Filomat 34:11 (2020), 3689–3704
https://doi.org/10.2298/FIL2011689K

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Convergence and Stability of an Iterative Algorithm for Strongly Accretive Lipschitzian Operator with Applications

Vivek Kumar[a], Nawab Hussain[b], Abdul Rahim Khan[c], Faik Gürsoy[d]

[a]*Department of Mathematics, K.L.P College, Rewari (INDIA)-123401*
[b]*Department of Mathematics, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia*
[c]*Department of Mathematics and Statistics, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia*
[d]*Department of Mathematics, Adiyaman University, Adiyaman 02040, Turkey*

Abstract. Using different technique and weaker restrictions on parameters, convergence and stability results of an SP iterative algorithm with errors for a strongly accretive Lipschitzian operator on a Banach space are established. Validity of new convergence results is verified through numerical examples and convergence comparison of various iterative algorithms is depicted. As applications of our convergence result, we solve a nonlinear operator equation and a variational inclusion problem. Our results are refinement and generalization of many classical results.

1. Introduction and Preliminaries

Let X be a real Banach space.

Definition 1.1. *An operator $T : X \to X$ is called:*

Definition 1.2. *(i) Lipschitizian, if there exists $L > 0$ such that for all $x, y \in X$, we have*

$$\left\|Tx - Ty\right\| \leq L\left\|x - y\right\|,$$

(ii) strongly pseudo-contractive [14] if for all $r > 0$, $k \in (0, 1)$ and $x, y \in X$, we have

$$\left\|x - y\right\| \leq \left\|x - y + r[(I - T - kI)\,x - (I - T - kI)\,y]\right\|, \tag{1}$$

(iii) strongly accretive iff for all $r > 0$, $x, y \in X$ and $k \in (0, 1)$, we have

$$\left\|x - y\right\| \leq \left\|x - y + r[(T - kI)\,x - (T - kI)\,y]\right\|. \tag{2}$$

2010 *Mathematics Subject Classification*. Primary 47H05; Secondary 47H06, 54H10
Keywords. Iterative algorithm, fixed point, stability, strongly accretive operator
Received: 22 November 2019; Accepted: 28 June 2020
Communicated by Vasile Berinde
The author A.R. Khan is grateful to King Fahd University of Petroleum and Minerals for supporting this research work.
Email addresses: ratheevivek15@yahoo.com (Vivek Kumar), nhusain@kau.edu.sa (Nawab Hussain), arahim@kfupm.edu.sa (Abdul Rahim Khan), faikgursoy02@hotmail.com (Faik Gürsoy)

From (1) and (2), it is obvious that an operator T is strongly pseudo-contractive iff $(I - T)$ is strongly accretive. Also, an operator T is strongly accretive iff $(T - kI)$ is accretive. Moreover, if T is accretive, then $(I + T)^{-1}$ is non-expansive (see [1]). It is well known that every strongly pseudo-contractive operator with a fixed point is strictly hemi-contractive but converse may not be true and class of strongly pseudo-contractive operators is a proper subclass of the class of ϕ- strongly pseudo-contractive operators (see [1] for details). So, strongly accretive operators are closely connected with strongly pseudo-contractive, accretive, non-expansive, strictly hemi-contractive and ϕ- strongly pseudo-contractive operators.

Definition 1.3. *([1]) Let $\{x_n\}_{n=0}^{\infty} \subset X$, be the sequence generated by an iterative algorithm of T defined by*

$$x_{n+1} = f(T, x_n), \tag{3}$$

where $x_0 \in X$ is the initial approximation and f is some function. Suppose $\{x_n\}_{n=0}^{\infty}$ converges to a fixed point p of T. Let $\{p_n\}_{n=0}^{\infty} \subset X$ be an arbitrary sequence and $k_n = \left\|p_{n+1} - f(T, p_n)\right\|$. Then, the iterative algorithm (3) is said to be T-stable if and only if $\lim_{n\to\infty} k_n = 0$ implies $\lim_{n\to\infty} p_n = p$. Moreover, if $\sum_{n=0}^{\infty} k_n < \infty$ implies that $\lim_{n\to\infty} p_n = p$, then the iterative algorithm defined by $x_{n+1} = f(T, x_n)$ is said to be almost T-stable.

Stability implies almost stability but converse may not be true (cf. [24]).

Fixed point results and iterative method for Lipschitizian operators have been considered by many authors [2, 15, 18].

The following iterative algorithms have been studied for approximating fixed points of nonlinear operators in Banach spaces:

Mann iterative algorithm with errors of Liu [20]:

$$x_{n+1} = (1 - \alpha_n)\, x_n + \alpha_n\, Tx_n + u_n, \tag{4}$$

where $0 \leq \alpha_n \leq 1$ and $\{u_n\}$ is a summable sequence in X.

Ishikawa iterative algorithm with errors of Liu [20]:

$$\begin{aligned} x_{n+1} &= (1 - \alpha_n)\, x_n + \alpha_n\, T\, y_n + u_n \\ y_n &= (1 - \beta_n)\, x_n + \beta_n\, Tx_n + v_n, \end{aligned} \tag{5}$$

where $0 \leq \alpha_n, \beta_n \leq 1$ and $\{u_n\}$, $\{v_n\}$ are summable sequences in X.

Noor iterative algorithm with errors [23]:

$$\begin{aligned} x_{n+1} &= (1 - \alpha_n)x_n + \alpha_n T y_n + \alpha_n\, u_n \\ y_n &= (1 - \beta_n)x_n + \beta_n T z_n + \beta_n\, v_n \\ z_n &= (1 - \gamma_n)x_n + \gamma_n\, Tx_n + w_n, \end{aligned} \tag{6}$$

where u_n, v_n, w_n are appropriate sequences in X and $0 \leq \alpha_n,\ \beta_n,\ \gamma_n \leq 1$.

SP iterative algorithm of Phuengrattana and Suantai [25]:

$$\begin{aligned} x_{n+1} &= (1 - \alpha_n)\, y_n + \alpha_n T y_n \\ y_n &= (1 - \beta_n)\, z_n + \beta_n T z_n \\ z_n &= (1 - \gamma_n)\, x_n + \gamma_n T x_n, \end{aligned} \tag{7}$$

where $0 \leq \alpha_n, \beta_n, \gamma_n \leq 1$.

Remark 1.4. *Putting $u_n = v_n = 0$ in (4) and (5), respectively, we get Mann [22] and Ishikawa [13] iterative algorithms, respectively. Also if we put $\beta_n = \gamma_n = 0$, then SP iterative algorithm (7) becomes Mann iterative algorithm [22].*

The convergence and stability problems for iterative algorithms involving various types of operators have been studied by many authors [5, 6, 9, 16, 17, 21, 23, 26–36]. Nonlinear maps act as models for many systems in different scientific disciplines (see [14] for details). Gu and Lu [11] studied Ishikawa iterative algorithm with mixed errors for solutions of variational inclusions for accretive type mappings in Banach spaces. Kim et al. [19] proved convergence of Ishikawa iterative algorithm with mixed errors using strong accretive Lipschitzian operators in Banach spaces. Chugh and Kumar [7] studied strong convergence and almost stability of SP iterative algorithm with mixed errors for the accretive Lipschitzian and strongly accretive Lipschitzian operators in Banach spaces. Recently, Chugh et al. [8] and Hussain et al. [12] proved some strong convergence results of iterative algorithms. In computational mathematics, a fixed point iterative algorithm is valuable and useful for applications if it satisfies the following conditions:

(i) it converges to a fixed point of an operator (ii) it is stable (iii) it is faster as compared to other iterative algorithms existing in the literature.

