



Fixed Point Theorems of Generalized Greguš Type in Quasi-Metric Spaces for Two Pairs of Mappings Satisfying Common Coincidence Range Property

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Abstract. The purpose of this paper is to prove some general fixed point theorems for two pairs of mappings satisfying implicit relations of generalized Greguš type in quasi-metric spaces without the notion of sequence and inequality. As applications we obtain new results for mappings satisfying contractive / extensive conditions of integral type and in G-metric spaces.

1. Introduction

Let X be a nonempty set and f, g be self mappings of X . We say that $x \in X$ is a coincidence point of f and g if $fx = gx$. The set of all coincidence points of f and g is denoted by $C(f, g)$. A point $w \in X$ is said to be a point of coincidence of f and g if there exists $x \in X$ such that $w = fx = gx$.

In [16] Jungck introduced the notion of compatible mappings. In [17] Jungck generalize to notion of compatible mappings and introduce the notion of weakly compatible mappings.

Definition 1.1. [17] Let f and g be self mappings of a nonempty set X . f and g are said to be weakly compatible if $fgu = gfu$ for $u \in C(f, g)$.

Definition 1.2. Let X be a nonempty set. A quasi-metric on X is a function $Q : X \times X \rightarrow \mathbb{R}_+$ such that
(Q₁) $Q(x, y) = 0$ if and only if $x = y$
(Q₂) $Q(x, y) \leq Q(x, z) + Q(z, y)$ for all $x, y, z \in X$.

A quasi-metric space is a nonempty set X with a quasi-metric and is denoted by (X, Q) .

Some fixed point theorems in quasi-metric spaces are proved in [10],[15],[27],[29],[30] and other papers. Several fixed point theorems and common fixed point theorems have been unified considering a general condition by an implicit relation in [22],[23] and other papers.

2. Preliminaries

Greguš [9] proved the following theorem:

Theorem 2.1 Let C be a nonempty closed subset in Banach space X and let T be a self mapping of X satisfying

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the inequality: $\|Tx - Ty\| \leq a\|x - y\| + b\|x - Tx\| + c\|y - Ty\|$ for all $x, y \in X$ where $a, b, c \geq 0$ and $a + b + c = 1$. Then T has a unique fixed point.

Some authors have generalized Theorem 2.1 in [3], [6], [7], [8], [21]. Some fixed point theorems for mappings satisfying implicit relations of Greguš type are proved in [27].

In 2007 Sintunavarat and Kuman introduced the notion of common limit range property for a pair of mappings.

Recently Imdad et al. [11] extend the notion of common limit range property for two pairs of self mappings.

Definition 2.1. The pairs (A, S) and (B, T) of self mappings of a metric spaces (X, d) are said to satisfy the $CLR_{(S,T)}$ - property [11] if there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \rightarrow \infty} (Ax_n) = \lim_{n \rightarrow \infty} (Sx_n) = \lim_{n \rightarrow \infty} (By_n) = \lim_{n \rightarrow \infty} (Ty_n) = t$$

for some $t \in S(X) \cap T(X)$.

Other results in this topic are obtained in [12],[13],[14] and other papers. In [13] and [14], using implicit relations, the authors unified some common fixed point theorems for pairs of mappings satisfying common limit range property.

In these results and others there exist convergent sequences in X .

Quite recently, the present authors introduced in [28] the notion of coincidence range property in metric spaces. Similar with Definition 1.5[28] we define coincidence range property in quasi metric spaces.

Definition 2.2. Let A, S and T be self mappings of a quasi metric space (X, Q) . (A, S) and T satisfy coincidence range property with respect to T , denoted by $CRP_{(A,S)T}$ - property, if there exists $u \in C(A, S)$ with $Au \in T(X)$.

Example 2.1. Let $X = [1, \infty)$ and $Ax = x^2 + 1/2, Sx = x + 1/2, Tx = x$. Then $Tx = [1, \infty)$. If $Ax = Sx$ then $x = 1$ and $z = 1 = A1 = S1; 1 \in T(X)$.

