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# New Hermite–Hadamard–Fejér Inequalities via $k$–Fractional Integrals for Differentiable Generalized Nonconvex Functions

**Artion Kashuri[a], Themistocles M. Rassias[b]**

[a]*Department of Mathematics, Faculty of Technical Science, University Ismail Qemali of Vlora, Vlora, Albania*
[b]*Department of Mathematics, National Technical University of Athens, Athens, Greece*

**Abstract.** The authors discover a new interesting generalized identity concerning differentiable functions via $k$–fractional integrals. By using the obtained identity as an auxiliary result, some new estimates with respect to Hermite–Hadamard–Fejér type inequalities via $k$–fractional integrals for a new class of function involving Raina's function, the so-called generalized $(h_1, h_2)$–nonconvex are presented. These inequalities have some connections with known integral inequalities. Also, some new special cases are provided as well from main results.

## 1. Introduction

**Definition 1.1.** *[13] A function $f : I \subset \mathbb{R} \longrightarrow \mathbb{R}$ is said to be convex on $I$, if*

$$f(\imath\mu + (1-\imath)\nu) \leq \imath f(\mu) + (1-\imath)f(\nu)$$

*holds for every $\mu, \nu \in I$ and $\imath \in [0,1]$.*

The following inequality, named Hermite–Hadamard inequality, is one of the most famous inequalities in the literature for convex functions.

**Theorem 1.2.** *Let $f : I \subset \mathbb{R} \longrightarrow \mathbb{R}$ be a convex function on $I$ and $\ell_1, \ell_2 \in I$ with $\ell_1 < \ell_2$. Then the following inequality holds:*

$$f\left(\frac{\ell_1+\ell_2}{2}\right) \leq \frac{1}{\ell_2-\ell_1}\int_{\ell_1}^{\ell_2} f(x)dx \leq \frac{f(\ell_1)+f(\ell_2)}{2}. \tag{1}$$

*This inequality (1) is also known as trapezium inequality.*

The trapezium type inequality has remained an area of great interest due to its wide applications in the field of mathematical analysis. For other recent results which generalize, improve and extend the inequality (1) through various classes of convex functions interested readers are referred to [3]–[8],[10],[16]–[18],[20]–[23].

The most well–known inequalities related to the integral mean of a convex function $f$ are the Hermite–Hadamard inequalities or its weighted versions, the so-called Hermite–Hadamard–Fejér inequalities.

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*Email addresses:* artionkashuri@gmail.com (Artion Kashuri), trassias@math.ntua.gr (Themistocles M. Rassias)

**Definition 1.3.** *[11] A function $g : [\ell_1, \ell_2] \subset \mathbb{R} \longrightarrow \mathbb{R}$ is said to be symmetric with respect to $\frac{\ell_1+\ell_2}{2}$, if $g(x) = g(\ell_1 + \ell_2 - x)$ holds for all $x \in [\ell_1, \ell_2]$.*

**Example 1.4.** *Assume that $g_1, g_2 : [\ell_1, \ell_2] \subset \mathbb{R} \longrightarrow \mathbb{R}$, $g_1(x) = c$ for $c \in \mathbb{R}$, $g_2(x) = \left(x - \frac{\ell_1+\ell_2}{2}\right)^2$, then $g_1, g_2$ are symmetric functions with respect to $\frac{\ell_1+\ell_2}{2}$.*

In [9], Fejér established the following Hermite–Hadamard–Fejér inequality which is the weighted generalization of the Hermite–Hadamard inequality (1).

**Theorem 1.5.** *[9] Let $f : [\ell_1, \ell_2] \subset \mathbb{R} \longrightarrow \mathbb{R}$ be a convex function. Then the following inequality holds:*

$$f\left(\frac{\ell_1+\ell_2}{2}\right)\int_{\ell_1}^{\ell_2} g(x)dx \leq \int_{\ell_1}^{\ell_2} f(x)g(x)dx \leq \frac{f(\ell_1)+f(\ell_2)}{2}\int_{\ell_1}^{\ell_2} g(x)dx, \tag{2}$$

*where $g : [\ell_1, \ell_2] \longrightarrow \mathbb{R}$ is non-negative, integrable and symmetric to $\frac{\ell_1+\ell_2}{2}$.*

In recent years, various generalizations, extensions and variants of such inequalities have been obtained. For other recent results which generalize, improve and extend the inequality (2) interested readers are referred to [1],[2],[9]–[12],[14],[15].

In [19], Raina introduced a class of functions defined formally by

$$\mathcal{F}^{\sigma}_{\rho,\lambda}(z) = \mathcal{F}^{\sigma(0),\sigma(1),\ldots}_{\rho,\lambda}(z) = \sum_{k=0}^{+\infty} \frac{\sigma(k)}{\Gamma(\rho k + \lambda)} z^k, \tag{3}$$

where $\rho, \lambda > 0$, $|z| < R$ and $\sigma = (\sigma(0), \ldots, \sigma(k), \ldots)$ is a bounded sequence of positive real numbers. Note that, if we take in (3) $\rho = 1, \lambda = 0$ and $\sigma(k) = \frac{((\alpha)_k(\beta)_k)}{(\gamma)_k}$ for $k = 0, 1, 2, \ldots$, where $\alpha, \beta$ and $\gamma$ are parameters which can take arbitrary real or complex values (provided that $\gamma \neq 0, -1, -2, \ldots$), and the symbol $(a)_k$ denote the quantity

$$(a)_k = \frac{\Gamma(a+k)}{\Gamma(a)} = a(a+1)\ldots(a+k-1), \quad k = 0, 1, 2, \ldots,$$

and restrict its domain to $|z| \leq 1$ (with $z \in \mathbb{C}$), then we have the classical hypergeometric function, that is

$$\mathcal{F}^{\sigma}_{\rho,\lambda}(z) = F(\alpha, \beta; \gamma; z) = \sum_{k=0}^{+\infty} \frac{(\alpha)_k(\beta)_k}{k!(\gamma)_k} z^k.$$

Also, if $\sigma = (1, 1, \ldots)$ with $\rho = \alpha$, $(Re(\alpha) > 0)$, $\lambda = 1$ and restricting its domain to $z \in \mathbb{C}$ in (3) then we have the classical Mittag–Leffler function

$$E_\alpha(z) = \sum_{k=0}^{+\infty} \frac{1}{\Gamma(1+\alpha k)} z^k.$$

Now we are able to define a new class of function involving Raina's function.

**Definition 1.6.** *Suppose $h_1, h_2 : [0, 1] \longrightarrow [0, +\infty)$ and $\rho_1, \rho_2, \lambda_1, \lambda_2 > 0$, where $\sigma_1 = (\sigma_1(0), \ldots, \sigma_1(k), \ldots)$, $\sigma_2 = (\sigma_2(0), \ldots, \sigma_2(k), \ldots)$ are bounded sequences of positive real numbers. If function $f : I \rightarrow \mathbb{R}$ satisfies the following inequality*

$$f(\ell_1 + \imath\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2 - \ell_1)) \leq h_1(\imath)f(\ell_1) + h_2(\imath)\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}(f(\ell_2) - f(\ell_1)), \tag{4}$$

*for all $\imath \in [0,1]$ and $\ell_1, \ell_2 \in I$, where $\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2 - \ell_1) > 0$, $\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}(f(\ell_2) - f(\ell_1)) > 0$, then $f$ is called generalized $(h_1, h_2)$–nonconvex. Taking $h_1(\imath) = 1 - \imath$, $h_2(\imath) = \imath$, where $\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2 - \ell_1) = \ell_2 - \ell_1 > 0$ and $\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}(f(\ell_2) - f(\ell_1)) = f(\ell_2) - f(\ell_1) > 0$ in our definition, then we obtain definition 1.1.*

**Remark 1.7.** *Let us discuss some special cases in definition 1.6 as follows:*

*(I) Taking $h_1(\imath) = h(1 - \imath)$ and $h_2(\imath) = h(\imath)$, then we get generalized $(h(1 - \imath), h(\imath))$–nonconvex functions.*
*(II) Taking $h_1(\imath) = (1 - \imath)^s$ and $h_2(\imath) = \imath^s$ for $s \in (0,1]$, then we get generalized $((1 - \imath)^s, \imath^s)$–nonconvex functions.*
*(III) Taking $h_1(\imath) = (1 - \imath)^{-s}$ and $h_2(\imath) = \imath^{-s}$ for $s \in (0,1]$, then we get generalized $((1 - \imath)^{-s}, \imath^{-s})$–nonconvex functions.*
*(IV) Taking $h_1(\imath) = h_2(\imath) = \imath(1 - \imath)$, then we get generalized $(\imath(1 - \imath), \imath(1 - \imath))$–nonconvex functions.*
*(V) Taking $h_1(\imath) = \dfrac{\sqrt{1-\imath}}{2\sqrt{\imath}}$ and $h_2(\imath) = \dfrac{\sqrt{\imath}}{2\sqrt{1-\imath}}$, then we get generalized $\left(\frac{\sqrt{1-\imath}}{2\sqrt{\imath}}, \frac{\sqrt{\imath}}{2\sqrt{1-\imath}}\right)$–nonconvex functions.*

It is worth to mention here that to the best of our knowledge all the special cases discussed above are new in the literature.

Motivated by the above literatures, the main objective of this paper is to establish in Section 2, some new Hermite–Hadamard–Fejér type inequalities via $k$–fractional integrals associated with generalized $(h_1, h_2)$–nonconvex functions. It is pointed out that some new special cases will be deduced from main results. Also we will see that these inequalities have some connections with known integral inequalities. In Section 3, a briefly conclusion will be given as well.

## 2. Main results

Throughout this section the following notation is used:

$$O = \left[\ell_1, \ell_1 + \mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2 - \ell_1)\right], \quad \text{where} \quad \mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2 - \ell_1) > 0.$$

For establishing our main results we need to prove the following lemma.

**Lemma 2.1.** *Let $g : O \subset \mathbb{R} \longrightarrow \mathbb{R}$ be a continuous function. Assume that $f : O \longrightarrow \mathbb{R}$ is a differentiable function on $O^\circ$ (the interior of $O$) such that $f' \in L(O)$. Then for $\alpha, k > 0$, the following equality for $k$–fractional integrals holds:*

$$\begin{aligned}
&\left(\int_{\ell_1}^{\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)} g(s)ds\right)^{\frac{\alpha}{k}} \left[f(\ell_1) + f\left(\ell_1 + \mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2 - \ell_1)\right)\right] \\
&-\frac{\alpha}{k}\int_{\ell_1}^{\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)} \left(\int_{\ell_1}^{\imath} g(s)ds\right)^{\frac{\alpha}{k}-1} g(\imath)f(\imath)d\imath \\
&-\frac{\alpha}{k}\int_{\ell_1}^{\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)} \left(\int_{\imath}^{\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)} g(s)ds\right)^{\frac{\alpha}{k}-1} g(\imath)f(\imath)d\imath \\
&= \int_{\ell_1}^{\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)} \left(\int_{\ell_1}^{\imath} g(s)ds\right)^{\frac{\alpha}{k}} f'(\imath)d\imath - \int_{\ell_1}^{\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)} \left(\int_{\imath}^{\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)} g(s)ds\right)^{\frac{\alpha}{k}} f'(\imath)d\imath. \qquad (5)
\end{aligned}$$

*We denote*

$$T^{\alpha,k}_{f,g}(\ell_1, \ell_2) = \int_{\ell_1}^{\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)} \left(\int_{\ell_1}^{\imath} g(s)ds\right)^{\frac{\alpha}{k}} f'(\imath)d\imath - \int_{\ell_1}^{\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)} \left(\int_{\imath}^{\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)} g(s)ds\right)^{\frac{\alpha}{k}} f'(\imath)d\imath. \qquad (6)$$

*Proof.* Integrating by parts (6), we get

$$T_{f,g}^{\alpha,k}(\ell_1,\ell_2)=\left(\int_{\ell_1}^{\imath}g(s)ds\right)^{\frac{\alpha}{k}}f(\imath)\Big|_{\ell_1}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}-\frac{\alpha}{k}\int_{\ell_1}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}\left(\int_{\ell_1}^{\imath}g(s)ds\right)^{\frac{\alpha}{k}-1}g(\imath)f(\imath)d\imath$$

$$-\left(\int_{\imath}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}g(s)ds\right)^{\frac{\alpha}{k}}f(\imath)\Big|_{\ell_1}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}-\frac{\alpha}{k}\int_{\ell_1}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}\left(\int_{\imath}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}g(s)ds\right)^{\frac{\alpha}{k}-1}g(\imath)f(\imath)d\imath$$

$$=\left(\int_{\ell_1}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}g(s)ds\right)^{\frac{\alpha}{k}}\left[f(\ell_1)+f\left(\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)\right)\right]$$

$$-\frac{\alpha}{k}\int_{\ell_1}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}\left(\int_{\ell_1}^{\imath}g(s)ds\right)^{\frac{\alpha}{k}-1}g(\imath)f(\imath)d\imath-\frac{\alpha}{k}\int_{\ell_1}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}\left(\int_{\imath}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}g(s)ds\right)^{\frac{\alpha}{k}-1}g(\imath)f(\imath)d\imath.$$

The proof of Lemma 2.1 is completed. □

**Remark 2.2.** *For $\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)=\ell_2-\ell_1$, where $\nu=\frac{\alpha}{k}$, we get ([8], Lemma 2.1).*

Using Lemma 2.1, we now state the following theorems for the corresponding version for power of first derivative.

**Theorem 2.3.** *Let $h_1,h_2:[0,1]\longrightarrow[0,+\infty)$ and $g:O\longrightarrow\mathbb{R}$ are continuous functions. Assume that $f:O\longrightarrow(0,+\infty)$ be a differentiable function on $O^\circ$ such that $f',g\in L(K)$. If $f'^q$ is generalized $(h_1,h_2)$–nonconvex function, where $q>1$, $p^{-1}+q^{-1}=1$ and $\|g\|_\infty=\sup_{s\in O}|g(s)|$, then for $\alpha$, $k>0$, the following inequality holds:*

$$\left|T_{f,g}^{\alpha,k}(\ell_1,\ell_2)\right|\le\frac{2\|g\|_\infty^{\frac{\alpha}{k}}\left[\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)\right]^{\frac{\alpha}{k}+1}}{\sqrt[p]{\frac{p\alpha}{k}+1}}\sqrt[q]{(f'(\ell_1))^qI(h_1(\imath))+\left[\mathcal{F}_{\rho_2,\lambda_2}^{\sigma_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]I(h_2(\imath))},\tag{7}$$

*where*

$$I(h_i(\imath))=\int_0^1h_i(\imath)d\imath,\quad\forall\,i=1,2.$$

*Proof.* From Lemma 2.1, generalized $(h_1,h_2)$–nonconvexity of $f'^q$, Hölder inequality, properties of the modulus, the fact that $g(s)\le\|g\|_\infty$, $\forall\,s\in O$ and changing the variable, we have

$$\left|T_{f,g}^{\alpha,k}(\ell_1,\ell_2)\right|\le\int_{\ell_1}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}\left|\int_{\ell_1}^{\imath}g(s)ds\right|^{\frac{\alpha}{k}}|f'(\imath)|d\imath+\int_{\ell_1}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}\left|\int_{\imath}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}g(s)ds\right|^{\frac{\alpha}{k}}|f'(\imath)|d\imath$$

$$\le\left(\int_{\ell_1}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}\left|\int_{\ell_1}^{\imath}g(s)ds\right|^{\frac{p\alpha}{k}}d\imath\right)^{\frac{1}{p}}\left(\int_{\ell_1}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}(f'(\imath))^qd\imath\right)^{\frac{1}{q}}$$

$$+\left(\int_{\ell_1}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}\left|\int_{\imath}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}g(s)ds\right|^{\frac{p\alpha}{k}}d\imath\right)^{\frac{1}{p}}\left(\int_{\ell_1}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}(f'(\imath))^qd\imath\right)^{\frac{1}{q}}$$

$$\le\|g\|_\infty^{\frac{\alpha}{k}}\times\left(\int_{\ell_1}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}(f'(\imath))^qd\imath\right)^{\frac{1}{q}}\left\{\left(\int_{\ell_1}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}(\imath-\ell_1)^{\frac{p\alpha}{k}}d\imath\right)^{\frac{1}{p}}\right.$$

$$+\left(\int_{\ell_1}^{\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)}(\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)-\imath)^{\frac{p\alpha}{k}}d\imath\right)^{\frac{1}{p}}\Bigg\}$$

$$=\frac{2\|g\|_\infty^{\frac{\alpha}{k}}\Big[\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\Big]^{\frac{\alpha}{k}+1}}{\sqrt[p]{\frac{p\alpha}{k}+1}}\left(\int_0^1\Big(f'(\ell_1+\imath\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1))\Big)^q d\imath\right)^{\frac{1}{q}}$$

$$\leq\frac{2\|g\|_\infty^{\frac{\alpha}{k}}\Big[\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\Big]^{\frac{\alpha}{k}+1}}{\sqrt[p]{\frac{p\alpha}{k}+1}}\left(\int_0^1\Big[(f'(\ell_1))^q h_1(\imath)+\Big[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\Big]h_2(\imath)\Big]d\imath\right)^{\frac{1}{q}}$$

$$=\frac{2\|g\|_\infty^{\frac{\alpha}{k}}\Big[\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\Big]^{\frac{\alpha}{k}+1}}{\sqrt[p]{\frac{p\alpha}{k}+1}}\sqrt[q]{(f'(\ell_1))^q I(h_1(\imath))+\Big[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\Big]I(h_2(\imath))}.$$

The proof of Theorem 2.3 is completed. □

**Remark 2.4.** *In Theorem 2.3 for $h_1(\imath)=\imath$, $h_2(\imath)=1-\imath$, if we choose $\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)=\ell_2-\ell_1$, where $\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)=(f'(\ell_2))^q-(f'(\ell_1))^q$, then*

**(1)** *If we put $\frac{\alpha}{k}=\nu$, we get ([8], Theorem 2.5).*

**(2)** *If we put $\frac{\alpha}{k}=\nu=1$, we obtain ([7], Corollary 3).*

We point out some special cases of Theorem 2.3.

**Corollary 2.5.** *In Theorem 2.3 for $p=q=2$, we have*

$$\left|T^{\alpha,k}_{f,g}(\ell_1,\ell_2)\right|\leq\frac{2\|g\|_\infty^{\frac{\alpha}{k}}\Big[\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\Big]^{\frac{\alpha}{k}+1}}{\sqrt{\frac{2\alpha}{k}+1}}\sqrt{(f'(\ell_1))^2 I(h_1(\imath))+\Big[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^2-(f'(\ell_1))^2)\Big]I(h_2(\imath))}.$$

**Corollary 2.6.** *In Theorem 2.3 for $g(s)\equiv 1$, we get*

$$\left|T^{\alpha,k}_{f}(\ell_1,\ell_2)\right|$$
$$=\left|\frac{f(\ell_1)+f\Big(\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\Big)}{2}-\frac{\Gamma_k(\alpha+k)}{2\Big[\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\Big]^{\frac{\alpha}{k}}}\Big[I^{\alpha,k}_{\ell_1^+}f\Big(\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\Big)+I^{\alpha,k}_{(\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1))^-}f(\ell_1)\Big]\right|$$
$$\leq\frac{\Big[\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\Big]^{\frac{\alpha}{k}+1}}{\sqrt[p]{\frac{p\alpha}{k}+1}}\sqrt[q]{(f'(\ell_1))^q I(h_1(\imath))+\Big[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\Big]I(h_2(\imath))},$$

*where $I^{\alpha,k}_{\ell_1^+}f(\cdot)$ and $I^{\alpha,k}_{\ell_2^-}f(\cdot)$ are denoted, respectively, the left and right k-fractional integrals.*

**Corollary 2.7.** *In Theorem 2.3 for $h_1(\imath)=h(1-\imath)$ and $h_2(\imath)=h(\imath)$, we obtain*

$$\left|T^{\alpha,k}_{f,g}(\ell_1,\ell_2)\right|\leq\frac{2\|g\|_\infty^{\frac{\alpha}{k}}\Big[\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\Big]^{\frac{\alpha}{k}+1}}{\sqrt[p]{\frac{p\alpha}{k}+1}}\sqrt[q]{(f'(\ell_1))^q I(h(1-\imath))+\Big[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\Big]I(h(\imath))}.$$

**Corollary 2.8.** *In Corollary 2.7 for $h_1(\iota) = (1-\iota)^s$ and $h_2(\iota) = \iota^s$, we have*

$$\left|T_{f,g}^{\alpha,k}(\ell_1,\ell_2)\right| \leq \frac{2\|g\|_\infty^{\frac{\alpha}{k}}\left[\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)\right]^{\frac{\alpha}{k}+1}}{\sqrt[p]{\frac{p\alpha}{k}+1}\sqrt[q]{s+1}}\sqrt[q]{(f'(\ell_1))^q+\left[\mathcal{F}_{\rho_2,\lambda_2}^{\sigma_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]}.$$

**Corollary 2.9.** *In Corollary 2.7 for $h_1(\iota) = (1-\iota)^{-s}$ and $h_2(\iota) = \iota^{-s}$, we get*

$$\left|T_{f,g}^{\alpha,k}(\ell_1,\ell_2)\right| \leq \frac{2\|g\|_\infty^{\frac{\alpha}{k}}\left[\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)\right]^{\frac{\alpha}{k}+1}}{\sqrt[p]{\frac{p\alpha}{k}+1}\sqrt[q]{1-s}}\sqrt[q]{(f'(\ell_1))^q+\left[\mathcal{F}_{\rho_2,\lambda_2}^{\sigma_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]}.$$

**Corollary 2.10.** *In Theorem 2.3 for $h_1(\iota) = h_2(\iota) = \iota(1-\iota)$, we obtain*

$$\left|T_{f,g}^{\alpha,k}(\ell_1,\ell_2)\right| \leq \frac{2\|g\|_\infty^{\frac{\alpha}{k}}\left[\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)\right]^{\frac{\alpha}{k}+1}}{\sqrt[q]{6}\sqrt[p]{\frac{p\alpha}{k}+1}}\sqrt[q]{(f'(\ell_1))^q+\left[\mathcal{F}_{\rho_2,\lambda_2}^{\sigma_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]}.$$

**Corollary 2.11.** *In Corollary 2.7 for $h_1(\iota) = \frac{\sqrt{1-\iota}}{2\sqrt{\iota}}$ and $h_2(\iota) = \frac{\sqrt{\iota}}{2\sqrt{1-\iota}}$, we have*

$$\left|T_{f,g}^{\alpha,k}(\ell_1,\ell_2)\right| \leq 2\sqrt[q]{\frac{\pi}{4}}\frac{\|g\|_\infty^{\frac{\alpha}{k}}\left[\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)\right]^{\frac{\alpha}{k}+1}}{\sqrt[p]{\frac{p\alpha}{k}+1}}\sqrt[q]{(f'(\ell_1))^q+\left[\mathcal{F}_{\rho_2,\lambda_2}^{\sigma_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]}.$$

**Theorem 2.12.** *Let $h_1, h_2 : [0,1] \longrightarrow [0,+\infty)$ and $g : O \longrightarrow \mathbb{R}$ are continuous functions. Assume that $f : O \longrightarrow (0,+\infty)$ be a differentiable function on $O^\circ$ such that $f', g \in L(K)$. If $f'^q$ is generalized $(h_1,h_2)$–nonconvex function, where $q \geq 1$ and $\|g\|_\infty = \sup_{s\in O}|g(s)|$, then for $\alpha$, $k > 0$, the following inequality holds:*

$$\left|T_{f,g}^{\alpha,k}(\ell_1,\ell_2)\right| \leq \frac{\|g\|_\infty^{\frac{\alpha}{k}}\left[\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)\right]^{\frac{\alpha}{k}+1}}{\left(\frac{\alpha}{k}+1\right)^{1-\frac{1}{q}}} \tag{8}$$

$$\times\left\{\sqrt[q]{(f'(\ell_1))^qF(h_1(\iota);\alpha,k)+\left[\mathcal{F}_{\rho_2,\lambda_2}^{\sigma_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]F(h_2(\iota);\alpha,k)}\right.$$

$$\left.+\sqrt[q]{(f'(\ell_1))^qG(h_1(\iota);\alpha,k)+\left[\mathcal{F}_{\rho_2,\lambda_2}^{\sigma_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]G(h_2(\iota);\alpha,k)}\right\},$$

*where*

$$F(h_i(\iota);\alpha,k) = \int_0^1 \iota^{\frac{\alpha}{k}}h_i(\iota)d\iota, \quad G(h_i(\iota);\alpha,k) = \int_0^1 (1-\iota)^{\frac{\alpha}{k}}h_i(\iota)d\iota, \quad \forall\, i = 1,2.$$

*Proof.* From Lemma 2.1, generalized $(h_1,h_2)$–nonconvexity of $f'^q$, the well–known power mean inequality, properties of the modulus, the fact that $g(s) \leq \|g\|_\infty$, $\forall\, s \in O$ and changing the variable, we have

$$\left|T_{f,g}^{\alpha,k}(\ell_1,\ell_2)\right| \leq \int_{\ell_1}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}\left|\int_{\ell_1}^{\iota}g(s)ds\right|^{\frac{\alpha}{k}}|f'(\iota)|d\iota + \int_{\ell_1}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}\left|\int_{\iota}^{\ell_1+\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)}g(s)ds\right|^{\frac{\alpha}{k}}|f'(\iota)|d\iota$$

$$\leq \left(\int_{\ell_1}^{\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)}\left|\int_{\ell_1}^{\imath} g(s)ds\right|^{\frac{\alpha}{k}} d\imath\right)^{1-\frac{1}{q}}\left(\int_{\ell_1}^{\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)}\left|\int_{\ell_1}^{\imath} g(s)ds\right|^{\frac{\alpha}{k}}(f'(\imath))^q d\imath\right)^{\frac{1}{q}}$$

$$+\left(\int_{\ell_1}^{\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)}\left|\int_{\imath}^{\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)} g(s)ds\right|^{\frac{\alpha}{k}} d\imath\right)^{1-\frac{1}{q}}\left(\int_{\ell_1}^{\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)}\left|\int_{\imath}^{\ell_1+\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)} g(s)ds\right|^{\frac{\alpha}{k}}(f'(\imath))^q d\imath\right)^{\frac{1}{q}}$$

$$\leq \frac{\|g\|_\infty^{\frac{\alpha}{k}}\left[\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\right]^{\frac{\alpha}{k}+1}}{\left(\frac{\alpha}{k}+1\right)^{1-\frac{1}{q}}}$$

$$\times\left\{\left[\int_0^1 \imath^{\frac{\alpha}{k}}\left(f'(\ell_1+\imath\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1))\right)^q d\imath\right]^{\frac{1}{q}}+\left[\int_0^1 (1-\imath)^{\frac{\alpha}{k}}\left(f'(\ell_1+\imath\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1))\right)^q d\imath\right]^{\frac{1}{q}}\right\}$$

$$\leq \frac{\|g\|_\infty^{\frac{\alpha}{k}}\left[\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\right]^{\frac{\alpha}{k}+1}}{\left(\frac{\alpha}{k}+1\right)^{1-\frac{1}{q}}}$$

$$\times\left\{\left[\int_0^1 \imath^{\frac{\alpha}{k}}\left[(f'(\ell_1))^q h_1(\imath)+\left[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]h_2(\imath)\right]d\imath\right]^{\frac{1}{q}}\right.$$

$$\left.+\left[\int_0^1 (1-\imath)^{\frac{\alpha}{k}}\left[(f'(\ell_1))^q h_1(\imath)+\left[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]h_2(\imath)\right]d\imath\right]^{\frac{1}{q}}\right\}$$

$$= \frac{\|g\|_\infty^{\frac{\alpha}{k}}\left[\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\right]^{\frac{\alpha}{k}+1}}{\left(\frac{\alpha}{k}+1\right)^{1-\frac{1}{q}}}$$

$$\times\left\{\sqrt[q]{(f'(\ell_1))^q F(h_1(\imath);\alpha,k)+\left[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]F(h_2(\imath);\alpha,k)}\right.$$

$$\left.+\sqrt[q]{(f'(\ell_1))^q G(h_1(\imath);\alpha,k)+\left[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]G(h_2(\imath);\alpha,k)}\right\}.$$

The proof of Theorem 2.12 is completed. □

We point out some special cases of Theorem 2.12.

**Corollary 2.13.** *In Theorem 2.12 for $q = 1$, we have*

$$\left|T^{\alpha,k}_{f,g}(\ell_1,\ell_2)\right| \leq \|g\|_\infty^{\frac{\alpha}{k}}\left[\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\right]^{\frac{\alpha}{k}+1}$$

$$\times\left\{f'(\ell_1)\Big(F(h_1(\imath);\alpha,k)+G(h_1(\imath);\alpha,k)\Big)+\left[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}(f'(\ell_2)-f'(\ell_1))\right]\Big(F(h_2(\imath);\alpha,k)+G(h_2(\imath);\alpha,k)\Big)\right\}.$$

**Corollary 2.14.** *In Theorem 2.12 for $g(s) \equiv 1$, we get*

$$\left|T^{\alpha,k}_{f}(\ell_1,\ell_2)\right| \leq \frac{\left[\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\right]^{\frac{\alpha}{k}+1}}{\left(\frac{\alpha}{k}+1\right)^{1-\frac{1}{q}}}$$

$$\times\left\{\sqrt[q]{(f'(\ell_1))^q F(h_1(\imath);\alpha,k)+\left[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]F(h_2(\imath);\alpha,k)}\right.$$

$$\left.+\sqrt[q]{(f'(\ell_1))^q G(h_1(\imath);\alpha,k)+\left[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]G(h_2(\imath);\alpha,k)}\right\}.$$

**Corollary 2.15.** *In Theorem 2.12 for* $h_1(\imath)=h(1-\imath)$ *and* $h_2(\imath)=h(\imath)$*, we obtain*

$$\left|T^{\alpha,k}_{f,g}(\ell_1,\ell_2)\right|\leq\frac{\|g\|_\infty^{\frac{\alpha}{k}}\left[\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\right]^{\frac{\alpha}{k}+1}}{\left(\frac{\alpha}{k}+1\right)^{1-\frac{1}{q}}}$$

$$\times\left\{\sqrt[q]{(f'(\ell_1))^q F(h(1-\imath);\alpha,k)+\left[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]F(h(\imath);\alpha,k)}\right.$$

$$\left.+\sqrt[q]{(f'(\ell_1))^q G(h(1-\imath);\alpha,k)+\left[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]G(h(\imath);\alpha,k)}\right\}.$$

**Corollary 2.16.** *In Corollary 2.15 for* $h_1(\imath)=(1-\imath)^s$ *and* $h_2(\imath)=\imath^s$*, we have*

$$\left|T^{\alpha,k}_{f,g}(\ell_1,\ell_2)\right|\leq\frac{\|g\|_\infty^{\frac{\alpha}{k}}\left[\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\right]^{\frac{\alpha}{k}+1}}{\left(\frac{\alpha}{k}+1\right)^{1-\frac{1}{q}}}$$

$$\times\left\{\sqrt[q]{(f'(\ell_1))^q\beta\left(s+1,\frac{\alpha}{k}+1\right)+\left[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]\left(\frac{1}{s+\frac{\alpha}{k}+1}\right)}\right.$$

$$\left.+\sqrt[q]{(f'(\ell_1))^q\left(\frac{1}{s+\frac{\alpha}{k}+1}\right)+\left[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]\beta\left(s+1,\frac{\alpha}{k}+1\right)}\right\}.$$

**Corollary 2.17.** *In Corollary 2.15 for* $h_1(\imath)=(1-\imath)^{-s}$ *and* $h_2(\imath)=\imath^{-s}$*, we get*

$$\left|T^{\alpha,k}_{f,g}(\ell_1,\ell_2)\right|\leq\frac{\|g\|_\infty^{\frac{\alpha}{k}}\left[\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\right]^{\frac{\alpha}{k}+1}}{\left(\frac{\alpha}{k}+1\right)^{1-\frac{1}{q}}}$$

$$\times\left\{\sqrt[q]{(f'(\ell_1))^q\beta\left(1-s,\frac{\alpha}{k}+1\right)+\left[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]\left(\frac{1}{1+\frac{\alpha}{k}-s}\right)}\right.$$

$$\left.+\sqrt[q]{(f'(\ell_1))^q\left(\frac{1}{1+\frac{\alpha}{k}-s}\right)+\left[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]\beta\left(1-s,\frac{\alpha}{k}+1\right)}\right\}.$$

**Corollary 2.18.** *In Theorem 2.12 for* $h_1(\imath)=h_2(\imath)=\imath(1-\imath)$*, we obtain*

$$\left|T^{\alpha,k}_{f,g}(\ell_1,\ell_2)\right|\leq\frac{2\|g\|_\infty^{\frac{\alpha}{k}}\left[\mathcal{F}^{\sigma_1}_{\rho_1,\lambda_1}(\ell_2-\ell_1)\right]^{\frac{\alpha}{k}+1}}{\left(\frac{\alpha}{k}+1\right)^{1-\frac{1}{q}}\sqrt[q]{\beta\left(2+\frac{\alpha}{k},2\right)}}\sqrt[q]{(f'(\ell_1))^q+\left[\mathcal{F}^{\sigma_2}_{\rho_2,\lambda_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]}.$$

**Corollary 2.19.** *In Corollary 2.15 for* $h_1(\imath) = \dfrac{\sqrt{1-\imath}}{2\sqrt{\imath}}$ *and* $h_2(\imath) = \dfrac{\sqrt{\imath}}{2\sqrt{1-\imath}}$, *we have*

$$\left|T_{f,g}^{\alpha,k}(\ell_1,\ell_2)\right| \leq \frac{\|g\|_\infty^{\frac{\alpha}{k}}\left[\mathcal{F}_{\rho_1,\lambda_1}^{\sigma_1}(\ell_2-\ell_1)\right]^{\frac{\alpha}{k}+1}}{\sqrt[q]{2}\left(\frac{\alpha}{k}+1\right)^{1-\frac{1}{q}}}$$

$$\times\left\{\sqrt[q]{(f'(\ell_1))^q\beta\left(\frac{\alpha}{k}+\frac{1}{2},\frac{3}{2}\right)+\left[\mathcal{F}_{\rho_2,\lambda_2}^{\sigma_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]\beta\left(\frac{\alpha}{k}+\frac{3}{2},\frac{1}{2}\right)}\right.$$

$$\left.+\sqrt[q]{(f'(\ell_1))^q\beta\left(\frac{\alpha}{k}+\frac{3}{2},\frac{1}{2}\right)+\left[\mathcal{F}_{\rho_2,\lambda_2}^{\sigma_2}((f'(\ell_2))^q-(f'(\ell_1))^q)\right]\beta\left(\frac{\alpha}{k}+\frac{1}{2},\frac{3}{2}\right)}\right\}.$$

**Remark 2.20.** *By taking particular values of positive parameters* $\alpha$ *and* $k$ *in above Theorems 2.3 and 2.12, several k–fractional integral inequalities associated with generalized* $(h_1,h_2)$*–nonconvex functions can be obtained. Also, for different choices of* $\rho_1,\rho_2,\lambda_1,\lambda_2 > 0$, *where* $\sigma_1 = (\sigma_1(0),\ldots,\sigma_1(k),\ldots)$, $\sigma_2 = (\sigma_2(0),\ldots,\sigma_2(k),\ldots)$ *are bounded sequences of positive real numbers, we can obtain several k–fractional integral inequalities. Also, for* $k = 1$, *we can get some new special Hermite–Hadamard–Fejér type inequalities via fractional integrals. