Generalized Kählerian spaces *

Svetislav M. Minčić, Mića S. Stanković and Ljubica S. Velimirović

Abstract

In this paper[1] we investigate generalized Kählerian spaces and find some relations for curvature tensors in these spaces. Also, we define holomorphically projective mappings of generalized Kählerian spaces and obtain an invariant geometric object for these mappings.

1 Introduction

A generalized Riemannian space GR_N in the sense of Eisenhart's definition [1] is a differentiable N-dimensional manifold, equipped with nonsymmetric basic tensor g_{ij}. Connection coefficients of this space are generalized Cristoffel's symbols of the second kind. Generally it is $\Gamma^i_{jk} \neq \Gamma^i_{kj}$.

In a generalized Riemannian space one can define four kinds of covariant derivatives [3], [4]. For example, for a tensor a^i_j in GR_N we have

$$a^i_{j\underset{1}{|}m} = a^i_{j,m} + \Gamma^i_{pm} a^p_j - \Gamma^p_{jm} a^i_p, \quad a^i_{j\underset{2}{|}m} = a^i_{j,m} + \Gamma^i_{mp} a^p_j - \Gamma^p_{mj} a^i_p,$$
$$a^i_{j\underset{3}{|}m} = a^i_{j,m} + \Gamma^i_{pm} a^p_j - \Gamma^p_{mj} a^i_p, \quad a^i_{j\underset{4}{|}m} = a^i_{j,m} + \Gamma^i_{mp} a^p_j - \Gamma^p_{jm} a^i_p.$$

In the case of the space GR_N we have five independent curvature tensors [5] (in [5] $\underset{5}{R}$ is denoted by $\underset{2}{\tilde{R}}$):

$$\underset{1}{R}^i{}_{jmn} = \Gamma^i_{jm,n} - \Gamma^i_{jn,m} + \Gamma^p_{jm}\Gamma^i_{pn} - \Gamma^p_{jn}\Gamma^i_{pm},$$
$$\underset{2}{R}^i{}_{jmn} = \Gamma^i_{mj,n} - \Gamma^i_{nj,m} + \Gamma^p_{mj}\Gamma^i_{np} - \Gamma^p_{nj}\Gamma^i_{mp},$$
$$\underset{3}{R}^i{}_{jmn} = \Gamma^i_{jm,n} - \Gamma^i_{nj,m} + \Gamma^p_{jm}\Gamma^i_{np} - \Gamma^p_{nj}\Gamma^i_{pm} + \Gamma^p_{nm}(\Gamma^i_{pj} - \Gamma^i_{jp}),$$
$$\underset{4}{R}^i{}_{jmn} = \Gamma^i_{jm,n} - \Gamma^i_{nj,m} + \Gamma^p_{jm}\Gamma^i_{np} - \Gamma^p_{nj}\Gamma^i_{pm} + \Gamma^p_{mn}(\Gamma^i_{pj} - \Gamma^i_{jp}),$$
$$\underset{5}{R}^i{}_{jmn} = \frac{1}{2}(\Gamma^i_{jm,n} + \Gamma^i_{mj,n} - \Gamma^i_{jn,m} - \Gamma^i_{nj,m} + \Gamma^p_{jm}\Gamma^i_{pn} + \Gamma^p_{mj}\Gamma^i_{np}$$
$$-\Gamma^p_{jn}\Gamma^i_{mp} - \Gamma^p_{nj}\Gamma^i_{pm}).$$

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The Kählerian spaces and their mappings were investigated by many authors, for example K. Yano [9],[10], M. Prvanović [7], N. S. Sinyukov [8], J. Mikeš [2] and many other authors.

An N-dimensional Riemannian space with basic metric tensor $g_{ij}(x)$ is a Kählerian space if there exists an almost complex structure $F^i_j(x)$, such that

$$F^h_p(x)F^p_i(x) = -\delta^h_i\,,$$
$$g_{pq}F^p_iF^q_j = g_{ij},$$
$$F^h_{i;j} = 0,$$

where ; denotes the covariant derivative with respect to the basic metric tensor g_{ij}. This paper is devoted to the generalized Kählerian spaces and their mappings.

2 Generalized Kählerian spaces

A generalized N-dimensional Riemannian space with (non-symmetric) metric tensor g_{ij}, is a generalized Kählerian space GK_N if there exists an almost complex structure $F^i_j(x)$, such that

$$F^h_p(x)F^p_i(x) = -\delta^h_i\,, \tag{2.1}$$
$$g_{\underline{pq}}F^p_iF^q_j = g_{\underline{ij}}, \tag{2.2}$$
$$F^h_{i\underset{\theta}{|}j} = 0, \quad (\theta = 1,2), \tag{2.3}$$

where $\underset{\theta}{|}$ denotes the covariant derivative of the kind θ with respect to the metric tensor g_{ij}. From (2.2), using (2.1), we get

$$g_{\underline{ip}}F^p_j + g_{\underline{pj}}F^p_i = 0, \tag{2.4}$$
$$g^{\underline{ip}}F^j_p + g^{\underline{jp}}F^i_p = 0. \tag{2.5}$$

Let us denote

$$F_{ji} = F^p_j g_{\underline{pi}}, \quad F^{ji} = F^j_p g^{\underline{pi}}. \tag{2.6}$$

Then from (2.4) and (2.5) one obtains

$$F_{ij} + F_{ji} = 0, \quad F^{ij} + F^{ji} = 0. \tag{2.7}$$

From here we prove the following theorems

Theorem 2.1. *For the torsion tensor of a generalized Kählerian space the next relation*

$$\Gamma^i_{\underset{\vee}{jm}} = -\Gamma^p_{\underset{\vee}{qm}}F^i_pF^q_j. \tag{2.8}$$

is valid.

Proof. From (2.3) we have

$$\Gamma^i_{\underset{\vee}{pm}}F^p_j = \Gamma^p_{\underset{\vee}{jm}}F^i_p,$$

from where follows (2.8). ∎

Theorem 2.2. *The curvature tensors* $\underset{\theta}{R}{}^h{}_{ijk}$ $(\theta = 1, \cdots, 4)$ *in the space* GK_N *satisfy the next relations*

$$F^p_i \underset{\alpha}{R}{}^h{}_{pjk} = F^h_p \underset{\alpha}{R}{}^p{}_{ijk}, \;\; \alpha = 1, 2, 3, \qquad (2.9-11)$$

$$F^p_i \underset{4}{R}{}^h{}_{pjk} + F^h_p \underset{3}{R}{}^p{}_{ikj} = 2(\Gamma^q_{\underset{\vee}{ij}|\underset{4}{k}} - \Gamma^q_{\underset{\vee}{ki}|\underset{3}{j}} + 2\Gamma^p_{\underset{\vee}{ij}}\Gamma^q_{\underset{\vee}{kp}} - 2\Gamma^p_{\underset{\vee}{ki}}\Gamma^q_{\underset{\vee}{pj}})F^h_q. \qquad (2.12)$$

Proof. a) From (2.3) we have $F^h_{i\underset{1}{|}jk} - F^h_{i\underset{1}{|}kj} = 0$, and then, using the first Ricci identity [3], [4] we have

$$-F^h_p \underset{1}{R}{}^p{}_{ijk} + F^p_i \underset{1}{R}{}^h{}_{pjk} - 2\Gamma^p_{\underset{\vee}{jk}} F^h_{i\underset{1}{|}p} = 0,$$

i.e.

$$F^p_i \underset{1}{R}{}^h{}_{pjk} - F^h_p \underset{1}{R}{}^p{}_{ijk} = 0. \qquad (2.13)$$

The relation (2.9) is proofed.
b) Analogously, using the Ricci identity for $F^h_{i\underset{2}{|}jk} - F^h_{i\underset{2}{|}kj}$ and (2.3) we get

$$F^p_i \underset{2}{R}{}^h{}_{pjk} - F^h_p \underset{2}{R}{}^p{}_{ijk} = 0, \qquad (2.14)$$

from where (2.10) follows.
c) From (2.3) we get $F^h_{i\underset{1}{|}j\underset{2}{|}k} - F^h_{i\underset{2}{|}k\underset{1}{|}j} = 0$. From another side is [3]

$$F^h_{i\underset{1}{|}j\underset{2}{|}k} - F^h_{i\underset{2}{|}k\underset{1}{|}j} = \underset{3}{R}{}^h{}_{pjk} F^p_i - \underset{3}{R}{}^p{}_{ijk} F^h_p.$$