An altering distance [18] is a mapping $\Psi : [0, \infty) \rightarrow [0, \infty)$ such that

(Ψ_1) : Ψ is increasing and continuous,

(Ψ_2) : $\Psi(t) = 0$ if and only if $t = 0$. Some fixed point theorems involving altering distance have been published in [26],[31] and other papers.

Definition 2.3 A weakly altering distance is a mapping $\Psi : [0, \infty) \rightarrow [0, \infty)$ which satisfy

(Ψ_1) : Ψ is increasing,

(Ψ_2) : $\Psi(t) = 0$ if and only if $t = 0$.

Remark 2.1. Every altering distance is a weak altering distance and conversely is not true.

Example 2.2.

$$\Psi(t) = \begin{cases} t & \text{if } t \in [0, 1) \\ e^t & \text{if } t \in [1, \infty) \end{cases}$$

The purpose of this paper is to prove some fixed point theorems for two pairs (A, S) and (B, T) of mappings in quasi-metric spaces satisfying $CRP_{(A,S)T}$ - property and an implicit relation of generalized Greguš type without the notions of sequence and inequality. As applications we obtain new results for mappings satisfying conditions of integral type and in G-metric spaces.

3. Implicit relations

Definition 3.1. Let F be the set of all functions $F : R_+^6 \rightarrow R$ satisfying the following conditions:

(F_G) : $F(t, 0, 0, t, t, 0) = 0$, for every $t > 0$

(F_C) : $F(t, t, 0, 0, t, t) = 0$, for every $t > 0$.

Example 3.1. $F(t_1, \dots, t_6) = t_1 - \max\{t_2, t_3, \dots, t_6\}$

Example 3.2. $F(t_1, \dots, t_6) = t_1 - \max\{t_2, t_3, t_4, \frac{t_5+t_6}{2}\}$

Example 3.3. $F(t_1, \dots, t_6) = t_1 - \max\{t_2, t_3 + t_4, \frac{t_5+t_6}{2}\}$

Example 3.4. $F(t_1, \dots, t_6) = t_1 - a \max\{t_2, t_3, t_4\} - b \max\{t_5, t_6\}$ where $a, b \geq 0$ and $a + b = 1$.

Example 3.5. $F(t_1, \dots, t_6) = t_1 - a \max\{t_2, t_4\} - b \max\{t_3, t_5\} - c \max\{t_5, t_6\}$ where $a, b, c \geq 0$ and $a + b + c = 1$.

Example 3.6. $F(t_1, \dots, t_6) = t_1 - \max\{\sqrt{t_2 t_3}, \sqrt{t_4 t_5}, \sqrt{t_5 t_6}\}$.

Example 3.7. $F(t_1, \dots, t_6) = t_1^2 - a t_1 \max\{t_2, t_3, t_4\} + b \max\{t_5^2, t_6^2\}$ where $a \geq 0$ and $a + b = 1$.

Example 3.8. $F(t_1, \dots, t_6) = t_1 - a t_2 - b t_3 - c t_4 - d t_5 - e t_6$, where $a, b, c, d, e > 0$ and $a + d + e = 1$ and $c + d = 1$.

4. Main result:

Lemma 4.1 [1] Let f, g be weakly compatible self mappings of a nonempty set X . If f and g have a unique point of coincidence $w = fx = gx$ for some $x \in X$ then w is the unique fixed point of f and g .

Theorem 4.1 Let A, B, S, T be self mappings of a quasi-metric space (X, Q) such that

(4.1) $F(\Psi(Q(Ax, By)), \Psi(Q(Sx, Ty)), \Psi(Q(Sx, Ax)), \Psi(Q(Ty, By)), \Psi(Q(Sx, By)), \Psi(Q(Ax, Ty))) \neq 0$ for all x, y with $Ax \neq By$ and some $F \in F_C \cap F_C$ and Ψ is a weakly altering distance.

If (A, S) and T satisfy $CRP_{(A,S)T}$ - property, then $C(B, T) \neq \Phi$.

Moreover, if (A, S) and (B, T) are weakly compatible, then A, B, S, T have a unique common fixed point.

Proof. Since (A, S) and T satisfy $CRP_{(A,S)T}$ - property, there exist $u \in X$ such that $z = Au = Su$ with $z \in T(X)$.

Hence, there exist $v \in X$ such that $z = Tv$. Suppose that $Au \neq Bv$, then by (4.1) we obtain

$F(\Psi(Q(Au, Bv)), \Psi(Q(Su, Tv)), \Psi(Q(Su, Au)), \Psi(Q(Tv, Bv)), \Psi(Q(Su, Bv)), \Psi(Q(Au, Tv))) \neq 0$,
 $F(\Psi(Q(z, Bu)), 0, 0, \Psi(Q(z, Bv)), \Psi(Q(z, Bv)), 0) \neq 0$

a contradiction of F_C .

Hence $z = Bv = Tv = Au = Su$.

If $Au = Bv$ then $z = Au = Su = Bv = Tv$. Hence z is a common point of coincidence of (A, S) and (B, T) .

Suppose that there exists an other point of coincidence for (A, S) $z' \neq z$ with $z' = Aw = Sw$, then $Aw \neq Bw$.

By (4.1) we obtain

$F(\Psi(Q(z, z')), \Psi(Q(z, z')), 0, 0, \Psi(Q(z, z')), \Psi(Q(z, z'))) \neq 0$, a contradiction of F_C .

Hence $\Psi(Q(z, z')) = 0$ which implies $z = z'$, and z is the unique point of coincidence of A and S . Similarly, z is the unique common fixed point of A, S , and B and T .

If $\Psi(t) = t$ by Theorem 4.1 we obtain

Theorem 4.2. Let A, B, S and T self mappings of quasi-metric space (X, Q) satisfying.

(4.2) $F(Q(Ax, By), Q(Sx, Ty), Q(Sx, Ax), Q(Ty, By), Q(Sx, By), Q(Ax, Ty)) \neq 0$. for all $x, y \in X$ with $Ax \neq By$ and some $F \in F_C \cap F_C$.

If (A, S) and T satisfy $CRP_{(A,S)T}$ - property, then $C(B, T) \neq \Phi$.

Moreover, if (A, S) and (B, T) are weakly compatible then A, B, S, T have a unique common fixed point.

By Example 3.1 and Theorem 4.1 we obtain

Theorem 4.3 Let A, B, S, T be self mappings of a quasi-metric space (X, Q) satisfying $\Psi(Q(Ax, By)) \neq \max\{\Psi(Q(Sx, Ty), \Psi(Q(Sx, Ax)), \Psi(Q(Ty, By)), \Psi(Q(Sx, By)), \Psi(Q(Ax, Ty))\}$ for all $x, y \in X$ and $Ax \neq By$ and Ψ is a weakly altering distance. If (A, S) and T satisfy $CRP_{(A,S)T}$ - property then $C(B, T) \neq \Phi$.

Moreover, if (A, S) and (B, T) are weakly compatible, then A, B, S, T have a unique common fixed point.

By Theorem 4.3 and Example 3.1 we obtain

Theorem 4.4 Let A, B, S and T be self mappings of a quasi-metric space (X, Q) satisfying $Q(Ax, By) \neq \max\{Q(Sx, Ty), Q(Sx, Ax), Q(Ty, By), Q(Sx, By), Q(Ax, Ty)\}$ for all $x, y \in X$ with $Ax \neq By$.

If (A, S) and T satisfy $CRP_{(A,S)T}$ - property then $C(B, T) \neq \Phi$.

Moreover, if (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point.

Remark 4.1. 1) By Theorem 4.1 and 4.2 for " $<$ " and " $>$ " instead of " \neq " we obtain new results for strict contractive and extensive mappings.

2) By Theorem 4.3 and 4.4 and Example 3.2 - 3.8 we obtain new particular results for contractive and extensive mappings.

