New knot tables

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Abstract

New knot tables based on the notion of knot families are given.[1] Using the methods of experimental mathematics, particular results obtained for knots with $n \leq 19$ crossings belonging to the families p, pq, $p1q$, $p11q$, $p111q$, pqr, $pq1r$ are extended, extrapolated and generalized to whole families. As the result, general formulas for Alexander polynomials, signatures, unknotting numbers, and data about symmetry properties of all knots belonging to the families mentioned, are derived and estimated.

1 Introduction

A possibility to study knots from the mathematical point of view was for the first time proposed by C.F. Gauss. Gauss formulated the "crossing problem", by assigning letters to the crossing points of a self-intersecting curve and trying to determine "words" defining a closed curve. J.B. Listing represented knots by their projections (diagrams) and made an attempt to derive and classify all projections having fewer than seven crossings using so-called Complexions-Symbols. Almost complete derivation of alternating knots having fewer than 11 crossings and non-alternating knots with $n \leq 10$ crossings was given by P.G. Tait, T.P. Kirkman, and C.N. Little till the end of 19^{th} century [23,24]. Kirkman's geometrical system for the systematic derivation of knot projections, closely connected with the enumeration of polyhedra, represented at the same time the geometrical method for the classification of knot projections [16].

In the 30-ties, after the appearance of the first modern polynomial knot invariant, discovered by J.W. Alexander, the knot theory was established as the part of topology, completely loosing connection with its roots - geometry. In K. Redmeister's book "Knotentheorie" (1932), each knot is represented by one projection, (randomly?) chosen from several possible ones. After Redmeister [20], all knot tables that can be found in knot theory books are simple copies of the first: sometimes, some projection is slightly changed, or turned upside down, and that's all. In order to compare them, the reader may consider knot tables from the books [1,4,14,15,19,21].

All knot tables are followed by the corresponding polynomial knot invariants: Alexander polynomials, Jones polynomials [1], Laurent polynomials [14], and

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data about some other knot invariants and properties – hyperbolic volumes [1], signatures [4,15], unknotting numbers [15], chirality and invertibility [4,15], symmetry groups of knots [15], etc.). Usually, knots are denoted in knot tables by their ordering numbers as 3_1, 4_1, 5_1, 5_2, 6_1, 6_2, 6_3, 7_1 - 7_7, 8_1 - 8_{21}, 9_1 - 9_{49}, 10_1 - 10_{166}, without any geometrical or topological "vertical" ordering principle connecting knots with n and $n+1$ crossings. This *classical notation*, giving no information about any knot or link (except its place in knot tables), is preserved till now in the most of knot theory books. The most of knots are alternating, and non-alternating knots will appear from $n \geq 8$: 8_{18} - 8_{21}, 9_{42} - 9_{49}, 10_{124} - 10_{166}, etc. The most complete printed knot tables contain knots with $n \leq 10$ crossings. The only tables containing links are given by D. Rolfsen [21].

Today, with the development of computers, the notation and enumeration of knots and links is very similar with the situation occurring in different unordered structures: prime numbers, polyominoes etc., giving no chance for any classification. This development made possible to construct all possible permutations of n even numbers, check their realizability as knot projections, and find the minimal Dowker sequence for every knot [9].

In Dowker notation every knot is given by its (minimal) Dowker sequence (e.g., 4 6 8 2 describing knot 4_1) and the signs of crossings (necessary only in the case of non-alternating knots), from which is possible to reconstruct the knot. Because Dowker code is dependent from a minimal projection and from the choice of beginning point, the mapping between knots and their Dowker sequences is one-to-many, so it is necessary to find a minimal Dowker sequence for each knot. Hence, Dowker codes are just minimal permutations representing certain knot projections, without carrying any other geometrical or topological information about knots, so they are absolutely non-useful in any attempt of knot classification.

Using computer enumeration and Dowker algorithm, M.B. Thistlethwaite (by the program "Knotscape" [18]) and H. Doll & J. Hoste [8], obtained the tables of knots with $n \leq 16$ crossings and non-isomorphic minimal link projections with $n \leq 9$ crossings. Similar program able to recognize all knot projections with $n \leq 10$ crossings was developed by the author and V. Veličković in 1995.