From last two equations we get the relation (2.11).
d) From (2.3) we get

$$F^h_{i\underset{3}{|}j} = 2\Gamma^p_{\underset{\vee}{ij}} F^h_p, \quad F^h_{i\underset{4}{|}k} = 2\Gamma^p_{\underset{\vee}{ki}} F^h_p,$$

$$F^h_{i\underset{3}{|}j\underset{4}{|}k} = 2\Gamma^p_{\underset{\vee}{ij}|\underset{4}{k}} F^h_p + 4\Gamma^p_{\underset{\vee}{ij}}\Gamma^q_{\underset{\vee}{np}} F^h_q, \quad F^h_{i\underset{4}{|}k\underset{3}{|}j} = 2\Gamma^p_{\underset{\vee}{ki}|\underset{3}{j}} F^h_p + 4\Gamma^p_{\underset{\vee}{ki}}\Gamma^q_{\underset{\vee}{pj}} F^h_q.$$

Using the Ricci type identity [4]

$$F^h_{i\underset{3}{|}j\underset{4}{|}k} - F^h_{i\underset{4}{|}k\underset{3}{|}j} = \underset{4}{R}{}^h{}_{pjk} F^p_i + \underset{3}{R}{}^p{}_{ikj} F^h_p,$$

we have the relation (2.12). ∎

Theorem 2.3. *For the curvature tensors* $\underset{\theta}{R}_{hijk}$ $(\theta = 1, \cdots, 4)$ *of the space* GK_N *are valid the next relations*

$$F^p_h \underset{\alpha}{R}_{pijk} = F^p_i \underset{\alpha}{R}_{phjk}, \;\; \alpha = 1, 2, 3, \qquad (2.15-17)$$

$$F^p_h \underset{4}{R}_{pijk} = -F^p_i \underset{3}{R}_{phkj} + 2F^p_i(\Gamma_{h.\underset{\vee}{ki}|\underset{3}{j}} - \Gamma_{h.\underset{\vee}{ij}|\underset{4}{k}} + 2\Gamma^p_{\underset{\vee}{ki}}\Gamma_{h.\underset{\vee}{pj}} - 2\Gamma^p_{\underset{\vee}{ij}}\Gamma_{h.\underset{\vee}{kp}}), \qquad (2.18)$$

$$F^p_h(\underset{4}{R}_{pijk} + \underset{3}{R}_{pikj}) = 2F^p_i(\Gamma_{h.\underset{\vee}{ki}|\underset{3}{j}} - \Gamma_{h.\underset{\vee}{ij}|\underset{4}{k}} + 2\Gamma^p_{\underset{\vee}{ki}}\Gamma_{h.\underset{\vee}{pj}} - 2\Gamma^p_{\underset{\vee}{ij}}\Gamma_{h.\underset{\vee}{kp}}). \qquad (2.19)$$

Proof. By composition in (2.9) with F^q_h we get

$$F^p_i F^h_q \underset{1}{R}{}^q{}_{pjk} + \underset{1}{R}{}^h{}_{ijk} = 0. \tag{2.20}$$

From here we have

$$F^p_h F^q_i \underset{1}{R}{}_{pqjk} - \underset{1}{R}{}_{hijk} = 0, \tag{2.21}$$

and by composition with F^i_r we get

$$F^p_h \underset{1}{R}{}_{pijk} + F^p_i \underset{1}{R}{}_{hpjk} = 0. \tag{2.22}$$

The first kind curvature tensor satisfy the relation $\underset{1}{R}{}_{hijk} = -\underset{1}{R}{}_{ihjk}$. Now from (2.22) we get the relation (2.15). The relations (2.16-18) we get in the same manner from (2.10-12) by using of anti-symmetry for the tensors $\underset{\theta}{R}{}_{hijk}$ $(\theta = 2,3,4)$ with respect to the two first indices. The relation (2.19) follows directly from (2.17,18). ■

Theorem 2.4. *The curvature tensors* $\underset{\theta}{R}{}^i{}_{jmn}$ $(\theta = 1,\cdots,5)$ *of the space* GK_N *satisfy the next relations*

$$\underset{\alpha}{R}{}_{(pq)} F^p_j F^q_m = \underset{\alpha}{R}{}_{(jm)} - 2\Gamma^p_{\underset{\vee}{rq}}\Gamma^q_{\underset{\vee}{ps}} F^r_j F^s_m + 2\Gamma^p_{\underset{\vee}{jq}}\Gamma^q_{\underset{\vee}{pm}},\ \alpha = 1,2,3, \tag{2.23 a, b, c}$$

$$\underset{4}{R}{}_{(pq)} F^p_j F^q_m = \underset{4}{R}{}_{(jm)} + 6\Gamma^p_{\underset{\vee}{rq}}\Gamma^q_{\underset{\vee}{ps}} F^r_j F^s_m - 6\Gamma^p_{\underset{\vee}{jq}}\Gamma^q_{\underset{\vee}{pm}}, \tag{2.23d}$$

$$\underset{5}{R}{}_{(pq)} F^p_j F^q_m = \underset{5}{R}{}_{(jm)} + 2\Gamma^p_{\underset{\vee}{rq}}\Gamma^q_{\underset{\vee}{ps}} F^r_j F^s_m - 2\Gamma^p_{\underset{\vee}{jq}}\Gamma^q_{\underset{\vee}{pm}}, \tag{2.23e}$$

where (jm) *denotes the symmetrization without division with respect to the indices* j, m.