In this paper, we improve results of Chidume [5], Chugh and Kumar [7], Gu and Lu [11], Kim et al. [19], Liu [20], Xu and Xie [27] and Xu [28] by using new and different convergence techniques for an SP iterative algorithm with mixed errors. We support our results with numerical examples and applications. Moreover, with the help of $C++$ programs, we show that SP algorithm with mixed errors converges faster and is stable instead of almost stable.

The following lemmas and definitions are needed to prove our main results:

Lemma 1.5. *([1]) Let $\{a_n\}$ be a sequence of nonnegative real numbers satisfying:*

$$a_{n+1} \le \delta a_n + b_n, \ n \ge 1,$$

where $b_n \ge 0$, $\lim_{n\to\infty} b_n = 0$ and $0 \le \delta < 1$. Then $a_n \to 0$ as $n \to \infty$.

Lemma 1.6. *([18]) Let $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ be nonnegative real sequences satisfying the condition:*

$$a_{n+1} \le (1-\lambda_n)a_n + b_n + c_n, \ (n \ge n_0),$$

where n_0 is some nonnegative integer and $\{\lambda_n\}$ is a sequence in $[0,1]$ such that $\sum_{n=0}^{\infty} \lambda_n = \infty$, $b_n = o(\lambda_n)$ and $\sum_{n=0}^{\infty} c_n < \infty$. Then $a_n \to 0$ as $n \to \infty$.

It is well known that a continuous strongly pseudo-contractive selfmap has a unique fixed point. Moreover, the equation $Tx = f$ has a unique solution for every continuous strongly accretive mapping $T : D(T) \subset X \to X$ for given $f \in X$ (for details see [10]).

For the solution by iteration of certain nonlinear functional equations in Banach spaces, the reader is referred to Browder and Petryshyn [3].

Variational inclusions, generalization of variational inequalities, have been widely studied [4, 11]. One of the most interesting and important problems in the theory of variational inclusions is the development of an efficient and implementable iterative algorithm [4, 11, 12].

Let $T, A : X \to X$, $g : X \to X^*$ be mappings on a real reflexive Banach space X, X^* is dual of X and $\langle \cdot, \cdot \rangle$ denotes pairing of X and X^*. Let $\varphi : X^* \to (-\infty, \infty)$ be a proper convex lower semicontinuous function with subdifferential $\partial\varphi : X^* \to 2^{X^*}$ defined by

$$(\partial\varphi)x = \{x^* \in X^* : \varphi(y) - \varphi(x) \geq \langle y - x, x^* \rangle, \ \forall y \in X\}.$$

If for any given $y \in X$, there exists an $x \in X$ such that $g(x) \in D(\partial\varphi)$ and

$$\langle Tx - Ax - y, f - g(x) \rangle \geq \varphi(g(x)) - \varphi(f), \forall f \in X^*, \tag{8}$$

holds then, we say that x is a solution of variational inclusion problem (8).

Lemma 1.7. *([4]) Let $\partial\varphi o g : X \to 2^X$ be a mapping on a real reflexive Banach space X. Then the following statements are equivalent:*

(i) $p \in X$ is a solution of variational inclusion problem (8);

(ii) $p \in X$ is a fixed point of the mapping $S : X \to 2^X$ defined as

$Sx = y - (Tx - Ax + \partial\varphi(g(x))) + x$; (that is, $p \in Sp$)

(iii) $p \in X$ is a solution of the equation $y = Tx - Ax + \partial\varphi(g(x))$.

2. Main Results

Theorem 2.1. *Let X be a real Banach space and $T : X \to X$ be a strongly pseudo-contractive Lipschitzian mapping with Lipschitz constant $L \geq 1$. For any given $x_0 \in X$, let $\{x_n\}$ be the SP iterative algorithm with mixed errors defined by*

$$\begin{aligned} x_{n+1} &= (1 - \alpha_n)\, y_n + \alpha_n\, T y_n + u_n \\ y_n &= (1 - \beta_n)\, z_n + \beta_n\, T z_n + v_n \\ z_n &= (1 - \gamma_n)\, x_n + \gamma_n\, T x_n + w_n, \end{aligned} \tag{9}$$

where $0 \leq \alpha_n, \beta_n, \gamma_n \leq 1$ and $\{u_n\}$, $\{v_n\}$, $\{w_n\}$ are sequences in X satisfying the following conditions:

(i) $0 < \alpha \leq \alpha_n \leq \frac{k}{(2-k)L(L+1)}$,

$\beta_n (L-1) + \gamma_n (L-1)^2 + \beta_n \gamma_n (L-1)^2 < \alpha_n\{k - (2-k)\alpha_n L(1+L)\}(1-t), \forall n \in \mathbb{N}$ *and* $t \in (0,1)$;

(ii) $u_n = u'_n + u''_n$, $\|u'_n\| = o(\alpha_n)$ $(n \geq 0)$ *and* $\sum_{n=0}^{\infty} \|u''_n\| < \infty$;

(iii) $\sum_{n=0}^{\infty} \|v_n\| < \infty$, $\sum_{n=0}^{\infty} \|w_n\| < \infty$.

Then

1. $\{x_n\}$ *in (9) converges strongly to a unique fixed point p of T.*
2. *for any sequence* $\{p_n\} \subset X$ $\lim_{n\to\infty} p_n = p$ *iff* $\lim_{n\to\infty} k_n = 0$, *where* $k_n = \|p_{n+1} - (1-\alpha_n)q_n - \alpha_n T q_n - u_n\|$, $q_n = (1-\beta_n)\, r_n + \beta_n T r_n + v_n$, $r_n = (1-\beta_n)p_n + \beta_n T p_n + w_n$, *that is,* $\{x_n\}$ *in (9) is* $T-$ *stable.*

Proof. By (9), we have

$$\begin{aligned} & (x_{n+1}-p)+\alpha_n[(1-T-kI)\,x_{n+1}-(I-T-kI)p] \\ = \; & (1-\alpha_n)(y_n-p)+\alpha_n[(I-T-kI)x_{n+1}+Ty_n]-\alpha_n(I-kI)p+u_n. \end{aligned} \tag{10}$$

