Neutrosophic Soft Continuity in Neutrosophic Soft Topological Spaces

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Abstract. In this study, using a more appreciate definition of a neutrosophic soft point, we introduce neutrosophic soft functions, and present the concepts of neutrosophic soft continuous, neutrosophic soft open, neutrosophic soft closed, and neutrosophic soft homeomorphic functions in a very different way from the source existing in the literature. In the investigation we prove theorems related to these concepts and provide some examples.

1. Introduction

The contribution of mathematics to the present-day technology in reaching to a fast trend cannot be ignored. The theories presented differently from classical methods in studies such as fuzzy set [18], intuitionistic set [7], soft set [12], neutrosophic set [15], etc. Many works have been done on these sets by mathematicians in many areas of mathematics [2–6, 8, 14, 17]. A number of theories have been proposed for dealing with uncertaintiesin an efficient way. However, these theories have their own difficulties. In 1999, Molodtsov [12] introduced the concept of soft set which is completely new approach for modelling uncertainties. He firstly applied a few directions for the applications of soft sets, such as game theory, smoothness of functions, and theory of probability. After then, soft set theory and its applications are progressing rapidly in different fields. Many people have observed that soft sets have many applications in both pure and applied sciences. Recently, F. Smarandache [15] generalized the concept of intuitionistic fuzzy sets and introduced the concept of neutrosophic sets. F. Smarandache also [16] extended several definitions neutro algebraic structures and anti algebraic structures. P.K. Maji [10] combined the neutrosophic set with soft sets. Bera [4] presented neutrosophic soft topological spaces. T.Y. Ozturk et al. [13] redefined some operations on neutrosophic soft set in contrast to the studies [4, 10] and examined the properties related to these operations. Considering these newly defined processes, they also reconstructed neutrosophic soft topology. C.A. Gunduz et al. [9] presented separation axioms on neutrosophic soft topological spaces.

In the present paper, based on the definition of a neutrosophic soft set, we introduce a kind of definition of neutrosophic soft continuity and give a comprehensive investigation of the neutrosophic soft continuous mapping. Moreover, we study the concepts of neutrosophic soft open, neutrosophic soft closed and neutrosophic soft homeomorphism, and present characterization theorems.

sectionPreliminaries

In this section, we will give some preliminary information for the present study.

Definition 1.1. ([15]) A neutrosophic set A on the universe of discourse X is defined as:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X\},$$

where $T, I, F : X \to]^-0, 1^+[$ and $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 1.2. ([12]) Let X be an initial universe, E be a set of all parameters and $P(X)$ denotes the power set of X. A pair (F, E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$.

In other words, the soft set is a parameterized family of subsets of the set X. For $e \in E$, $F(e)$ may be considered as the set of $e-$elements of the soft set (F, E), or as the set of $e-$approximate elements of the soft set, i.e.,

$$(F, E) = \{(e, F(e)) : e \in E, F : E \to P(X)\}.$$

Firstly, neutrosophic soft set defined by Maji [10] and later this concept has been modified by Deli and Bromi [8] as given below:

Definition 1.3. Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denote the set of all neutrosophic sets of X. Then, a neutrosophic soft set $\left(\widetilde{F}, E\right)$ over X is a set defined by a set valued function $\widetilde{F}$ representing a mapping $\widetilde{F} : E \to P(X)$ where $\widetilde{F}$ is called approximate function of the neutrosophic soft set $\left(\widetilde{F}, E\right)$. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $P(X)$ and therefore it can be written as a set of ordered pairs,

$$\left(\widetilde{F}, E\right) = \left\{\left(e, \left\langle x, T_{\widetilde{F}(e)}(x), I_{\widetilde{F}(e)}(x), F_{\widetilde{F}(e)}(x)\right\rangle : x \in X\right) : e \in E\right\}$$

where $T_{\widetilde{F}(e)}(x), I_{\widetilde{F}(e)}(x), F_{\widetilde{F}(e)}(x) \in [0, 1]$, respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $\widetilde{F}(e)$. Since supremum of each T, I, F is 1 so the inequality $0 \le T_{\widetilde{F}(e)}(x) + I_{\widetilde{F}(e)}(x) + F_{\widetilde{F}(e)}(x) \le 3$ is obvious.

Definition 1.4. ([4]) Let $\left(\widetilde{F}, E\right)$ be neutrosophic soft set over the universe set X. The complement of $\left(\widetilde{F}, E\right)$ is denoted by $\left(\widetilde{F}, E\right)^c$ and is defined by:

$$\left(\widetilde{F}, E\right)^c = \left\{\left(e, \left\langle x, F_{\widetilde{F}(e)}(x), 1 - I_{\widetilde{F}(e)}(x), T_{\widetilde{F}(e)}(x)\right\rangle : x \in X\right) : e \in E\right\}.$$

Obvious that, $\left(\left(\widetilde{F}, E\right)^c\right)^c = \left(\widetilde{F}, E\right)$.

Definition 1.5. ([10]) Let $\left(\widetilde{F}, E\right)$ and $\left(\widetilde{G}, E\right)$ be two neutrosophic soft sets over the universe set X. $\left(\widetilde{F}, E\right)$ is said to be neutrosophic soft subset of $\left(\widetilde{G}, E\right)$ if $T_{\widetilde{F}(e)}(x) \le T_{\widetilde{G}(e)}(x)$, $I_{\widetilde{F}(e)}(x) \le I_{\widetilde{G}(e)}(x)$, $F_{\widetilde{F}(e)}(x) \ge F_{\widetilde{G}(e)}(x)$, $\forall e \in E$, $\forall x \in X$. It is denoted by $\left(\widetilde{F}, E\right) \subseteq \left(\widetilde{G}, E\right)$.

$\left(\widetilde{F}, E\right)$ is said to be neutrosophic soft equal to $\left(\widetilde{G}, E\right)$ if $\left(\widetilde{F}, E\right)$ is neutrosophic soft subset of $\left(\widetilde{G}, E\right)$ and $\left(\widetilde{G}, E\right)$ is neutrosophic soft subset of $\left(\widetilde{F}, E\right)$. It is denoted by $\left(\widetilde{F}, E\right) = \left(\widetilde{G}, E\right)$.

Definition 1.6. ([13]) Let $\left(\widetilde{F}_1, E\right)$ and $\left(\widetilde{F}_2, E\right)$ be two neutrosophic soft sets over the universe set X. Then their union is denoted by $\left(\widetilde{F}_1, E\right) \cup \left(\widetilde{F}_2, E\right) = \left(\widetilde{F}_3, E\right)$ and is defined by:

$$\left(\widetilde{F}_3, E\right) = \left\{\left(e, \left\langle x, T_{\widetilde{F}_3(e)}(x), I_{\widetilde{F}_3(e)}(x), F_{\widetilde{F}_3(e)}(x)\right\rangle : x \in X\right) : e \in E\right\}$$

where

$$
\begin{aligned}
T_{\widetilde{F}_3(e)}(x) &= \max\left\{T_{\widetilde{F}_1(e)}(x), T_{\widetilde{F}_2(e)}(x)\right\}, \\
I_{\widetilde{F}_3(e)}(x) &= \max\left\{I_{\widetilde{F}_1(e)}(x), I_{\widetilde{F}_2(e)}(x)\right\}, \\
F_{\widetilde{F}_3(e)}(x) &= \min\left\{F_{\widetilde{F}_1(e)}(x), F_{\widetilde{F}_2(e)}(x)\right\}.
\end{aligned}
$$

Definition 1.7. ([13]) Let $(\widetilde{F}_1, E)$ and $(\widetilde{F}_2, E)$ be two neutrosophic soft sets over the universe set X. Then their intersection is denoted by $(\widetilde{F}_1, E) \cap (\widetilde{F}_2, E) = (\widetilde{F}_3, E)$ and is defined by:

$$(\widetilde{F}_3, E) = \left\{ \left(e, \left\langle x, T_{\widetilde{F}_3(e)}(x), I_{\widetilde{F}_3(e)}(x), F_{\widetilde{F}_3(e)}(x) \right\rangle : x \in X \right) : e \in E \right\}$$

where

$$
\begin{aligned}
T_{\widetilde{F}_3(e)}(x) &= \min \left\{ T_{\widetilde{F}_1(e)}(x), T_{\widetilde{F}_2(e)}(x) \right\}, \\
I_{\widetilde{F}_3(e)}(x) &= \min \left\{ I_{\widetilde{F}_1(e)}(x), I_{\widetilde{F}_2(e)}(x) \right\}, \\
F_{\widetilde{F}_3(e)}(x) &= \max \left\{ F_{\widetilde{F}_1(e)}(x), F_{\widetilde{F}_2(e)}(x) \right\}.
\end{aligned}
$$

Definition 1.8. ([13]) Let $(\widetilde{F}_1, E)$ and $(\widetilde{F}_2, E)$ be two neutrosophic soft sets over the universe set X. Then "$(\widetilde{F}_1, E)$ difference $(\widetilde{F}_2, E)$" operation on them is denoted by $(\widetilde{F}_1, E) \setminus (\widetilde{F}_2, E) = (\widetilde{F}_3, E)$ and is defined by $(\widetilde{F}_3, E) = (\widetilde{F}_1, E) \cap (\widetilde{F}_2, E)^c$ as follows:

$$(\widetilde{F}_3, E) = \left\{ \left(e, \left\langle x, T_{\widetilde{F}_3(e)}(x), I_{\widetilde{F}_3(e)}(x), F_{\widetilde{F}_3(e)}(x) \right\rangle : x \in X \right) : e \in E \right\}$$

where

$$
\begin{aligned}
T_{\widetilde{F}_3(e)}(x) &= \min \left\{ T_{\widetilde{F}_1(e)}(x), F_{\widetilde{F}_2(e)}(x) \right\}, \\
I_{\widetilde{F}_3(e)}(x) &= \min \left\{ I_{\widetilde{F}_1(e)}(x), 1 - I_{\widetilde{F}_2(e)}(x) \right\}, \\
F_{\widetilde{F}_3(e)}(x) &= \max \left\{ F_{\widetilde{F}_1(e)}(x), T_{\widetilde{F}_2(e)}(x) \right\}.
\end{aligned}
$$

Definition 1.9. ([13]) Let $\left\{ (\widetilde{F}_i, E) \middle| i \in I \right\}$ be a family of neutrosophic soft sets over the universe set X. Then

$$
\begin{aligned}
\underset{i \in I}{\cup} (\widetilde{F}_i, E) &= \left\{ \left(e, \left\langle x, \sup \left[T_{\widetilde{F}_i(e)}(x) \right]_{i \in I}, \sup \left[I_{\widetilde{F}_i(e)}(x) \right]_{i \in I}, \inf \left[F_{\widetilde{F}_i(e)}(x) \right]_{i \in I} \right\rangle : x \in X \right) : e \in E \right\}, \\
\underset{i \in I}{\cap} (\widetilde{F}_i, E) &= \left\{ \left(e, \left\langle x, \inf \left[T_{\widetilde{F}_i(e)}(x) \right]_{i \in I}, \inf \left[I_{\widetilde{F}_i(e)}(x) \right]_{i \in I}, \sup \left[F_{\widetilde{F}_i(e)}(x) \right]_{i \in I} \right\rangle : x \in X \right) : e \in E \right\}.
\end{aligned}
$$

Definition 1.10. ([13]) A neutrosophic soft set $(\widetilde{F}, E)$ over the universe set X is said to be null neutrosophic soft set if $T_{\widetilde{F}(e)}(x) = 0$, $I_{\widetilde{F}(e)}(x) = 0$, $F_{\widetilde{F}(e)}(x) = 1$; $\forall e \in E, \forall x \in X$. It is denoted by $0_{(X,E)}$.

Definition 1.11. ([13]) A neutrosophic soft set $(\widetilde{F}, E)$ over the universe set X is said to be absolute neutrosophic soft set if $T_{\widetilde{F}(e)}(x) = 1$, $I_{\widetilde{F}(e)}(x) = 1$, $F_{\widetilde{F}(e)}(x) = 0$; $\forall e \in E, \forall x \in X$. It is denoted by $1_{(X,E)}$.
   Clearly, $0_{(X,E)}^c = 1_{(X,E)}$ and $1_{(X,E)}^c = 0_{(X,E)}$.

Definition 1.12. ([13]) Let $NSS(X, E)$ be the family of all neutrosophic soft sets over the universe set X and $\tau \subset NSS(X, E)$. Then τ is said to be a neutrosophic soft topology on X if

1. $0_{(X,E)}$ and $1_{(X,E)}$ belongs to τ
2. the union of any number of neutrosophic soft sets in τ belongs to τ
3. the intersection of finite number of neutrosophic soft sets in τ belongs to τ.

   Then (X, τ, E) is said to be a neutrosophic soft topological space over X. Each members of τ is said to be neutrosophic soft open set.

Definition 1.13. ([13]) Let (X, τ, E) be a neutrosophic soft topological space over X and $(\widetilde{F}, E)$ be a neutrosophic soft set over X. Then $(\widetilde{F}, E)$ is said to be neutrosophic soft closed set if and only if its complement is a neutrosophic soft open set.

Proposition 1.14. ([13]) *Let (X, τ, E) be a neutrosophic soft topological space over X. Then:*
1. *$0_{(X,E)}$ and $1_{(X,E)}$ are neutrosophic soft closed sets over X*
2. *the intersection of any number of neutrosophic soft closed sets is a neutrosophic soft closed set over X*
3. *the union of finite number of neutrosophic soft closed sets is a neutrosophic soft closed set over X.*

Definition 1.15. ([13]) Let $NSS(X, E)$ be the family of all neutrosophic soft sets over the universe set X.
1. If $\tau = \left\{ 0_{(X,E)}, 1_{(X,E)} \right\}$, then τ is said to be the neutrosophic soft indiscrete topology and (X, τ, E) is said to be a neutrosophic soft indiscrete topological space over X.
2. If $\tau = NSS(X, E)$, then τ is said to be the neutrosophic soft discrete topology and (X, τ, E) is said to be a neutrosophic soft discrete topological space over X.

Definition 1.16. ([13]) Let (X, τ, E) be a neutrosophic soft topological space over X and $\left(\widetilde{F}, E \right) \in NSS(X, E)$ be a neutrosophic soft set. Then the neutrosophic soft interior of $\left(\widetilde{F}, E \right)$, denoted $\left(\widetilde{F}, E \right)^{\circ}$, is defined as the neutrosophic soft union of all neutrosophic soft open subsets of $\left(\widetilde{F}, E \right)$.

Clearly, $\left(\widetilde{F}, E \right)^{\circ}$ is the biggest neutrosophic soft open set that is contained by $\left(\widetilde{F}, E \right)$.

Definition 1.17. ([13]) Let (X, τ, E) be a neutrosophic soft topological space over (X, E) and $\left(\widetilde{F}, E \right) \in NSS(X, E)$ be a neutrosophic soft set. Then the neutrosophic soft closure of $\left(\widetilde{F}, E \right)$, denoted $\overline{\left(\widetilde{F}, E \right)}$, is defined as the neutrosophic soft intersection of all neutrosophic soft closed supersets of $\left(\widetilde{F}, E \right)$.

