Filomat 34:10 (2020), 3441–3458
https://doi.org/10.2298/FIL2010441A

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Neutrosophic Soft δ-Topology and Neutrosophic Soft δ-Compactness

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Abstract. We introduce the concepts of neutrosophic soft $\delta-$interior, neutrosophic soft quasi-coincidence, neutrosophic soft q-neighbourhood, neutrosophic soft regular open set, neutrosophic soft $\delta-$closure, neutrosophic soft $\theta-$closure and neutrosophic soft semi open set. It is also shown that the set of all neutrosophic soft $\delta-$open sets is a neutrosophic soft topology, which is called the neutrosophic soft $\delta-$topology. We obtain equivalent forms of neutrosophic soft $\delta-$continuity. Moreover, the notions of neutrosophic soft $\delta-$compactness and neutrosophic soft locally $\delta-$compactness are defined and their basic properties under neutrosophic soft $\delta-$continuous mappings are investigated.

1. Introduction

In 2005, the concept of neutrosophic set was introduced by Smarandache as a generalization of classical sets, fuzzy set theory [25], intuitionistic fuzzy set theory [4], etc. By using the theory of neutrosophic set, many researches were made by mathematicians in subbranches of mathematics [7, 21]. There are many inherent difficulties in classical methods for the inadequacy of the theories of parametrization tools. So, classical methods are insufficient in dealing with several practical problems in some other disciplines such as economics, engineering, environment, social science, medical science, etc. In 1999, Molodtsov pointed out the inherent difficulties of these theories [18]. A different approach was initiated by Molodtsov for modeling uncertainties. This approach was applied in some other directions such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration and so on. The theory of soft topological spaces was introduced by Shabir and Naz [22] for the first time in 2011. Soft topological spaces were defined over an initial universe with a fixed set of parameters and it was showed that a soft topological space gave a parameterized family of topological spaces. In [1, 2, 5, 6, 13, 14, 16, 19], some scientists made researches and did theoretical studies in soft topological spaces. In 2013, Maji [17] defined the concept of neutrosophic soft sets for the first time. Then, Deli and Broumi [15] modified this concept. In 2017, Bera presented neutrosophic soft topological spaces in [8]. Then, he focused on this space and made researches as in [7,9,10,11,12].

In this paper, we introduce the concepts of neutrosophic soft $\delta-$interior, neutrosophic soft quasi-coincidence, neutrosophic soft q-neighbourhood, neutrosophic soft regular open set, neurosophic soft

2010 *Mathematics Subject Classification*. Primary 54A05; Secondary 54A05, 54C10, 54D30, 54D45

Keywords. Neutrosophic soft quasi-coincidence, neutrosophic soft regular open set, neutrosophic soft δ-closed, neutrosophic soft semi open, neutrosophic soft δ-topology

Received: 12 November 2019; Revised: 04 April 2020; Accepted: 08 April 2020

Communicated by Ljubiša D.R. Kočinac

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$\delta-$cluster point, neutrosophic soft $\delta-$closure, neurosophic soft $\theta-$cluster point, neutrosophic soft $\theta-$closure, neutrosophic soft $\delta-$neighbourhood and neutrosophic soft semi open set. It is also shown that the set of all neutrosophic soft $\delta-$open sets is also a neutrosophic soft topology, which is called the neutrosophic soft $\delta-$topology. We obtain equivalent forms of neutrosophic soft $\delta-$continuity. Moreover, the notions of neutrosophic soft $\delta-$compactness and neutrosophic soft locally $\delta-$compactness are defined and their basic properties under neutrosophic soft $\delta-$continuous mappings are investigated.

2. Preliminaries

In this section, we present the basic definitions and theorems related to neutrosophic soft set theory.

Definition 2.1. ([23]) A neutrosophic set A on the universe set X is defined as:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\},$$

where $T, I, F : X \to]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Scientifically, membership functions, indeterminacy functions and non-membership functions of a neutrosophic set take value from real standard or nonstandard subsets of $]^-0, 1^+[$. However, these subsets are sometimes inconvenient to be used in real life applications such as economical and engineering problems. On account of this fact, we consider the neutrosophic sets, whose membership function, indeterminacy functions and non-membership functions take values from subsets of $[0, 1]$.

Definition 2.2. ([18]) Let X be an initial universe, E be a set of all parameters and $P(X)$ denote the power set of X. A pair (F, E) is called a soft set over X, where F is a mapping given by $F : E \longrightarrow P(X)$. In other words, the soft set is a parameterized family of subsets of the set X. For $e \in E$, $F(e)$ may be considered as the set of e-elements of the soft set (F, E) or as the set of e-approximate elements of the soft set, i.e. $(F, E) = \{(e, F(e)) : e \in E, F : E \longrightarrow P(X)\}$.

After the neutrosophic soft set was defined by Maji [16], this concept was modified by Deli and Broumi [15] as given below.

Definition 2.3. ([15]) Let X be an initial universe set and E be a set of parameters. Let $NS(X)$ denote the set of all neutrosophic sets of X. Then, a neutrosophic soft set $(\widetilde{F}, E)$ over X is a set defined by a set valued function $\widetilde{F}$ representing a mapping $\widetilde{F} : E \to NS(X)$, where $\widetilde{F}$ is called the approximate function of the neutrosophic soft set $(\widetilde{F}, E)$. In other words, the neutrosophic soft set is a parametrized family of some elements of the set $NS(X)$ and therefore it can be written as a set of ordered pairs:

$$\left(\widetilde{F}, E\right) = \left\{\left(e, \left\langle x, T_{\widetilde{F}(e)}(x), I_{\widetilde{F}(e)}(x), F_{\widetilde{F}(e)}(x)\right\rangle : x \in X\right) : e \in E\right\},$$

where $T_{\widetilde{F}(e)}(x), I_{\widetilde{F}(e)}(x), F_{\widetilde{F}(e)}(x) \in [0, 1]$ are respectively called the truth-membership, indeterminacy-membership and falsity-membership function of $\widetilde{F}(e)$. Since the supremum of each T, I, F is 1, the inequality $0 \leq T_{\widetilde{F}(e)}(x) + I_{\widetilde{F}(e)}(x) + F_{\widetilde{F}(e)}(x) \leq 3$ is obvious.

Definition 2.4. ([8]) Let $(\widetilde{F}, E)$ be a neutrosophic soft set over the universe set X. The complement of $(\widetilde{F}, E)$ is denoted by $(\widetilde{F}, E)^c$ and is defined by

$$\left(\widetilde{F}, E\right)^c = \left\{\left(e, \left\langle x,\ F_{\widetilde{F}(e)}(x), 1 - I_{\widetilde{F}(e)}(x),\ T_{\widetilde{F}(e)}(x)\right\rangle : x \in X\right) : e \in E\right\}.$$

It is obvious that $\left[\left(\widetilde{F}, E\right)^c\right]^c = \left(\widetilde{F}, E\right)$.

Definition 2.5. ([17]) Let $\left(\widetilde{F}, E\right)$ and $\left(\widetilde{G}, E\right)$ be two neutrosophic soft sets over the universe set X. $\left(\widetilde{F}, E\right)$ is said to be a neutrosophic soft subset of $\left(\widetilde{G}, E\right)$ if

$$T_{\widetilde{F}(e)}(x) \leq T_{\widetilde{G}(e)}(x), I_{\widetilde{F}(e)}(x) \leq I(x), F_{\widetilde{F}(e)}(x) \geq F_{\widetilde{G}(e)}(x), \forall e \in E, \forall x \in X.$$

It is denoted by $\left(\widetilde{F}, E\right) \subseteq \left(\widetilde{G}, E\right)$. $\left(\widetilde{F}, E\right)$ is said to be neutrosophic soft equal to $\left(\widetilde{G}, E\right)$ if $\left(\widetilde{F}, E\right) \subseteq \left(\widetilde{G}, E\right)$ and $\left(\widetilde{G}, E\right) \subseteq \left(\widetilde{F}, E\right)$. It is denoted by $\left(\widetilde{F}, E\right) = \left(\widetilde{G}, E\right)$.

Definition 2.6. ([3]) Let $\left(\widetilde{F}_1, E\right)$ and $\left(\widetilde{F}_2, E\right)$ be two neutrosophic soft sets over the universe set X. Then, their union is denoted by $\left(\widetilde{F}_1, E\right) \cup \left(\widetilde{F}_2, E\right) = \left(\widetilde{F}_3, E\right)$ and is defined by

$$\left(\widetilde{F}_3, E\right) = \left\{\left(e, \left\langle x,\ T_{\widetilde{F}_3(e)}(x),\ I(x),\ F_{\widetilde{F}_3(e)}(x)\right\rangle : x \in X\right) e \in E\right\},$$

where

$$T_{\widetilde{F}_3(e)}(x) = max\left\{T_{\widetilde{F}_1(e)}(x), T_{\widetilde{F}_2(e)}(x)\right\},$$

$$I_{\widetilde{F}_3(e)}(x) = max\left\{I_{\widetilde{F}_1(e)}(x), I_{\widetilde{F}_2(e)}(x)\right\},$$

$$F_{\widetilde{F}_3(e)}(x) = min\left\{F_{\widetilde{F}_1(e)}(x), F_{\widetilde{F}_2(e)}(x)\right\}.$$

Definition 2.7. ([3]) Let $\left(\widetilde{F}_1, E\right)$ and $\left(\widetilde{F}_2, E\right)$ be two neutrosophic soft sets over the universe set X. Then, their intersection is denoted by $\left(\widetilde{F}_1, E\right) \cap \left(\widetilde{F}_2, E\right) = \left(\widetilde{F}_4, E\right)$ and is defined by

$$\left(\widetilde{F}_4, E\right) = \left\{\left(e, \left\langle x,\ T_{\widetilde{F}_4(e)}(x),\ I_{\widetilde{F}_4(e)}(x),\ F_{\widetilde{F}_4(e)}(x)\right\rangle : x \in X\right) e \in E\right\},$$

where

$$T_{\widetilde{F}_4(e)}(x) = min\left\{T_{\widetilde{F}_1(e)}(x), T_{\widetilde{F}_2(e)}(x)\right\},$$

$$I_{\widetilde{F}_4(e)}(x) = min\left\{I_{\widetilde{F}_1(e)}(x), I_{\widetilde{F}_2(e)}(x)\right\},$$

$$F_{\widetilde{F}_4(e)}(x) = max\left\{F_{\widetilde{F}_1(e)}(x), F_{\widetilde{F}_2(e)}(x)\right\}.$$

Definition 2.8. ([3]) A neutrosophic soft set $\left(\widetilde{F}, E\right)$ over the universe set X is said to be a null neutrosophic soft set if $T_{\widetilde{F}(e)}(x) = 0,\ I_{\widetilde{F}(e)}(x) = 0,\ F_{\widetilde{F}(e)}(x) = 1; \forall e \in E,\ \forall x \in X$. It is denoted by $0_{(X,E)}$.

Definition 2.9. ([3]) A neutrosophic soft set $\left(\widetilde{F}, E\right)$ over the universe set X is said to be an absolute neutrosophic soft set if $T_{\widetilde{F}(e)}(x) = 1,\ I_{\widetilde{F}(e)}(x) = 1,\ F_{\widetilde{F}(e)}(x) = 0; \forall e \in E,\ \forall x \in X$. It is denoted by $1_{(X,E)}$. Clearly $0^c_{(X,E)} = 1_{(X,E)}$ and $1^c_{(X,E)} = 0_{(X,E)}$.

Definition 2.10. ([3]) Let $NSS(X, E)$ be the family of all neutrosophic soft sets over the universe set X and $\tau \subset NSS(X, E)$. Then, τ is said to be a neutrosophic soft topology on X if

1. $0_{(X,E)}$ and $1_{(X,E)}$ belong to τ,
2. the union of any number of neutrosophic soft sets in τ belongs to τ ,
3. the intersection of a finite number of neutrosophic soft sets in τ belongs to τ.