Proof. (a) From $F^h_{i\underset{1}{|}j} = 0$, $\quad F^h_{i\underset{2}{|}j} = 0$ by addition and division with 2 we get

$$F^h_{i;j} = 0, \tag{2.24}$$

where ; denotes covariant derivative with respect to $g_{\underline{ij}}$. The integrability conditions of the equation (2.24) give the relation

$$F^h_p R^p{}_{ijk} - F^p_i R^h{}_{pjk} = 0,$$

where $R^h{}_{ijk}$ is a curvature tensor with respect to symmetric basic tensor $g_{\underline{ij}}$. Using the condition (2.1) we get

$$F^h_p F^q_i R^p{}_{ijk} + R^h{}_{ijk} = 0,$$

and from here

$$F^p_h F^q_i R_{pqjk} - R_{hijk} = 0.$$

With respect to the condition (2.1), we get

$$F^p_h R_{pijk} - F^p_i R_{phjk} = 0.$$

By composition with $g^{\underline{ij}}$ and contraction by virtue of indices i,j, we get

$$F^p_h R_{pk} = F^p_q R_{ph.k}^{\ \ \ \ q}.$$

By symmetrization with respect to h, k we get

$$R_{hk} = F^p_h F^q_k R_{pq}. \tag{2.25}$$

We can express the tensor $\underset{1}{R}{}^i{}_{jmn}$ in the form [5]:

$$\underset{1}{R}{}^i{}_{jmn} = R^i{}_{jmn} + \underset{\vee}{\Gamma}{}^i_{jm;n} - \underset{\vee}{\Gamma}{}^i_{jn;m} + \underset{\vee}{\Gamma}{}^p_{jm}\underset{\vee}{\Gamma}{}^i_{pn} - \underset{\vee}{\Gamma}{}^p_{jn}\underset{\vee}{\Gamma}{}^i_{pm}.$$

By contraction with respect to indices i, n, and by symmetrization with respect to j, m, we get

$$\underset{1}{R}_{(jm)} = R_{(jm)} - 2\underset{\vee}{\Gamma}{}^p_{jq}\underset{\vee}{\Gamma}{}^q_{pm}. \tag{2.26}$$

From (2.25) and (2.26) we have (2.23a).
(b) The tensor $\underset{2}{R}{}^i{}_{jmn}$ we can express in the form [5]:

$$\underset{2}{R}{}^i{}_{jmn} = R^i{}_{jmn} - \underset{\vee}{\Gamma}{}^i_{jm;n} + \underset{\vee}{\Gamma}{}^i_{jn;m} - \underset{\vee}{\Gamma}{}^p_{jm}\underset{\vee}{\Gamma}{}^i_{pn} + \underset{\vee}{\Gamma}{}^p_{jn}\underset{\vee}{\Gamma}{}^i_{pm}.$$

By contraction with respect to i, n, and then by symmetrization with respect to j, m, we get

$$\underset{2}{R}_{(jm)} = R_{(jm)} - 2\underset{\vee}{\Gamma}{}^p_{jq}\underset{\vee}{\Gamma}{}^q_{pm},$$

from where, using (2.25), we get the relation (2.23b).
(c) For the tensor $\underset{3}{R}{}^i{}_{jmn}$ we have [5]:

$$\underset{3}{R}{}^i{}_{jmn} = R^i{}_{jmn} + \underset{\vee}{\Gamma}{}^i_{jm;n} + \underset{\vee}{\Gamma}{}^i_{jn;m} - \underset{\vee}{\Gamma}{}^p_{jm}\underset{\vee}{\Gamma}{}^i_{pn} + \underset{\vee}{\Gamma}{}^p_{jn}\underset{\vee}{\Gamma}{}^i_{pm} - 2\underset{\vee}{\Gamma}{}^p_{mn}\underset{\vee}{\Gamma}{}^i_{pj}.$$