Continuing the "geometrical" line (Kirkman-Conway-Caudron) [5,7,16] and the classification of knots and links proposed in [12,13], in this paper we will introduce new knot tables, based on the notion of knot families. Till now, such new tables are completed only for prime knots with $n \leq 8$ crossings.

2 Notation

In the tables, knots are denoted by *Conway notation* [21]. This unique symbolical notation for knots and links was introduced by J.Conway in 1967 [7]. From Conway symbols it is possible to read directly many of important knot or link properties: their symmetry, to recognize the world [5] to which they belong, to prove the equality of rational knots or links using very simple calculation, and even to derive some general conclusions: for example, all rational knots with symmetrical Conway symbol are amphicheiral [5].

A prime knot or link with singular digons, expressed by a Conway symbol, is called generating, and a knot or link without digons is called a basic polyhedron [7,12,13]. Any other knot or link can be derived from some generating knot or link, by replacing singular digons by chains of digons. All knots and links that can be derived from a generating knot or link by such replacement make a *family* [12,13]. All knots and links are distributed into disjoint sets, called by A.Caudron worlds [5].

It is interesting that the term "family" is very rarely mentioned and used in knot theory: its description can be found only in [10], where a family of knots is introduced as an "informal term used to describe a list of knots where each successive knot is obtained from the previous one by a simple process. The twist knots are an example, as are the knots 3_1, 5_1, 7_1,...". The other recent use of knot and link families can be found in the CD-R "Raising Public Awareness of Mathematics" by R. Brown [3].

3 New knot tables

In the new knot tables based on knot families, every family is given by its general Conway symbol and existential conditions (i.e. conditions necessary that a given Conway symbol represents a knot, and not a link). In each "Notation" subsection it is given a comparative classical notation of knots with $n \leq 10$ crossings belonging to certain family and their corresponding Conway symbols. For each knot with $n \leq 19$ crossings, its Dowker sequence is given. After the list of particular Alexander polynomials for the knots with $n \leq 19$ crossings, it is given a general formula for the Alexander polynomial of a family considered, and the list of particular Jones polynomials for knots with $n \leq 19$ crossings. For every family they are determined in the general form the symmetry group, symmetry type, signatures, and unknotting numbers. All that data are calculated for the knots with $n \leq 19$ crossings first by using program "Knotplot" [17] and its tangle calculator for calculating Dowker sequences. After that, Alexander polynomials given in the form proposed by Rolfsen [21], Jones polynomials, symmetry groups, symmetry types and signatures are calculated by putting Dowker sequences mentioned in the program "Knotscape" [18]. Unknotting numbers are calculated for the knots with $n \leq 19$ crossings mentioned according to *Bernhard-Jablan Conjecture* [11,13,22] by using a program developed by the author and V. Veličković. Finally, particular results obtained for the knots with $n \leq 19$ crossings are extrapolated to whole families in order to derive general formulas for the Alexander polynomials, symmetry groups, symmetry types, signatures, and unknotting numbers.

This way, all the general formulas in this paper belong to the *experimental mathematics*: they represent the results that are *estimated*, *extrapolated* and *conjectured*, and need to be proved (or disproved!). The general Alexander polynomials derived that way coincide with the general Alexander polynomials for the family p ($p = 2k+1$) and subfamilies $p2$, $p12$ ($p = 2k+1$) proved in [6].

As an example, complete results are given only for the family p. Because the complete material is to large for publishing it in printed form, for all the other families beginning for $n \leq 7$, it is given only comparative notation and the general formulas for Alexander polynomials, symmetry groups, symmetry type, signature, and unknotting numbers. The complete tables reader can find at the addresses:

http://www.mi.sanu.ac.yu/vismath/

http://members.tripod.com/vismath7/knotab/

Knot family: p $(p = 2k + 1)$

3	3_1
5	5_1
7	7_1
9	9_1

Dowker codes:

3	4 6 2
5	6 8 10 2 4
7	8 10 12 14 2 4 6
9	10 12 14 16 18 2 4 6 8
11	12 14 16 18 20 22 2 4 6 8 10
13	14 16 18 20 22 24 26 2 4 6 8 10 12
15	16 18 20 22 24 26 28 30 2 4 6 8 10 12 14
17	18 20 22 24 26 28 30 32 34 2 4 6 8 10 12 14 16
19	20 22 24 26 28 30 32 34 36 38 2 4 6 8 10 12 14 16 18

Alexander polynomials:

3	[1 −1
5	[1 −1 1
7	[1 −1 1 −1
9	[1 −1 1 −1 1
11	[1 −1 1 −1 1 −1
13	[1 −1 1 −1 1 −1 1
15	[1 −1 1 −1 1 −1 1 −1
17	[1 −1 1 −1 1 −1 1 −1 1
19	[1 −1 1 −1 1 −1 1 −1 1 −1

$$\Delta(p) = \sum_{i=0}^{2k} (-1)^i t^i$$

Jones polynomials:

3	1	4	1 0 1 −1
5	2	7	1 0 1 −1 1 −1
7	3	10	1 0 1 −1 1 −1 1 −1
9	4	13	1 0 1 −1 1 −1 1 −1 1 −1
11	5	16	1 0 1 −1 1 −1 1 −1 1 −1 1 −1
13	6	19	1 0 1 −1 1 −1 1 −1 1 −1 1 −1 1 −1

15	7	22	1 0 1 −1 1 −1 1 −1 1 −1 1 −1 1 −1 1 −1
17	8	25	1 0 1 −1 1 −1 1 −1 1 −1 1 −1 1 −1 1 −1 1 −1
19	9	28	1 0 1 −1 1 −1 1 −1 1 −1 1 −1 1 −1 1 −1 1 −1 1 −1

Symmetry groups: D_1

Symmetry type: chiral, reversible.

Signatures: $2k$

Unknotting numbers: k

Knot family: pq ($pq = 0 \pmod 2$; $p \geq q$; $pq = qp$)

Notation:

22	4_1				
32	5_2				
42	6_1				
52	7_2	43	7_3	44	8_3
62	8_1			54	9_4
72	9_2	63	9_3	64	10_3
42	10_1				

Alexander polynomials:

$$\Delta((2m)(2n)) = mn - (2mn+1)t + mnt^2$$

$$\Delta((2m+1)(2n)) = (m+1) + (2m+1)\sum_{i=1}^{2n-1}(-1)^i t^i + (m+1)t^{2n}$$

$$\Delta((2m)(2n+1)) = (n+1) + (2n+1)\sum_{i=1}^{2m-1}(-1)^i t^i + (n+1)t^{2m}$$

Symmetry groups: D_2 if $p = q$; otherwise D_1.

Symmetry type: fully amphicheiral for $p = q$; otherwise cheiral and reversible.

Signatures:
- 0 if $p = q = 0 \pmod 2$;
- p if $p = 0 \pmod 2$ and $q = 1 \pmod 2$;
- q if $q = 0 \pmod 2$ and $p = 1 \pmod 2$.

Unknotting numbers:
- $u((2m)(2n)) = n$
- $u((2m+1)(2n)) = n$
- $u((2m)(2n+1)) = m$

Knot family: $p1q$ ($p = 1 \pmod 2$ or $q = 1 \pmod 2$; $p \geq q$; $p1q = q1p$)

Notation:

312 6_2		
	313 7_4	
512 8_2	413 8_4	
	513 9_5	
712 10_2	613 10_4	514 10_8

Alexander polynomials:

$$\Delta((2m+1)1(2n+1)) = (m+1)(n+1) - (2mn+2m+2n+1)t + (m+1)(n+1)t^2$$

$$\Delta((2m)1(2n+1)) = m + (2m+1)\sum_{i=1}^{2n+2}(-1)^i t^i + mt^{2n+3}$$

$$\Delta((2m+1)1(2n)) = n + (2n+1)\sum_{i=1}^{2m+2}(-1)^i t^i + nt^{2m+3}$$

Symmetry groups: D_4 if $p = q$; otherwise D_2.