Then, (X, τ, E) is said to be a neutrosophic soft topological space over X. Each member of τ is said to be a neutrosophic soft open set [3].

Definition 2.11. ([3]) Let (X, τ, E) be a neutrosophic soft topological space over X and $\left(\widetilde{F}, E\right)$ be a neutrosophic soft set over X. Then $\left(\widetilde{F}, E\right)$ is said to be a neutrosophic soft closed set iff its complement is a neutrosophic soft open set.

Definition 2.12. ([3]) Let $NSS(X, E)$ be the family of all neutrosophic soft sets over the universe set X. Then, neutrosophic soft set $x^{e}_{(\alpha,\beta,\gamma)}$ is called a neutrosophic soft point for every $x \in X$, $0 < \alpha, \beta, \gamma \leq 1$,$e \in E$ and is defined as

$$x^{e}_{(\alpha,\beta,\gamma)}(e')(y) = \begin{cases} (\alpha,\beta,\gamma), & \text{if } e' = e \text{ and } y = x \\ (0,0,1), & \text{if } e' \neq e \text{ or } y \neq x \end{cases}$$

It is clear that every neutrosophic soft set is the union of its neutrosophic soft points.

Definition 2.13. ([3]) Let $\left(\widetilde{F}, E\right)$ be a neutrosophic soft set over the universe set X. We say that $x^{e}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F}, E\right)$ is read as belonging to the neutrosophic soft set $\left(\widetilde{F}, E\right)$, whenever

$$\alpha \leq T_{\widetilde{F}(e)}(x), \beta \leq I_{\widetilde{F}(e)}(x) \text{ and } \gamma \geq F_{\widetilde{F}(e)}(x).$$

Definition 2.14. ([3]) Let $x^{e}_{(\alpha,\beta,\gamma)}$ and $y^{e'}_{(\alpha',\beta',\gamma')}$ be two neutrosophic soft points. For the neutrosophic soft points $x^{e}_{(\alpha,\beta,\gamma)}$ and $y^{e'}_{(\alpha',\beta',\gamma')}$ over a common universe X, we say that the neutrosophic soft points are distinct points if $x^{e}_{(\alpha,\beta,\gamma)} \cap y^{e'}_{(\alpha',\beta',\gamma')} = 0_{(X,E)}$. It is clear that $x^{e}_{(\alpha,\beta,\gamma)}$ and $y^{e'}_{(\alpha',\beta',\gamma')}$ are distinct neutrosophic soft points if and only if $x \neq y$ or $e \neq e'$.

Definition 2.15. ([7]) Let $\left(\widetilde{F}, E_1\right)$, $\left(\widetilde{G}, E_2\right)$ be two neutrosophic soft sets over the universal set X. Then, their cartesian product is another neutrosophic soft set $\left(\widetilde{K}, E_3\right) = \left(\widetilde{F}, E_1\right) \times \left(\widetilde{G}, E_2\right)$, where $E_3 = E_1 \times E_2$ and $\widetilde{K}(e_1, e_2) = \widetilde{F}(e_1) \times \widetilde{G}(e_2)$. The truth, indeterminacy and falsity membership of $\left(\widetilde{K}, E_3\right)$ are given by $\forall e_1 \in E_1, \forall e_2 \in E_2, \forall x \in X$,

$$\begin{aligned} T_{\widetilde{K}(e_1,e_2)}(x) &= min\left\{T_{\widetilde{F}(e_1)}(x), T_{\widetilde{G}(e_2)}(x)\right\}, \\ I_{\widetilde{K}(e_1,e_2)}(x) &= I_{\widetilde{F}(e_1)}(x) \cdot I_{\widetilde{G}(e_2)}(x), \\ F_{\widetilde{K}(e_1,e_2)}(x) &= max\left\{F_{\widetilde{F}(e_1)}(x), F_{\widetilde{G}(e_2)}(x)\right\} \end{aligned}$$

This definition can be extended for more than two neutrosophic soft sets.

Definition 2.16. ([7]) A neutrosophic soft relation $\widetilde{R}$ between two neutrosophic soft sets $\left(\widetilde{F}, E_1\right)$ and $\left(\widetilde{G}, E_2\right)$ over the common universe X is the neutrosophic soft subset of $\left(\widetilde{F}, E_1\right) \times \left(\widetilde{G}, E_2\right)$. Clearly, it is another neutrosophic soft set $\left(\widetilde{R}, E_3\right)$, where $E_3 \subseteq E_1 \times E_2$ and $\widetilde{R}(e_1, e_2) = \widetilde{F}(e_1) \times \widetilde{G}(e_2)$ for $(e_1, e_2) \in E_3$.

Definition 2.17. ([7]) Let $\left(\widetilde{F}, E_1\right)$, $\left(\widetilde{G}, E_2\right)$ be two neutrosophic soft sets over the universal set X and f be a neutrosophic soft relation defined on $\left(\widetilde{F}, E_1\right) \times \left(\widetilde{G}, E_2\right)$. Then, f is called neutrosophic soft function, if f associates each element of $\left(\widetilde{F}, E_1\right)$ with the unique element of $\left(\widetilde{G}, E_2\right)$. We write $f : \left(\widetilde{F}, E_1\right) \rightarrow \left(\widetilde{G}, E_2\right)$ as a neutrosophic soft function or a mapping. For $x^{e}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F}, E_1\right)$ and $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{G}, E_2\right)$, when $x^{e}_{(\alpha,\beta,\gamma)} \times y^{e'}_{(\alpha',\beta',\gamma')} \in f$, we denote it by $f\left(x^{e}_{(\alpha,\beta,\gamma)}\right) = y^{e'}_{(\alpha',\beta',\gamma')}$. Here, $\left(\widetilde{F}, E_1\right)$ and $\left(\widetilde{G}, E_2\right)$ are called domain and codomain respectively and $y^{e'}_{(\alpha',\beta',\gamma')}$ is the image of $x^{e}_{(\alpha,\beta,\gamma)}$ under f.

Definition 2.18. ([7]) Let $f : (F, A) \to (G, B)$ be a neutrosophic soft function over the universal set U. If there exists another neutrosophic soft function $g : (G, B) \to (F, A)$ with $g \circ f : (F, A) \to (F, A)$ and $f \circ g : (G, B) \to (G, B)$ such that $g \circ f = I_{(F,A)}$ and $f \circ g = I_{(G,B)}$ then g is called the inverse neutrosophic soft function of f. It is denoted by f^{-1} and is defined as $F(a) \times G(b) \in f^{-1}$ iff $G(b) \times F(a) \in f$.

Definition 2.19. ([8]) Let (X, τ, E) be a neutrosophic soft topological space and $\left(\widetilde{F}, E\right) \in NSS(X, E)$ be arbitrary. Then, the interior of $\left(\widetilde{F}, E\right)$ is denoted by $\left(\widetilde{F}, E\right)^{\circ}$ and is defined as :

$$\left(\widetilde{F},E\right)^{\circ} = \bigcup\{\left(\widetilde{G},E\right) : \left(\widetilde{G},E\right) \subset \left(\widetilde{F},E\right) \ , \left(\widetilde{G},E\right) \in \tau\},$$

i.e. it is the union of all open neutrosophic soft subsets of $\left(\widetilde{F}, E\right)$.

Definition 2.20. ([8]) Let (X, τ, E) be a neutrosophic soft topological space and $\left(\widetilde{F}, E\right) \in NSS(X, E)$ be arbitrary. Then the closure of $\left(\widetilde{F}, E\right)$ is denoted by $\overline{\left(\widetilde{F}, E\right)}$ and is defined as:

$$\overline{\left(\widetilde{F},E\right)} = \bigcap\{\left(\widetilde{G},E\right) : \left(\widetilde{G},E\right) \subset \left(\widetilde{F},E\right) \ , \left(\widetilde{G},E\right)^{c} \in \tau\},$$

i.e. it is the intersection of all closed neutrosophic soft super sets of $\left(\widetilde{F}, E\right)$.

3. Some Definitions

Definition 3.1. A neutrosophic soft point $x^{e}_{(\alpha,\beta,\gamma)}$ is said to be neutrosophic soft quasi-coincident (neutrosophic soft q-coincident, for short) with $\left(\widetilde{F}, E\right)$, denoted by $x^{e}_{(\alpha,\beta,\gamma)} q \left(\widetilde{F}, E\right)$, if and only if $x^{e}_{(\alpha,\beta,\gamma)} \not\subseteq \left(\widetilde{F}, E\right)^{c}$. If $x^{e}_{(\alpha,\beta,\gamma)}$ is not neutrosophic soft quasi-coincident with $\left(\widetilde{F}, E\right)$, we denote by $x^{e}_{(\alpha,\beta,\gamma)} \widetilde{q} \left(\widetilde{F}, E\right)$.

Example 3.2. Let $X = \{x, y\}$ be a universe, $E = \{a, b\}$ be a parameteric set. Consider the neutrosophic soft set $\left(\widetilde{F}, E\right)$ defined as

$$\widetilde{F}(a) = \{\langle x, 0.7, 0.3, 0.3\rangle, \langle y, 0.3, 0.3, 0.7\rangle\},$$
$$\widetilde{F}(b) = \{\langle x, 0.3, 0.3, 0.7\rangle, \langle y, 0.3, 0.3, 0.7\rangle\}.$$

The family $\tau = \left\{0_{(X,E)}, 1_{(X,E)}, \left(\widetilde{F}, E\right)\right\}$ is a neutrosophic soft topology over X. Then, $x^{a}_{(0,5,0,5,0,5)}$ is a neutrosophic soft point in (X, τ, E) and $x^{a}_{(0,5,0,5,0,5)} \not\subseteq \left(\widetilde{F}, E\right)^{c}$. So, $x^{a}_{(0,5,0,5,0,5)}\ q \left(\widetilde{F}, E\right)$.

Definition 3.3. A neutrosophic soft set $\left(\widetilde{F}, E\right)$ in a neutrosophic soft topological space (X, τ, E) is said to be a neutrosophic soft q-neighbourhood of a neutrosophic soft point $x^{e}_{(\alpha,\beta,\gamma)}$ if and only if there exists a neutrosophic soft open set $\left(\widetilde{G}, E\right)$ such that $x^{e}_{(\alpha,\beta,\gamma)} q \left(\widetilde{G}, E\right) \subset \left(\widetilde{F}, E\right)$.

Example 3.4. Let $X = \{x, y\}$ be a universe, $E = \{a, b\}$ be a parameteric set. Consider the neutrosophic soft sets $\left(\widetilde{F}, E\right)$ and $\left(\widetilde{G}, E\right)$ defined as

$$\widetilde{F}(a) = \{\langle x, 0.3, 0.3, 0.7\rangle, \langle y, 0.3, 0.3, 0.7\rangle\},$$
$$\widetilde{F}(b) = \{\langle x, 0.3, 0.3, 0.7\rangle, \langle y, 0.3, 0.3, 0.7\rangle\},$$
$$\widetilde{G}(a) = \{\langle x, 0.8, 0.8, 0.2\rangle, \langle y, 0.8, 0.8, 0.2\rangle\},$$
$$\widetilde{G}(b) = \{\langle x, 0.8, 0.8, 0.2\rangle, \langle y, 0.8, 0.8, 0.2\rangle\}.$$

The family $\tau = \left\{0_{(X,E)}, 1_{(X,E)}, \left(\widetilde{F}, E\right)\right\}$ is a neutrosophic soft topology over X. Then, $x^a_{(0{,}2,0{,}2,0{,}2)}$ is a neutrosophic soft point in (X, τ, E), where $x^a_{(0{,}2,0{,}2,0{,}2)} q \left(\widetilde{F}, E\right) \subset \left(\widetilde{G}, E\right)$. So, $\left(\widetilde{G}, E\right)$ is a neutrosophic soft q-neighbourhood of $x^a_{(0{,}2,0{,}2,0{,}2)}$.

Definition 3.5. A neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)} \in \overline{\left(\widetilde{F}, E\right)}$ if and only if each neutrosophic soft q-neighbourhood of $x^e_{(\alpha,\beta,\gamma)}$ is neutrosophic soft q-coincident with $\left(\widetilde{F}, E\right)$.

Definition 3.6. A neutrosophic soft set $\left(\widetilde{F}, E\right)$ in a neutrosophic soft topological space (X, τ, E) is called a neutrosophic soft regular open set if and only if $\left(\widetilde{F}, E\right) = \left[\overline{\left(\widetilde{F}, E\right)}\right]^\circ$. The complement of a neutrosophic soft regular open set is called a neutrosophic soft regular closed set.

Definition 3.7. A neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$ is said to be a neurosophic soft $\delta-$cluster point of a neutrosophic soft set $\left(\widetilde{F}, E\right)$ if and only if every neutrosophic soft regular open q-neighbourhood $\left(\widetilde{G}, E\right)$ of $x^e_{(\alpha,\beta,\gamma)}$ is q-coincident with $\left(\widetilde{F}, E\right)$. The set of all neutrosophic soft $\delta-$cluster points of $\left(\widetilde{F}, E\right)$ is called the neutrosophic soft $\delta-$closure of $\left(\widetilde{F}, E\right)$ and denoted by $NScl_\delta \left(\widetilde{F}, E\right)$.

Definition 3.8. A neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$ is said to be a neurosophic soft $\theta-$cluster point of a neutrosophic soft set $\left(\widetilde{F}, E\right)$ if and only if, for every neutrosophic soft open q-neighbourhood $\left(\widetilde{G}, E\right)$ of $x^e_{(\alpha,\beta,\gamma)}$, $\overline{\left(\widetilde{G}, E\right)}$ is q-coincident with $\left(\widetilde{F}, E\right)$. The set of all neutrosophic soft $\theta-$cluster points of $\left(\widetilde{F}, E\right)$ is called the neutrosophic soft $\theta-$closure of $\left(\widetilde{F}, E\right)$ and denoted by $NScl_\theta \left(\widetilde{F}, E\right)$.

Definition 3.9. A neutrosophic soft set $\left(\widetilde{F}, E\right)$ is said to be a neutrosophic soft $\delta-$ neighbourhood of a neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$ if and only if there exists a neutrosophic soft regular open q-neighbourhood $\left(\widetilde{G}, E\right)$ of $x^e_{(\alpha,\beta,\gamma)}$ such that $\left(\widetilde{G}, E\right) \subset \left(\widetilde{F}, E\right)$.

4. Neutrosophic Soft δ-Topology

In this section, we will define the notion of neutrosophic soft $\delta-$interior by using neutrosophic soft $\delta-$closure. Moreover, it will be shown that the set of all neutrosophic soft $\delta-$open sets is also a neutrosophic soft topology on (X, τ, E).

In Definition 3.7, the concept of neutrosophic soft $\delta-$closure is introduced by using neutrosophic soft $\delta-$cluster points. Now, we give an equivalent definition for this concept by using neutrosophic soft sets.

Definition 4.1. Let $\left(\widetilde{F}, E\right)$ be a neutrosophic soft set in a neutrosophic soft topological space (X, τ, E). Then

$$NScl_\delta \left(\widetilde{F}, E\right) = \bigcap \left\{\left(\widetilde{G}, E\right) \in NSS(X, E) : \left(\widetilde{F}, E\right) \subset \left(\widetilde{G}, E\right), \left(\widetilde{G}, E\right) = \overline{\left[\left(\widetilde{G}, E\right)^\circ\right]}\right\}$$

Definition 4.2. A neutrosophic soft set $\left(\widetilde{F}, E\right)$ is said to be neutrosophic soft $\delta-$closed if and only if $\left(\widetilde{F}, E\right) = NScl_\delta \left(\widetilde{F}, E\right)$. The complement of a neutrosophic soft $\delta-$closed set is called a neutrosophic soft $\delta-$open set.

Definition 4.3. For a neutrosophic soft subset $\left(\widetilde{F}, E\right)$ in a neutrosophic soft topological space (X, τ, E), the neutrosophic soft $\delta-$interior is defined as follows:

$$NSint_\delta\left(\widetilde{F},E\right)=\left[NScl_\delta\left[\left(\widetilde{F},E\right)^c\right]\right]^c$$

It is clear that for any neutrosophic soft set $\left(\widetilde{F},E\right)$,

$$NScl\left(NScl_\delta\left(\widetilde{F},E\right)\right)=NScl_\delta\left(\widetilde{F},E\right).$$

We have the following equality;

$$NSint_\delta\left(\widetilde{F},E\right)=\left[NScl_\delta\left[\left(\widetilde{F},E\right)^c\right]\right]^c$$

$$=\left[\bigcap\left\{\left(\widetilde{G},E\right)\in NSS\left(X,E\right):\left(\widetilde{F},E\right)^c\subset\left(\widetilde{G},E\right),\left(\widetilde{G},E\right)=\overline{\left[\left(\widetilde{G},E\right)^\circ\right]}\right\}\right]^c$$

(Using De Morgan's Law in [8])

$$=\bigcup\left\{\left(\widetilde{G},E\right)^c\in NSS\left(X,E\right):\left(\widetilde{G},E\right)^c\subset\left(\widetilde{F},E\right),\left(\widetilde{G},E\right)^c=\left[\overline{\left[\left(\widetilde{G},E\right)^\circ\right]}\right]^c\right\}$$

(Replacing $\left(\widetilde{G},E\right)^c$ by $\left(\widetilde{K},E\right)$ and using the result of Theorem 3.8.6 in [8])

$$=\bigcup\left\{\left(\widetilde{K},E\right)\in NSS\left(X,E\right):\left(\widetilde{K},E\right)\subset\left(\widetilde{F},E\right),\left(\widetilde{K},E\right)=\left[\overline{\left(\widetilde{K},E\right)}\right]^\circ\right\}$$

That is, the neutrosophic soft $\delta-$interior of $\left(\widetilde{F},E\right)$ is the union of all neutrosophic soft regular open subsets of $\left(\widetilde{F},E\right)$. Since any neutrosophic soft $\delta-$open set is the complement of a neutrosophic soft $\delta-$closed set, $\left(\widetilde{G},E\right)$ is a neutrosophic soft $\delta-$open set if and only if $\left(\widetilde{G},E\right)=NSint_\delta\left(\widetilde{G},E\right)$.

Clearly, a neutrosophic soft set $\left(\widetilde{F},E\right)$ *i*s neutrosophic soft δ-open in a neutrosophic soft topological space (X,τ,E) if and only if, for each neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$ with $x^e_{(\alpha,\beta,\gamma)}\ q\left(\widetilde{F},E\right)$, $\left(\widetilde{F},E\right)$ is a neutrosophic soft δ-neighbourhood of $x^e_{(\alpha,\beta,\gamma)}$.

Definition 4.4. A subset $\left(\widetilde{F},E\right)$ of a neutrosophic soft topological space (X,τ,E) is said to be neutrosophic soft semi open if $\left(\widetilde{F},E\right)\subset\left[\overline{\left(\widetilde{F},E\right)}\right]^\circ$. The family of all neutrosophic soft semi open sets of (X,τ,E) is denoted by $NSSO(X)$. The family of all neutrosophic soft semi open sets of (X,τ,E) containing a neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$ is denoted by $NSSO\left(X,x^e_{(\alpha,\beta,\gamma)}\right)$.

Definition 4.5. A neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$ of a neutrosophic soft topological space (X,τ,E) is said to be neutrosophic soft semi interior point of a neutrosophic soft set $\left(\widetilde{F},E\right)$, if there exists $\left(\widetilde{G},E\right)\in NSSO\left(X,x^e_{(\alpha,\beta,\gamma)}\right)$ such that $x^e_{(\alpha,\beta,\gamma)}\not\subset\left(\widetilde{G},E\right)^c$ and $\left(\widetilde{G},E\right)\subset\left(\widetilde{F},E\right)$.

It is easy to show that $\overline{\left(\widetilde{F},E\right)}\subset NScl_\delta\left(\widetilde{F},E\right)\subset NScl_\theta\left(\widetilde{F},E\right)$ for any neutrosophic soft set $\left(\widetilde{F},E\right)$ in a neutrosophic soft topological space (X,τ,E). Hence, for any neutrosophic soft set $\left(\widetilde{F},E\right)$ in a neutrosophic soft topological space (X,τ,E),

$$NSint_\theta\left(\widetilde{F},E\right)\subset NSint_\delta\left(\widetilde{F},E\right)\subset\left(\widetilde{F},E\right)^\circ.$$

It is clear that any neutrosophic soft regular open set is neutrosophic soft δ-open and any neutrosophic soft δ-open set is neutrosophic soft open. Furthermore, if a neutrosophic soft set $\left(\widetilde{F},E\right)$ is neutrosophic soft semi open in a neutrosophic soft topological space (X,τ,E) then $\overline{\left(\widetilde{F},E\right)}=NScl_\delta\left(\widetilde{F},E\right)$.

Theorem 4.6. *The finite union of neutrosophic soft $\delta-$closed sets is also neutrosophic soft $\delta-$closed. That is, if* $\left(\widetilde{F},E\right) = NScl_\delta\left(\widetilde{F},E\right)$ *and* $\left(\widetilde{G},E\right) = NScl_\delta\left(\widetilde{G},E\right)$ *then*

$$\left(\widetilde{F},E\right)\cup\left(\widetilde{G},E\right) = NScl_\delta\left[\left(\widetilde{F},E\right)\cup\left(\widetilde{G},E\right)\right].$$

Proof. Clearly, $\left(\widetilde{F},E\right)\cup\left(\widetilde{G},E\right)\subset NScl_\delta\left[\left(\widetilde{F},E\right)\cup\left(\widetilde{G},E\right)\right]$. We will show that $NScl_\delta\left[\left(\widetilde{F},E\right)\cup\left(\widetilde{G},E\right)\right]\subset\left(\widetilde{F},E\right)\cup\left(\widetilde{G},E\right)$. Let $x^e_{(\alpha,\beta,\gamma)}$ be a neutrosophic soft point. Suppose that $x^e_{(\alpha,\beta,\gamma)}\in NScl_\delta\left[\left(\widetilde{F},E\right)\cup\left(\widetilde{G},E\right)\right]$. Then, for any regular open q-neighbourhood $\left(\widetilde{K},E\right)$ of $x^e_{(\alpha,\beta,\gamma)}$, $\left(\widetilde{K},E\right)q\left[\left(\widetilde{F},E\right)\cup\left(\widetilde{G},E\right)\right]$. Thus, $\left(\widetilde{K},E\right)q\left(\widetilde{F},E\right)$ or $\left(\widetilde{K},E\right)q\left(\widetilde{G},E\right)$. Hence, $x^e_{(\alpha,\beta,\gamma)}\in NScl_\delta\left(\widetilde{F},E\right)\cup NScl_\delta\left(\widetilde{G},E\right)$. That is, $x^e_{(\alpha,\beta,\gamma)}\in\left(\widetilde{F},E\right)\cup\left(\widetilde{G},E\right)$. □

Furthermore, the finite intersection of neutrosophic soft regular open sets is also neutrosophic soft regular open. That is, if $\left(\widetilde{F},E\right)=\left[\overline{\left(\widetilde{F},E\right)}\right]^\circ$ and $\left(\widetilde{G},E\right)=\left[\overline{\left(\widetilde{G},E\right)}\right]^\circ$, then $\left(\widetilde{F},E\right)\cap\left(\widetilde{G},E\right)=\left[\overline{\left(\widetilde{F},E\right)\cap\left(\widetilde{G},E\right)}\right]^\circ$.

