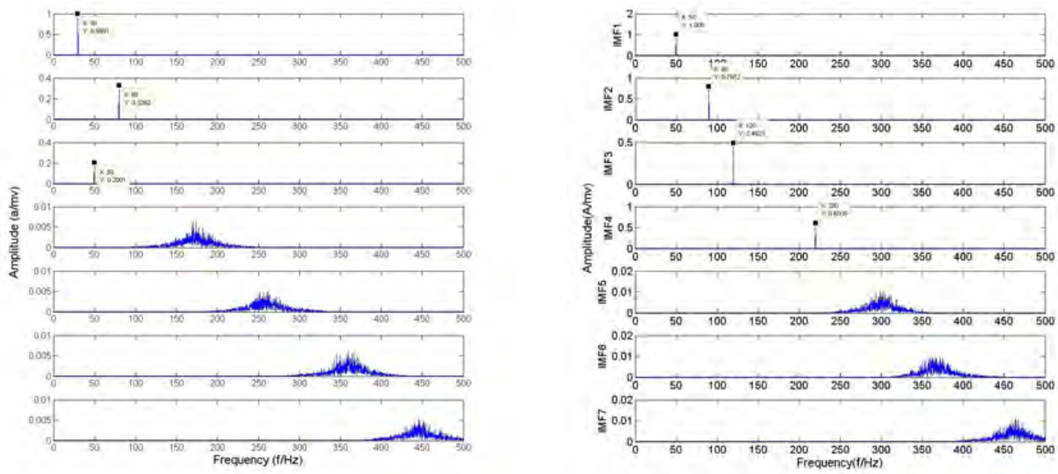
When $\alpha$ is 2000, $\varepsilon$ is $10^{-7}$, and K is 1-11, VMD is performed on the simulation signal 1 and signal 2. The relationships between leaked energy and K are shown in Fig.1 .




a signal 1 b signal 2

Figure1: The relationships between leaked energy and K

According to Fig.1, K is selected as 7, VMD is performed on simulation signal 1 and signal 2, the frequency spectrums of the IMFs are shown in Fig.2. For signal 1, IMF1-IMF3 correspond to 30Hz, 80Hz and 50Hz signal components respectively, IMF4-IMF7 are noise IMFs. For signal 2, IMF1-IMF4 correspond to 30Hz, 90Hz, 120Hz and 220Hz signal components respectively, IMF5-IMF7 are noise IMFs. Both siganl 1 and signal 2 have no mode aliasing, which indicate that it is feasible to determine K base on the leaked energy.




a signal 1 b signal 2

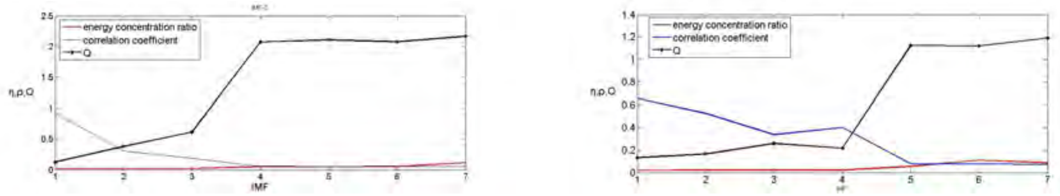
Figure 2: The frequency spectrums of IMFs

### 5.2. noise IMFs analysis

When $K = 7$, the $\eta$ ( $[-n, n]$ is 2% of the entire time period), $\rho$ and $Q$ of IMFs are shown in Table 1 and Fig.6 . According to the $\eta$, $\rho$ and $Q$ . For signal 1, IMF1-IMF3 are IMFs containing the signal components, IMF4-IMF7 are noise IMFs. For signal 2, IMF1-IMF4 are IMFs containing the signal components, IMF5-IMF7 are noise IMFs. Both signal 1 and signal 2 are consistent with the results of Figure 2. And $Q$ have stronger ability to judge noise IMFs than or . The noise IMFs are removed. The signal is reconstructed by remaining IMFs.

Table 1.The $\eta$,$\rho$,$Q$ of IMFs

| | IMF | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 1 | $\eta$ | 0.020 | 0.021 | 0.021 | 0.055 | 0.052 | 0.053 | 0.117 |
| Signal | $\rho$ | 0.926 | 0.308 | 0.190 | 0.056 | 0.056 | 0.056 | 0.0544 |
| | $Q$ | 0.126 | 0.380 | 0.616 | 2.074 | 2.109 | 2.076 | 2.168 |
| 2 | $\eta$ | 0.020 | 0.021 | 0.021 | 0.021 | 0.048 | 0.094 | 0.087 |
| Siganl | $\rho$ | 0.662 | 0.5293 | 0.332 | 0.398 | 0.076 | 0.072 | 0.067 |
| | $Q$ | 0.132 | 0.165 | 0.263 | 0.219 | 1.140 | 1.209 | 1.300 |

