Vector and scalar variables laminar natural convection in 2D geometry arbitrary angle of inclination

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Abstract

The aim of this study[1] is to analyze vector and scalar depending variables in parallelogram enclosures which changing angle of inclination measured from horizontal plane in laminar natural convection conditions. Parallelogram enclosure has two isothermal opposite walls with different temperatures, and two adiabatic mating walls. Mathematical model described by the system of partial differential equations which solved by numerical procedure, control volume method. Results of numerical procedure, treated by original software demonstrate values of variables in estimate domain, as velocities, pressures and temperatures. There are comprehensive graphic presentation obtained by named results.

1 Description

The study represents investigation of the laminar natural convection phenomena in enclosed rectangular spaces. Therefore, realization of this subject has been done through number of phases which have to make better understanding and configuration: velocity, temperature and pressure fields in enclosures. The real physical model of the enclosure, which represents two dimensional rectangular object with differentially heated side and adiabatic horizontal walls, has been defined in order to predict good enough results. Rotation of the enclosures is one of the aim of this study too. Physical model represent base for mathematical model which define valid parameters for temperature flow regime. Solution of the defined mathematical model, with respect the nature of the equation has been done with numerical control volume method. Due to presence of the numerical procedure results there is made a computer code, which contain SIMPLE procedure in essence, and which contain routine for solving variable fields. Results of the numerical experiment has been compared with literature ones, and it gives a satisfaction. Results of this study with some other analysis and research present good base for definition of the object parameters which can find in engineering practice, and which essentially contain natural convection

[1]Presented at the IMC "Filomat 2001", Niš, August 26–30, 2001

2000 Mathematics Subject Classification: 80
Keywords: Laminar natural convection, FV method, Gauss-Zeidel method

phenomena. There are so many phenomena like: cooling electronic equipment, solar panel, different modes of heating, cooling nuclear reactors, global natural phenomena etc.

2 Basic

In order to get some quality values of named phenomena there are defined physical model of enclosure, $2D$ geometry with differentially heated opposite walls and two sided isothermal walls as on the figure 1.

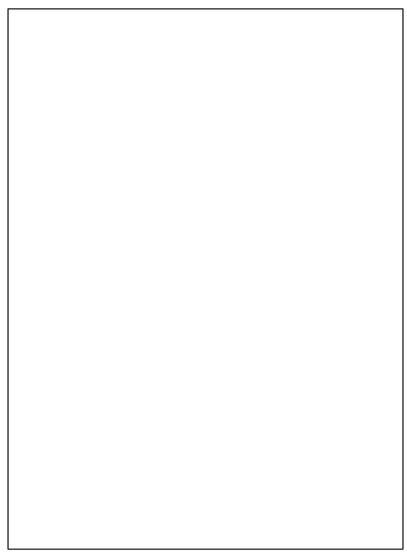

Figure 1: $2D$ enclosed geometry

In fact, named enclosure represent well enough base for $3D$ investigation because of similarity of flow pattern in cross section of enclosed space if there is enough depth of $3D$ enclosure. One more element is used as very important in this investigation and it is rotation of the enclosure. Dimension are: L for length, H for width. There were used Cartesian coordinate system fixed on the one corner point of the enclosure, and it rotate with object around z axe. There is defined geometrical scale as $A = H/L$. For $x = 0$ temperature of the hot wall is T_H, and for $x = L$ temperature of the cold wall is T_C. Temperature difference of the isothermal walls defined as: $\Delta T = T_H - T_C$. The other sided walls are adiabatic. Because of the rotate hot wall always makes some angle by the horizontal plane. Laminar convection achieved by the fixed of the Ra number on 106 value, as the Pr number is about 0,73 for air. The other physical characteristic: dynamic viscosity, thermal conductivity, specific heat on the constant pressure and density (excluded buoyancy therm) assumed to be constant for averaged rate of temperature T_0. In investigation excluded radiation heat transfer because of low intensity of heat transfer. We used Boussinesq approximation in Buoyancy therm in equations of momentum because of small temperature differences. Therefore the density in buoyancy therm of momentum equations linearized as:

$$\rho(T) = \rho(T_0) - \beta\rho(T_0)(T - T_0) \tag{1}$$

were (β represent volume thermal expansion coefficient for averaged temperature T_0.

3 Modelling

There are defined mathematical model by physical model, and it define parameters which we are interesting for.

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \beta g(T - T_0)\cos\varphi$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial v} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \beta g(T - T_0)\sin\varphi$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{2}$$

In order to get some results we are defined boundary conditions. Velocities on the walls are zero as well as temperature gradients on the adiabatic walls. The temperatures of the isothermal walls defined as T_H and T_C.

$$0 \le x \le L,\ y = 0,\ u = v = 0,\ \frac{\partial T}{\partial y} = 0,$$

$$0 \le x \le L,\ y = H,\ u = v = 0,\ \frac{\partial T}{\partial y} = 0,$$

$$x = 0,\ 0 \le y \le H,\ u = v = 0,\ T = T_H,$$

$$x = L,\ 0 \le y \le H,\ u = v = 0,\ T = T_C \tag{3}$$

4 Numerical Investigation

Mathematical model solved by the control volume numerical investigation. Numerical procedure, based on FV method, transforming system of the partial differential equations in system of algebraic equation by the discretisation method. Named algebra equation can be solved easily by the some well known method of solving as Gauss-Zeidel method General discretizated equation:

$$\frac{\partial}{\partial t}(\rho\Phi) + \mathrm{div}(\rho\vec{w}\Phi) = \mathrm{div}(\Gamma_\Phi\ \mathrm{grad}\Phi) + S_\Phi \tag{4}$$

were: Φ - dependent variable, $\vec{w}$ - velocity vector, S_Φ - source therm for dependent variable.

Enclosure is divided by orthogonal grid. The essence of the investigation is in quality choice of the discretization scheme, and good balancing between convective and diffusive therms through the walls of the enclosed space. We used hybrid scheme which gives well enough results. The hybrid scheme developed for east side of the enclosure is:

$$\left(\rho u - \Gamma \frac{\partial \Phi}{\partial x}\right)_e = \begin{cases} (\rho u)_e \Phi_E, & Pe_e \leq -2 \\ (\rho u)_e \frac{\Phi_P - \Phi_E}{2} - \Gamma_e \frac{\Phi_E - \Phi_P}{\Delta x_e}, & -2 < Pe_e < 2 \\ (\rho u)_e \Phi_P, & Pe_e \geq 2 \end{cases} \tag{5}$$

other words:

$$\begin{gathered} Pe_e \leq -2 J_e = F_e \Phi_E, \\ -2 < Pe_e < 2 J_e = 0,5 F_e (\Phi_P + \Phi_E) - D_e \frac{\Phi_E - \Phi_P}{\Delta x_e} \\ Pe_e \geq 2 J_e = F_e \Phi_P \end{gathered} \tag{6}$$

where: F represent fluxes, and Γ_e diffusion coefficients. By the integration of the named differential equation for control volume, using Gauss theorem of divergation we find balance of the source therms and fluxes for each side of control volume:

$$\int_{V_P} \frac{\partial}{\partial t}(\rho \Phi) dV + \int_{S_P} (\rho \vec{w} \Phi - \Gamma_\Phi \ \mathrm{grad} \Phi) \, \vec{n} dS = \int_{V_P} S_\Phi dV. \tag{7}$$

Total fluxes, convective and diffusive for one dimension are:

$$J_x = \rho u \Phi - \Gamma_\Phi \frac{\partial \Phi}{\partial x} \tag{8}$$

as they are constant for side of neighbor control volume

$$J_e = \int J_x dy = \left(\rho u \Phi - \Gamma_\Phi \frac{\partial \Phi}{\partial x}\right)_e \Delta y. \tag{9}$$

Generalize for other side of the control volume:

$$(J_e - J_w) + (J_n - J_S) = \int S_\Phi dV. \tag{10}$$

Source therm on the right side of the eq. (4) can be write as:

$$\int S_\Phi dV = (S_c + S_p \Phi_P) \, \Delta x \Delta y \tag{11}$$

and S_c i S_p represent constants, independent of the Φ_p value. The S_p therm must be positive to achieve numerical stability and result convergation. Final results presented by the general discretizated equation:

$$a_P \Phi_P = a_E \Phi_E + a_W \Phi_W + a_N \Phi_N + a_S \Phi_S + b \tag{12}$$

where: a_p, a_e, a_w, a_n, a_s are coefficients for point of discretization.

There where used SIMPLE procedure in order to get results of the numerical investigation and therefore we made original source code which including routine for solving independent values. By adding of some Mathlab routines there were made wide graphical presentation independent values.

5 Results

6 Closure

Results represent very complex dependance of the parameters. The angle of rotation below 20^0 gives unstable results which can be describes as time dependent or $3D$ trend. The patterns with angle of rotation between 20^0 and 180^0 are stabile. Increasing angle of rotation makes increasing of diffusion mechanism, particulary for angle close to 180^0 where is pure diffusion. The highest intensity of heat transfer noticeable near by $70^0 - 80^0$. It can be conclude that the enclosure act as some kind of thermal diode. Increasing of the Ra number results increasing Nu number. There should be emphasized that for increasing Ra number gives Nu number as linear function of Ra number characterized for boundary layer. The Nu number achieve maximum value on the different height of hot wall depending of the angle of rotation. The results, generally can be good base for further investigation, in air conditioning, cooling phenomena, wherever natural convection represent base phenomena in heat transfer and flowing. there are a numerous ways of hating spaces, solar cells, cooling electronic equipment, even for cooling of nuclear reactor or global natural phenomena.

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Figure 2: Temperature (isotherms): angle 60^0, 90^0, 160^0; $\Delta T = 10^0 C$; $Ra = 1,28x10^6$,$Pr = 0,73$

Figure 3: Pressure (isobars): angle 60^0, 90^0, 160^0; $\Delta T = 10^0 C$; $Ra = 1,28x10^6$, $Pr = 0,73$

Figure 4: Velocity vectors (isovels): angle 60^0, 90^0, 160^0; scale 1:3; geometr. scale (1:4:62)