Lemma 4.7. *Let (X,τ,E) be a neutrosophic soft topological space. If $\left(\widetilde{F},E\right)$ is neutrosophic soft open then $\overline{\left(\widetilde{F},E\right)}$ is neutrosophic soft regular closed.*

Proof. We know that $\left(\widetilde{F},E\right)\subset\overline{\left(\widetilde{F},E\right)}$. Thus, $\left(\widetilde{F},E\right)=\left(\widetilde{F},E\right)^\circ\subset\left[\overline{\left(\widetilde{F},E\right)}\right]^\circ$ and $\overline{\left(\widetilde{F},E\right)}\subset\overline{\left[\overline{\left(\widetilde{F},E\right)}\right]^\circ}$.

Conversely, we know that $\left[\overline{\left(\widetilde{F},E\right)}\right]^\circ\subset\overline{\left(\widetilde{F},E\right)}$. Thus,

$$\overline{\left[\overline{\left(\widetilde{F},E\right)}\right]^\circ}\subset\overline{\overline{\left(\widetilde{F},E\right)}}=\overline{\left(\widetilde{F},E\right)}. \text{ Hence, } \overline{\left(\widetilde{F},E\right)}=\overline{\left[\overline{\left(\widetilde{F},E\right)}\right]^\circ}.$$

□

Lemma 4.8. *Let (X,τ,E) be a neutrosophic soft topological space. Then*

$$\left\{\overline{\left(\widetilde{F},E\right)}\mid\left(\widetilde{F},E\right)\in\tau\right\}=\left\{\overline{\left(\widetilde{F},E\right)}\mid\left(\widetilde{F},E\right)\text{ is neutrosophic soft regular closed in }(X,\tau,E)\right\}.$$

Proof. We know that for any neutrosophic soft open set $\left(\widetilde{F},E\right)$ in (X,τ,E), $\overline{\left(\widetilde{F},E\right)}$ is neutrosophic soft regular closed. Conversely, take any neutrosophic soft regular closed set $\left(\widetilde{G},E\right)$ in (X,τ,E). Then,

$$\begin{aligned}\left(\widetilde{G},E\right) &= \overline{\left[\left(\widetilde{G},E\right)^\circ\right]}\\ &= \overline{\left[\bigcup\left\{\left(\widetilde{K},E\right)\mid\left(\widetilde{K},E\right)\subset\left(\widetilde{G},E\right),\left(\widetilde{K},E\right)\in\tau\right\}\right]}\in\left\{\left(\widetilde{K},E\right)\mid\left(\widetilde{K},E\right)\in\tau\right\}.\end{aligned}$$

It may be difficult to find the neutrosophic soft $\delta-$closure of any neutrosophic soft set. By the help of above lemmas, we have the clue to find it. □

Theorem 4.9. *For any neutrosophic soft set $\left(\widetilde{F},E\right)$ in a neutrosophic soft topological space (X,τ,E), $NScl_\delta\left(\widetilde{F},E\right)=\bigcap\left\{\overline{\left(\widetilde{K},E\right)}\mid\left(\widetilde{F},E\right)\subset\overline{\left(\widetilde{K},E\right)},\left(\widetilde{K},E\right)\in\tau\right\}$.*

Proof. The proof is straightforward. □

Corollary 4.10. *For any neutrosophic soft set $\left\{\overline{\left(\widetilde{G},E\right)}\mid NScl_\delta\left(\widetilde{F},E\right)\subseteq\overline{\left(\widetilde{G},E\right)},\left(\widetilde{G},E\right)\in\tau\right\}$, in a neutrosophic soft topological space (X,τ,E), $NScl_\delta\left(\widetilde{F},E\right)$ is a neutrosophic soft δ-closed set. That is, $NScl_\delta\left(NScl_\delta\left(\widetilde{F},E\right)\right)=NScl_\delta\left(\widetilde{F},E\right)$*
.

Proof. It is sufficient to show that

$$\left\{\overline{\left(\widetilde{K},E\right)} \mid \left(\widetilde{F},E\right) \subseteq \overline{\left(\widetilde{K},E\right)}, \left(\widetilde{K},E\right) \in \tau\right\} = \left\{\overline{\left(\widetilde{G},E\right)} \mid NScl_\delta\left(\widetilde{F},E\right) \subseteq \overline{\left(\widetilde{G},E\right)}, \left(\widetilde{G},E\right) \in \tau\right\}.$$

Suppose that there is a neutrosophic soft open set $\left(\widetilde{H},E\right)$ such that

$$\overline{\left(\widetilde{H},E\right)} \in \left\{\overline{\left(\widetilde{K},E\right)} \mid \left(\widetilde{F},E\right) \subseteq \overline{\left(\widetilde{K},E\right)}, \left(\widetilde{K},E\right) \in \tau\right\} \text{ and } \overline{\left(\widetilde{H},E\right)} \neq \left\{\overline{\left(\widetilde{G},E\right)} \mid NScl_\delta\left(\widetilde{F},E\right) \subseteq \overline{\left(\widetilde{G},E\right)}, \left(\widetilde{G},E\right) \in \tau\right\}.$$

Then $\left(\widetilde{F},E\right) \subseteq \overline{\left(\widetilde{H},E\right)}$ and $NScl_\delta\left(\widetilde{F},E\right) \nsubseteq \overline{\left(\widetilde{H},E\right)}$. But, since $\left(\widetilde{F},E\right) \subseteq \overline{\left(\widetilde{H},E\right)}$, $NScl_\delta\left(\widetilde{F},E\right) \subseteq \overline{\left(\widetilde{H},E\right)}$. This is a contradiction. So, the equality holds.

Clearly, $NScl_\delta\left(0_{(X,E)}\right) = 0_{(X,E)}$. And, for any neutrosophic soft subsets $\left(\widetilde{F},E\right)$ and $\left(\widetilde{G},E\right)$, if $\left(\widetilde{F},E\right) \subseteq \left(\widetilde{G},E\right)$ then $NScl_\delta\left(\widetilde{F},E\right) \subseteq NScl_\delta\left(\widetilde{G},E\right)$.

Therefore, by Theorem 4.6 and Corollary 4.10, the neutrosophic soft δ-closure operation on a neutrosophic soft topological space (X,τ,E) satisfies the Kuratowski Closure Axioms. So, there exists one and only one topology on X. We will define the topology as follows. □

Definition 4.11. The set of all neutrosophic soft $\delta-$open sets of (X,τ,E) is also a neutrosophic soft topology on X. We denote it by τ_δ and it is called a neutrosophic soft $\delta-$topology on X. An ordered pair (X,τ_δ) is called a neutrosophic soft $\delta-$topological space.

5. Neutrosophic Soft δ-Continuous Mappings

Now, we will find some equivalent conditions of neutrosophic soft δ-continuity and will show that neutrosophic soft $\delta-$continuity is a standard continuity in neutrosophic soft $\delta-$topology introduced in the previous section.

Definition 5.1. Let $f : (X,\tau_1,E_1) \to (Y,\tau_2,E_2)$ be a neutrosophic soft mapping.

(1) f is said to be neutrosophic soft continuous, if, for each neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$ in (X,τ_1,E_1) and for any neutrosophic soft open q-neighbourhood $\left(\widetilde{G},E\right)$ of $f\left(x^e_{(\alpha,\beta,\gamma)}\right)$ in (Y,τ_2,E_2), there exists a neutrosophic soft open q-neighbourhood $\left(\widetilde{F},E\right)$ of $x^e_{(\alpha,\beta,\gamma)}$ such that $f\left(\left(\widetilde{F},E\right)\right) \subseteq \left(\widetilde{G},E\right)$.

(2) f is said to be neutrosophic soft $\delta-$continuous, if, for each neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$ in (X,τ_1,E_1) and for any neutrosophic soft regular open q-neighbourhood $\left(\widetilde{G},E\right)$ of $f\left(x^e_{(\alpha,\beta,\gamma)}\right)$ in (Y,τ_2,E_2), there exists a neutrosophic soft regular open q-neighbourhood $\left(\widetilde{F},E\right)$ of $x^e_{(\alpha,\beta,\gamma)}$ such that $f\left(\left(\widetilde{F},E\right)\right) \subseteq \left(\widetilde{G},E\right)$.

The neutrosophic soft continuity and the neutrosophic soft $\delta-$continuity are independent notions as we can see in the following examples.

Example 5.2. $X = \{x,y\}$ be a universe, $E = \{a,b\}$ be a parameteric set. Consider the neutrosophic soft sets $\left(\widetilde{F},E\right)$ and $\left(\widetilde{G},E\right)$ defined as $\widetilde{F}(a) = \{\langle x,0.3,0.3,0.7\rangle, \langle y,0.3,0.3,0.7\rangle\}$, $\widetilde{F}(b) = \{\langle x,0.3,0.3,0.7\rangle, \langle y,0.3,0.3,0.7\rangle\}$, $\widetilde{G}(a) = \{\langle x,0.8,0.8,0.2\rangle, \langle y,0.8,0.8,0.2\rangle\}$ and $\widetilde{G}(b) = \{\langle x,0.8,0.8,0.2\rangle, \langle y,0.8,0.8,0.2\rangle\}$. The families $\tau_1 = \left\{0_{(X,E)}, 1_{(X,E)}, \left(\widetilde{F},E\right)\right\}$ and $\tau_2 = \left\{0_{(X,E)}, 1_{(X,E)}, \left(\widetilde{F},E\right), \left(\widetilde{G},E\right)\right\}$ are neutrosophic soft topologies over X. So, (X,τ_1,E_1) and (X,τ_2,E_2) are neutrosophic soft topological spaces. Then, the identity map $idX : (X,\tau_1,E_1) \to (X,\tau_2,E_2)$ is neutrosophic soft $\delta-$continuous but not neutrosophic soft continuous.

Example 5.3. $X = \{x, y\}$ be a universe, $E = \{a, b\}$ be a parameteric set. Consider the neutrosophic soft sets $\left(\widetilde{F}, E\right)$ and $\left(\widetilde{G}, E\right)$ defined as $\widetilde{F}(a) = \{\langle x, 0.3, 0.3, 0.7\rangle, \langle y, 0.3, 0.3, 0.7\rangle\}$, $\widetilde{F}(b) = \{\langle x, 0.3, 0.3, 0.7\rangle, \langle y, 0.3, 0.3, 0.7\rangle\}$, $\widetilde{G}(a) = \{\langle x, 0.5, 0.5, 0.5\rangle, \langle y, 0.5, 0.5, 0.5\rangle\}$ and $\widetilde{G}(b) = \{\langle x, 0.5, 0.5, 0.5\rangle, \langle y, 0.5, 0.5, 0.5\rangle\}$. The families $\tau_1 = \left\{0_{(X,E)}, 1_{(X,E)}, \left(\widetilde{F}, E\right), \left(\widetilde{G}, E\right)\right\}$ and $\tau_2 = \left\{0_{(X,E)}, 1_{(X,E)}, \left(\widetilde{F}, E\right)\right\}$ are neutrosophic soft topologies over X. So, (X, τ_1, E_1) and (X, τ_2, E_2) are neutrosophic soft topological spaces. Then, the identity map $idX : (X, \tau_1, E_1) \to (X, \tau_2, E_2)$ is neutrosophic soft continuous but not neutrosophic soft $\delta-$ continuous.

The concept of neutrosophic soft $\delta-$continuity is described by using neutrosophic soft $\delta-$ neighbourhoods and by using neutrosophic soft $\delta-$open sets as follows.

Theorem 5.4. *Let $f : (X, \tau_1, E_1) \to (Y, \tau_2, E_2)$ be a bijective neutrosophic soft function. Then, f is neutrosophic soft $\delta-$continuous if and only if, for each neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$ in (X, τ_1, E_1) and each neutrosophic soft $\delta-$neighbourhood $\left(\widetilde{G}, E\right)$ of $f\left(x^e_{(\alpha,\beta,\gamma)}\right)$, $f^{-1}\left(\left(\widetilde{G}, E\right)\right)$ is a neutrosophic soft $\delta-$neighbourhood of $x^e_{(\alpha,\beta,\gamma)}$.*

Proof. Let $x^e_{(\alpha,\beta,\gamma)}$ be a neutrosophic soft point in (X, τ_1, E_1) and $\left(\widetilde{G}, E\right)$ be a neutrosophic soft $\delta-$neighbourhood of $f\left(x^e_{(\alpha,\beta,\gamma)}\right)$. Then, there exists a neutrosophic soft regular open q-neighbourhood $\left(\widetilde{K}, E\right)$ of $f\left(x^e_{(\alpha,\beta,\gamma)}\right)$ such that $\left(\widetilde{K}, E\right) \subseteq \left(\widetilde{G}, E\right)$. From Theorem 6.3 in [7], f is invertible. Since f is neutrosophic soft $\delta-$continuous, there exists a neutrosophic soft regular open q-neighbourhood $\left(\widetilde{H}, E\right)$ of $x^e_{(\alpha,\beta,\gamma)}$ such that $f\left(\left(\widetilde{H}, E\right)\right) \subseteq \left(\widetilde{K}, E\right)$ and $\left(\widetilde{H}, E\right) \subseteq f^{-1}\left(f\left(\left(\widetilde{H}, E\right)\right)\right) \subseteq f^{-1}\left(\left(\widetilde{K}, E\right)\right)$. Therefore, since $f^{-1}\left(\left(\widetilde{K}, E\right)\right) \subseteq f^{-1}\left(\left(\widetilde{G}, E\right)\right)$, $f^{-1}\left(\left(\widetilde{G}, E\right)\right)$ is a neutrosophic soft $\delta-$neighbourhood of $x^e_{(\alpha,\beta,\gamma)}$.

Conversely, let $x^e_{(\alpha,\beta,\gamma)}$ be a neutrosophic soft point in (X, τ_1, E_1) and $\left(\widetilde{G}, E\right)$ be a neutrosophic soft regular open q-neighbourhood of $f\left(x^e_{(\alpha,\beta,\gamma)}\right)$. Then, $\left(\widetilde{G}, E\right)$ is a neutrosophic soft $\delta-$neighbourhood of $f\left(x^e_{(\alpha,\beta,\gamma)}\right)$. By the hypothesis, $f^{-1}\left(\left(\widetilde{G}, E\right)\right)$ is a neutrosophic soft $\delta-$neighbourhood of $x^e_{(\alpha,\beta,\gamma)}$. Therefore, there exists a neutrosophic soft regular open q-neighbourhood $\left(\widetilde{K}, E\right)$ of $x^e_{(\alpha,\beta,\gamma)}$ such that $\left(\widetilde{K}, E\right) \subseteq f^{-1}\left(\left(\widetilde{G}, E\right)\right)$ and $f\left(\left(\widetilde{K}, E\right)\right) \subseteq f\left(f^{-1}\left(\left(\widetilde{G}, E\right)\right)\right) \subseteq \left(\widetilde{G}, E\right)$. Hence, f is neutrosophic soft $\delta-$continuous. □

Corollary 5.5. *$f : (X, \tau_1, E_1) \to (Y, \tau_2, E_2)$ is a neutrosophic soft $\delta-$continuous mapping if and only if for each neutrosophic soft $\delta-$open set $\left(\widetilde{G}, E\right)$ in (Y, τ_2, E_2), $f^{-1}\left(\left(\widetilde{G}, E\right)\right)$ is neutrosophic soft $\delta-$open in (X, τ_1, E_1).*

Definition 5.6. Let $f : (X, \tau_1, E_1) \to (Y, \tau_2, E_2)$ be a neutrosophic soft mapping.

(1) f is said to be neutrosophic soft $\delta-$open, if, for each neutrosophic soft $\delta-$open set $\left(\widetilde{F}, E\right)$ in (X, τ_1, E_1), $f\left(\left(\widetilde{F}, E\right)\right)$ is neutrosophic soft $\delta-$open in (Y, τ_2, E_2).

(2) f is said to be neutrosophic soft $\delta-$closed, if, for each neutrosophic soft $\delta-$closed set $\left(\widetilde{G}, E\right)$ in (X, τ_1, E_1), $f\left(\left(\widetilde{G}, E\right)\right)$ is neutrosophic soft $\delta-$closed in (Y, τ_2, E_2).

Definition 5.7. (X, τ, E) is called a neutrosophic soft semiregular space if and only if, for each neutrosophic soft open q-neighbourhood $\left(\widetilde{F}, E\right)$ of $x^e_{(\alpha,\beta,\gamma)}$, there exists another neutrosophic soft open q-neighbourhood $\left(\widetilde{G}, E\right)$ of $x^e_{(\alpha,\beta,\gamma)}$ such that $\left(\widetilde{G}, E\right) \subseteq \left[\overline{\left(\widetilde{G}, E\right)}\right]^\circ \subseteq \left(\widetilde{F}, E\right)$.

Lemma 5.8. *For a neutrosophic soft bijective function $f : (X, \tau_1, E_1) \to (Y, \tau_2, E_2)$, the followings are true:*

(a) if $f : (X, \tau_1, E_1) \to (Y, \tau_2, E_2)$ is neutrosophic soft continuous and (X, τ_1, E_1) is neutrosophic soft semiregular then f is neutrosophic soft $\delta-$continuous.

(b) if $f : (X, \tau_1, E_1) \to (Y, \tau_2, E_2)$ is neutrosophic soft $\delta-$continuous and (Y, τ_2, E_2) is neutrosophic soft semiregular then f is neutrosophic soft continuous.

Proof. (a) Let $x^e_{(\alpha,\beta,\gamma)}$ be a neutrosophic soft point in (X,τ_1,E_1) and $\left(\widetilde{U},E\right)$ be any neutrosophic soft regular open q-neighbourhood of $f\left(x^e_{(\alpha,\beta,\gamma)}\right)$. As f is neutrosophic soft continuous, $f^{-1}\left(\left(\widetilde{U},E\right)\right)$ is a neutrosophic soft open q-neighbourhood of $x^e_{(\alpha,\beta,\gamma)}$ and, by neutrosophic soft semiregularity of (X,τ_1,E_1), there exists a neutrosophic soft open q-neighbourhood $\left(\widetilde{V},E\right)$ of $x^e_{(\alpha,\beta,\gamma)}$ such that $\left[\overline{\left(\widetilde{V},E\right)}\right]^\circ \subseteq f^{-1}\left(\left(\widetilde{U},E\right)\right)$. This implies that $f\left(\left[\overline{\left(\widetilde{V},E\right)}\right]^\circ\right) \subseteq \left(\widetilde{U},E\right)$. So, f is neutrosophic soft $\delta-$continuous.

(b) Let $x^e_{(\alpha,\beta,\gamma)}$ be a neutrosophic soft point in (X,τ_1,E_1) and $\left(\widetilde{U},E\right)$ be any neutrosophic soft open q-neighbourhood of $f\left(x^e_{(\alpha,\beta,\gamma)}\right)$. By neutrosophic soft semiregularity of (Y,τ_2,E_2), there exists a neutrosophic soft open q-neighbourhood $\left(\widetilde{V},E\right)$ of $f\left(x^e_{(\alpha,\beta,\gamma)}\right)$ such that $\left[\overline{\left(\widetilde{V},E\right)}\right]^\circ \subseteq \left(\widetilde{U},E\right)$. As f is neutrosophic soft $\delta-$continuous, $f^{-1}\left(\left[\overline{\left(\widetilde{V},E\right)}\right]^\circ\right)$ is a neutrosophic soft $\delta-$neighbourhood of $x^e_{(\alpha,\beta,\gamma)}$. Then, there exists a neutrosophic soft regular open q-neighbourhood $\left(\widetilde{G},E\right)$ of $x^e_{(\alpha,\beta,\gamma)}$ such that $\left(\widetilde{G},E\right) \subseteq f^{-1}\left(\left[\overline{\left(\widetilde{V},E\right)}\right]^\circ\right)$. This implies that $\left(\widetilde{G},E\right) \subseteq f^{-1}\left(\left(\widetilde{U},E\right)\right)$. So, f is neutrosophic soft continuous. □

Theorem 5.9. *If $f:(X,\tau_1,E_1)\to(Y,\tau_2,E_2)$ is a neutrosophic soft mapping then the following are equivalent:*
(a) f is neutrosophic soft $\delta-$continuous,
(b) For each neutrosophic soft set $\left(\widetilde{F},E\right)$ in (X,τ_1,E_1), $f\left(NScl_\delta\left(\widetilde{F},E\right)\right) \subseteq NScl_\delta\left(f\left(\left(\widetilde{F},E\right)\right)\right)$,
(c) For each neutrosophic soft set $\left(\widetilde{G},E\right)$ in (Y,τ_2,E_2), $NScl_\delta\left(f^{-1}\left(\left(\widetilde{G},E\right)\right)\right) \subseteq f^{-1}\left(NScl_\delta\left(\widetilde{G},E\right)\right)$,
(d) For each neutrosophic soft $\delta-$closed set $\left(\widetilde{G},E\right)$ in (Y,τ_2,E_2), $f^{-1}\left(\left(\widetilde{G},E\right)\right)$ is a neutrosophic soft $\delta-$closed set in (X,τ_1,E_1),
(e) For each neutrosophic soft $\delta-$open set $\left(\widetilde{H},E\right)$ in (Y,τ_2,E_2), $f^{-1}\left(\left(\widetilde{H},E\right)\right)$ is a neutrosophic soft $\delta-$open set in (X,τ_1,E_1).

Proof. (a) ⇒ (b) Let $x^e_{(\alpha,\beta,\gamma)} \in NScl_\delta\left(\widetilde{F},E\right)$ and $\left(\widetilde{G},E\right)$ be a neutrosophic soft regular open q-neighbourhood of $f\left(x^e_{(\alpha,\beta,\gamma)}\right)$. Then, there exists a neutrosophic soft regular open q-neighbourhood $\left(\widetilde{H},E\right)$ of $x^e_{(\alpha,\beta,\gamma)}$ such that $f\left(\left(\widetilde{H},E\right)\right) \subseteq \left(\widetilde{G},E\right)$. Since $x^e_{(\alpha,\beta,\gamma)} \in NScl_\delta\left(\widetilde{F},E\right)$, we have $\left(\widetilde{H},E\right) q \left(\widetilde{F},E\right)$. Then, $f\left(\left(\widetilde{H},E\right)\right) q f\left(\left(\widetilde{F},E\right)\right)$. Thus, $\left(\widetilde{G},E\right) q f\left(\left(\widetilde{F},E\right)\right)$ and $f\left(x^e_{(\alpha,\beta,\gamma)}\right) \in NScl_\delta\left(f\left(\left(\widetilde{F},E\right)\right)\right)$. So, $f\left(NScl_\delta\left(\widetilde{F},E\right)\right) \subseteq NScl_\delta\left(f\left(\left(\widetilde{F},E\right)\right)\right)$.

(b) ⇒(c) Let $\left(\widetilde{G},E\right)$ be a neutrosophic soft set in (Y,τ_2,E_2). From (b),

$$f\left(NScl_\delta\left(f^{-1}\left(\left(\widetilde{G},E\right)\right)\right)\right) \subseteq NScl_\delta\left(f\left(f^{-1}\left(\left(\widetilde{G},E\right)\right)\right)\right) \subseteq NScl_\delta\left(\widetilde{G},E\right)$$

Hence, $NScl_\delta f^{-1}\left(\left(\widetilde{G},E\right)\right) \subseteq f^{-1}\left(NScl_\delta\left(\widetilde{G},E\right)\right)$.