As T is strongly pseudo-contractive and a Lipschitzian mapping, so by (10), we get

$$\begin{aligned} \left\|x_{n+1}-p\right\| & \leq \left\|x_{n+1}-p+\alpha_n[(I-T-kI)x_{n+1}-(I-T-kI)p]\right\| \\ & \leq (1-\alpha_n)\left\|y_n-p\right\|+\alpha_n\left\|Ty_n-Tx_{n+1}\right\|+\alpha_n I(1-k)\left\|x_{n+1}-p\right\|+\|u_n\| \end{aligned}$$

which implies

$$[1-\alpha_n(1-k)]\left\|x_{n+1}-p\right\| \leq (1-\alpha_n)\left\|y_n-p\right\|+\alpha_n\left\|Ty_n-Tx_{n+1}\right\|+\|u_n\|$$

or

$$\left\|x_{n+1}-p\right\| \leq \frac{(1-\alpha_n)}{[1-\alpha_n(1-k)]}\left\|y_n-p\right\|+\frac{\alpha_n}{[1-\alpha_n(1-k)]}\left\|Ty_n-Tx_{n+1}\right\|+\frac{1}{[1-\alpha_n(1-k)]}\,\|u_n\|. \tag{11}$$

Now, $1-\frac{1-\alpha_n}{1-\alpha_n(1-k)}=\frac{1-(1-\alpha_n k)}{1-\alpha_n(1-k)} \geq 1-(1-\alpha_n k)$
implies

$$\frac{1-\alpha_n}{1-\alpha_n(1-k)} \leq 1-\alpha_n k \tag{12}$$

and

$$1-\frac{\alpha_n}{1-\alpha_n(1-k)}=\frac{1-\alpha_n(2-k)}{1-\alpha_n(1-k)} \geq 1-\alpha_n\,(2-k)$$

implies

$$\frac{\alpha_n}{1-\alpha_n(1-k)} \leq \alpha_n\,(2-k). \tag{13}$$

Also

$$\frac{1}{1-\alpha_n(1-k)} \leq \frac{1}{1-(1-k)}=\frac{1}{k}. \tag{14}$$

Using (12)-(14), (11) yields

$$\left\|x_{n+1}-p\right\| \leq (1-\alpha_n k)\left\|y_n-p\right\|+\alpha_n(2-k)\left\|Ty_n-Tx_{n+1}\right\|+\frac{\|u_n\|}{k}. \tag{15}$$

Now, using Lipschitz condition on T, (9) implies

$$\begin{aligned} \left\|Tx_{n+1}-Ty_n\right\| & \leq L\left\|x_{n+1}-y_n\right\| \\ & \leq L\alpha_n\left\|y_n-Ty_n\right\|+L\|u_n\| \\ & \leq L\alpha_n\left\|y_n-p\right\|+L\alpha_n\left\|Ty_n-p\right\|+L\|u_n\| \\ & \leq L\alpha_n(1+L)\left\|y_n-p\right\|+L\|u_n\|. \end{aligned} \tag{16}$$

Also, by (9), we have the following estimate:

$$\begin{aligned}
\left\|y_n - p\right\| &\leq (1-\beta_n)\left\|z_n - p\right\| + \beta_n\left\|Tz_n - p\right\| + \|v_n\| \\
&\leq (1-\beta_n)\left\|z_n - p\right\| + \beta_n L\left\|z_n - p\right\| + \|v_n\| \\
&= [1+\beta_n(L-1)]\left\|z_n - p\right\| + \|v_n\| \\
&= [1+\beta_n(L-1)]\left\|(1-\gamma_n)x_n + \gamma_n Tx_n + w_n - p\right\| + \|v_n\| \\
&= [1+\beta_n(L-1)][(1-\gamma_n)\left\|x_n - p\right\| + \gamma_n\left\|Tx_n - p\right\|] \\
&\quad +[1+\beta_n(L-1)]\|w_n\| + \|v_n\| \\
&\leq [1+\beta_n(L-1)][(1-\gamma_n)\left\|x_n - p\right\| + L\gamma_n\left\|x_n - p\right\|] \\
&\quad +\|v_n\| + [1+\beta_n(L-1)]\|w_n\| \\
&= [1+\beta_n(L-1)](1-\gamma_n + L\gamma_n)\left\|x_n - p\right\| + \|v_n\| + [1+\beta_n(L-1)]\|w_n\|.
\end{aligned} \tag{17}$$

Using estimate (17), (16) becomes

$$\left\|Ty_n - Tx_{n+1}\right\| \leq L\alpha_n(1+L)[1+\beta_n(L-1)][(1-\gamma_n+L\gamma_n)\left\|x_n - p\right\|] \tag{18}$$

$$+L\alpha_n(1+L)\|v_n\| + L\|u_n\| + L\alpha_n(1+L)[1+\beta_n(L-1)]\|w_n\|.$$

Estimates (15), (17) and (18) yield

$$\begin{aligned}
\left\|x_{n+1} - p\right\| &\leq (1-\alpha_n k)[1+\beta_n(L-1)]\left[(1-\gamma_n+L\gamma_n)\left\|x_n - p\right\|\right] \\
&\quad +\alpha_n(2-k)L\alpha_n(1+L)[1+\beta_n(L-1)]\left[(1-\gamma_n+L\gamma_n)\left\|x_n - p\right\|\right] \\
&\quad +\left[1-\alpha_n k + L\alpha_n^2(2-k)(1+L)\right]\|v_n\| + L\alpha_n\left(2-k+\frac{1}{k}\right)\|u_n\| \\
&\quad +[1-\alpha_n k + L\alpha_n^2(2-k)(1+L)]\|v_n\| + L\alpha_n\left(2-k+\frac{1}{k}\right)\|u_n\| \\
&\quad +[1-\alpha_n k + L(1+L)\alpha_n^2(2-k)][1+\beta_n(L-1)]\|w_n\| \\
&= \left\{\begin{array}{c} (1-\alpha_n k)[1+\beta_n(L-1)](1-\gamma_n+L\gamma_n) \\ +(2-k)L\alpha_n^2(1+L)[1+\beta_n(L-1)](1-\gamma_n+L\gamma_n) \end{array}\right\}\left\|x_n - p\right\| \\
&\quad +[1-\alpha_n k + L\alpha_n^2(2-k)(1+L)]\|v_n\| + L\alpha_n\left(2-k+\frac{1}{k}\right)\|u_n\| \\
&\quad +[1-\alpha_n k + L(1+L)\alpha_n^2(2-k)][1+\beta_n(L-1)]\|w_n\| \\
&= [1+\beta_n(L-1)](1-\gamma_n+L\gamma_n)[1-\alpha_n\{k-(2-k)\alpha_n L(1+L)\}]\left\|x_n - p\right\| \\
&\quad +[1-\alpha_n k + L\alpha_n^2(2-k)(1+L)]\|v_n\| + L\alpha_n\left(2-k+\frac{1}{k}\right)\|u_n\| \\
&\quad +[1-\alpha_n k + L(1+L)\alpha_n^2(2-k)][1+\beta_n(L-1)]\|w_n\| \\
&\leq 1-[\alpha_n\{k-(2-k)\alpha_n L(1+L)\} - \gamma_n(L-1) - \beta_n(L-1) - \gamma_n\beta_n(L-1)^2]\left\|x_n - p\right\| \\
&\quad +[1+2L(1+L)]\|v_n\| + L\left(2+\frac{1}{k}\right)\|u_n\| + L[1+2L(1+L)]\|w_n\|.
\end{aligned} \tag{19}$$

By condition (i), the inequality (19) becomes

$$\begin{aligned}\left\|x_{n+1}-p\right\| &\leq 1-\alpha_n\{k-(2-k)\alpha_n L(1+L)\}+\alpha_n\{k-(2-k)\alpha_n L(1+L)\}(1-t)\left\|x_n-p\right\| \\ &\quad +[1+2L(1+L)]\,\|v_n\|+L\left(2+\frac{1}{k}\right)\|u_n\|+L[1+2L(1+L)]\,\|w_n\| \\ &= [1-\alpha_n\{k-(2-k)\alpha_n L(1+L)\}t]\,\|\,x_n-p\,\| \\ &\quad +L\left(2+\frac{1}{k}\right)\|u_n\|+[1+2L(1+L)]\,\|v_n\|+L[1+2L(1+L)]\,\|w_n\| \\ &\leq [1-\alpha\{k-(2-k)\alpha L(1+L)\}t]\left\|x_n-p\right\| \\ &\quad +L\left(2+\frac{1}{k}\right)\|u_n\|+[1+2L(1+L)]\,\|v_n\|+L[1+2L(1+L)]\,\|w_n\|. \end{aligned} \tag{20}$$

Also, by condition (ii), we have $u_n' = \delta_n\alpha_n$, where $\{\delta_n\}$ is a sequence of nonnegative numbers tending to 0. Hence

$$\|u_n\| \leq \delta_n\alpha_n + \left\|u_n''\right\|. \tag{21}$$

Using (21), (20) becomes

$$\begin{aligned}\left\|x_{n+1}-p\right\| &\leq [1-\alpha\{k-(2-k)\alpha L(1+L)\}t]\left\|x_n-p\right\| \\ &\quad +L\left(2+\frac{1}{k}\right)(\delta_n\alpha_n+\left\|u_n''\right\|)+[1+2L(1+L)](1+L)(\|v_n\|+\|w_n\|).\end{aligned} \tag{22}$$

Now, if we put $[1-\alpha\{k-(2-k)\alpha L(1+L)\}\,t]=\delta$ and $L\left(2+\frac{1}{k}\right)(\delta_n\,\alpha_n+\left\|u_n''\right\|)+[1+2L(1+L)](1+L)(\|v_n\|+\|w_n\|)=\sigma_n$, then (22) reduces to

$$\left\|x_{n+1}-p\right\| \leq \delta\left\|x_n-p\right\|+\sigma_n. \tag{23}$$

By conditions (ii)-(iii) and Lemma 1.5, the inequality (23) yields strong convergence of $\{x_n\}$ in (9) to a fixed point p of T.
To prove uniqueness of fixed point p, let q be an another fixed point of T. With $r=1$, in the definition of strongly pseudo-contractive mapping, we have

$$\left\|p-q\right\| \leq \left\|p-q+[(I-T-kI)p-(I-T-kI)q]\right\| = \left\|p-q+k(q-p)\right\| = (1-k)\left\|p-q\right\|,$$

which is possible only if $p=q$.
(2) Suppose that $\{p_n\}\subset X$, is an arbitrary sequence and $\lim_{n\to\infty} k_n = 0$.
Then

$$\begin{aligned}\left\|p_{n+1}-Tp\right\| &= \left\|p_{n+1}-(1-\alpha_n)q_n-\alpha_n Tq_n-u_n\right\|+\left\|(1-\alpha_n)q_n+\alpha_n Tq_n+u_n-Tp\right\| \\ &= k_n+\left\|s_n-Tp\right\|,\end{aligned} \tag{24}$$

where

$$s_n=(1-\alpha_n)q_n+\alpha_n\,Tq_n+u_n. \tag{25}$$

By (25), we have

$$\begin{aligned}&(s_n-p)+\alpha_n[(I-T-kI)\,s_n-(I-T-kI)\,p] \\ =\;&(1-\alpha_n)(q_n-p)+\alpha_n[(I-T-kI)s_n+Tq_n]-\alpha_n(I-kI)p+u_n\end{aligned}$$

which further implies

$$\begin{aligned}\left\|s_n - p\right\| &\leq \left\|s_n - p + \alpha_n[(I - T - kI)s_n - (I - T - kI)p]\right\| \\ &\leq (1-\alpha_n)\left\|q_n - p\right\| + \alpha_n \left\|Tq_n - Ts_n\right\| + \alpha_n(1-k)\left\|s_n - p\right\| + \|u_n\|.\end{aligned} \tag{26}$$

Rearranging terms in (26), as in (15), we get

$$\left\|s_n - p\right\| \leq (1-\alpha_n k)\left\|q_n - p\right\| + \alpha_n(2-k)\left\|Tq_n - Ts_n\right\| + \frac{\|u_n\|}{k}. \tag{27}$$

Following the procedure to get (22), we have the following estimate

$$\begin{aligned}\left\|s_n - p\right\| &\leq [1 - \alpha\{k - (2-k)\alpha L(1+L)\}t]\left\|p_n - p\right\| \\ &+L\left(2 + \frac{1}{k}\right)(\delta_n\alpha_n + \left\|u''_n\right\|) + [1 + 2L(1+L)](\|v_n\| + L\|w_n\|).\end{aligned} \tag{28}$$

The inequality (28) together with the inequality (24) yields

$$\begin{aligned}\left\|p_{n+1} - Tp\right\| &\leq [1 - \alpha\{k - (2-k)\alpha L(1+L)\}t]\left\|p_n - p\right\| \\ &+L\left(2 + \frac{1}{k}\right)(\delta_n\alpha_n + \left\|u''_n\right\|) + [1 + 2L(1+L)](\|v_n\| + L\|w_n\|) + k_n.\end{aligned} \tag{29}$$

Set

$$\begin{aligned}\delta &= 1 - \alpha\{k - (2-k)\alpha L(1+L)\}t, \\ \sigma_n &= L\left(2 + \frac{1}{k}\right)(\delta_n\,\alpha_n + \left\|u''_n\right\|) + [1 + 2L(1+L)](\|v_n\| + L\|w_n\|) + k_n.\end{aligned}$$

Using condition (iii) and Lemma 1.5, the inequality (29) yields $\lim_{n\to\infty}\left\|p_{n+1} - p\right\| = 0$, that is, $\lim_{n\to\infty} p_{n+1} = p$.