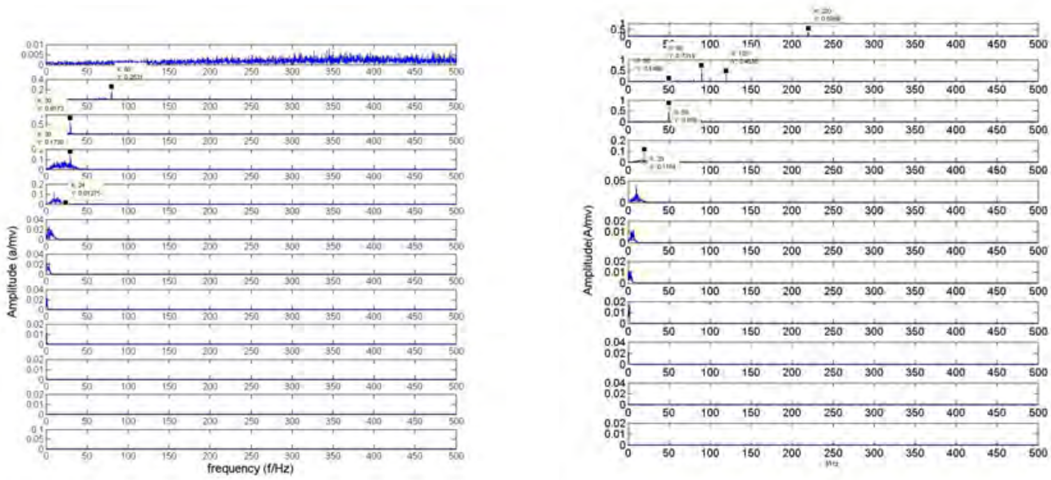



a signal 1 b signal 2

Figure 3: The $\eta$,$\rho$,$Q$ of IMFs

*5.3. comparison between VMD Nnoise reduction method and EMD noise reduction method*

EMD is performed on simulation signal 1 and signal 2 , the frequency spectrums of the IMFs are shown in Fig.5. For the siganl 1, IMF1 is noise IMF according to [12], IMF2 corresponds to 80Hz signal component, 50Hz signal component is decomposed into IMF3, both IMF3 and IMF4 contain 30Hz signal component, mode aliasing occurr in IMF3 and IMF4, IMF5-IMF12 are fault modes. For the signal 2, IMF1 corresponds to 220Hz signal component, 50Hz, 90Hz and 120Hz signal components are decomposed into IMF2, both IMF2 and IMF3 contain 50Hz signal component, mode aliasing occurr in IMF2 and IMF3, IMF4-IMF11 are fault modes. According to the EMD noise reduction method in [13], IMF5-IMF12 and IMF1 are removed in signal 1, IMF4-IMF11 are removed in signal 2. The signals are reconstructed by remaining IMFs.




a signal 1 b signal 2

Figure 4: frequency spectrums of IMFs

In order to compare the noise reduction method proposed in this paper with EMD noise reduction method. The signal-to-noise ratio (SNR) is introduced as an evaluation index. The SNR of original signals, EMD reconstruction signals and VMD reconstruction signals are shown in Table 2.

Table 2 .SNR of EMD and VMD

| | reduction method | original signal | EMD reconstruction signal | VMD reconstruction signal |
|---|---|---|---|---|
| Signal 1 | SNR(DB) | 34.860 | 36.444 | 42.8034 |
| Signal 2 | SNR(DB) | 28.9298 | 29.2921 | 33.4019 |

## 6. Experimental analysis

The signal acquisition target is a brand of inverter air conditioner. The main vibration sources of the air conditioner are the fan and the compressor. The fan speed is 850 rad/s and the compressor operating frequency is 50 Hz. The signal sampling rate is 9600. The frequency spectrum of the vibration signal is shown in Fig.5, and the logarithmic spectrum is shown in Fig.6. According to Fig.3, the frequency of the main components of the vibration signal are lower than 300 Hz.

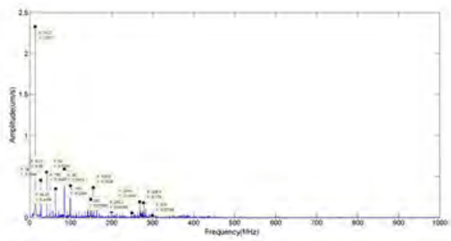


Figure 5: Frequency spectrum of the vibration signal

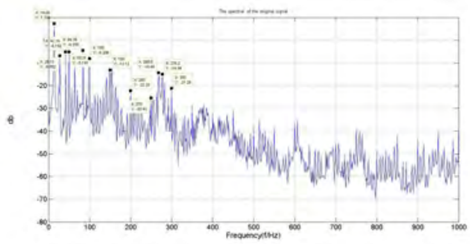


Figure 6: Logarithmic spectrum of the vibration signal

When the $\alpha$ is 2000, the $\varepsilon$ is $10^{-7}$, VMD is performed on vibration signal. The relationship between leaked energy and K is shown in Fig.7. K is selected as 6 according to Fig.7. VMD is performed on vibration signal, frequency spectrums of the IMFs decomposed by VMD are shown in Fig.8. The frequency range of IMF4, IMF5 and IMF6 are distributed above 300HZ, so IMF4, IMF5 and IMF6 are noise IMFs.

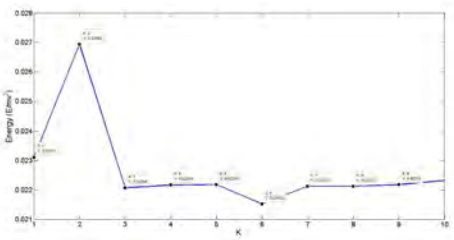


Figure 7: The relationship between leaked energy and the K

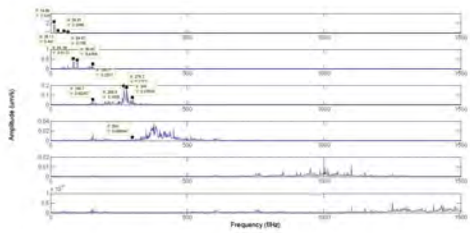
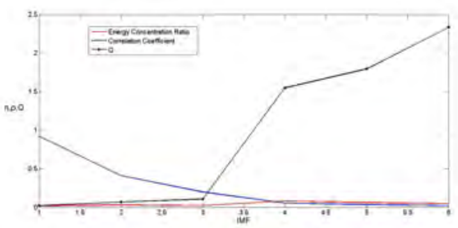


Figure 8: Frequency spectrums of IMFs

The $\eta$,$\rho$ and $Q$ of IMFs decomposed by VMD are shown in Fig.9. According to the meaning of Q, IMF4, IMF5 and IMF6 are removed. The vibration signal is reconstructed by remaining IMFs. The logarithmic spectrum of VMD reconstruction signal is shown in Fig.10. In the VMD reconstruction signal, the frequencies of the vibration sources and their multiplication are not reduced relative to the original vibration signal.



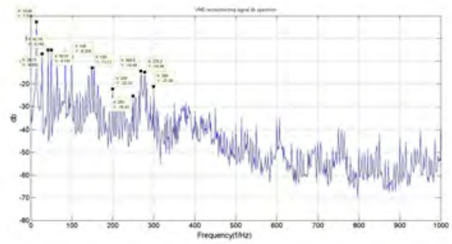


Figure 9: The $\eta$,$\rho$ and $Q$ of IMFs

Figure 10: Logarithmic spectrum of the signal reconstructed by VMD

The EMD is performed on vibration signal, the frequency spectrums of the IMFs decomposed by EMD are shown in Fig.11. Mode aliasing occurr in IMF2 and IMF3, IMF4 and IMF5, IMF5 and IMF6, IMF7 and IMF8. According to [13]. IMF1 and IMF9-IMF14 are removed.