Symmetry type: chiral and reversible.

Signatures:
- 2 if $p = q = 1 \pmod 2$;
- $p-1$ if $p = 1 \pmod 2$ and $q = 0 \pmod 2$;
- $q-1$ if $q = 1 \pmod 2$ and $p = 0 \pmod 2$;

Unknotting numbers:
- $u((2m+1)1(2n+1)) = n+1$
- $u((2m+1)(2n)) = m$
- $u((2m)1(2n+1)) = n+1$

Knot family: $p11q$ ($p = q \pmod 2$; $p \geq q$; $p11q = q11p$)

Notation:

2112 6_3		
4112 8_7	3113 8_9	
6112 10_5	5113 10_9	4114 10_{17}

Alexander polynomials:

$$\Delta(p11q)) = \sum_{i=0}^{q-1}(2i+1)(-1)^i t^i +$$

$$(2q+1)\sum_{i=q}^{p}(-1)^i t^i + \sum_{i=p+1}^{p+q}(2p+2q-2i+1)(-1)^i t^i$$

Symmetry groups: D_4 if $p = q$; otherwise D_2.

Symmetry type: fully amphicheiral for $p = q$; otherwise chiral and reversible.

Signatures: $p - q$.

Unknotting numbers:
$u(p11q) = 1$ if $p = q$;
$u(p11q) = \frac{p-q}{2}$ if $p > q$.

Knot family: pqr ($p \neq r \pmod 2$ or $p = q = r = 1 \pmod 2$; $p \geq r$; $pqr = rpq$)

Notation:

322 7_5				
	332 8_6			
522 9_6	333 9_{10}	342 9_7	423 9_9	
		352 10_{20}	433 10_{11}	532 10_6

Alexander polynomials:

$$\Delta(p(2m)r) = \sum_{i=0}^{r-1}((2i+1)m+1)((-t)^i + (-t)^{p+r-i-1}) + \sum_{i=r}^{p-1}(2rm+1)(-t)^i$$

$$\Delta((2k+1)(2m+1)(2n)) = (m+1)n(1+t^{2k+2}) -$$

$$-(3mn+2n+1)(t+t^{2k+1}) + (4mn+2n+1)\sum_{i=1}^{2k-1}(-t)^{i+1}$$

$$\Delta((2k+1)(2m+1)(2n+1)) = (k+1)(n+1)(1+t^{2m+2}) -$$

$$-(3kn+2k+2n+1)(t+t^{2m+1}) + (2k+1)(2n+1)\sum_{i=1}^{2m-1}(-t)^{i+1}$$

Symmetry groups: D_4 if $p = r$; otherwise D_2.

Symmetry type: chiral and reversible.

Signatures:
$q - 1$ if $p = q = r = 1 \pmod 2$;
$p - 1$ if $p = q = 1 \pmod 2$ and $r = 0 \pmod 2$;
$p + r - 1$ if $p = 1 \pmod 2$ and $q = r = 0 \pmod 2$.

Unknotting numbers:
$u((2k+1)(2m+1)(2n+1)) = m + n + 1$
$u(p(2m)r) = \frac{p+r-1}{2}$
$u((2k)(2m+1)(2n+1)) = min(k, m+1) + n$

Knot family: $p111q$ ($pq = 0 \pmod 2$; $p \geq q$; $p111q = q111p$)

Notation:

21112 7_7
31112 8_{13}
41112 9_{14}
51112 10_{10} 41113 10_{19}

Alexander polynomials:

$$\Delta((2m)111(2n)) = mn - (3mn + m + n)t +$$
$$+(2m+1)(2n+1)t^2 - (3mn + m + n)t^3 + mnt^4$$

$$\Delta((2m+1)111(2n)) = m + 1 - (4m+3)t +$$
$$+(6m+5)\sum_{i=2}^{2n}(-1)^i t^i - (4m-3)t^{2n+1} + (m+1)t^{2n+2}$$

$$\Delta((2m)111(2n+1)) = n + 1 - (4n+3)t$$
$$+(6n+5)\sum_{i=2}^{2m}(-1)^i t^i - (4n-3)t^{2m+1} + (n+1)t^{2m+2}$$

Symmetry groups: D_4 if $p = q$; otherwise D_2.

Symmetry type: reversible.

Signatures:
0 if $p = q = 0 \pmod 2$;
$q - 2$ if $p = 1 \pmod 2$ and $q = 0 \pmod 2$;
$p - 2$ if $q = 1 \pmod 2$ and $p = 0 \pmod 2$;

Unknotting numbers:
$u((2m)111(2n)) = n$
$u((2m)111(2n+1)) = m$
$u((2m+1)111(2n)) = n$

Knot family: $pq1r$ ($r = 0 \pmod 2$ or $p = r = 1 \pmod 2$; $pq1r = r1pq$)

Notation:
2212 7_6
3212 8_{11} 2312 8_8
4212 9_{12} 2412 9_8 2214 9_{11}
5212 10_7 2512 10_{34}

4312 10_{12} 3412 10_{21}

3213 9_{13}
3214 10_{16} 2314 10_{15}

3313 10_{22}

Alexander polynomials:

$$\Delta((2k)(2l)1(2m)) = kl - (3kl+k+1)t+$$

$$+(4kl+2k+1)\sum_{i=2}^{2m}(-t)^i - (3kl+k+1)t^{2m+1} + klt^{2m+2}$$

$$\Delta((2k+1)(2l)1(2m)) = (km+m) - (6km-2k-m+4)t+$$

$$+(2k+1)(2m+1)\sum_{i=2}^{2m}(-t)^i - (6km-2k-m+4)t^{2m+1} + (km+m)t^{2m+2}$$

$$\Delta((2k)(2l+1)1(2m)) = (l+1)\sum_{i=0}^{2k-1}(2i+1)(-t)^i+$$

$$+(4kl+4k+1)\sum_{i=2k}^{2m}(-t)^i + (l+1)\sum_{i=0}^{2k-1}(2i+1)(-t)^{2k+2m-i}, \quad k \le m$$

$$\Delta((2k)(2l+1)1(2m)) = (l+1)\sum_{i=0}^{2m}(2i+1)(-t)^i+$$

$$+(2ml+2m+2l+1)\sum_{i=2m+1}^{2k-1}(-t)^i + (l+1)\sum_{i=0}^{2m}(2i+1)(-t)^{2k+2m-i}, \quad k > m$$

$$\Delta((2k+1)(2l)1(2m+1)) = (l+1)(m+1) - (3lm+3l+2m+1)t+$$

$$+(4lm+4l+2m+1)\sum_{i=2}^{2k}(-t)^i - (3lm+3l+2m+1)t^{2k+1} + (l+1)(m+1)t^{2k+2}$$

$$\Delta((2k+1)(2l+1)1(2m+1)) = (l+1)\sum_{i=0}^{2k}(2i+1)(-t)^i+$$

$$+(4kl+4k+2l+3)\sum_{i=2k+1}^{2m+1}(-t)^i + (l+1)\sum_{i=0}^{2k}(2i+1)(-t)^{2k+2m+1-i}, \quad k \le m$$

$$\Delta((2k+1)(2l+1)1(2m+1)) = (l+1)\sum_{i=0}^{2m+1}(2i+1)(-t)^i+$$

$$+(4lm+4l+4m+3)\sum_{i=2m+2}^{2k}(-t)^i + (l+1)\sum_{i=0}^{2k}(2i+1)(-t)^{2k+2m+1-i}, \quad k > m$$

Symmetry group: D_2.

Symmetry type: reversible.

Signatures:
$|p-r|$ if $q=1 \pmod 2$;
$p+1$ if $p=r=1 \pmod 2$ and $q=0 \pmod 2$;
q if $q=r=0 \pmod 2$ and $p=1 \pmod 2$;
r if $p=q=r=0 \pmod 2$.