Example 4.1 Let $X = [0, 1]$ and $Q(x, y) = |x - y|$. Then (X, Q) is a quasi-metric space. Let be the following mappings: $Ax = 0, Sx = \frac{x}{x+2}, Bx = \frac{x}{5}, Tx = x$ with $T(X) = [0, 1]$. $Sx = Ax$ implies $x = 0$ and $0 \in [0, 1] = T(X)$. Hence $(A, S), T$ satisfy $CRP_{(A,S)T}$ - property. On the other hand $AS0 = SA0$ and $BT0 = TB0$, hence (A, S) and (B, T) are weakly compatible.

On the other hand

$(Q(Ax, By)) = |0 - \frac{y}{5}| = \frac{y}{5}$ and $Q(Ty, By) = |y - \frac{y}{5}| = \frac{4y}{5}$ If $Ax \neq By$ implies $y \neq 0$, then $Q(Ax, By) < Q(Ty, By)$ which implies $Q(Ax, By) < \max\{Q(Sx, Ty), Q(Sx, Ax), Q(Ty, By), Q(Sx, By), Q(Ax, Ty)\}$,
 $Q(Ax, By) - \max\{Q(Sx, Ty), Q(Sx, Ax), Q(Ty, By), Q(Sx, By), Q(Ax, Ty)\} \neq 0$.

By Theorem 4.4 and Example 3.1, A, B, S, T have a unique common fixed point $z = 0$.

5. Applications

5.1. Fixed point results for two pairs of mappings in G-metric spaces

In [4], [5] Dhage introduced a new class of generalized metric space named D-metric space. Mustafa and Sims [19],[20] proved that most of claims concerning the fundamental topological structure on D-metric spaces are incorrect and introduced an appropriate notion of generalized metric space named G-metric spaces.

Definition 5.1 [20] Let X be a nonempty set and $G : X^3 \rightarrow \mathbb{R}_+$ be a function satisfying the following properties:

- $(G_1) : G(x, y, z) = 0$ if $x = y = z$
- $(G_2) : 0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$.
- $(G_3) : G(x, y, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $y \neq z$.
- $(G_4) : G(x, y, z) = G(y, z, x)$... simetry in all three variable).
- $(G_5) G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$.

The function $G(x, y, z)$ is called a $G - metric$ on X and the pair (X, G) is called a G -metric space.

Remark 5.1. If $G(x, y, z) = 0$ then $x = y = z$.

Lemma 5.1 ([25]) If (X, G) is a G -metric space and $Q(x, y) = G(x, y, y)$, then $Q(x, y)$ is a quasi-metric on X .

Theorem 5.1 Let (X, G) be a G -metric space and A, B, S and T be self mappings of X such that

(5.1) $F(G(Ax, By, By), G(Sx, Ty, Ty), G(Sx, Ax, Ax), G(Ty, By, By), G(Sx, By, By), G(Ax, Ty, Ty)) \neq 0$ for all $x, y \in X$ with $Ax \neq By$ and $F \in F_G \cap F_C$.

If (A, S) and T satisfy $CRP_{(A,S)T}$ - property, the $C(B, T) \neq 0$.

Moreover, if (A, S) and (B, T) are weakly compatible, then A, B, S, T have a unique common fixed point.

Proof. As in Lemma 5.1 (X, Q) is a quasi-metric space with $Q(x, y) = G(x, y, y)$ Then

$G(Ax, By, By) = Q(Ax, By), G(Sx, Ty, Ty) = Q(Sx, Ty), G(Sx, Ax, Ax) = Q(Sx, Ax),$
 $G(Ty, By, By) = Q(Ty, By), G(Sx, By, By) = Q(Sx, By), G(Ax, Ty, Ty) = Q(Ax, Ty).$

By 5.1 we obtain

$F(Q(Ax, By), Q(Sx, Ty), Q(Sx, Ax), Q(Ty, By), Q(Sx, By), Q(Ax, Ty)) \neq 0$ which is inequality 4.2 by Theorem 4.2. Hence all conditions of Theorem 4.3 are satisfied and Theorem 5.1 follows by Theorem 4.2.

Remark 5.2

1) Similarly, by Theorem 4.2 and 4.3 we obtain new results.

2) If in Theorem 5.1 we have " $<$ " or " $>$ " instead of " \neq " we obtain new results for contractive and extensive mappings.

3) By Examples 3.1-3.8 we obtain new particular results.

5.2. Fixed point results for two pairs of mappings satisfying a condition of integral type

In [2] Branciari established the following fixed point theorem which opened the way to the study of fixed points for mappings satisfying a condition of integral type.

Theorem 5.2 Let (X, d) be a complete metric space $c \in (0, 1)$ and $f : (X, d) \rightarrow (X, d)$ be a mapping such that

$$\int_0^{d(fx, fy)} h(t)dt \leq c \int_0^{d(x, y)} h(t)dt$$

where $h : [0, \infty) \rightarrow [0, \infty)$ is a Lebesgue measurable mappings which is summable (i.e. with finite integral) on each compact subset of $[0, \infty)$ such that for $\varepsilon > 0$, $\int_0^\varepsilon h(t)dt > 0$. Then f has a unique fixed point. Recently, there exists a vast literature in this topic.

Lemma 5.2 Let $h : (0, \infty) \rightarrow (0, \infty)$ as in Theorem 5.2. Then $\Psi(t) = \int_0^t h(x)dx$ is a weakly altering distance.

Proof. The proof follows by the first part of Lemma 2.5[25].

Let (X, Q) be a quasi-metric spaces and $\Psi(t)$ as in Lemma 5.2, then

$$\begin{aligned}
 (5.2) \quad \Psi(Q(Ax, By)) &= \int_0^{Q(Ax,By)} h(t)dt, \quad \Psi(Q(Sx, Ty)) = \int_0^{Q(Sx,Ty)} h(t)dt \\
 \Psi(Q(Sx, Ax)) &= \int_0^{Q(Sx,Ax)} h(t)dt, \quad \Psi(Q(Ty, By)) = \int_0^{Q(Ty,By)} h(t)dt \\
 \Psi(Q(Sx, By)) &= \int_0^{Q(Sx,By)} h(t)dt, \quad \Psi(Q(Ax, Ty)) = \int_0^{Q(Ax,Ty)} h(t)dt
 \end{aligned}$$

Theorem 5.3. Let (X, Q) be a quasi-metric space and A, B, S, T self mappings of X such that

$$(5.3) \quad F\left(\int_0^{Q(Ax,By)} h(t)dt, \int_0^{Q(Sx,Ty)} h(t)dt, \int_0^{Q(Sx,Ax)} h(t)dt, \int_0^{Q(Ty,By)} h(t)dt, \int_0^{Q(Sx,By)} h(t)dt, \int_0^{Q(Ax,Ty)} h(t)dt\right) \neq 0$$

for all $x, y \in X$ with $Ax \neq By$ and some $F \in F_G \cap F_C$.

If (A, S) and T satisfy $CRP_{(A,S)T}$ – property, then $C(B, T) \neq 0$.

Moreover, if (A, S) and (B, T) are weakly compatible then A, B, S, T have a unique common fixed point.

Proof. By (5.2) and (5.3) we obtain

$$(5.3) \quad F(\Psi(Q(Ax, By)), \Psi(Q(Sx, Ty)), \Psi(Q(Sx, Ax)), \Psi(Q(Ty, By)), \Psi(Q(Sx, By)), \Psi(Q(Ax, Ty))) \neq 0$$

for all $x, y \in X$ with $Ax \neq By$ and $F \in F_G \cap F_C$ and Ψ is a weakly altering distance. By Theorem 4.1 we have obtained Theorem 5.2.

Remark 5.3

- 1) Similarly, by Theorem 4.4, we obtain a new result.
- 2) If in Theorem 5.3 we have " $<$ " or " $>$ " instead of " \neq " we obtain new results for strict contractive and strict extensive pairs of mappings.
- 3) By Examples 3.1-3.8 we have obtained new particular results.

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