For the converse, let $\lim_{n\to\infty} p_n = p$. Then by (28), we have

$$\begin{aligned}k_n &= \left\|p_{n+1} - (1-\alpha_n)\,q_n - \alpha_n\,Tq_n - u_n\right\| \\ &= \left\|p_{n+1} - s_n\right\| \\ &\leq \left\|p_{n+1} - p\right\| + \left\|s_n - p\right\| \\ &\leq \left\|p_{n+1} - p\right\| + [1 - \alpha\{k - (2-k)\alpha L(1+L)\}t]\left\|p_n - p\right\| \\ &\quad +L\left(2 + \frac{1}{k}\right)(\delta_n\,\alpha_n + \left\|u''_n\right\|) + [1 + 2L(1+L)]\,(\|v_n\| + L\|w_n\|),\end{aligned}$$

which implies $\lim_{n\to\infty} k_n = 0$. Hence $\{x_n\}$ in (9) is T-stable.
□

Corollary 2.2. *Let X be a real Banach space and $T : X \to X$ be a strongly pseudo-contractive Lipschitzian mapping with Lipschitz constant $L \geq 1$. For any given $x_0 \in X$, let $\{x_n\}$ be the Mann iterative algorithm with mixed errors defined by (4) with the following conditions:*

(i) $0 < \alpha \leq \alpha_n \leq \frac{k}{(2-k)L(L+1)}$ $(n \geq 0)$;
(ii) $u_n = u'_n + u''_n$, $\left\|u'_n\right\| = o(\alpha_n)$ $(n \geq 0)$ *and* $\sum_{n=0}^{\infty}\left\|u''_n\right\| < \infty$.

Then

1. $\{x_n\}$ *in (4) converges strongly to the unique fixed point* p *of* T.
2. *For any sequence* $\{p_n\} \subset X, \lim_{n\to\infty} p_n = p$ *implies* $\lim_{n\to\infty} k_n = 0$, *where* $k_n = \left\|p_{n+1} - (1-\alpha_n)p_n - \alpha_n T p_n - u_n\right\|$, *that is,* $\{x_n\}$ *in (4) is* $T-$ *stable.*

Proof. Take $\beta_n = 0, \gamma_n = 0, v_n = w_n = 0$ in Theorem *2.1*. □

By taking $u_n = c_n(r_n - x_n)$, in Corollary *2.2*, we can obtain the results similar to ([28, Theorem 1] and [27, Corollary 3.4]), by using the iterative algorithm:

$$x_{n+1} = (1 - \alpha_n - c_n)\, x_n + \alpha_n\, T x_n + c_n r_n, \tag{30}$$

where $\{r_n\}$ is a bounded sequence in X and $\{\alpha_n\}, \{c_n\}$ are sequences in $[0,1]$ satisfying the following conditions:

(i) $0 < \alpha \le \alpha_n \le \frac{k}{2L(L+1)}$ $(n \ge 0)$;

(ii) $\lim_{n\to\infty} c_n = 0$.

Noor iterative algorithm with mixed errors in view of SP iterative algorithm with mixed errors in *(9)* is given by:

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)\, x_n + \alpha_n\, T y_n + u_n \\ y_n &= (1-\beta_n)\, x_n + \beta_n\, T z_n + v_n \\ z_n &= (1-\gamma_n)\, x_n + \gamma_n T x_n + w_n. \end{aligned} \tag{31}$$

The following examples validate our results.

Example 2.3. *Let* $X = \left[\frac{1}{2}, 3\right]$. *Define an operator T from X to X as* $Tx = \frac{1}{x}$ *with fixed point* $p = 1$. *It is easy to check that T is a Lipschitz strongly pseudo-contractive operator for any* $k \in (0,1)$ *with Lipschitz constant* $L = 4$. *Put* $\alpha = 0.002, \alpha_n = 0.008$, $k = 0.9$, $t = 0.5$,

$$\beta_n = \frac{1}{(1+L)^6}, \gamma_n = \frac{1}{(1+L)^7}, \|u_n\| = \frac{0.008}{(n+1)^2} + \frac{1}{(n+1)^2}, \|v_n\| = \frac{1}{(n+2)^2} \text{ and } \|w_n\| = \frac{1}{(n+3)^2}.$$

All the conditions of Theorem 2.1 are satisfied. So, the sequence $\{x_n\}$ *in (9) converges strongly to the fixed point* 1 *and is T-stable. Taking initial value* $x_0 = 3$, *convergence comparison of different iterative algorithms to the fixed point* 1 *is shown in Table 1 (Figure 1).*

Example 2.4. *Let* $X = [0,2]$ *and define an operator T from X to X as* $Tx = |1 - 2\sin x|$ *with fixed point* $p = 0.33759$. *It is easy to check that the operator T is a Lipschitz strongly pseudo-contractive operator with Lipschitz constant* $L = 2$. *With* $\alpha = 0.001$, $\alpha_n = 0.009$, $k = 0.8$, $t = 0.6$, $\beta_n = \frac{1}{(1+L)^6}, \gamma_n = \frac{1}{(1+L)^7}, \|u_n\| = \frac{0.009}{(n+1)^2} + \frac{1}{(n+1)^2}, \|v_n\| = \frac{1}{(n+2)^2}, \|w_n\| = \frac{1}{(n+3)^2}$, *all the conditions of Theorem 2.1 are satisfied. So* $\{x_n\}$ *in (9) converges strongly to the fixed point* 0.33759 *and is T-stable. Taking initial value* $x_0 = 2$, *convergence of different iterative algorithms to fixed point* 0.33759 *is shown in Table 2 (Figure 2).*

Table 1 (Figure 1) and Table 2 (Figure 2) show that SP iterative algorithm converges faster to the fixed point of given operator as compared to other iterative algorithms. Near initial approximation, only Mann algorithm shows resemblance with SP algorithm but as we move away from it, SP algorithm shows fast convergence as compared to Mann iterative algorithm. Moreover, from the beginning to the end, SP iterative algorithm converges much faster than Ishikawa and Noor iterative algorithms. Hence, we conclude that due to faster convergence rate, SP iterative algorithm has better potential for further applications.

Table 1: Convergence behavior of various iterative algorithms for Example 2.3.

No of iterations (n)	Algorithm (4)	Algorithm (5)	Algorithm (31)	Algorithm (9)
1	2.97867	2.99983	2.99997	2.97846
2	2.95752	2.99966	2.99993	2.95712
3	2.93657	2.99949	2.9999	2.93597
4	2.91580	2.99932	2.99986	2.91501
5	2.89522	2.99915	2.99983	2.89424
-	-	-	-	-
833	1.00001	2.8621	2.97173	1.00001
834	1.00001	2.86194	2.9717	1.00001
835	1.00001	2.86178	2.97167	1.00001
836	1.00001	2.86162	2.97163	**1**
837	1.00001	2.86145	2.9716	**1**
838	1.00001	2.86129	2.97157	**1**
839	1.00001	2.86113	2.97153	**1**
840	1.00001	2.86097	2.9715	**1**
841	1.00001	2.86081	2.97146	**1**
842	1.00001	2.86065	2.97143	**1**
843	1.00001	2.86049	2.9714	**1**
844	**1**	2.86033	2.97136	**1**
845	**1**	2.86017	2.97133	**1**
-	-	-	-	-
65455	**1**	1.00097	1.58059	**1**
65456	**1**	1.00096	1.58058	**1**
65457	**1**	1.00096	1.58057	**1**
65458	**1**	1.00096	1.58055	**1**
65459	**1**	1.00096	1.58054	**1**

Table 2: Convergence behavior of various iterative algorithms for Example 2.4.