Unknotting numbers:
$u((2k)(2l)1(2m)) = min(k, l-m)+m$ if $l \geq m$
$u((2k)(2l)1(2m)) = m$ if $l < m$
$u((2k+1)(2l)1(2m)) = l$ if $l \geq m$
$u((2k+1)(2l)1(2m)) = min(k+1, m-l)+l$ if $l < m$
$u((2k)(2l+1)1(2m)) = l+1$ if $l = m$, $k = 1$
$u((2k)(2l+1)1(2m)) = k+l-1$ if $l = m$, $k \neq 1$
$u((2k)(2l+1)1(2m)) = k+m$ if $l > m$
$u((2k)(2l+1)1(2m)) = l+1$ if $k = m$, $l < m$
$u((2k)(2l+1)1(2m)) = k+l-m$ if $l < m < k$, $k+l \neq m$
$u((2k)(2l+1)1(2m)) = l+1$ if $k+l = m$
$u((2k)(2l+1)1(2m)) = |k+l-m|+l$ if $k+l \neq m$, $l < m$
$u((2k+1)(2l)1(2m+1)) = min(k+l+1, k+m+1)$
$u((2k+1)(2l+1)1(2m+1)) = l+1$ if $m = k+l$
$u((2k+1)(2l+1)1(2m+1)) = k+l$ if $l \geq m$
$u((2k+1)(2l+1)1(2m+1)) = |m-k-l|+l$ if $m \neq k+l$, $l < m$

4 Conclusions

The concept of new knot tables based on knot families can be naturally extended to links, in the spirit of [5,12,13]. For that, it is necessary to develop programs able to work with links and calculate polynomial invariants, and other data already calculated for knots.

Because the complete concept of new knot tables is based on the notion of generating knots and links and families originating from them, one of the possible future aims can be a search for new knot and link invariants that will be the invariants of families. If we will be able for a given knot to recognize a family to which it belongs, even Alexander polynomial maybe can be sufficient for the recognition of particular knots.

From the results obtained, it looks that all properties of knots or links belonging to some family are well-ordered, so it is possible to extend them to some general form. It works for Alexander polynomials, Jones polynomials, symmetry properties, unknotting numbers, and even for Dowker sequences.

Next interesting question is a possibility to try to establish connections between coefficients of polynomial invariants and other knot or link invariants and understand the topological meaning of certain coefficients.

One of main open questions is *Bernhard-Jablan Conjecture* on unknotting number. According to that *Conjecture*, a number $u(k)$ is defined in the following way:

(a) $u(1) = 0$, where 1 is the unknot;

(b) $u(k) = min(u(k') + 1)$, where the minimum is taken over all the knots k' obtained from a minimal projection of k by a crossing change.

It is conjectured that the number $u(k)$ will be the unknotting number of the knot k.

For all alternating knots with $n \leq 10$ crossings, the obtained numbers $u(k)$ coincide with the unknotting numbers of those knots [2], calculated (or even estimated by giving two or more expected values for the unknotting number [15]) by other methods. Because the unknotting number of every knot is greater or equal to the half of signature $\sigma(k)$, for a lot of knots with $n \leq 19$ crossings used in the experimental work we namely proved that the numbers $u(k)$ obtained from their minimal projections by recursive unknotting process are really the unknotting numbers of those knots, because they are equal to $\frac{\sigma(k)}{2}$. This property holds as well for the general formulas for $\sigma(k)$ and $u(k)$, extrapolated for several families of knots, where $u(k) = \frac{\sigma(k)}{2}$, so we strongly believe that at least for such families we succeeded to find the correct unknotting number, even in its general form. Certainly, the results obtained in experimenting with all knots of the families discussed with $n \leq 19$ crossings are out of the question, so for those knots with $u(k) = \frac{\sigma(k)}{2}$ we are sure that the unknotting numbers are determined correctly.

The present work was restricted to a very small part of knots: only to several families of rational knots, because for $n \leq 7$ there are no other generating knots except rational ones. Thanks to that, out of consideration remained non-alternating knots, so it is possible that some of the conjectures or estimations need to be restricted to alternating knots or even only to rational knots. For example, till now it is proved that *Bernhard-Jablan Conjecture* holds for rational knots with the unknotting number one [22], so all the general formulas presented in this work expect very serious proving (or disproving) procedure and verification.

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