Contracting according to i, n, and then symmetrizing in relation to j, m, we get

$$\underset{3}{R}_{(jm)} = R_{(jm)} - 2\underset{\vee}{\Gamma}{}^p_{jq}\underset{\vee}{\Gamma}{}^q_{pm},$$

from where, using (2.25), we can see that the relation (2.23c) is valid.
(d) The tensor $\underset{4}{R}{}^i{}_{jmn}$ we can express in the form [5]:

$$\underset{4}{R}{}^i{}_{jmn} = R^i{}_{jmn} + \underset{\vee}{\Gamma}{}^i_{jm;n} + \underset{\vee}{\Gamma}{}^i_{jn;m} - \underset{\vee}{\Gamma}{}^p_{jm}\underset{\vee}{\Gamma}{}^i_{pn} + \underset{\vee}{\Gamma}{}^p_{jn}\underset{\vee}{\Gamma}{}^i_{pm} + 2\underset{\vee}{\Gamma}{}^p_{mn}\underset{\vee}{\Gamma}{}^i_{pj}.$$

Contracting according to i, n, and symmetrizing with respect to j, m, we get

$$\underset{4}{R}_{(jm)} = R_{(jm)} + 6\underset{\vee}{\Gamma}{}^p_{jq}\underset{\vee}{\Gamma}{}^q_{pm}.$$

Using (2.25) we get the relation (2.23d).
(e) The tensor $\underset{5}{R}^i{}_{jmn}$ satisfies the relation [5]:

$$\underset{5}{R}^i{}_{jmn} = R^i{}_{jmn} + \Gamma^p_{\underset{\vee}{jm}}\Gamma^i_{\underset{\vee}{pn}} + \Gamma^p_{\underset{\vee}{jn}}\Gamma^i_{\underset{\vee}{pm}}.$$

Contracting by virtue of indices i, n, and than symmetrizing by virtue of j, m, we get

$$\underset{5}{R}_{(jm)} = R_{(jm)} + 2\Gamma^p_{\underset{\vee}{jq}}\Gamma^q_{\underset{\vee}{pm}},$$

from where, using (2.25) we get (2.23e). ■

For generalized Kählerian spaces also the next theorem is valid.
Theorem 2.5. *If the almost complex structure F^h_i of the space GK_N satisfies the condition $F^h_{i\underset{\theta}{|}j} = 0$, $(\theta = 3, 4)$ then $\Gamma^h_{\underset{\vee}{ij}} = 0$.*

Proof. The proof follows directly from $F^h_{i\underset{\theta}{|}j} = 0$, $(\theta = 1, \cdots, 4)$. ■

3 Holomorphically projective mappings

Generalizing the concept of analytic planar curve in a Kählerian space [6], [8] we get an analog notion for a generalized Kählerian space.
Definition 3.1. A curve

$$l : x^h = x^h(t), \quad (h = 1, 2, \cdots, N) \tag{3.1}$$

is *an analytic planar* if for it is satisfied the relation

$$\lambda^h{}_{\underset{\theta}{|}p}\lambda^p = a(t)\lambda^h + b(t)F^h_p\lambda^p, \quad (\theta = 1, 2) \tag{3.2}$$

where $\lambda^h = dx^h/dt$, and $a(t)$ and $b(t)$ are same function of a parameter t.
In GK_N is

$$\lambda^h{}_{\underset{1}{|}p}\lambda^p = \frac{d\lambda^h}{dt} + \Gamma^h_{pq}\lambda^p\lambda^q = \lambda^h{}_{\underset{2}{|}p}\lambda^p.$$

Then the expression on the left side in (3.2) is self-same with respect to the booth kind of covariant derivative, from where we can define analytic planar curve in the space GK_N by one relation

$$\frac{d\lambda^h}{dt} + \Gamma^h_{pq}\lambda^p\lambda^q = a(t)\lambda^h + b(t)F^h_p\lambda^p. \tag{3.3}$$

Consider two N-dimensional generalized Kählerian spaces GK_N and $G\overline{K}_N$ with almost complex structures F^h_i and $\overline{F}^h_i$ respectively, where

$$F^h_i = \overline{F}^h_i \tag{3.4}$$

in common by mapping $f : GK_N \to G\overline{K}_N$ coordinate system.