No of iterations (n)	Algorithm (4)	Algorithm (5)	Algorithm (31)	Algorithm (9)
1	1.97463	1.99611	1.99871	1.96949
2	1.94931	1.99221	1.99742	1.93904
3	1.92404	1.98832	1.99613	1.90868
4	1.89883	1.98443	1.99484	1.87842
5	1.8737	1.98054	1.999355	1.84829
6	1.84866	1.97665	1.99226	1.8183
7	1.82371	1.97276	1.99097	1.78846
8	1.79887	1.96888	1.98968	1.75881
9	1.77415	1.96499	1.98839	1.72934
10	1.74955	1.96111	1.9871	1.70009
-	-	-	-	-
400	0.337651	0.81795	1.50643	0.337592
401	0.337649	0.81622	1.50529	0.337592
402	0.337648	0.814495	1.50415	0.337591
403	0.337646	0.812775	1.503	0.337591
404	0.337644	0.811061	1.50186	0.337591
405	0.337643	0.809353	1.50072	0.337591
406	0.337641	0.80765	1.49958	**0.33759**
407	0.33764	0.805953	1.49844	**0.33759**
-	-	-	-	-
480	0.337592	0696122	1.41686	**0.33759**
481	0.337592	0.694798	1.41577	**0.33759**
482	0.337592	0.693479	1.41468	**0.33759**
483	0.337591	0.692164	1.41358	**0.33759**
484	0.337591	0.690854	1.41249	**0.33759**
485	0.337591	0.689549	1.4114	**0.33759**
486	0.337591	0.688548	1.41031	**0.33759**
487	0.337591	0.686951	1.40922	**0.33759**
488	**0.33759**	0.685659	1.40813	**0.33759**
489	**0.33759**	0.684371	1.40704	**0.33759**
490	**0.33759**	0.642801	1.40690	**0.33759**
-	-	-	-	-
3275	**0.33759**	0.337591	0.370747	**0.33759**
3276	**0.33759**	0.337591	0.370703	**0.33759**
3277	**0.33759**	0.337591	0.37066	**0.33759**
3278	**0.33759**	0.337591	0.370616	**0.33759**
3279	**0.33759**	0.337591	0.370573	**0.33759**
3280	**0.33759**	**0.33759**	0.37053	**0.33759**
3281	**0.33759**	**0.33759**	0.370487	**0.33759**
3282	**0.33759**	**0.33759**	0.370443	**0.33759**
-	-	-	-	-
9716	**0.33759**	**0.33759**	0.337591	**0.33759**
9717	**0.33759**	**0.33759**	0.337591	**0.33759**
9718	**0.33759**	**0.33759**	0.337591	**0.33759**
9719	**0.33759**	**0.33759**	0.337591	**0.33759**
9720	**0.33759**	**0.33759**	0.337591	**0.33759**
9721	**0.33759**	**0.33759**	0.337591	**0.33759**
9722	**0.33759**	**0.33759**	0.337591	**0.33759**
9723	**0.33759**	**0.33759**	0.337591	**0.33759**
9724	**0.33759**	**0.33759**	**0.33759**	**0.33759**
9725	**0.33759**	**0.33759**	**0.33759**	**0.33759**

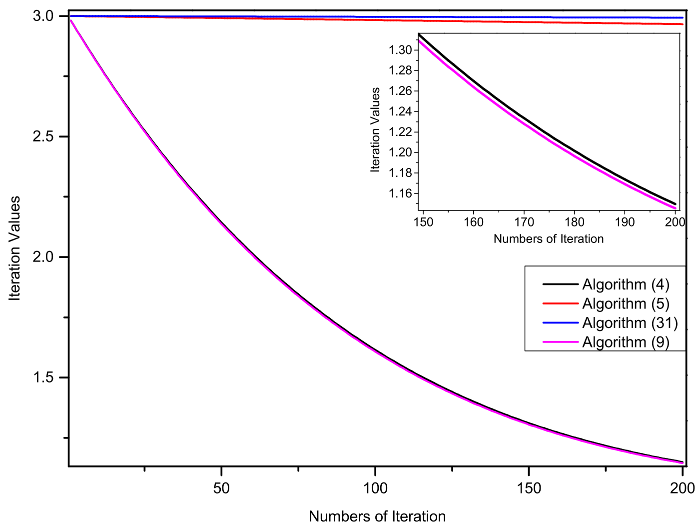

Figure 1: Convergence behavior of different iterative algorithms for Example 2.3 with initial guess $x_0 = 3$.

3. Applications

In this section, we use our convergence result to solve a nonlinear operator equation and a variational inclusion problem.

Theorem 3.1. *Suppose that X is a real Banach space and $A : X \to X$ is a Lipschitz strongly accretive mapping. Let x^* be a solution of equation $Ax = f$, where $f \in X$ is any given point.*
Define $R : X \to X$ by $Rx = f + x - Ax, \forall x \in X$. For arbitrary $x_0 \in X$, let $\{x_n\}$ be the SP iterative algorithm with mixed errors defined by

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)\, y_n + \alpha_n\, Ry_n + u_n \\ y_n &= (1-\beta_n)\, z_n + \beta_n\, Rz_n + v_n \\ z_n &= (1-\gamma_n)\, x_n + \gamma_n Rx_n + w_n,\ n \geq 0, \end{aligned} \tag{32}$$

where $0 \leq \alpha_n, \beta_n, \gamma_n \leq 1$ and $\{u_n\}, \{v_n\}, \{w_n\}$ are sequences in X satisfying the following conditions:

(i) $0 < \alpha \leq \alpha_n \leq \frac{k}{(2-k)L(L+1)}$,
$\beta_n(L-1) + \gamma_n(L-1)^2 + \beta_n\,\gamma_n(L-1)^2 < \alpha_n\,\{k - (2-k)\alpha_n L(1+L)\}(1-t), \forall n \in \mathbb{N}$ *and* $t \in (0,1)$;

(ii) $u_n = u'_n + u''_n$, $\left\|u'_n\right\| = O(\alpha_n)\ (n \geq 0)$ *and* $\sum_{n=0}^{\infty} \left\|u''_n\right\| < \infty$;

(iii) $\sum_{n=0}^{\infty} \|v_n\| < \infty, \sum_{n=0}^{\infty} \|w_n\| < \infty$.

Then $\{x_n\}$ in (32) converges strongly to the unique solution x^* of $Ax = f$ and is $R-$ stable to approximate this solution of $Ax = f$.