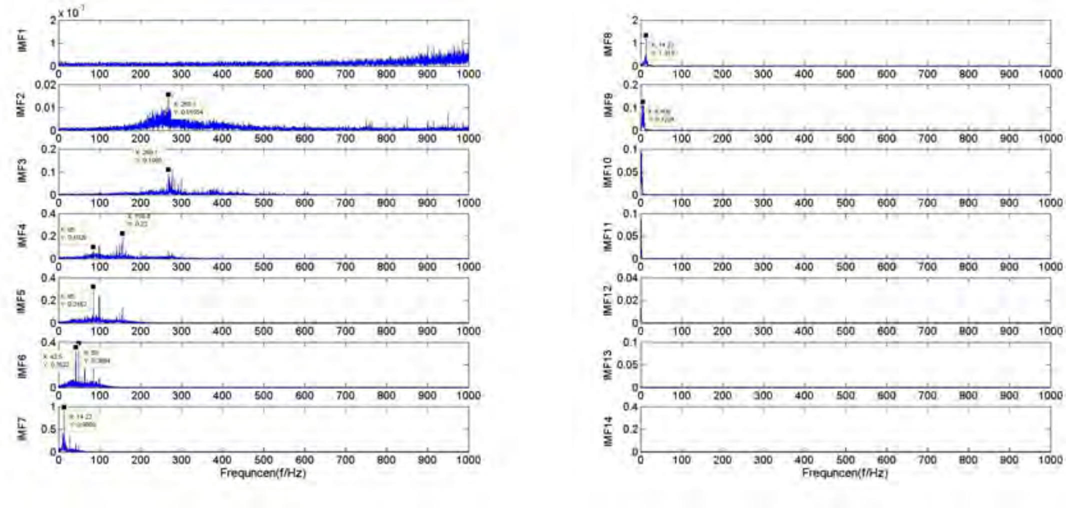



Figure 11: Frequency spectrums of IMFs

The vibration signal is reconstructed by remaining IMFs. The logarithmic spectrum of the EMD reconstruction signal is shown in Fig.12. In the VMD reconstruction signal, the frequencies of the vibration sources and its multiplication are different from the original vibration signal. This means that useful signal components are lost in the EMD reconstruction signal. The EMD reconstruction signal is distorted.

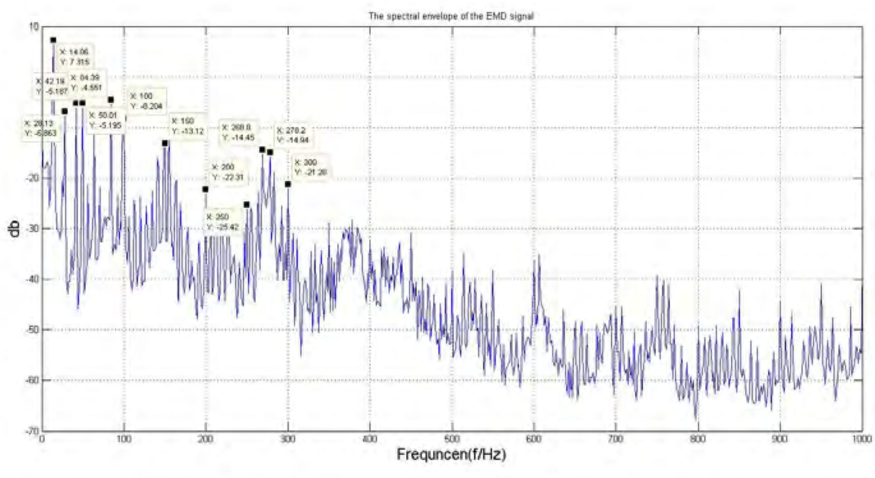



Figure 12: Logarithmic spectrum of the EMD reconstruction signal

Finally, the logarithmic spectrums of the VMD reconstruction signal and the EMD reconstruction signal are compared. In the low frequency band, the VMD reconstruction signal retains a more useful fundamental frequencies and frequency multiplications. In the noise band, the VMD reconstruction signal have less noise components. This means that, the effect of the VMD noise reduction method proposed in this paper is better than that of EMD noise reduction method.

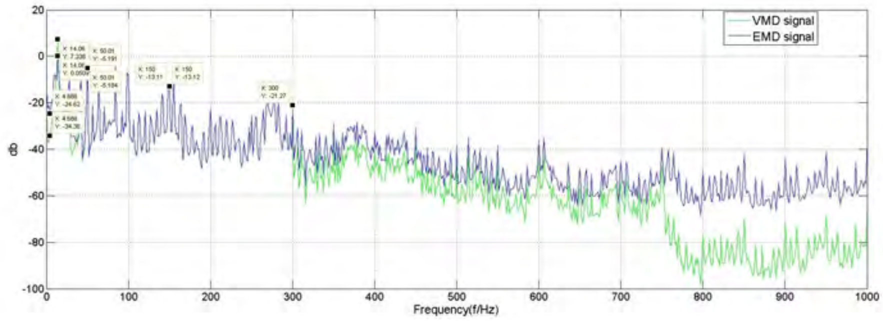



Figure 13: the logarithmic spectrums of the VMD reconstruction signal and the EMD reconstruction signal

## 7. CONCLUSION

We use the leaked energy to determine the number K of VMD decomposition, which making up for the problem of choosing K based on experience. In the simulation, The IMFs decomposed by VMD have no mode aliasing, but the IMFs decomposed by EMD have mode aliasing. And, we propose a new decision method to determine the noise IMFs. Q have stronger ability to judge noise IMFs than energy concentration ratio or correlation coefficient. In the simulation part, the SNR of the signal reconstructed by VMD is better than the the signal reconstructed by EMD. In the experiment part, the VMD reconstruction signal save more useful information than the VMD reconstruction signal, and the noise reduction effect of the VMD noise reduction method proposed in this parper is better than the EMD noise reduction method. It shows that the method proposed in this paper has certain superiority and practicability.

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