Definition 3.2. Diffeomorfism $f : GK_N \to G\overline{K}_N$ is *holomorphically projective* or *analytic planar* if by this mapping analytic planar curves of the space GK_N map into analytic planar curves of the space $G\overline{K}_N$. Let us denote

$$P^h_{ij} = \overline{\Gamma}^h_{ij} - \Gamma^h_{ij} \tag{3.5}$$

the deformation tensor of connection at analytic planar mapping, where Γ^h_{ij} and $\overline{\Gamma}^h_{ij}$ are the second kind Cristophell's symbols of the space GK_N and $G\overline{K}_N$ respectively.

Analytic planar curves of the space GK_N and $G\overline{K}_N$ are given by relations

$$\frac{d\lambda^h}{dt} + \Gamma^h_{pq}\lambda^p\lambda^q = a(t)\lambda^h + b(t)F^h_p\lambda^p, \quad \frac{d\lambda^h}{dt} + \overline{\Gamma}^h_{pq}\lambda^p\lambda^q = \overline{a}(t)\lambda^h + \overline{b}(t)F^h_p\lambda^p,$$

respectively. From these relations we get $(\overline{\Gamma}^h_{pq} - \Gamma^h_{pq})\lambda^p\lambda^q = \psi(t)\lambda^h + \sigma(t)F^h_p\lambda^p$, where we denote $\psi(t) = \overline{a}(t) - a(t), \quad \sigma(t) = \overline{b}(t) - b(t)$. Putting $\psi(t) = \psi_p\lambda^p, \quad \sigma(t) = \sigma_q\lambda^q$, we have $(\overline{\Gamma}^h_{pq} - \Gamma^h_{pq} - \psi_p\delta^h_q - \sigma_p F^h_q)\lambda^p\lambda^q = 0$, from where is

$$\overline{\Gamma}^h_{ij} = \Gamma^h_{ij} + \psi_{(i}\delta^h_{j)} + \sigma_{(i}F^h_{j)} + \xi^h_{ij}, \tag{3.6}$$

where (ij) denotes a symmetrization without division by indices i, j and ξ^h_{ij} is an anti-symmetric tensor. In (3.6) vector σ_i we can select such that $\sigma_i = -\psi_p F^p_i$. Then we have

$$\overline{\Gamma}^h_{ij} = \Gamma^h_{ij} + \psi_{(i}\delta^h_{j)} - \psi_p F^p_{(i}F^h_{j)} + \xi^h_{ij}. \tag{3.7}$$

Contracting by indices h, i in (3.7) and using $F^p_p = 0, \quad \xi^p_{pj} = 0$ we have

$$\overline{\Gamma}^p_{pj} - \Gamma^p_{pj} = (N+2)\psi_j. \tag{3.8}$$

From (3.8) we can see that ψ_j is a gradient vector. By substitution from (3.8) in to (3.7) we get

$$\begin{aligned} &\overline{\Gamma}^h_{ij} - \frac{1}{N+2}(\overline{\Gamma}^p_{pi}\delta^h_j + \overline{\Gamma}^p_{pj}\delta^h_i - \overline{\Gamma}^q_{qp}\overline{F}^p_{(i}\overline{F}^h_{j)}) - \overline{\Gamma}^h_{\underset{\vee}{ij}} \\ &\quad = \Gamma^h_{ij} - \frac{1}{N+2}(\Gamma^p_{pi}\delta^h_j + \Gamma^p_{pj}\delta^h_i - \Gamma^q_{qp}F^p_{(i}F^h_{j)}) - \Gamma^h_{\underset{\vee}{ij}}. \end{aligned} \tag{3.9}$$

Let us denote

$$HT^h_{ij} = \Gamma^h_{\underline{ij}} - \frac{1}{N+2}(\Gamma^p_{p(i}\delta^h_{j)} - \Gamma^q_{qp}F^p_{(i}F^h_{j)}). \tag{3.10}$$

Then (3.9) we can present in the form

$$H\overline{T}^h_{ij} = HT^h_{ij}, \tag{3.11}$$

where $H\overline{T}^h_{ij}$ denotes an object of the form (3.10) in the space $G\overline{K}_N$. The magnitude HT^h_{ij} is not a tensor. We shall call it *holomorphically projective parameter of the type of Thomas's projective parameter*. From the facts given above, we have

Theorem 3.1. *Geometric objects* (3.10) *of the space* GK_N *are invariant of holomorphically projective mappings.*

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Faculty of Science and Mathematics
Višegradska 33, 18000 Niš
smica@pmf.pmf.ni.ac.yu, stmica@ptt.yu
vljubica@pmf.pmf.ni.ac.yu