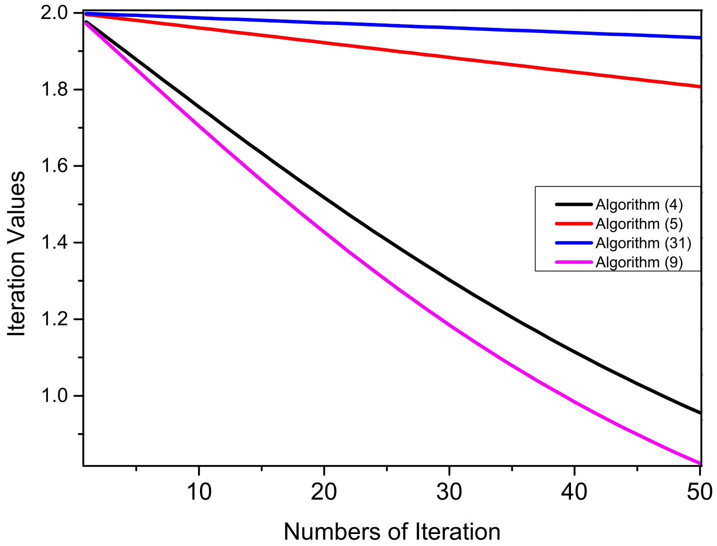

Figure 2: Convergence behavior of different iterative algorithms for Example 2.4 with initial guess $x_0 = 2$.

Proof. As A is Lipschitz strongly accretive mapping, so $Rx = f + x - Ax$ is Lipschitz strongly pseudo-contractive mapping with Lipschitz constant $L^* = 1 + L > 1$. If we replace T by R in (*9*), then convergence of iterative algorithm (*32*) to the fixed point of mapping R is obvious from Theorem *2.1* and it is easy to see that x^* is a unique fixed point of R iff x^* is a solution of equation $Ax = f$. Stability of iterative algorithm (*32*) follows on the same lines as stability of iterative algorithm (*9*) in the proof of Theorem *2.1*.
□

Putting $v_n = w_n = 0$, $\beta_n = \gamma_n = 0$, in Theorem *3.1*, we can obtain result for Mann iterative algorithm with mixed errors [27, Corollary 3.2].

Also, putting $u_n = v_n = w_n = 0$, in Theorem *3.1*, we can obtain corresponding result for SP iterative algorithm (*7*).

Remark 3.2. *Theorem 3.1 generalizes: (i) and improves [7, Theorem 2.4] as "almost stability" of SP iterative algorithm with mixed errors is replaced by "stability",*
(ii) corresponding results in [28] as our sequence $\{\alpha_n\}$ need not converge to zero and boundness condition on domain or range of the mapping R is waived,
(iii) the results in [5] as L_p space is replaced by more general Banach space,
(iv) and improves some results in [19, 20, 24, 27].

Theorem 3.3. *Suppose that X is a real reflexive Banach space, $T, A : X \to X$, $g : X \to X^*$ are mappings and $\varphi : X^* \to (-\infty, \infty)$ is a proper convex lower semicontinuous function with subdifferential $\partial\varphi$ such that $T - A + \partial\varphi o\, g : X \to X$ is a Lipschitzian strongly accretive operator with a Lipschitz constant $L \geq 1$. Define an operator $S : X \to X$ by $Sx = f + (I - (T - A + \partial\varphi o g))\,x$, where $f \in X$ is any given point. For arbitrary $x_0 \in X$, let $\{x_n\}$ be the SP iterative algorithm with mixed errors defined by*

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)\, y_n + \alpha_n S y_n + u_n \\ y_n &= (1-\beta_n)\, z_n + \beta_n\, S z_n + v_n \\ z_n &= (1-\gamma_n) x_n + \gamma_n\, S x_n + w_n,\ n \geq 0, \end{aligned} \tag{33}$$

where $0 \leq \alpha_n, \beta_n, \gamma_n \leq 1$ *and* $\{u_n\}$, $\{v_n\}$, $\{w_n\}$ *are sequences in* X *satisfying the following conditions:*

(i) $0 < \alpha \leq \alpha_n \leq \frac{k}{(2-k)L(L+1)}$,
$\beta_n(L-1) + \gamma_n (L-1)^2 + \beta_n\gamma_n(L-1)^2 < \alpha_n\{k-(2-k)\alpha_n L(1+L)\}(1-t)$, $\forall n \in \mathbb{N}$ *and* $t \in (0,1)$;

(ii) $u_n = u_n' + u_n''$, $\left\|u_n'\right\| = O(\alpha_n)$ $(n \geq 0)$ *and* $\sum_{n=0}^{\infty} \left\|u_n''\right\| < \infty$;

(iii) $\sum_{n=0}^{\infty} \|v_n\| < \infty, \sum_{n=0}^{\infty} \|w_n\| < \infty$.

Then the iterative algorithm (33) converges to the fixed point x^* *of* S *and* x^* *is the unique solution of nonlinear variational inclusion problem (8).*

Proof. As $T - A + \partial\varphi o g$ is a Lipschitzian strongly accretive operator, so for any $f \in X$, the equation $f = (T - A + \partial\varphi o g)\, x$ has a unique solution $x^* \in X$. Using Lemma *1.7*, $x^* \in X$ is a solution of nonlinear variational inclusion problem (*8)* and it is fixed point of the operator S. Since $T - A + \partial\varphi o g : X \to X$ is a Lipschitzian strongly accretive operator, therefore by (*2)*, we have

$$\left\|x-y\right\| \leq \left\|x - y + r[((T - A + \partial\varphi o\, g) - kI)x - ((T - A + \partial\varphi o g) - kI)y]\right\|. \tag{34}$$

Using $Sx = f + (I - (T - A + \partial\varphi o g))\, x$, (*34)* yields

$$\left\|x-y\right\| \leq \left\|x - y + r[(I - S - kI)\, x - (I - S - kI)\, y]\right\|. \tag{35}$$

So that S is a Lipschitzian strongly pseudo-contractive operator with Lipscitz constant $L^* = 1+L$. Replacing T by S in (*9)*, L by L^* in condition (i) of Theorem *2.1* and following the procedure of the proof of Theorem *2.1*, it is easy to prove that the iterative algorithm (*33)* converges to the unique solution x^* of nonlinear variational inclusion problem (*8)*.
□

Putting $v_n = w_n = 0$, $\beta_n = \gamma_n = 0$, in Theorem *3.3*, we can obtain the corresponding result for Mann iterative algorithm with mixed errors (*4)*.

Also, taking $\varphi = 0, u_n = v_n = w_n = 0$, in Theorem *3.3*, we can obtain the solution of variational inequaltiy

$$\langle Tx - Ax - y, f - gx\rangle \geq 0,\ \ \forall f \in X^*, \tag{36}$$

using SP iterative algorithm (*7)*.

Remark 3.4. *Theorem 3.1 is a special case of Theorem 3.3 when* $\varphi \equiv 0$, $A \equiv 0$, $g \equiv 0$ *and* $S \equiv R$.

Remark 3.5. *Theorem 3.3: (i) improves and modifies results of Gu and Lu [11] as faster SP iterative algorithm has been used instead of slow Mann and Ishikawa iterative algorithms,*
(ii) improves results of Gu and Lu [11] for strongly accretive maps.

