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Approximation for Difference of Lupaş and Some Classical Operators

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Abstract. The approximation of difference of two linear positive operators having different basis functions is discussed in the present article. The quantitative estimates in terms of weighted modulus of continuity for the difference of Lupaş operators and the classical ones are obtained, viz. Lupaş and Baskakov operators, Lupaş and Szász operators, Lupaş and Baskakov-Kantorovich operators, Lupaş and Szász-Kantorovich operators.

Dedicated to Prof. Hari M. Srivastava

1. Introduction

Varied approximation properties for the difference of linear positive operators having same/different basis functions have been extensively studied and investigated (cf. [1], [2], [3], [4], [5], [6], [8], [9], [16], etc.). In the present article, we discuss the difference of Lupaş operators and Baskakov operators, Lupaş and Szász operators, Lupaş and Baskakov-Kantorovich operators, Lupaş and Szász-Kantorovich operators. We also refer some references here, wherein researchers have studied approximation properties of classical linear positive operators (cf. [7], [10], [11], [12], [13], [14], [17], [18], [19], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35]).
Throughout the paper, let $C_2[0,\infty)$ denote the class of all continuous functions on positive real axis and $f(x) = \mathcal{O}(1+x^2)$. N. Ispir [20] considered the following weighted modulus of continuity:

$$\Omega(f,\delta) = \sup_{x\geqslant 0, |m|<\delta} |f(x+m) - f(x)| \, [(1+x^2)(1+m^2)]^{-1}.$$

Also, let $\tilde{C}_2[0,\infty)$ denote the closed subspace of $C_2[0,\infty)$, for which, $\lim_{x\to\infty} |f(x)| \, (1+x^2)^{-1} < C$, for some constant C and $\|.\|_2 = \sup_{x\in[0,\infty)} |f(x)| \, (1+x^2)^{-1}$.
Very recently, Gupta [9] established a general estimate for the difference of operators having different basis functions for A_n and B_n, where

$$A_n(f,x) = \sum_{k\in Z^+} c_{n,k}(x) \, F_k^n(f)$$

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and

$$B_n(f,x)=\sum_{k\in Z^+} d_{n,k}(x)\, G_k^n(f)$$

Theorem A.[9] Let $f\in C_2[0,\infty)$ with $f''\in \tilde{C}_2[0,\infty)$. Then, for any two positive linear operators A_n and B_n, we have

$$\begin{aligned}|(A_n-B_n)(f,x)| &\leqslant \frac{1}{2}\|f''\|_2\,(\beta_1(x)+\beta_2(x))+8\,\Omega(f'',\delta_1)\,(1+\beta_1(x))+8\,\Omega(f'',\delta_2)\,(1+\beta_2(x))\\ &\quad +16\,\Omega(f,\delta_3)\,(1+\gamma_1(x))+16\,\Omega(f,\delta_4)\,(1+\gamma_2(x)),\end{aligned}$$

where $\beta_1(x)=\sum\limits_{k\in Z^+} c_{n,k}(x)\,[1+(F_k^n(e_1))^2]\,T_2^{F_k^n}$,
$\beta_2(x)=\sum\limits_{k\in Z^+} d_{n,k}(x)\,[1+(G_k^n(e_1))^2]\,T_2^{G_k^n}$,
$\delta_1^4(x)=\sum\limits_{k\in Z^+} c_{n,k}(x)\,[1+(F_k^n(e_1))^2]\,T_6^{F_k^n}$,
$\delta_2^4(x)=\sum\limits_{k\in Z^+} d_{n,k}(x)\,[1+(G_k^n(e_1))^2]\,T_6^{G_k^n}$,
$\delta_3^4(x)=\sum\limits_{k\in Z^+} c_{n,k}(x)\,[1+(F_k^n(e_1))^2]\,[F_k^n(e_1)-x]^4$,
$\delta_4^4(x)=\sum\limits_{k\in Z^+} d_{n,k}(x)\,[1+(G_k^n(e_1))^2]\,[G_k^n(e_1)-x]^4$,
$\gamma_1(x)=\sum\limits_{k\in Z^+} c_{n,k}(x)\,[1+(F_k^n(e_1))^2]$ and $\gamma_2(x)=\sum\limits_{k\in Z^+} d_{n,k}(x)\,[1+(G_k^n(e_1))^2]$.
Here, $e_r(t)=t^r$, $r=0,1,2,\dots$; $T_r^{F_k^n}=F_k^n[e_1-F_k^n(e_1)]^r$ and $\delta_i(x)\leqslant 1$, $i=1,2,3,4$.

We extend the studies of [9] as we study quantitative estimates in terms of weighted modulus of continuity and obtain the difference between Lupaş operators and certain classical ones.

2. Preliminaries

In the year 1995, A. Lupaş [21] proposed the following discrete operators:

$$\begin{aligned}L_n(f,x) &= \sum_{k=0}^{\infty} v_{n,k}(x)\, f\left(\frac{k}{n}\right)\\ &:= \sum_{k=0}^{\infty} 2^{-nx}\,\frac{(nx)_k}{k!\,2^k}\, f\left(\frac{k}{n}\right),\quad f\in C[0,\infty).\end{aligned}\tag{1}$$

Lemma 2.1. *[15] First few moments of the Lupaş operators* (1) *are given by*

$$\begin{aligned}L_n(e_0,x) &= 1,\\ L_n(e_1,x) &= x,\\ L_n(e_2,x) &= x^2+\frac{2x}{n},\\ L_n(e_3,x) &= x^3+\frac{6x^2}{n}+\frac{6x}{n^2},\\ L_n(e_4,x) &= x^4+\frac{12x^3}{n}+\frac{36x^2}{n^2}+\frac{26x}{n^3},\\ L_n(e_5,x) &= x^5+\frac{20x^4}{n}+\frac{120x^3}{n^2}+\frac{250x^2}{n^3}+\frac{150x}{n^4},\\ L_n(e_6,x) &= x^6+\frac{30x^5}{n}+\frac{300x^4}{n^2}+\frac{1230x^3}{n^3}+\frac{2040x^2}{n^4}+\frac{1082x}{n^5}.\end{aligned}$$

Remark 2.2. *Denote* $F_k^n(f) := f\left(\frac{k}{n}\right)$. *Then,* $F_k^n(e_1) = \frac{k}{n}$ *and for* $m \in \mathbb{N}$, *we have*

$$T_m^{F_k^n} := F_k^n[e_1 - F_k^n(e_1)]^m = 0.$$

In the year 1957, V. A. Baskakov proposed a generalization of the Bernstein polynomials based on negative binomial distribution. For $f \in C[0,\infty)$, the Baskakov operators are defined as

$$\begin{aligned} V_n(f,x) &= \sum_{k=0}^{\infty} b_{n,k}(x)\, f\left(\frac{k}{n}\right) \\ &:= \sum_{k=0}^{\infty} \binom{n+k-1}{k} \frac{x^k}{(1+x)^{n+k}}\, f\left(\frac{k}{n}\right). \end{aligned} \tag{2}$$

Lemma 2.3. *[15] First few moments of the Baskakov operators* (2) *are given by*

$$\begin{aligned} V_n(e_0,x) &= 1, \\ V_n(e_1,x) &= x, \\ V_n(e_2,x) &= \frac{nx^2 + x^2 + x}{n}, \\ V_n(e_3,x) &= \frac{n^2x^3 + 3nx^3 + 2x^3 + 3nx^2 + 3x^2 + x}{n^2}, \\ V_n(e_4,x) &= \frac{n^3x^4 + 6n^2x^4 + 11nx^4 + 6x^4 + 6n^2x^3 + 18nx^3 + 12x^3 + 7nx^2 + 7x^2 + x}{n^3}, \\ V_n(e_5,x) &= \frac{\begin{bmatrix} x^5(n+1)(n+2)(n+3)(n+4) + 10x^4(n+1)(n+2)(n+3) \\ +25x^3(n+1)(n+2) + 15x^2(n+1) + x \end{bmatrix}}{n^4}, \\ V_n(e_6,x) &= \frac{\begin{bmatrix} x^6(n+1)(n+2)(n+3)(n+4)(n+5) + 15x^5(n+1)(n+2)(n+3)(n+4) \\ +65x^4(n+1)(n+2)(n+3) + 90x^3(n+1)(n+2) + 31x^2(n+1) + x \end{bmatrix}}{n^5}. \end{aligned}$$

Remark 2.4. *For Baskakov operators, denote* $G_k^n(f) := f\left(\frac{k}{n}\right)$. *Then,* $G_k^n(e_1) = \frac{k}{n}$ *and for* $m \in \mathbb{N}$, *we have*

$$T_m^{G_k^n} := G_k^n[e_1 - G_k^n(e_1)]^m = 0.$$

The Szász-Mirakyan operators are generalizations of the Bernstein polynomials to infinite intervals. For $f \in C[0,\infty)$, the Szász operators are defined as

$$\begin{aligned} S_n(f,x) &= \sum_{k=0}^{\infty} s_{n,k}(x)\, f\left(\frac{k}{n}\right) \\ &:= \sum_{k=0}^{\infty} e^{-nx} \frac{(nx)^k}{k!}\, f\left(\frac{k}{n}\right). \end{aligned} \tag{3}$$

Lemma 2.5. *[15] First few moments of the Szász-Mirakyan operators* (3) *are given by*

$$\begin{aligned} S_n(e_0,x) &= 1,\\ S_n(e_1,x) &= x,\\ S_n(e_2,x) &= x^2+\frac{x}{n},\\ S_n(e_3,x) &= x^3+\frac{3x^2}{n}+\frac{x}{n^2},\\ S_n(e_4,x) &= x^4+\frac{6x^3}{n}+\frac{7x^2}{n^2}+\frac{x}{n^3},\\ S_n(e_5,x) &= x^5+\frac{10x^4}{n}+\frac{25x^3}{n^2}+\frac{15x^2}{n^3}+\frac{x}{n^4},\\ S_n(e_6,x) &= x^6+\frac{15x^5}{n}+\frac{65x^4}{n^2}+\frac{90x^3}{n^3}+\frac{31x^2}{n^4}+\frac{x}{n^5}. \end{aligned}$$

Remark 2.6. *For Szász-Mirakyan operators, denote $H_k^n(f) := f\left(\frac{k}{n}\right)$. Then, $H_k^n(e_1)=\frac{k}{n}$ and for $m\in\mathbb{N}$, we have*

$$T_m^{H_k^n} := H_k^n[e_1-H_k^n(e_1)]^m = 0.$$

The Baskakov-Kantorovich operators are defined as

$$\begin{aligned} R_n(f,x) &= \sum_{k=0}^{\infty} b_{n,k}(x)\, n\int_{\frac{k}{n}}^{\frac{k+1}{n}} f(t)\,dt\\ &:= \sum_{k=0}^{\infty} b_{n,k}(x)\, J_k^n(f), \end{aligned} \tag{4}$$

where $b_{n,k}(x)$ is the Baskakov basis function defined in (2).

Lemma 2.7. *By simple computation, first few moments of the Baskakov-Kantorovich operators* (4) *can be obtained as*

$$\begin{aligned} R_n(e_0,x) &= 1,\\ R_n(e_1,x) &= x+\frac{1}{2n},\\ R_n(e_2,x) &= x^2+\frac{x(2+x)}{n}+\frac{1}{3n^2}. \end{aligned}$$

Remark 2.8. *We have $J_k^n(f) := n\int_{\frac{k}{n}}^{\frac{k+1}{n}} f(t)\,dt$. Then, $J_k^n(e_1)=\frac{2k+1}{2n}$ and by simple computation, we have*

$$\begin{aligned} T_2^{J_k^n} &:= J_k^n[e_1-J_k^n(e_1)]^2 = \frac{1}{12n^2},\\ T_6^{J_k^n} &:= J_k^n[e_1-J_k^n(e_1)]^6 = \frac{1}{448n^6}. \end{aligned}$$

The Szász-Kantorovich operators are defined as

$$\begin{aligned} U_n(f,x) &= \sum_{k=0}^{\infty} s_{n,k}(x)\, n\int_{\frac{k}{n}}^{\frac{k+1}{n}} f(t)\,dt\\ &:= \sum_{k=0}^{\infty} s_{n,k}(x)\, I_k^n(f), \end{aligned} \tag{5}$$

where $s_{n,k}(x)$ is the Szász basis function defined in (3).

Lemma 2.9. *By simple computation, first few moments of the Szász-Kantorovich operators* (5) *can be obtained as*

$$\begin{aligned} U_n(e_0,x) &= 1, \\ U_n(e_1,x) &= x+\frac{1}{2n}, \\ U_n(e_2,x) &= x^2+\frac{2x}{n}+\frac{1}{3n^2}. \end{aligned}$$

Remark 2.10. *We have* $I_k^n(f) := n\int_{\frac{k}{n}}^{\frac{k+1}{n}} f(t)\,dt$. *Then,* $I_k^n(e_1)=\frac{2k+1}{2n}$ *and by simple computation, we have*

$$T_2^{I_k^n} := I_k^n[e_1 - I_k^n(e_1)]^2 = \frac{1}{12n^2},$$

$$T_6^{I_k^n} := I_k^n[e_1 - I_k^n(e_1)]^6 = \frac{1}{448n^6}.$$

3. Difference of operators/Quantitative Estimates

We compute the magnitude of difference of the two operators having the different basis functions. As an application of Theorem A, we have the following quantitative estimates for the difference between the operators.

Theorem 3.1. *Let* $f\in C_2[0,\infty)$ *with* $f''\in \tilde{C}_2[0,\infty)$. *Then, we have*

1.

$$|(L_n - V_n)(f,x)| \leqslant 16\,\Omega(f,\delta_3)\,(1+\gamma_1(x)) + 16\,\Omega(f,\delta_4)\,(1+\gamma_2(x)),$$

where

$$\delta_3^4(x) = \frac{12x^4}{n^2}+\frac{386x^3}{n^3}+\frac{1440x^2}{n^4}+\frac{1082x}{n^5}+\frac{12x^2}{n^2}+\frac{26x}{n^3},$$

$$\begin{aligned} \delta_4^4(x) = {} & \frac{x}{n^5}+\frac{x}{n^3}+\frac{31x^2}{n^5}+\frac{27x^2}{n^4}+\frac{7x^2}{n^3}+\frac{3x^2}{n^2}+\frac{180x^3}{n^5} \\ & +\frac{210x^3}{n^4}+\frac{48x^3}{n^3}+\frac{6x^3}{n^2}+\frac{390x^4}{n^5}+\frac{515x^4}{n^4}+\frac{138x^4}{n^3} \\ & +\frac{6x^4}{n^2}+\frac{360x^5}{n^5}+\frac{510x^5}{n^4}+\frac{157x^5}{n^3}+\frac{6x^5}{n^2}+\frac{120x^6}{n^5} \\ & +\frac{178x^6}{n^4}+\frac{61x^6}{n^3}+\frac{3x^6}{n^2}, \end{aligned}$$

$$\gamma_1(x) = 1+x^2+\frac{2x}{n}$$

and

$$\gamma_2(x) = 1+x^2+\frac{x(1+x)}{n}.$$

2.

$$|(L_n - S_n)(f,x)| \leqslant 16\,\Omega(f,\delta_3)\,(1+\gamma_1(x)) + 16\,\Omega(f,\delta_4)\,(1+\gamma_2(x)),$$

where

$$\delta_3^4(x) = \frac{12x^4}{n^2} + \frac{386x^3}{n^3} + \frac{1440x^2}{n^4} + \frac{1082x}{n^5} + \frac{12x^2}{n^2} + \frac{26x}{n^3},$$

$$\delta_4^4(x) = \frac{3x^4}{n^2} + \frac{36x^3}{n^3} + \frac{27x^2}{n^4} + \frac{x}{n^5} + \frac{3x^2}{n^2} + \frac{x}{n^3},$$

$$\gamma_1(x) = 1 + x^2 + \frac{2x}{n}$$

and

$$\gamma_2(x) = 1 + x^2 + \frac{x}{n}\cdot$$

3.

$$\begin{aligned}|(L_n - R_n)(f,x)| \leqslant\ & \frac{1}{2}\|f''\|_2\,\beta_2(x) + 8\,\Omega(f'',\delta_2)\,(1+\beta_2(x))\\ & +16\,\Omega(f,\delta_3)\,(1+\gamma_1(x)) + 16\,\Omega(f,\delta_4)\,(1+\gamma_2(x)),\end{aligned}$$

where

$$\beta_2(x) = \frac{1}{12n^2}\left\{1 + x^2\left(1+\frac{1}{n}\right) + \frac{2x}{n} + \frac{1}{4n^2}\right\},$$

$$\delta_2^4(x) = \frac{1}{448n^6}\left\{1 + x^2\left(1+\frac{1}{n}\right) + \frac{2x}{n} + \frac{1}{4n^2}\right\},$$

$$\delta_3^4(x) = \frac{12x^4}{n^2} + \frac{386x^3}{n^3} + \frac{1440x^2}{n^4} + \frac{1082x}{n^5} + \frac{12x^2}{n^2} + \frac{26x}{n^3},$$

$$\begin{aligned}\delta_4^4(x) =\ & \frac{1}{64n^6} + \frac{1}{16n^4} + \frac{45x}{4n^5} + \frac{9x}{2n^3} + \frac{1771x^2}{16n^5} + \frac{1293x^2}{16n^4}\\ & + \frac{29x^2}{2n^3} + \frac{3x^2}{n^2} + \frac{380x^3}{n^5} + \frac{405x^3}{n^4} + \frac{141x^3}{2n^3} + \frac{6x^3}{n^2}\\ & + \frac{1185x^4}{2n^5} + \frac{2985x^4}{4n^4} + \frac{351x^4}{2n^3} + \frac{23x^4}{4n^2} + \frac{432x^5}{n^5} + \frac{600x^5}{n^4}\\ & + \frac{176x^5}{n^3} + \frac{6x^5}{n^2} + \frac{120x^6}{n^5} + \frac{178x^6}{n^4} + \frac{61x^6}{n^3} + \frac{3x^6}{n^2},\end{aligned}$$

$$\gamma_1(x) = 1 + x^2 + \frac{2x}{n}$$

and

$$\gamma_2(x) = 1 + x^2\left(1+\frac{1}{n}\right) + \frac{2x}{n} + \frac{1}{4n^2}\cdot$$

4.

$$
\begin{aligned}
|(L_n - U_n)(f,x)| &\leqslant \frac{1}{2}\|f''\|_2\, \beta_2(x) + 8\,\Omega(f'',\delta_2)\,(1+\beta_2(x)) \\
&\quad +16\,\Omega(f,\delta_3)\,(1+\gamma_1(x)) + 16\,\Omega(f,\delta_4)\,(1+\gamma_2(x)),
\end{aligned}
$$

where

$$\beta_2(x) = \frac{1}{12n^2}\left\{1 + x^2 + \frac{2x}{n} + \frac{1}{4n^2}\right\},$$

$$\delta_2^4(x) = \frac{1}{448n^6}\left\{1 + x^2 + \frac{2x}{n} + \frac{1}{4n^2}\right\},$$

$$\delta_3^4(x) = \frac{12x^4}{n^2} + \frac{386x^3}{n^3} + \frac{1440x^2}{n^4} + \frac{1082x}{n^5} + \frac{12x^2}{n^2} + \frac{26x}{n^3},$$

$$\delta_4^4(x) = \frac{1}{64n^6} + \frac{1}{16n^4} + \frac{45x}{4n^5} + \frac{9x}{2n^3} + \frac{1293x^2}{16n^4} + \frac{3x^2}{n^2} + \frac{109x^3}{2n^3} + \frac{11x^4}{4n^2},$$

$$\gamma_1(x) = 1 + x^2 + \frac{2x}{n}$$

and

$$\gamma_2(x) = 1 + x^2 + \frac{2x}{n} + \frac{1}{4n^2}.$$

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