(c) ⇒(d) Let $\left(\widetilde{G},E\right)$ be a neutrosophic soft $\delta-$closed set in (Y,τ_2,E_2). Then, $\left(\widetilde{G},E\right) = NScl_\delta\left(\widetilde{G},E\right)$. From (c),

$$NScl_\delta\left(f^{-1}\left(\left(\widetilde{G},E\right)\right)\right) \subseteq f^{-1}\left(NScl_\delta\left(\widetilde{G},E\right)\right) = f^{-1}\left(\left(\widetilde{G},E\right)\right).$$

Therefore, $NScl_\delta f^{-1}\left(\left(\widetilde{G},E\right)\right) = f^{-1}((G,E))$. This means that $f^{-1}\left(\left(\widetilde{G},E\right)\right)$ is a neutrosophic soft $\delta-$closed set in (X,τ_1,E_1).

(d) ⇒(e) Let $\left(\widetilde{H}, E\right)$ be a neutrosophic soft $\delta-$open set in (Y, τ_2, E_2). Then, $\left(\widetilde{H}, E\right)^c$ is a neutrosophic soft $\delta-$closed set in (Y, τ_2, E_2). From (d), $f^{-1}\left(\left(\widetilde{H}, E\right)^c\right)$ is a neutrosophic soft $\delta-$closed set in (X, τ_1, E_1). Since $f^{-1}\left(\left(\widetilde{H}, E\right)^c\right) = \left[f^{-1}\left(\left(\widetilde{H}, E\right)\right)\right]^c$, $f^{-1}\left(\left(\widetilde{H}, E\right)\right)$ is a neutrosophic soft $\delta-$open set in (X, τ_1, E_1).

(e) ⇒(a) The proof is clear. □

Corollary 5.10. *If $f : (X, \tau_1, E_1) \to (Y, \tau_2, E_2)$ is a neutrosophic soft $\delta-$continuous mapping then, for each neutrosophic soft open set $\left(\widetilde{G}, E\right)$ in (Y, τ_2, E_2), $NScl_\delta\left(f^{-1}\left(\left(\widetilde{G}, E\right)\right)\right) \subseteq f^{-1}\left(NScl\left(\widetilde{G}, E\right)\right)$.*

Proof. Since $\left(\widetilde{G}, E\right)$ is neutrosophic soft open in (Y, τ_2, E_2), $NScl\left(\widetilde{G}, E\right) = NScl_\delta\left(\widetilde{G}, E\right)$. By (c) of the above theorem, $NScl_\delta\left(f^{-1}\left(\left(\widetilde{G}, E\right)\right)\right) \subseteq f^{-1}\left(NScl_\delta\left(\widetilde{G}, E\right)\right) = f^{-1}\left(\overline{\left(\widetilde{G}, E\right)}\right)$. □

Theorem 5.11. *Let (X, τ_1, E_1) and (Y, τ_2, E_2) be neutrosophic soft topological spaces. Then, $f : (X, \tau_1, E_1) \to (Y, \tau_2, E_2)$ is neutrosophic soft $\delta-$continuous if and only if $f : (X, \tau_{1_\delta}, E_1) \to (Y, \tau_{2_\delta}, E_2)$ is neutrosophic soft $\delta-$continuous.*

Proof. The proof is clear. □

6. Neutrosophic Soft δ-Compact and Neutrosophic Soft Locally δ-Compact Spaces

In this section, we will introduce the notion of neutrosophic soft $\delta-$compactness and neutrosophic soft locally $\delta-$compactness. Furthermore, we will study the properties of neutrosophic soft $\delta-$compactness and neutrosophic soft locally $\delta-$compactness under the neutrosophic soft $\delta-$continuous mappings.

Definition 6.1. A collection $\left\{\left(\widetilde{G}, E\right)_i \mid i \in I\right\}$ of neutrosophic soft $\delta-$open sets in a neutrosophic soft topological space (X, τ, E) is called a neutrosophic soft $\delta-$open cover of a neutrosophic soft set $\left(\widetilde{F}, E\right)$, if $\left(\widetilde{F}, E\right) \subseteq \bigcup\left\{\left(\widetilde{G}, E\right)_i \mid i \in I\right\}$ holds.

Definition 6.2. A neutrosophic soft topological space (X, τ, E) is said to be a neutrosophic soft $\delta-$compact space, if every neutrosophic soft $\delta-$open cover of $1_{(X,E)}$ has a finite subcover. A neutrosophic soft subset $\left(\widetilde{F}, E\right)$ of a neutrosophic soft topological space (X, τ, E) is said to be neutrosophic soft $\delta-$compact in (X, τ, E), provided that, for every collection $\left\{\left(\widetilde{G}, E\right)_i \mid i \in I\right\}$ of neutrosophic soft $\delta-$open sets in (X, τ, E) such that $\left(\widetilde{F}, E\right) \subseteq \bigcup\left\{\left(\widetilde{G}, E\right)_i \mid i \in I\right\}$, there exists a finite subset I_0 of I such that $\left(\widetilde{F}, E\right) \subseteq \bigcup\left\{\left(\widetilde{G}, E\right)_i \mid i \in I_0\right\}$.

Theorem 6.3. *Every neutrosophic soft compact space is neutrosophic soft $\delta-$compact.*

Proof. Let $\left\{\left(\widetilde{G}, E\right)_i \mid i \in I\right\}$ be a neutrosophic soft $\delta-$open cover of a neutrosophic soft topological space (X, τ, E). Since any neutrosophic soft $\delta-$open set is neutrosophic soft open, $\left\{\left(\widetilde{G}, E\right)_i \mid i \in I\right\}$ is a neutrosophic soft open cover of the neutrosophic soft topological space (X, τ, E). Since (X, τ, E) is neutrosophic soft compact, there exists a finite subset I_0 of I such that $1_{(X,E)} \subseteq \bigcup\left\{\left(\widetilde{G}, E\right)_i \mid i \in I_0\right\}$. Hence, (X, τ, E) is neutrosophic soft $\delta-$compact.

But, the converse statement is not always true as shown in the following example. □

Example 6.4. Let $X = [0, 1]$ be a universe, $E = \{a, b\}$ be a parameteric set and $U_n(x)$ be a neutrosophic set on X defined as below:

$$U_n(x) = \begin{cases} < x, 1, 1, 0 >, & \text{if } x = 0 \\ < x, n.x, n.x, 1 - n.x >, & \text{if } 0 < x \leq \frac{1}{n} \\ < x, 1, 1, 0 >, & \text{if } \frac{1}{n} < x \leq 1 \end{cases}$$

Consider the neutrosophic soft set $\left(\widetilde{F}_n, E\right)$ defined as $\widetilde{F}_n(a) = \widetilde{F}_n(b) = \{U_n(x) : x \in [0,1]\}$ for each $n \in N$. Then, the family $\tau = \left\{0_{(X,E)}, 1_{(X,E)}\right\} \bigcup \left\{\left(\widetilde{F}_n, E\right) : n \in N\right\}$ is a neutrosophic soft topology over X. Since $\left\{\left(\widetilde{F}_n, E\right) : n \in N\right\}$ is a neutrosophic soft open cover of (X, τ, E), which does not have a finite subcover, (X, τ, E) is not neutrosophic soft compact. As $\left\{\overline{\left(\widetilde{U}, E\right)} : \left(\widetilde{U}, E\right) \in \tau\right\} = \left\{\left(\widetilde{F}, E\right) : \left(\widetilde{F}, E\right) \text{ is neutrosophic soft regular closed in } (X, \tau, E)\right\}$ and for any neutrosophic soft set $\left(\widetilde{G}, E\right)$ in a neutrosophic soft topological space,

$$NScl_\delta\left(\widetilde{G}, E\right) = \bigcap\left\{\left(\widetilde{H}, E\right) : \left(\widetilde{G}, E\right) \subseteq \left(\widetilde{H}, E\right) \text{ and } \left(\widetilde{H}, E\right) \text{ is neutrosophic soft regular closed } (X, \tau, E)\right\}.$$

Furthermore, $\overline{\left(\widetilde{F}_n, E\right)} = 1_{(X,E)}$, $\overline{1_{(X,E)}} = 1_{(X,E)}$, $\overline{0_{(X,E)}} = 0_{(X,E)}$. So, the set of all neutrosophic soft regular closed sets in (X, τ, E) is $\left\{0_{(X,E)}, 1_{(X,E)}\right\}$. Hence, the set of all neutrosophic soft $\delta-$closed sets in (X, τ, E) is $\left\{0_{(X,E)}, 1_{(X,E)}\right\}$. Since the only neutrosophic soft $\delta-$open cover of $1_{(X,E)}$ is $\left\{0_{(X,E)}, 1_{(X,E)}\right\}$, (X, τ, E) is neutrosophic soft $\delta-$compact.

Theorem 6.5. *(X, τ, E) is neutrosophic soft $\delta-$compact if and only if every family of neutrosophic soft $\delta-$closed subsets of X, which has the finite intersection property, has a nonempty intersection.*

Proof. Let (X, τ, E) be neutrosophic soft $\delta-$compact and $\left\{\left(\widetilde{F}_i, E\right) : i \in I\right\}$ be a family of neutrosophic soft $\delta-$closed subsets of (X, τ, E) with the finite intersection property. Suppose

$$\bigcap\left\{\left(\widetilde{F}_i, E\right) : i \in I\right\} = 0_{(X,E)}. \text{ Then, } \left\{\left(\widetilde{F}_i, E\right)^c : i \in I\right\}$$

is a neutrosophic soft $\delta-$ open cover of (X, τ, E). Since (X, τ, E) is neutrosophic soft $\delta-$compact, it contains a finite subcover

$$\left\{\left(\widetilde{F}_i, E\right)^c : i = i_1, i_2, i_3, ..., i_n\right\}$$

for (X, τ, E). This implies that

$$\bigcap\left\{\left(\widetilde{F}_i, E\right)^c : i = i_1, i_2, i_3, ..., i_n\right\} = 0_{(X,E)}.$$

This contradicts that $\left\{\left(\widetilde{F}_i, E\right) : i \in I\right\}$ has the finite intersection property.

Conversely, let $\left\{\left(\widetilde{U}_i, E\right) : i \in I\right\}$ be a neutrosophic soft $\delta-$open cover of (X, τ, E). Consider the family $\left\{\left(\widetilde{U}_i, E\right)^c : i \in I\right\}$ of neutrosophic soft $\delta-$closed sets. Since $\left\{\left(\widetilde{U}_i, E\right) : i \in I\right\}$ is a cover of (X, τ, E), the intersection of all members of $\left\{\left(\widetilde{U}_i, E\right)^c : i \in I\right\}$ is null. Hence, $\left\{\left(\widetilde{U}_i, E\right) : i \in I\right\}$ does not have the finite intersection property. In other words, there are finite number of neutrosophic soft $\delta-$open sets $\left(\widetilde{U}_{i_1}, E\right)$,$\left(\widetilde{U}_{i_2}, E\right)$,...,$\left(\widetilde{U}_{i_n}, E\right)$ such that

$$\left(\widetilde{U}_{i_1}, E\right)^c \cap \left(\widetilde{U}_{i_2}, E\right)^c \cap, ..., \cap \left(\widetilde{U}_{i_n}, E\right)^c = 0_{(X,E)}.$$

This implies that $\left\{\left(\widetilde{U}_{i_1}, E\right), \left(\widetilde{U}_{i_2}, E\right), ..., \left(\widetilde{U}_{i_n}, E\right)\right\}$ is a finite subcover of (X, τ, E). Hence, (X, τ, E) is neutrosophic soft $\delta-$compact. □

Corollary 6.6. *A neutrosophic soft topological space (X, τ_δ, E) is neutrosophic soft compact if and only if every family of neutrosophic soft $\tau_\delta-$closed subsets in (X, τ_δ, E) with the finite intersection property has a nonempty intersection.*

Therefore, we can notice that neutrosophic soft $\delta-$compactness of a neutrosophic soft topological space is equivalent to neutrosophic soft compactness of a smaller space, namely the collection of all neutrosophic soft $\delta-$open subsets.

Remark 6.7. (X, τ, E) is neutrosophic soft $\delta-$compact if and only if (X, τ_δ, E) is neutrosophic soft compact.

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Theorem 6.8. *Let $\left(\widetilde{F}, E\right)$ be a neutrosophic soft $\delta-$closed subset of a neutrosophic soft $\delta-$ compact space (X, τ, E). Then, $\left(\widetilde{F}, E\right)$ is also neutrosophic soft $\delta-$compact in (X, τ, E).*

Proof. Let $\left(\widetilde{F}, E\right)$ be any neutrosophic soft $\delta-$closed subset of (X, τ, E) and $\left\{\left(\widetilde{U}_i, E\right) : i \in I\right\}$ be a neutrosophic soft $\delta-$open cover of (X, τ, E). Since $\left(\widetilde{F}, E\right)^c$ is neutrosophic soft $\delta-$open, $\left\{\left(\widetilde{U}_i, E\right) : i \in I\right\} \bigcup \left\{\left(\widetilde{F}, E\right)^c\right\}$ is a neutrosophic soft $\delta-$open cover of (X, τ, E). Since (X, τ, E) is neutrosophic soft $\delta-$compact, there exists a finite subset $I_0 \subseteq I$ such that $1_{(X,E)} \subseteq \bigcup \left\{\left(\widetilde{U}_i, E\right) : i \in I_0\right\} \cup \left(\widetilde{F}, E\right)^c$. But, $\left(\widetilde{F}, E\right) \cup \left(\widetilde{F}, E\right)^c \neq 1_{(X,E)}$. Hence, $\left(\widetilde{F}, E\right) \subseteq \bigcup \left\{\left(\widetilde{U}_i, E\right) : i \in I_0\right\}$. Therefore, $\left(\widetilde{F}, E\right)$ is neutrosophic soft $\delta-$compact in (X, τ, E) . □

Theorem 6.9. *Let $\left(\widetilde{F}, E\right)$ and $\left(\widetilde{G}, E\right)$ be neutrosophic soft subsets of a neutrosophic soft topological space (X, τ, E) such that $\left(\widetilde{F}, E\right)$ is neutrosophic soft $\delta-$compact in (X, τ, E) and $\left(\widetilde{G}, E\right)$ is neutrosophic soft $\delta-$closed in (X, τ, E). Then, $\left(\widetilde{F}, E\right) \cap \left(\widetilde{G}, E\right)$ is neutrosophic soft $\delta-$compact in (X, τ, E).*

Proof. Let $\left\{\left(\widetilde{U}_i, E\right) : i \in I\right\}$ be a cover of $\left(\widetilde{F}, E\right) \cap \left(\widetilde{G}, E\right)$ consisting of neutrosophic soft $\delta-$open subsets in (X, τ, E). Since $\left(\widetilde{G}, E\right)^c$ is a neutrosophic soft $\delta-$open set,

$$\left\{\left(\widetilde{U}_i, E\right) : i \in I\right\} \bigcup \left\{\left(\widetilde{G}, E\right)^c\right\}$$

is a neutrosophic soft $\delta-$open cover of $\left(\widetilde{F}, E\right)$. Since $\left(\widetilde{F}, E\right)$ is neutrosophic soft $\delta-$compact in (X, τ, E), there exists a finite subset $I_0 \subseteq I$ such that

$$\left(\widetilde{F}, E\right) \subseteq \bigcup \left\{\left(\widetilde{U}_i, E\right) : i \in I_0\right\} \bigcup \left\{\left(\widetilde{G}, E\right)^c\right\}.$$

Therefore,

$$\left(\widetilde{F}, E\right) \bigcap \left(\widetilde{G}, E\right) \subseteq \bigcup \left\{\left(\widetilde{U}_i, E\right) : i \in I_0\right\}.$$

Hence, $\left(\widetilde{F}, E\right) \bigcap \left(\widetilde{G}, E\right)$ is neutrosophic soft $\delta-$compact in (X, τ, E). □

Theorem 6.10. *Let $f : (X, \tau_1, E_1) \to (Y, \tau_2, E_2)$ be a neutrosophic soft $\delta-$continuous and surjective mapping. If (X, τ_1, E_1) is a neutrosophic soft $\delta-$compact space then (Y, τ_2, E_2) is also a neutrosophic soft $\delta-$compact space.*

Proof. Let $\left\{\left(\widetilde{U}_i, E\right) : i \in I\right\}$ be a neutrosophic soft $\delta-$open cover in (Y, τ_2, E_2). Then, $\left\{f^{-1}\left(\left(\widetilde{U}_i, E\right)\right) : i \in I\right\}$ is a cover in (X, τ_1, E_1). Since f is neutrosophic soft $\delta-$continuous, $f^{-1}\left(\left(\widetilde{U}_i, E\right)\right)$ is neutrosophic soft $\delta-$open and $\left\{f^{-1}\left(\left(\widetilde{U}_i, E\right)\right) : i \in I\right\}$ is a neutrosophic soft $\delta-$open cover in (X, τ_1, E_1). Since (X, τ_1, E_1) is neutrosophic soft $\delta-$compact, there exists a finite subset $I_0 \subseteq I$ such that $1_{(X,E_1)} \subseteq \bigcup \left\{f^{-1}\left(\left(\widetilde{U}_i, E\right)\right) : i \in I_0\right\}$. Thus,

$$f\left(1_{(X,E_1)}\right) \subseteq f\left(\bigcup \left\{f^{-1}\left(\left(\widetilde{U}_i, E\right)\right) : i \in I_0\right\}\right)$$

$$= \bigcup \left\{f\left(f^{-1}\left(\left(\widetilde{U}_i, E\right)\right)\right) : i \in I_0\right\} = \bigcup \left\{\left(\widetilde{U}_i, E\right) : i \in I_0\right\}.$$

Since f is surjective, $1_{(Y,E_2)} = f\left(1_{(x,E_1)}\right) \subseteq \bigcup \left\{\left(\widetilde{U}_i, E\right) : i \in I_0\right\}$. Hence, (Y, τ_2, E_2) is neutrosophic soft $\delta-$compact. □

Theorem 6.11. *Let $f : (X, \tau_1, E_1) \to (Y, \tau_2, E_2)$ be neutrosophic soft $\delta-$continuous. If a neutrosophic soft subset $\left(\widetilde{F}, E\right)$ is neutrosophic soft $\delta-$compact in (X, τ_1, E_1) then the image $f\left(\left(\widetilde{F}, E\right)\right)$ is neutrosophic soft $\delta-$compact in (Y, τ_2, E_2).*

Proof. Let $\left\{\left(\widetilde{U}_i, E\right) : i \in I\right\}$ be a neutrosophic soft $\delta-$open cover of $f\left(\left(\widetilde{F}, E\right)\right)$. Since f is neutrosophic soft $\delta-$continuous, $f^{-1}\left(\left(\widetilde{U}_i, E\right)\right)$ is a neutrosophic soft $\delta-$open set in (X, τ_1, E_1) for all $i \in I$. Thus, $\left\{f^{-1}\left(\left(\widetilde{U}_i, E\right)\right) : i \in I\right\}$ is a cover of $\left(\widetilde{F}, E\right)$ by neutrosophic soft $\delta-$open sets in (X, τ_1, E_1). Since $\left(\widetilde{F}, E\right)$ is neutrosophic soft $\delta-$compact in (X, τ_1, E_1), there is a finite subset $I_0 \subseteq I$ such that $\left(\widetilde{F}, E\right) \subseteq \bigcup\left\{f^{-1}\left(\widetilde{U}_i, E\right) : i \in I_0\right\}$. So,

$$f\left(\left(\widetilde{F}, E\right)\right) \subseteq f\left(\bigcup\left\{f^{-1}\left(\widetilde{U}_i, E\right) : i \in I_0\right\}\right) \text{ and } f\left(\left(\widetilde{F}, E\right)\right) \subseteq \bigcup\left\{\left(\widetilde{U}_i, E\right) : i \in I_0\right\}.$$

Therefore, $f\left(\left(\widetilde{F}, E\right)\right)$is neutrosophic soft $\delta-$compact in (Y, τ_2, E_2). □

Theorem 6.12. *Let $f : (X, \tau_1, E_1) \to (Y, \tau_2, E_2)$ be a neutrosophic soft $\delta-$continuous, neutrosophic soft $\delta-$open and injective mapping. If a neutrosophic soft subset $\left(\widetilde{G}, E\right)$ in (Y, τ_2, E_2) is neutrosophic soft $\delta-$compact in (Y, τ_2, E_2) then $f^{-1}\left(\left(\widetilde{G}, E\right)\right)$ is neutrosophic soft $\delta-$compact in (X, τ_1, E_1).*

Proof. Let $\left\{\left(\widetilde{V}_i, E\right) : i \in I\right\}$ be a neutrosophic soft $\delta-$open cover of $f^{-1}\left(\left(\widetilde{G}, E\right)\right)$ in (X, τ_1, E_1). Then, $f^{-1}\left(\left(\widetilde{G}, E\right)\right) \subseteq \bigcup\left\{\left(\widetilde{V}_i, E\right) : i \in I\right\}$ and $\left(\widetilde{G}, E\right) \subseteq f\left(f^{-1}\left(\left(\widetilde{G}, E\right)\right)\right) \subseteq f\left(\bigcup\left\{\left(\widetilde{V}_i, E\right) : i \in I\right\}\right)$. Since $\left(\widetilde{G}, E\right)$ is neutrosophic soft $\delta-$compact in (Y, τ_2, E_2), there is a finite subset $I_0 \subseteq I$ such that $\left(\widetilde{G}, E\right) \subseteq \bigcup\left\{f\left(\left(\widetilde{V}_i, E\right)\right) : i \in I_0\right\}$. So,

$$f^{-1}\left(\left(\widetilde{G}, E\right)\right) \subseteq f^{-1}\left(\bigcup\left\{f\left(\left(\widetilde{V}_i, E\right)\right) : i \in I_0\right\}\right) = \bigcup\left\{f^{-1}\left(f\left(\left(\widetilde{V}_i, E\right)\right) : i \in I_0\right)\right\} = \bigcup\left\{\left(\widetilde{V}_i, E\right) : i \in I_0\right\}.$$

The proof is completed. □

Definition 6.13. A neutrosophic soft topological space (X, τ, E) is said to be neutrosophic soft locally $\delta-$compact at a neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$, if there is a neutrosophic soft $\delta-$open subset $\left(\widetilde{U}, E\right)$ and a neutrosophic soft subset $\left(\widetilde{F}, E\right)$, which is neutrosophic soft $\delta-$compact in (X, τ, E) such that $x^e_{(\alpha,\beta,\gamma)} \subseteq \left(\widetilde{F}, E\right) \subseteq \left(\widetilde{U}, E\right)$. If (X, τ, E) is neutrosophic soft locally $\delta-$compact at each of its neutrosophic soft point, (X, τ, E) is said to be a neutrosophic soft locally $\delta-$compact space.

It is clear that each neutrosophic soft $\delta-$compact space is a neutrosophic soft locally $\delta-$ compact space. But, the converse is not true.

Example 6.14. Let $X = \{4, 5, 6, ...\}$, $E = \{a, b\}$ and for each $n \in X$

$$U_n(x) = \begin{cases} < x, 1, 1, 0 >, & x = n \\ < x, \frac{1}{2} - \frac{1}{n}, \frac{1}{2} - \frac{1}{n}, \frac{1}{2} + \frac{1}{n} > & x \neq n \end{cases}$$

$$V_n(x) = \begin{cases} < x, 1, 1, 0 >, & x = n \\ < x, \frac{1}{2} + \frac{1}{n}, \frac{1}{2} + \frac{1}{n}, \frac{1}{2} - \frac{1}{n} > & x \neq n \end{cases}$$

Consider the neutrosophic soft sets $\left(\widetilde{F}_n, E\right)$ and $\left(\widetilde{G}_n, E\right)$ defined as $\widetilde{F}_n(a) = \widetilde{F}_n(b) = \{U_n(x) : x \in X\}$, $\widetilde{G}_n(a) = \widetilde{G}_n(b) = \{V_n(x) : x \in X\}$. Let τ be a neutrosophic soft topology on X generated by $\left\{\left(\widetilde{F}_n, E\right), \left(\widetilde{G}_n, E\right) : n \in X\right\}$. Then, $\left[\overline{\left(\widetilde{F}_n, E\right)}\right]^\circ = \left(\widetilde{F}_n, E\right)$ for all $n \in X$. So, every $\left(\widetilde{F}_n, E\right)$ is neutrosophic soft $\delta-$open. Therefore, $\left\{\left(\widetilde{F}_n, E\right) : n \in X\right\}$ is a neutrosophic soft $\delta-$open cover of $1_{(X,E)}$, which does not have a finite subcover. Hence, (X, τ, E) is not neutrosophic soft $\delta-$compact. But, for any neutrosophic soft point $n^e_{(\alpha,\beta,\gamma)}$ in X, where $e \in E$, $n^e_{(\alpha,\beta,\gamma)} \subseteq n^e_{(1,1,0)} \subseteq \left(\widetilde{F}_n, E\right)$. Note that $n^e_{(1,1,0)}$ is neutrosophic soft $\delta-$compact and $\left(\widetilde{F}_n, E\right)$ is neutrosophic soft $\delta-$open. Hence, (X, τ, E) is neutrosophic soft locally $\delta-$compact.

Definition 6.15. A neutrosophic soft subset $\left(\widetilde{A}, E\right)$ of a neutrosophic soft topological space (X, τ, E) is said to be neutrosophic soft locally $\delta-$compact in (X, τ, E), provided that, for each neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$ in $\left(\widetilde{A}, E\right)$, there is a neutrosophic soft $\delta-$open subset $\left(\widetilde{U}, E\right)$ and a neutrosophic soft subset $\left(\widetilde{F}, E\right)$, which is neutrosophic soft $\delta-$compact in (X, τ, E) such that $x^e_{(\alpha,\beta,\gamma)} \subseteq \left(\widetilde{F}, E\right) \subseteq \left(\widetilde{U}, E\right)$.

Theorem 6.16. *Let (X, τ, E) be a neutrosophic soft locally $\delta-$compact space and $\left(\widetilde{A}, E\right)$ be a neutrosophic soft subset in (X, τ, E). If $\left(\widetilde{A}, E\right)$ is neutrosophic soft $\delta-$closed in (X, τ, E) then $\left(\widetilde{A}, E\right)$ is neutrosophic soft locally $\delta-$compact in (X, τ, E).*

Proof. Take any neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$ in $\left(\widetilde{A}, E\right)$. Since (X, τ, E) is neutrosophic soft locally $\delta-$compact, there exist a neutrosophic soft $\delta-$open subset $\left(\widetilde{U}, E\right)$ and a neutrosophic soft subset $\left(\widetilde{F}, E\right)$, which is neutrosophic soft $\delta-$compact in (X, τ, E) such that $x^e_{(\alpha,\beta,\gamma)} \subseteq \left(\widetilde{F}, E\right) \subseteq \left(\widetilde{U}, E\right)$. Then, $\left(\widetilde{F}, E\right) \cap \left(\widetilde{A}, E\right)$ is neutrosophic soft $\delta-$compact in (X, τ, E). Because, $\left(\widetilde{A}, E\right)$ is neutrosophic soft $\delta-$closed in (X, τ, E). Therefore, $\left(\widetilde{U}, E\right)$ is a neutrosophic soft $\delta-$open subset containing a neutrosophic soft $\delta-$compact subset $\left(\widetilde{F}, E\right) \cap \left(\widetilde{A}, E\right)$ with $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F}, E\right) \cap \left(\widetilde{A}, E\right)$. Hence, $\left(\widetilde{A}, E\right)$ is neutrosophic soft locally $\delta-$compact in (X, τ, E). □

Theorem 6.17. *Let a neutrosophic soft topological space (X, τ, E) be neutrosophic soft locally $\delta-$compact and $\left(\widetilde{A}, E\right)$ be a neutrosophic soft open subset in (X, τ, E). Then, $\left(\widetilde{A}, E\right)$ is neutrosophic soft locally $\delta-$compact in (X, τ, E).*

Proof. Take any neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$ in $\left(\widetilde{A}, E\right)$. Since (X, τ, E) is neutrosophic soft locally $\delta-$compact, there exist a neutrosophic soft $\delta-$open subset $\left(\widetilde{U}, E\right)$ and a neutrosophic soft subset $\left(\widetilde{F}, E\right)$, which is neutrosophic soft $\delta-$compact in (X, τ, E) such that $x^e_{(\alpha,\beta,\gamma)} \subseteq \left(\widetilde{F}, E\right) \subseteq \left(\widetilde{U}, E\right)$. We know that $\overline{\left(\widetilde{A}, E\right)}$ is neutrosophic soft regular closed and neutrosophic soft $\delta-$closed. So, $\left(\widetilde{F}, E\right) \cap \overline{\left(\widetilde{A}, E\right)}$ is neutrosophic soft $\delta-$compact in (X, τ, E). Therefore, $\left(\widetilde{U}, E\right)$ is a neutrosophic soft $\delta-$open subset containing a neutrosophic soft $\delta-$compact subset $\left(\widetilde{F}, E\right) \cap \overline{\left(\widetilde{A}, E\right)}$ with $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F}, E\right) \cap \overline{\left(\widetilde{A}, E\right)}$. Hence, $\left(\widetilde{A}, E\right)$ is neutrosophic soft locally $\delta-$compact in (X, τ, E). □

Theorem 6.18. *Let (X, τ_1, E_1) and (Y, τ_2, E_2) be neutrosophic soft topological spaces and $f : (X, \tau_1, E_1) \to (Y, \tau_2, E_2)$ be a neutrosophic soft $\delta-$continuous, neutrosophic soft $\delta-$open and surjective function. If (X, τ_1, E_1) is neutrosophic soft locally $\delta-$compact then (Y, τ_2, E_2) is also neutrosophic soft locally $\delta-$compact.*

Proof. Let $y^{e'}_{(\alpha',\beta',\gamma')}$ be a neutrosophic soft point in (Y, τ_2, E_2). Since f is onto, there is a neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$ in (X, τ_1, E_1) such that $y^{e'}_{(\alpha',\beta',\gamma')} = f\left(x^e_{(\alpha,\beta,\gamma)}\right)$. Since (X, τ_1, E_1) is neutrosophic soft locally $\delta-$compact, there exist a neutrosophic soft $\delta-$open subset $\left(\widetilde{U}, E\right)$ in (X, τ_1, E_1) and a neutrosophic soft $\delta-$compact subset $\left(\widetilde{F}, E\right)$ in (X, τ_1, E_1) such that $x^e_{(\alpha,\beta,\gamma)} \subseteq \left(\widetilde{F}, E\right) \subseteq \left(\widetilde{U}, E\right)$. Since f is neutrosophic soft $\delta-$open, $f\left(\left(\widetilde{U}, E\right)\right)$ is a neutrosophic soft $\delta-$open subset in (Y, τ_2, E_2) containing $y^{e'}_{(\alpha',\beta',\gamma')}$ and since f is neutrosophic soft $\delta-$continuous, $f\left(\left(\widetilde{F}, E\right)\right)$ is neutrosophic soft $\delta-$compact in (Y, τ_2, E_2). Therefore, $y^{e'}_{(\alpha',\beta',\gamma')} \subseteq f\left(\left(\widetilde{F}, E\right)\right) \subseteq f\left(\left(\widetilde{U}, E\right)\right)$. Hence, (Y, τ_2, E_2) is neutrosophic soft locally $\delta-$ compact. □

Corollary 6.19. *Let (X, τ_1, E_1) be a semiregular neutrosophic soft topological space and $f : (X, \tau_1, E_1) \to (Y, \tau_2, E_2)$ be a neutrosophic soft continuous, neutrosophic soft $\delta-$open and surjective function. If (X, τ_1, E_1) is neutrosophic soft locally $\delta-$compact then (Y, τ_2, E_2) is also neutrosophic soft locally $\delta-$compact.*

Theorem 6.20. *Let (X,τ_1,E_1) and (Y,τ_2,E_2) be neutrosophic soft topological spaces and $f:(X,\tau_1,E_1)\to(Y,\tau_2,E_2)$ be a neutrosophic soft $\delta-$continuous, neutrosophic soft $\delta-$open and injective function. If (Y,τ_2,E_2) is neutrosophic soft locally $\delta-$compact then (X,τ_1,E_1) is also neutrosophic soft locally $\delta-$compact.*

Proof. Take $x^e_{(\alpha,\beta,\gamma)}$ in (X,τ_1,E_1). Then since f is injective, there is a neutrosophic soft point $y^{e'}_{(\alpha',\beta',\gamma')}$ in (Y,τ_2,E_2) such that $y^{e'}_{(\alpha',\beta',\gamma')}=f\left(x^e_{(\alpha,\beta,\gamma)}\right)$. Since (Y,τ_2,E_2) is neutrosophic soft locally $\delta-$compact, there exist a neutrosophic soft $\delta-$open subset $\left(\widetilde{U},E\right)$ and a neutrosophic soft subset $\left(\widetilde{F},E\right)$, which is neutrosophic soft $\delta-$compact in (Y,τ_2,E_2) such that $y^{e'}_{(\alpha',\beta',\gamma')}\subseteq\left(\widetilde{F},E\right)\subseteq\left(\widetilde{U},E\right)$. Since f is neutrosophic soft $\delta-$continuous, $f^{-1}\left(\left(\widetilde{U},E\right)\right)$ is a neutrosophic soft $\delta-$open subset in (X,τ_1,E_1) containing $x^e_{(\alpha,\beta,\gamma)}$. Since f is neutrosophic soft $\delta-$continuous and injective, $f^{-1}\left(\left(\widetilde{F},E\right)\right)$ is neutrosophic soft $\delta-$compact in (X,τ_1,E_1). Therefore, $x^e_{(\alpha,\beta,\gamma)}\subseteq f^{-1}\left(\left(\widetilde{F},E\right)\right)\subseteq f^{-1}\left(\left(\widetilde{U},E\right)\right)$. The proof is completed. □

Corollary 6.21. *Let (X,τ_1,E_1) be a neutrosophic soft topological space, (Y,τ_2,E_2) be a neutrosophic soft semiregular topological space and $f:(X,\tau_1,E_1)\to(Y,\tau_2,E_2)$ be a neutrosophic soft continuous, neutrosophic soft $\delta-$open, injective function. If (Y,τ_2,E_2) is neutrosophic soft locally $\delta-$compact then (X,τ_1,E_1) is also neutrosophic soft locally $\delta-$compact.*

7. Conclusion

Some properties of the notions of neutrosophic soft $\delta-$open sets, neutrosophic soft $\delta-$closed sets, neutrosophic soft $\delta-$interior, neutrosophic soft $\delta-$closure, neutrosophic soft $\delta-$interior point, neutrosophic soft $\delta-$cluster point and neutrosophic soft $\delta-$topology are introduced. Also, the notions of neutrosophic soft $\delta-$compactness and neutrosophic soft locally $\delta-$compactness are introduced. Furthermore, the properties of neutrosophic soft $\delta-$ compactness and neutrosophic soft locally $\delta-$compactness are analized under the neutrosophic soft $\delta-$ continuous mappings. Additionally, a new approach is made to the concept of quasi-coincidence in neutrosophic soft topology. Since topological structures on neutrosophic soft sets have been introduced by many scientists, we generalize the $\delta-$topological properties to the neutrosophic soft sets, which may be useful in some other disciplines. For the existence of compact connections between soft sets and information systems [20, 24], the results obtained from the studies on neutrosophic soft topological space can be used to develop these connections. We hope that many researchers will benefit from the findings in this document to further their studies on neutrosophic soft topology to carry out a general framework for their applications in practical life.

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