References

[1] V. Berinde,Iterative Approximation of Fixed Points, Springer, 2007.
[2] V. Berinde, A.R. Khan, H. Fukhar-Ud-Din, Convergence theorems for admissible perturbations of generalized pseudo-contractive operators, J. Nonlinear Convex Anal. 16 (2015) 563–572.
[3] F.E. Browder, W.V. Petryshyn, The solution by iteration of nonlinear functional equations in Banach spaces, Bull. Am. Math. Soc. 72 (1966) 571–575.
[4] S.S. Chang, The Mann and Ishikawa iterative approximation of solutions to variational inclusions with accretive type mappings, Comput. Math. Appl. 37 (1999) 17–24.
[5] C.E. Chidume, An iterative process for nonlinear Lipschitz strongly accretive mappings in L_p spaces, J. Math. Anal. Appl. 151 (1990) 453–461.
[6] Y.J. Cho, H. Zhou, G. Guo, Weak and strong convergence theorems for three step iteration with errors for asymptotically non-expansive mappings, Comput. Math. Appl. 47 (2004) 707–714.
[7] R. Chugh, V. Kumar, Convergence of SP iterative scheme with mixed errors for accretive Lipschitzian and strongly accretive Lipschitzian operators in Banach space, Int. J. Computer Math, 90 (2013) 1865–1880.
[8] R. Chugh, P. Malik, V. Kumar, On analytical and numerical study of implicit fixed point iterations, Cogent Mathematics, 2 (2015) 1–14. ID :1021623.
[9] L.B. Ciric, J.S. Ume, S.N. Ješić, M.M. Arandjelović-Milovanović, Modified Ishikawa iteration process for nonlinear Lipschitz generalized strongly pseudo-contractive operators in arbitrary Banach spaces, Numer. Funct. Anal. Optim. 28 (2007) 1231–1243.
[10] K. Dimling, Zeros of accretive operators, Manuscripta Math. 13 (1974) 365–374.
[11] F. Gu, J. Lu, Stability of Mann and Ishikawa iterative processes with errors for a class of nonlinear inclusion problem, Math.Commun. 9 (2004) 149–159.
[12] N. Hussain, S. Narwal, R. Chugh, V. Kumar, On convergence of random iterative schemes with errors for strongly pseudo-contractive Lipschitzian maps in real Banach spaces, J. Nonlinear Sci. Appl.9 (2016) 3157–3168.
[13] S. Ishikawa, Fixed point by a new iteration method, Proc. Am. Math. Soc. 44 (1974) 147–150.
[14] T. Kato, Nonlinear semigroups and evolution equations, J. Math. Soc. Japan 19 (1967) 508–520.
[15] M.A. Khamsi, W.A. Kirk, On Uniformly Lipschitzian multivalued mapping in Banach and metric spaces, Nonlinear Anal. 72 (2010) 2080–2085.
[16] A.R. Khan, V. Kumar, S. Narwal, R. Chugh, Random iterative algorithms and almost sure stability in Banach spaces, Filomat 31 (2017) 3611–3626.
[17] A.R. Khan, V. Kumar, N. Hussain, Analytical and numerical treatment of Jungck-Type iterative schemes, Appl. Math. Comput. 231 (2014) 521–535.
[18] J.K. Kim, Convergence of Ishikawa iterative sequences for accretive Lipschitzian mappings in Banach spaces, Taiwanese J. Math. 10 (2006) 553–561.
[19] J.K. Kim, S.M. Jang, Z. Liu, Convergence theorems and stability problems of Ishikawa iterative sequences for nonlinear operator equations of the accretive and strong accretive operators, Comm. Appl. Nonlinear Anal. 10 (2003) 285–294.
[20] L.S. Liu, Ishikawa-type and Mann-type iterative process with errors for constructing solutions of nonlinear equations involving m-accretive operators in Banach spaces, Nonlinear Anal. 34 (1998) 307–317.
[21] Z. Liu, J.S. Ume, Stable and almost stable iteration schemes for strictly hemi-contractive operators in arbitrary Banach spaces, Numer. Funct. Anal. Optim. 23 (2002) 833–848.
[22] W. R. Mann, Mean value methods in iteration, Proc. Am. Math. Soc. 4 (1953) 506–510.
[23] M.A. Noor, New approximation schemes for general variational inequalities, J. Math. Anal. Appl. 215 (2000) 217–229.
[24] M.O. Osilike, Stability of the Mann and Ishikawa iteration procedures for ϕ-strong pseudo-contractions and nonlinear equations of the ϕ-strongly accretive type, J. Math. Anal. Appl. 227 (1998) 319–334.
[25] W. Phuengrattana, S. Suantai, On the rate of convergence of Mann, Ishikawa, Noor and SP iterations for continuous functions on an arbitrary interval, J. Comput. Appl. Math. 235 (2011) 3006–3014.
[26] K.P.R. Sastry, G.V.R. Babu, Approximation of fixed point of strictly pseudo-contractive mappings on arbitrary closed convex sets in a Banach space, Proc. Am. Math. Soc. 128 (2000) 2907–2909.
[27] Y. Xu, F. Xie, Stability of Mann iterative process with random errors for the fixed point of strongly pseudo-contractive mapping in arbitrary Banach spaces, Rostock. Math. Kollooq. 58 (2004) 93–100.
[28] Y. Xu, Ishikawa and Mann iterative process with errors for nonlinear strongly accretive operator equations, J. Math. Anal. Appl. 224 (1998) 91–101.
[29] L. Yang, Strong convergence theorems of an iterative scheme for strictly pseudo-contractive mappings in Banach spaces, Optimization 67 (2018) 855–863.
[30] L.C. Zeng, Iterative solutions of nonlinear equations involving m-accretive operators in Banach spaces, J. Math Anal. Appl. 188 (1998) 410–415.
[31] F. Gürsoy, A. R. Khan, M. Ertürk, V. Karakaya, Convergence and data dependency of normal-S iterative method for discontinuous operators on Banach space, Numer. Funct. Anal. Optim. 39 (2018) 322–345.
[32] F. Gürsoy, A. R. Khan, H. Fukhar-ud-din, Convergence and data dependence results for quasi-contractive type operators in hyperbolic spaces, Hacet. J. Math. Stat. 46 (2017) 391–406.
[33] V. Karakaya, Y. Atalan, K. Doğan, N. E. H. Bouzara, Some fixed point results for a new three steps iteration process in Banach spaces, Fixed Point Theory 18 (2017) 625–640.
[34] Y. Atalan, V. Karakaya, ,Iterative solution of functional Volterra-Fredholm integral equation with deviating argument, J. Nonlinear Convex Anal. 18 (2017) 675–684.

[35] V. Karakaya, K. Doğan, Y. Atalan, N. E. H. Bouzara, The local and semilocal convergence analysis of new Newton-like iteration methods, Turk J. Math. 42 (2018) 735–751.

[36] K. Doğan, V. Karakaya, On the convergence and stability results for a new general iterative process,The Scientific World Journal, 2014 (2014) 1–8. Article ID 852475.