Definition 1.18. ([9]) Let NS be the family of all neutrosophic sets over the universe set X and $x \in X$. The neutrosophic set $x_{(\alpha,\beta,\gamma)}$ is called a neutrosophic point, for $0 < \alpha, \beta, \gamma \le 1$, and defined as follows:

$$x_{(\alpha,\beta,\gamma)}(y) = \begin{cases} (\alpha, \beta, \gamma), & \text{if } y = x \\ (0, 0, 1), & \text{if } y \ne x. \end{cases}$$

It is clear that every neutrosophic set is the union of its neutrosophic points.

Definition 1.19. ([9]) Let $NSS(X, E)$ be the family of all neutrosophic soft sets over the universe set X. Then neutrosophic soft set $x_{(\alpha,\beta,\gamma)}^{e}$ is called a neutrosophic soft point, for every $x \in X, 0 < \alpha, \beta, \gamma \le 1, e \in E$, and defined as follows:

$$x_{(\alpha,\beta,\gamma)}^{e}(e')(y) = \begin{cases} (\alpha, \beta, \gamma) & \text{if } e' = e \text{ and } y = x, \\ (0, 0, 1) & \text{if } e' \ne e \text{ or } y \ne x. \end{cases}$$

Definition 1.20. ([9]) Let $\left(\widetilde{F}, E \right)$ be a neutrosophic soft set over the universe set X. We say that $x_{(\alpha,\beta,\gamma)}^{e} \in \left(\widetilde{F}, E \right)$ read as belongs to the neutrosophic soft set $\left(\widetilde{F}, E \right)$, whenever $\alpha \le F_{\widetilde{F}(e)}(x), \beta \le I_{\widetilde{F}(e)}(x)$ and $\gamma \ge T_{\widetilde{F}(e)}(x)$.

Definition 1.21. ([9]) Let (X, τ, E) be a neutrosophic soft topological space over X. A neutrosophic soft set $\left(\widetilde{F}, E \right)$ in (X, τ, E) is called a neutrosophic soft neighborhood of the neutrosophic soft point $x_{(\alpha,\beta,\gamma)}^{e} \in \left(\widetilde{F}, E \right)$, if there exists a neutrosophic soft open set $\left(\widetilde{G}, E \right)$ such that $x_{(\alpha,\beta,\gamma)}^{e} \in \left(\widetilde{G}, E \right) \subset \left(\widetilde{F}, E \right)$.

Proposition 1.22. ([9]) *Let (X, τ, E) be a neutrosophic soft topological space and $\left(\widetilde{F}, E \right)$ be a neutrosophic soft set over X. Then $\left(\widetilde{F}, E \right)$ is a neutrosophic soft open set if and only if $\left(\widetilde{F}, E \right)$ is a neutrosophic soft neighborhood of its neutrosophic soft points.*

Definition 1.23. ([9]) Let $x_{(\alpha,\beta,\gamma)}^{e}$ and $y_{(\alpha',\beta',\gamma')}^{e'}$ be two neutrosophic soft points. For the neutrosophic soft points $x_{(\alpha,\beta,\gamma)}^{e}$ and $y_{(\alpha',\beta',\gamma')}^{e'}$ over a common universe X, we say that the neutrosophic soft points are distinct points if $x_{(\alpha,\beta,\gamma)}^{e} \cap y_{(\alpha',\beta',\gamma')}^{e'} = 0_{(X,E)}$.

It is clear that $x_{(\alpha,\beta,\gamma)}^{e}$ and $y_{(\alpha_1,\beta_1,\gamma_1)}^{e'}$ are distinct neutrosophic soft points if and only if $x \ne y$ or $e' \ne e$.

2. Neutrosophic Soft Conrtinuity

Based on the definition of a neutrosophic soft set, we introduce a kind of definition of neutrosophic soft continuity, and give a comprehensive investigation of the neutrosophic soft continuous mapping.

Definition 2.1. Let $(X, \tau, E), (Y, \tau', E')$ be two neutrosophic soft topological spaces, $(f, \varphi) : (X, \tau, E) \rightarrow (Y, \tau', E')$ be a soft mapping and $(\widetilde{F}, E)$ be a neutrosophic soft set over X. Then the image of $(\widetilde{F}, E)$ under the mapping (f, φ), denoted by $(f, \varphi)\left((\widetilde{F}, E)\right)$, is a neutrosophic soft set over Y, defined by

$$(f, \varphi)\left((\widetilde{F}, E)\right) = \left((f, \varphi)(F), (f, \varphi)(I), (f, \varphi)(T)\right),$$

$$(f, \varphi)(F)(e')(y) = \bigvee_{\varphi(e)=e'} \bigvee_{f(x)=y} F(e)(x),$$

$$(f, \varphi)(I)(e')(y) = \bigvee_{\varphi(e)=e'} \bigvee_{f(x)=y} I(e)(x),$$

$$(f, \varphi)(F)(e')(y) = \bigwedge_{\varphi(e)=e'} \bigwedge_{f(x)=y} T(e)(x), \text{ for each } e \in E.$$

Definition 2.2. Let $(X, \tau, E), (Y, \tau', E')$ be two neutrosophic soft topological spaces, $(f, \varphi) : (X, \tau, E) \rightarrow (Y, \tau', E')$ be a mapping and

$$\left(\widetilde{G}, E'\right) = \left\{ \left(e', \left\langle y, G_{\widetilde{G}(e')}(y), J_{\widetilde{G}(e')}(y), P_{\widetilde{G}(e')}(y) \right\rangle : y \in Y \right) : e' \in E' \right\}$$

be a neutrosophic soft set over Y. Then the inverse image of $\left(\widetilde{G}, E'\right)$ under the mapping (f, φ), denoted by $(f, \varphi)^{-1}\left(\left(\widetilde{G}, E'\right)\right)$, is a neutrosophic soft set over X, defined by

$$(f, \varphi)^{-1}\left(\left(\widetilde{G}, E'\right)\right) = \left((f, \varphi)^{-1}(G), (f, \varphi)^{-1}(J), (f, \varphi)^{-1}(P)\right),$$

$$(f, \varphi)^{-1}(G)(e)(x) = G(\varphi(e))(f(x)),$$

$$(f, \varphi)^{-1}(J)(e)(x) = J(\varphi(e))(f(x)),$$

$$(f, \varphi)^{-1}(P)(e)(x) = P(\varphi(e))(f(x)).$$

Note that the image of neutrosophic soft point under the mapping (f, φ) is a neutrosophic soft point. It is clear that,

$$(f, \varphi)\left(x^e_{(\alpha, \beta, \gamma)}\right) = f(x)^{\varphi(e)}_{(\alpha, \beta, \gamma)}.$$

Example 2.3. Suppose that $X = \{x, y, z\}, Y = \{a, b\}$ and the set of parameters sets $E = \{e_1, e_2, e_3\}$ and $E' = \{e'_1, e'_2\}$. Let us consider the neutrosophic soft point $x^{e_1}_{\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)}$. If we get the mapping

$$f : X \rightarrow Y, \; \varphi : E \rightarrow E'$$

defined as

$$\varphi(e_1) = \varphi(e_2) = e'_1, \; \varphi(e_3) = e'_2 \text{ and } f(x) = f(y) = f(z) = a,$$

then

$$(f, \varphi)\left(x^{e_1}_{\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)}\right) = f(x)^{e'_1}_{\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)} = a^{e'_1}_{\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)}.$$

Proposition 2.4. *Let* (X, τ, E), (Y, τ', E') *be two neutrosophic soft topological spaces,* $(f, \varphi) : (X, \tau, E) \to (Y, \tau', E')$ *be a soft mapping and* $\left(\widetilde{G_1}, E'\right)$, $\left(\widetilde{G_2}, E'\right)$ *be two neutrosophic soft sets over Y. Then:*

(1) $(f, \varphi)^{-1}\left(\left(\widetilde{G_1}, E'\right) \cup \left(\widetilde{G_2}, E'\right)\right) = (f, \varphi)^{-1}\left(\left(\widetilde{G_1}, E'\right)\right) \cup (f, \varphi)^{-1}\left(\left(\widetilde{G_2}, E'\right)\right)$,

(2) $(f, \varphi)^{-1}\left(\left(\widetilde{G_1}, E'\right) \cap \left(\widetilde{G_2}, E'\right)\right) = (f, \varphi)^{-1}\left(\left(\widetilde{G_1}, E'\right)\right) \cap (f, \varphi)^{-1}\left(\left(\widetilde{G_2}, E'\right)\right)$,

(3) $(f, \varphi)^{-1}\left(\left(\widetilde{G_1}, E'\right)^c\right) = \left((f, \varphi)^{-1}\left(\widetilde{G_1}, E'\right)\right)^c$.

Proof. The proof is clear. □

Note that this proposition is provided in an arbitrary number of intersections and joints.

Definition 2.5. Let (X, τ, E), (Y, τ', E') be two neutrosophic soft topological spaces, $(f, \varphi) : (X, \tau, E) \to (Y, \tau', E')$ be a soft mapping. For each neutrosophic soft neighborhood $\left(\widetilde{G}, E'\right)$ of $(f, \varphi)\left(x^e_{(\alpha, \beta, \gamma)}\right)$, if there exists a neutrosophic soft neighborhood $\left(\widetilde{F}, E\right)$ of $x^e_{(\alpha, \beta, \gamma)}$ such that $(f, \varphi)\left(\left(\widetilde{F}, E\right)\right) \subset \left(\widetilde{G}, E'\right)$, then (f, φ) is said to be a neutrosophic soft continuous mapping at $x^e_{(\alpha, \beta, \gamma)}$.

If (f, φ) is neutrosophic soft continuous for all $x^e_{(\alpha, \beta, \gamma)}$, then (f, φ) is called neutrosophic soft continuous on X.

In the following theorem, we give characterizations of neutrosophic soft continuity.

Theorem 2.6. *Let* (X, τ, E), (Y, τ', E') *be two neutrosophic soft topological spaces,* $(f, \varphi) : (X, \tau, E) \to (Y, \tau', E')$ *be a mapping. Then the following conditions are equivalent:*

(1) $(f, \varphi) : (X, \tau, E) \to (Y, \tau', E')$ *is a neutrosophic soft continuous mapping,*

(2) For each neutrosophic soft open set $\left(\widetilde{G}, E'\right)$ *over Y,* $(f, \varphi)^{-1}\left(\left(\widetilde{G}, E'\right)\right)$ *is a neutrosophic soft open set over X,*

(3) For each neutrosophic soft closed set $\left(\widetilde{H}, E'\right)$ *over Y,* $(f, \varphi)^{-1}\left(\left(\widetilde{H}, E'\right)\right)$ *is a neutrosophic soft closed set over X,*

(4) For each neutrosophic soft set $\left(\widetilde{F}, E\right)$ *over X,* $(f, \varphi)\overline{\left(\left(\widetilde{F}, E\right)\right)} \subset \overline{(f, \varphi)\left(\left(\widetilde{F}, E\right)\right)}$,

(5) For each neutrosophic soft set $\left(\widetilde{G}, E'\right)$ *over Y,* $\overline{(f, \varphi)^{-1}\left(\left(\widetilde{G}, E'\right)\right)} \subset (f, \varphi)^{-1}\left(\overline{\left(\widetilde{G}, E'\right)}\right)$,

(6) For each neutrosophic soft set $\left(\widetilde{G}, E'\right)$ *over Y,* $(f, \varphi)^{-1}\left(\left(\widetilde{G}, E'\right)^\circ\right) \subset \left((f, \varphi)^{-1}\left(\widetilde{G}, E'\right)\right)^\circ$.

Proof. (1)⇒(2) Let $\left(\widetilde{G}, E'\right)$ be a neutrosophic soft open set over Y and $x^e_{(\alpha, \beta, \gamma)} \in (f, \varphi)^{-1}\left(\left(\widetilde{G}, E'\right)\right)$ be an arbitrary neutrosophic soft point. Then $(f, \varphi)\left(x^e_{(\alpha, \beta, \gamma)}\right) = f(x)^{\varphi(e)}_{(\alpha, \beta, \gamma)} \in \left(\widetilde{G}, E'\right)$. Since (f, φ) is a neutrosophic soft continuous mapping, there exists $x^e_{(\alpha, \beta, \gamma)} \in \left(\widetilde{F}, E\right) \in \tau$ such that $(f, \varphi)\left(\left(\widetilde{F}, E\right)\right) \subset \left(\widetilde{G}, E'\right)$. This implies that $x^e_{(\alpha, \beta, \gamma)} \in \left(\widetilde{F}, E\right) \subset (f, \varphi)^{-1}\left(\left(\widetilde{G}, E'\right)\right)$, so $(f, \varphi)^{-1}\left(\left(\widetilde{G}, E'\right)\right)$ is a neutrosophic soft open set over X.

(2)⇒(1) Let $x^e_{(\alpha, \beta, \gamma)}$ be a neutrosophic soft point and $(f, \varphi)\left(x^e_{(\alpha, \beta, \gamma)}\right) \in \left(\widetilde{G}, E'\right)$ be an arbitrary soft neighborhood. Then $x^e_{(\alpha, \beta, \gamma)} \in (f, \varphi)^{-1}\left(\left(\widetilde{G}, E'\right)\right)$ is a soft neighborhood and $(f, \varphi)\left((f, \varphi)^{-1}\left(\left(\widetilde{G}, E'\right)\right)\right) \subset \left(\widetilde{G}, E'\right)$.

(2)⇒(3) The proof is obtained definition of complement of neutrosophic soft set.

(3)⇒(4) Let $\left(\widetilde{F}, E\right)$ be a neutrosophic soft set over X.

Since $\left(\widetilde{F}, E\right) \subset (f, \varphi)^{-1}\left((f, \varphi)\left(\left(\widetilde{F}, E\right)\right)\right)$ and $(f, \varphi)\left(\left(\widetilde{F}, E\right)\right) \subset \overline{(f, \varphi)\left(\left(\widetilde{F}, E\right)\right)}$, we have

$$\left(\widetilde{F}, E\right) \subset (f, \varphi)^{-1}\left((f, \varphi)\left(\left(\widetilde{F}, E\right)\right)\right) \subset (f, \varphi)^{-1}\overline{\left((f, \varphi)\left(\left(\widetilde{F}, E\right)\right)\right)}.$$

From (3), since $(f, \varphi)^{-1} \overline{\left((f, \varphi) \left(\overline{\left(\widetilde{F}, E \right)} \right) \right)}$ is a neutrosophic soft closed set over X,

$$\overline{\left(\widetilde{F}, E \right)} \subset (f, \varphi)^{-1} \overline{\left((f, \varphi) \left(\overline{\left(\widetilde{F}, E \right)} \right) \right)}.$$

Thus

$$(f, \varphi) \overline{\left(\overline{\left(\widetilde{F}, E \right)} \right)} \subset (f, \varphi) \left((f, \varphi)^{-1} \overline{\left((f, \varphi) \left(\widetilde{F}, E \right) \right)} \right) \subset \overline{\left((f, \varphi) \left(\widetilde{F}, E \right) \right)}$$

is obtained.

(4)$\Rightarrow$(5) Let $\left(\widetilde{G}, E' \right)$ be a neutrosophic soft set over Y and $(f, \varphi)^{-1} \left(\left(\widetilde{G}, E' \right) \right) = \left(\widetilde{F}, E \right)$. From (4), we have

$$(f, \varphi) \overline{\left(\left(\widetilde{F}, E \right) \right)} = (f, \varphi) \overline{\left((f, \varphi)^{-1} \left(\left(\widetilde{G}, E' \right) \right) \right)} \subset \overline{\left((f, \varphi) \left((f, \varphi)^{-1} \left(\left(\widetilde{G}, E' \right) \right) \right) \right)} \subset \overline{\left(\widetilde{G}, E' \right)}.$$

Then

$$\overline{(f, \varphi)^{-1} \left(\left(\widetilde{G}, E' \right) \right)} = \overline{\left(\widetilde{F}, E \right)} \subset (f, \varphi)^{-1} \left((f, \varphi) \left(\overline{\left(\widetilde{F}, E \right)} \right) \right) \subset (f, \varphi)^{-1} \left(\overline{\left(\widetilde{G}, E' \right)} \right)$$

is obtained.

(5)$\Rightarrow$(6) Let $\left(\widetilde{G}, E' \right)$ be a neutrosophic soft set over Y. Substituting $\left(\widetilde{G}, E' \right)^c$ for condition in (5). Then

$$\overline{(f, \varphi)^{-1} \left(\left(\widetilde{G}, E' \right) \right)^c} \subset (f, \varphi)^{-1} \left(\overline{\left(\widetilde{G}, E' \right)^c} \right).$$

It is clear that $\left(\widetilde{G}, E' \right)^\circ = \left(\overline{\left(\widetilde{G}, E' \right)^c} \right)^c$. Then we have

$$
\begin{aligned}
(f, \varphi)^{-1} \left(\left(\widetilde{G}, E' \right)^\circ \right) &= (f, \varphi)^{-1} \left(\left(\overline{\left(\widetilde{G}, E' \right)^c} \right)^c \right) = \left((f, \varphi)^{-1} \left(\overline{\left(\widetilde{G}, E' \right)^c} \right) \right)^c \\
&\subset \left(\overline{(f, \varphi)^{-1} \left(\widetilde{G}, E' \right)^c} \right)^c = \left((f, \varphi)^{-1} \left(\left(\widetilde{G}, E' \right) \right) \right)^\circ.
\end{aligned}
$$

(6)$\Rightarrow$(2) Let $\left(\widetilde{G}, E' \right)$ be a neutrosophic soft open set over Y. Since

$$
\begin{aligned}
\left((f, \varphi)^{-1} \left(\left(\widetilde{G}, E' \right) \right) \right)^\circ &\subset (f, \varphi)^{-1} \left(\left(\widetilde{G}, E' \right) \right) \\
&= (f, \varphi)^{-1} \left(\left(\widetilde{G}, E' \right)^\circ \right) \subset \left((f, \varphi)^{-1} \left(\left(\widetilde{G}, E' \right) \right) \right)^\circ,
\end{aligned}
$$

then $\left((f, \varphi)^{-1} \left(\left(\widetilde{G}, E' \right) \right) \right)^\circ = \left((f, \varphi)^{-1} \left(\left(\widetilde{G}, E' \right) \right) \right)$ is obtained. This implies that $(f, \varphi)^{-1} \left(\left(\widetilde{G}, E' \right) \right)$ is a neutrosophic soft open set over X. $\quad \square$

Example 2.7. Let $(X, \tau, E), (Y, \tau', E')$ be two neutrosophic soft topological spaces, $(f, \varphi) : (X, \tau, E) \to (Y, \tau', E')$ be a soft mapping. If τ' is neutrosophic soft indiscrete topology on Y, then (f, φ) is a neutrosophic soft continuous mapping.

Example 2.8. Let $(X, \tau, E), (Y, \tau', E')$ be two neutrosophic soft topological spaces, $(f, \varphi) : (X, \tau, E) \to (Y, \tau', E')$ be a mapping. If τ is neutrosophic soft discrete topology on Y, then (f, φ) is a neutrosophic soft continuous mapping.

Theorem 2.9. *If $(f, \varphi) : (X, \tau, E) \to (Y, \tau', E')$ is a neutrosophic soft continuous mapping, then for each $e \in E$, $f_e : (X, \tau_e) \to \left(Y, \tau'_{\varphi(e)} \right)$ is a neutrosophic continuous mapping.*

Proof. Let $U \in \tau'_{\varphi(e)}$. Then there exists a neutrosophic soft open set $\left(\widetilde{G}, E'\right)$ over Y such that

$$U = \{\langle y, G\left(\varphi\left(e\right)\right)\left(y\right), J\left(\varphi\left(e\right)\right)\left(y\right), P\left(\varphi\left(e\right)\right)\left(y\right)\rangle : y \in Y\}.$$

Since $(f, \varphi) : (X, \tau, E) \rightarrow (Y, \tau', E')$ is a neutrosophic soft continuous mapping, $(f, \varphi)^{-1}\left(\left(\widetilde{G}, E'\right)\right)$ is a neutrosophic soft open set over X and,

$$\begin{aligned}
(f, \varphi)^{-1}\left(\left(\widetilde{G}, E'\right)\right)(e)(x) &= \left\{\left\langle x, (f, \varphi)^{-1}(G)(e)(x), (f, \varphi)^{-1}(J)(e)(x), (f, \varphi)^{-1}(P)(e)(x)\right\rangle : x \in X\right\} \\
&= f_e^{-1}(U)
\end{aligned}$$

is a neutrosophic open set. This implies that $f_e : (X, \tau_e) \rightarrow \left(Y, \tau'_{\varphi(e)}\right)$ is a neutrosophic continuous mapping. $\square$

Now we give an example to show that the converse of the above theorem does not hold.

Example 2.10. Let $X = \{x, y, z\}, Y = \{a, b, c\}$ and $E = \{e_1, e_2\}$. Here $\left\{\left(\widetilde{F}_i, E\right) : 1 \le i \le 9\right\}$ are neutrosophic soft sets over X and $\left(\widetilde{G}_1, E\right), \left(\widetilde{G}_2, E\right)$ are neutrosophic soft sets over Y, defined as follows:

$$\left(\widetilde{F}_1, E\right) = \left\{\begin{array}{l} e_1 = \left\{\left\langle x, \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right\rangle, \left\langle y, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle, \left\langle z, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle\right\}, \\ e_2 = \left\{\left\langle x, \frac{5}{7}, \frac{5}{7}, \frac{2}{7}\right\rangle, \left\langle y, \frac{1}{5}, \frac{1}{5}, \frac{4}{5}\right\rangle, \left\langle z, \frac{1}{5}, \frac{1}{5}, \frac{4}{5}\right\rangle\right\} \end{array}\right\},$$

$$\left(\widetilde{F}_2, E\right) = \left\{\begin{array}{l} e_1 = \left\{\left\langle x, \frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right\rangle, \left\langle y, \frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right\rangle, \left\langle z, \frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right\rangle\right\}, \\ e_2 = \left\{\langle x, 0, 0, 1\rangle, \langle y, 1, 1, 0\rangle, \left\langle z, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle\right\} \end{array}\right\},$$

$$\left(\widetilde{F}_3, E\right) = \left\{\begin{array}{l} e_1 = \left\{\left\langle x, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle, \left\langle y, \frac{1}{5}, \frac{1}{5}, \frac{4}{5}\right\rangle, \left\langle z, \frac{1}{5}, \frac{1}{5}, \frac{4}{5}\right\rangle\right\}, \\ e_2 = \left\{\left\langle x, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle, \langle y, 0, 0, 1\rangle, \langle z, 0, 0, 1\rangle\right\} \end{array}\right\},$$

$$\left(\widetilde{F}_4, E\right) = \left\{\begin{array}{l} e_1 = \left\{\left\langle x, \frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right\rangle, \left\langle y, \frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right\rangle, \left\langle z, \frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right\rangle\right\}, \\ e_2 = \left\{\langle x, 0, 0, 1\rangle, \left\langle y, \frac{1}{5}, \frac{1}{5}, \frac{4}{5}\right\rangle, \left\langle z, \frac{1}{5}, \frac{1}{5}, \frac{4}{5}\right\rangle\right\} \end{array}\right\},$$

$$\left(\widetilde{F}_5, E\right) = \left\{\begin{array}{l} e_1 = \left\{\left\langle x, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle, \left\langle y, \frac{1}{5}, \frac{1}{5}, \frac{4}{5}\right\rangle, \left\langle z, \frac{1}{5}, \frac{1}{5}, \frac{4}{5}\right\rangle\right\}, \\ e_2 = \left\{\left\langle x, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle, \langle y, 0, 0, 1\rangle, \langle z, 0, 0, 1\rangle\right\} \end{array}\right\},$$

$$\left(\widetilde{F}_6, E\right) = \left\{\begin{array}{l} e_1 = \left\{\left\langle x, \frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right\rangle, \left\langle y, \frac{1}{5}, \frac{1}{5}, \frac{4}{5}\right\rangle, \left\langle z, \frac{1}{5}, \frac{1}{5}, \frac{4}{5}\right\rangle\right\}, \\ e_2 = \{\langle x, 0, 0, 1\rangle, \langle y, 0, 0, 1\rangle, \langle z, 0, 0, 1\rangle\} \end{array}\right\},$$

$$\left(\widetilde{F}_7, E\right) = \left\{\begin{array}{l} e_1 = \left\{\left\langle x, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle, \left\langle y, \frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right\rangle, \left\langle z, \frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right\rangle\right\}, \\ e_2 = \left\{\left\langle x, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle, \langle y, 1, 1, 0\rangle, \left\langle z, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle\right\} \end{array}\right\},$$

$$\left(\widetilde{F}_8, E\right) = \left\{\begin{array}{l} e_1 = \left\{\left\langle x, \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right\rangle, \left\langle y, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle, \left\langle z, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle\right\}, \\ e_2 = \left\{\left\langle x, \frac{5}{7}, \frac{5}{7}, \frac{2}{7}\right\rangle, \langle y, 1, 1, 0\rangle, \left\langle z, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle\right\} \end{array}\right\},$$

$$\left(\widetilde{F}_9, E\right) = \left\{\begin{array}{l} e_1 = \left\{\left\langle x, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle, \left\langle y, \frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right\rangle, \left\langle z, \frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right\rangle\right\}, \\ e_2 = \left\{\left\langle x, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle, \left\langle y, \frac{1}{5}, \frac{1}{5}, \frac{4}{5}\right\rangle, \left\langle z, \frac{1}{5}, \frac{1}{5}, \frac{4}{5}\right\rangle\right\} \end{array}\right\}.$$

and

$$\left(\widetilde{G}_1, E\right) = \left\{\begin{array}{l} e_1 = \left\{\left\langle a, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle, \left\langle b, \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right\rangle, \left\langle c, \frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right\rangle\right\}, \\ e_2 = \left\{\left\langle a, \frac{1}{5}, \frac{1}{5}, \frac{4}{5}\right\rangle, \left\langle b, \frac{5}{7}, \frac{5}{7}, \frac{2}{7}\right\rangle, \langle c, 1, 1, 0\rangle\right\} \end{array}\right\},$$

$$\left(\widetilde{G}_2, E\right) = \left\{\begin{array}{l} e_1 = \left\{\left\langle a, \frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right\rangle, \left\langle b, \frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right\rangle, \left\langle c, \frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right\rangle\right\}, \\ e_2 = \left\{\langle a, 0, 0, 1\rangle, \left\langle b, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle, \left\langle c, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}\right\rangle\right\} \end{array}\right\}.$$

Then the family $\tau = \left\{ \left(\widetilde{F}_i, E \right) : \left(\widetilde{F}_i, E \right) \in NSS\left(X, E\right), 1 \le i \le 9 \right\}$ is a neutrosophic soft topology over X and $\tau' = \left\{ \left(\widetilde{G}_i, E \right) : \left(\widetilde{G}_i, E \right) \in NSS\left(Y, E\right), 1 \le i \le 2 \right\}$ is a neutrosophic soft topology over Y. If we get the mapping

$$f : X \to Y, \ \varphi : E \to E$$

defined as

$$f(x) = b, \ f(y) = f(z) = a, \ \varphi(e_i) = e_i, \ i = 1, 2,$$

then (f, φ) is not a neutrosophic soft continuous mapping. Since

$$(f, \varphi)^{-1}\left(\widetilde{G}_2, E \right) = \left\{ \begin{array}{l} e_1 = \left\{ \left\langle x, \frac{1}{4}, \frac{1}{4}, \frac{3}{4} \right\rangle, \left\langle y, \frac{1}{4}, \frac{1}{4}, \frac{3}{4} \right\rangle, \left\langle z, \frac{1}{4}, \frac{1}{4}, \frac{3}{4} \right\rangle \right\}, \\ e_2 = \left\{ \left\langle x, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\rangle, \left\langle y, 0, 0, 1 \right\rangle, \left\langle z, 0, 0, 1 \right\rangle \right\} \end{array} \right\}$$

we have $(f, \varphi)^{-1}\left(\widetilde{G}_2, E \right) \notin \tau$. Also,

$$\tau_{e_1} = \left\{ \widetilde{0}, \widetilde{1}, \widetilde{F}_1(e_1), \widetilde{F}_2(e_1), \widetilde{F}_3(e_1), \widetilde{F}_6(e_1), \widetilde{F}_7(e_1) \right\},$$
$$\tau_{e_2} = \left\{ \widetilde{0}, \widetilde{1}, \widetilde{F}_1(e_2), \widetilde{F}_2(e_2), \widetilde{F}_3(e_2), \widetilde{F}_4(e_2), \widetilde{F}_7(e_2), \widetilde{F}_8(e_2), \widetilde{F}_9(e_2) \right\}$$

and

$$\tau'_{e_1} = \left\{ \widetilde{0}, \widetilde{1}, \widetilde{G}_1(e_1), \widetilde{G}_2(e_1) \right\},$$
$$\tau'_{e_2} = \left\{ \widetilde{0}, \widetilde{1}, \widetilde{G}_1(e_2), \widetilde{G}_2(e_2) \right\}$$

are neutrosophic topologies over X and over Y, respectively. Then the mapping

$$f_{e_1} : (X, \tau_{e_1}) \to \left(Y, \tau'_{e_1} \right)$$

is a neutrosophic continuous mapping. Because,

$$f_{e_1}^{-1}\left(\widetilde{G}_1 \right) = \left\{ \left\langle x, \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle, \left\langle y, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\rangle, \left\langle z, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\rangle \right\},$$
$$f_{e_1}^{-1}\left(\widetilde{G}_2 \right) = \left\{ \left\langle x, \frac{1}{4}, \frac{1}{4}, \frac{3}{4} \right\rangle, \left\langle y, \frac{1}{4}, \frac{1}{4}, \frac{3}{4} \right\rangle, \left\langle z, \frac{1}{4}, \frac{1}{4}, \frac{3}{4} \right\rangle \right\},$$

$f_{e_1}^{-1}\left(\widetilde{G}_1 \right) = \widetilde{F}_1(e_1), f_{e_1}^{-1}\left(\widetilde{G}_2 \right) = \widetilde{F}_2(e_1)$ is obtained.

Similarly, the mapping

$$f_{e_2} : (X, \tau_{e_2}) \to \left(Y, \tau'_{e_2} \right)$$

is a neutrosophic continuous mapping. Because

$$f_{e_2}^{-1}\left(\widetilde{G}_1 \right) = \left\{ \left\langle x, \frac{5}{7}, \frac{5}{7}, \frac{2}{7} \right\rangle, \left\langle y, \frac{1}{5}, \frac{1}{5}, \frac{4}{5} \right\rangle, \left\langle z, \frac{1}{5}, \frac{1}{5}, \frac{4}{5} \right\rangle \right\},$$
$$f_{e_2}^{-1}\left(\widetilde{G}_2 \right) = \left\{ \left\langle x, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\rangle, \left\langle y, 0, 0, 1 \right\rangle, \left\langle z, 0, 0, 1 \right\rangle \right\},$$

$f_{e_2}^{-1}\left(\widetilde{G}_1 \right) = \widetilde{F}_1(e_2), f_{e_2}^{-1}\left(\widetilde{G}_2 \right) = \widetilde{F}_3(e_2)$ is obtained.

Theorem 2.11. *Let $(X, \tau, E), (Y, \tau', E')$ and (Z, τ^*, E'') be neutrosophic soft topological spaces. If $(f, \varphi) : (X, \tau, E) \to (Y, \tau', E')$ and $(g, \psi) : (Y, \tau', E') \to (Z, \tau^*, E'')$ are neutrosophic soft continuous mappings, then $(g \circ f, \psi \circ \varphi) : (X, \tau, E) \to (Z, \tau^*, E'')$ is a neutrosophic soft continuous mapping.*

Proof. Let $(\widetilde{W}, E) \in \tau^*$ be a neutrosophic soft open set and let us show that $((g \circ f, \psi \circ \varphi))^{-1} (\widetilde{W}, E'') \in \tau$. Since $((g \circ f, \psi \circ \varphi))^{-1} (\widetilde{W}, E'') = (f^{-1} \circ g^{-1}, \varphi^{-1} \circ \psi^{-1}) (\widetilde{W}, E'')$ and the mapping (g, ψ) is a neutrosophic soft continuous mapping, then $(g, \psi)^{-1} (\widetilde{W}, E'')$ is a neutrosophic soft open set in Y. Other hand, since (f, φ) is a neutrosophic soft continuous mapping, then $(f^{-1} \circ g^{-1}, \varphi^{-1} \circ \psi^{-1}) (\widetilde{W}, E'')$ is a neutrosophic soft open set in X. That is, $((g \circ f, \psi \circ \varphi))^{-1} (\widetilde{W}, E'')$ is a neutrosophic soft open set in X and $(g \circ f, \psi \circ \varphi)$ is a neutrosophic soft continuous mapping. $\square$

Definition 2.12. Let $(X, \tau, E), (Y, \tau', E')$ be two neutrosophic soft topological spaces, $(f, \varphi) : (X, \tau, E) \rightarrow (Y, \tau', E')$ be a soft mapping. If the image $(f, \varphi) ((\widetilde{F}, E))$ of each neutrosophic soft open set $(\widetilde{F}, E)$ over X is a neutrosophic soft open set in Y, then (f, φ) is said to be a neutrosophic soft open mapping.

Definition 2.13. Let $(X, \tau, E), (Y, \tau', E')$ be two neutrosophic soft topological spaces, $(f, \varphi) : (X, \tau, E) \rightarrow (Y, \tau', E')$ be a soft mapping. If the image $(f, \varphi) ((\widetilde{H}, E))$ of each neutrosophic soft closed set $(\widetilde{H}, E)$ over X is a neutrosophic soft closed set in Y, then (f, φ) is said to be a neutrosophic soft closed mapping.

Proposition 2.14. *If $(f, \varphi) : (X, \tau, E) \rightarrow (Y, \tau', E')$ is a neutrosophic soft open (closed) mapping, then for each $e \in E$, $f_e : (X, \tau_e) \rightarrow \left(Y, \tau'_{\varphi(e)}\right)$ is a neutrosophic open (closed) mapping.*

Proof. The proof of the proposition is straightforward and it is left to the reader. $\square$

It is clear that the concepts of neutrosophic soft continuous, neutrosophic soft open and neutrosophic soft closed mappings are all independent of each other.

Example 2.15. Let $X = \{x, y\}, Y = \{a, b\}$ and $E = \{e_1, e_2\}$. We define the mapping

$$f : X \rightarrow Y, \ \varphi : E \rightarrow E$$

defined as

$$f(x) = b = f(y), \ \varphi(e_i) = e_i, \ i = 1, 2,$$

then (f, φ) is not a neutrosophic soft continuous mapping.

Here $\left\{(\widetilde{F}_i, E) : 1 \le i \le 4\right\}$ are neutrosophic soft sets over X and $\left\{(\widetilde{G}_i, E) : 1 \le i \le 6\right\}$ are neutrosophic soft sets over Y, defined as follows:

$$
(\widetilde{F}_1, E) = \left\{ \begin{array}{l} e_1 = \left\{\left\langle x, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}\right\rangle, \left\langle y, \frac{2}{5}, \frac{2}{5}, \frac{3}{5}\right\rangle\right\}, \\ e_2 = \left\{\left\langle x, \frac{2}{7}, \frac{2}{7}, \frac{5}{7}\right\rangle, \left\langle y, \frac{3}{4}, \frac{3}{4}, \frac{1}{4}\right\rangle\right\} \end{array} \right\},
$$

$$
(\widetilde{F}_2, E) = \left\{ \begin{array}{l} e_1 = \left\{\left\langle x, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}\right\rangle, \left\langle y, \frac{1}{5}, \frac{1}{5}, \frac{4}{5}\right\rangle\right\}, \\ e_2 = \left\{\left\langle x, \frac{4}{5}, \frac{4}{5}, \frac{1}{5}\right\rangle, \left\langle y, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle\right\} \end{array} \right\},
$$

$$
(\widetilde{F}_3, E) = \left\{ \begin{array}{l} e_1 = \left\{\left\langle x, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}\right\rangle, \left\langle y, \frac{1}{5}, \frac{1}{5}, \frac{4}{5}\right\rangle\right\}, \\ e_2 = \left\{\left\langle x, \frac{2}{7}, \frac{2}{7}, \frac{5}{7}\right\rangle, \left\langle y, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle\right\} \end{array} \right\},
$$

$$
(\widetilde{F}_4, E) = \left\{ \begin{array}{l} e_1 = \left\{\left\langle x, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}\right\rangle, \left\langle y, \frac{2}{5}, \frac{2}{5}, \frac{3}{5}\right\rangle\right\}, \\ e_2 = \left\langle x, \frac{4}{5}, \frac{4}{5}, \frac{1}{5}\right\rangle, \left\langle y, \frac{3}{4}, \frac{3}{4}, \frac{1}{4}\right\rangle \end{array} \right\},
$$

$$
(\widetilde{G}_1, E) = \left\{ \begin{array}{l} e_1 = \left\{\left\langle a, \frac{2}{5}, \frac{2}{5}, \frac{3}{5}\right\rangle, \langle b, 0, 0, 1\rangle\right\}, \\ e_2 = \left\{\left\langle a, \frac{3}{4}, \frac{3}{4}, \frac{1}{4}\right\rangle, \langle b, 0, 0, 1\rangle\right\} \end{array} \right\},
$$

$$
(\widetilde{G}_2, E) = \left\{ \begin{array}{l} e_1 = \left\{\left\langle a, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}\right\rangle, \langle b, 0, 0, 1\rangle\right\}, \\ e_2 = \left\{\left\langle a, \frac{4}{5}, \frac{4}{5}, \frac{1}{5}\right\rangle, \langle b, 0, 0, 1\rangle\right\} \end{array} \right\},
$$

$$\left(\widetilde{G_3}, E\right) = \left\{ \begin{array}{l} e_1 = \left\{\left\langle a, \frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right\rangle, \langle b, 0, 0, 1\rangle\right\}, \\ e_2 = \left\{\left\langle a, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle, \langle b, 0, 0, 1\rangle\right\} \end{array} \right\},$$

$$\left(\widetilde{G_4}, E\right) = \left\{ \begin{array}{l} e_1 = \left\{\left\langle a, \frac{2}{5}, \frac{2}{5}, \frac{3}{5}\right\rangle, \langle b, 0, 0, 1\rangle\right\}, \\ e_2 = \left\{\left\langle a, \frac{4}{5}, \frac{4}{5}, \frac{1}{5}\right\rangle, \langle b, 0, 0, 1\rangle\right\} \end{array} \right\},$$

$$\left(\widetilde{G_5}, E\right) = \left\{ \begin{array}{l} e_1 = \left\{\left\langle a, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}\right\rangle, \langle b, 0, 0, 1\rangle\right\}, \\ e_2 = \left\{\left\langle a, \frac{3}{4}, \frac{3}{4}, \frac{1}{4}\right\rangle, \langle b, 0, 0, 1\rangle\right\} \end{array} \right\},$$

$$\left(\widetilde{G_6}, E\right) = \left\{ \begin{array}{l} e_1 = \left\{\left\langle a, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}\right\rangle, \langle b, 0, 0, 1\rangle\right\}, \\ e_2 = \left\{\left\langle a, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\rangle, \langle b, 0, 0, 1\rangle\right\} \end{array} \right\}.$$

Here $(f, \varphi)^{-1}\left(\widetilde{G_5}, E\right) \notin \tau$ and $(f, \varphi)\left(\left(\widetilde{F_3}, E\right)^c\right) \notin \tau$ is not a neutrosophic soft closed set. Thus the soft mapping is not neutrosophic soft continuous mapping. Also it is not neutrosophic soft closed mapping. But it is neutrosophic soft open mapping.

Theorem 2.16. *Let* $(X, \tau, E), (Y, \tau', E')$ *be two neutrosophic soft topological spaces,* $(f, \varphi) : (X, \tau, E) \to (Y, \tau', E')$ *be a soft mapping.*

(1) (f, φ) *is a neutrosophic soft open mapping if and only if for each neutrosophic soft set* $\left(\widetilde{F}, E\right)$ *over* X, $(f, \varphi)\left(\left(\widetilde{F}, E\right)^\circ\right) \subset \left((f, \varphi)\left(\left(\widetilde{F}, E\right)\right)\right)^\circ$ *is satisfied.*

(2) (f, φ) *is a neutrosophic soft closed mapping if and only if for each neutrosophic soft set* $\left(\widetilde{F}, E\right)$ *over* X, $\overline{\left((f, \varphi)\left(\left(\widetilde{F}, E\right)\right)\right)} \subset (f, \varphi)\left(\overline{\left(\widetilde{F}, E\right)}\right)$ *is satisfied.*

Proof. (1) Let (f, φ) be a neutrosophic soft open mapping and $\left(\widetilde{F}, E\right)$ be a neutrosophic soft set over X. $\left(\widetilde{F}, E\right)^\circ$ is a neutrosophic soft open set and $\left(\widetilde{F}, E\right)^\circ \subset \left(\widetilde{F}, E\right)$. Since (f, φ) is a neutrosophic soft open mapping, $(f, \varphi)\left(\left(\widetilde{F}, E\right)^\circ\right)$ is a neutrosophic soft open set in Y and $(f, \varphi)\left(\left(\widetilde{F}, E\right)^\circ\right) \subset (f, \varphi)\left(\left(\widetilde{F}, E\right)\right)$. Thus $(f, \varphi)\left(\left(\widetilde{F}, E\right)^\circ\right) \subset \left((f, \varphi)\left(\left(\widetilde{F}, E\right)\right)\right)^\circ$ is obtained.

Conversely, let $\left(\widetilde{F}, E\right)$ be any neutrosophic soft open set over X. Then $\left(\widetilde{F}, E\right) = \left(\widetilde{F}, E\right)^\circ$. From the condition of theorem, we have $(f, \varphi)\left(\left(\widetilde{F}, E\right)^\circ\right) \subset \left((f, \varphi)\left(\left(\widetilde{F}, E\right)\right)\right)^\circ$. Then $(f, \varphi)\left(\left(\widetilde{F}, E\right)\right) = (f, \varphi)\left(\left(\widetilde{F}, E\right)^\circ\right) \subset \left((f, \varphi)\left(\left(\widetilde{F}, E\right)\right)\right)^\circ \subset (f, \varphi)\left(\left(\widetilde{F}, E\right)\right)$. This implies that $(f, \varphi)\left(\left(\widetilde{F}, E\right)\right) = (f, \varphi)\left(\left(\widetilde{F}, E\right)\right)^\circ$. This completes the proof.

(2) Let (f, φ) be a neutrosophic soft closed mapping and $\left(\widetilde{F}, E\right)$ be any neutrosophic soft set over X. Since (f, φ) is a neutrosophic soft closed mapping, $(f, \varphi)\left(\overline{\left(\widetilde{F}, E\right)}\right)$ is a neutrosophic soft closed set in Y and $(f, \varphi)\left(\left(\widetilde{F}, E\right)\right) \subset (f, \varphi)\left(\overline{\left(\widetilde{F}, E\right)}\right)$. Hence $\overline{(f, \varphi)\left(\left(\widetilde{F}, E\right)\right)} \subset \left((f, \varphi)\left(\overline{\left(\widetilde{F}, E\right)}\right)\right)$ is obtained.

Conversely, let $\left(\widetilde{F}, E\right)$ be any neutrosophic soft closed set over X. Then $\left(\widetilde{F}, E\right) = \overline{\left(\widetilde{F}, E\right)}$. From the condition of theorem, we have $\overline{(f, \varphi)\left(\left(\widetilde{F}, E\right)\right)} \subset \left((f, \varphi)\left(\overline{\left(\widetilde{F}, E\right)}\right)\right) = (f, \varphi)\left(\left(\widetilde{F}, E\right)\right) \subset \overline{(f, \varphi)\left(\left(\widetilde{F}, E\right)\right)}$. This implies that $\overline{(f, \varphi)\left(\left(\widetilde{F}, E\right)\right)} = (f, \varphi)\left(\left(\widetilde{F}, E\right)\right)$. This completes the proof. $\square$

Definition 2.17. Let $(X, \tau, E), (Y, \tau', E')$ be two neutrosophic soft topological spaces, $(f, \varphi) : (X, \tau, E) \to (Y, \tau', E')$ be a mapping. If (f, φ) is a bijection, neutrosophic soft continuous anf $(f, \varphi)^{-1}$ is a neutrosophic soft continuous mapping, then (f, φ) is said to be a neutrosophic soft homeomorphism from X to Y. When a neutrosophic soft homeomorphism (f, φ) exists between X and Y, we say that X is neutrosophic soft homeomorphic to Y.

Theorem 2.18. *Let* (X, τ, E), (Y, τ', E') *be two neutrosophic soft topological spaces,* $(f, \varphi) : (X, \tau, E) \rightarrow (Y, \tau', E')$ *be bijective mapping. Then the following conditions are equivalent:*

(1) (f, φ) *is neutrosophic soft homeomorphism,*
(2) (f, φ) *is neutrosophic soft continuous and neutrosophic soft closed mapping,*
(3) (f, φ) *is neutrosophic soft continuous and neutrosophic soft open mapping.*

Proof. The proof can be easily obtained by using the theorems on continuity, openness and closedness, so are omitted. □

3. Conclusion

In the present paper, based on the definition of a neutrosophic soft set, we introduce a kind of definition of neutrosophic soft continuity and give a comprehensive investigation of the neutrosophic soft continuous mapping. Moreover we study the concepts of neutrosophic soft open, neutrosophic soft closed and neutrosophic soft homeomorphism, and present characterization theorems. We hope that, the results of this study may help in the investigation of neutrosophic soft continuous function spaces and in many researches.

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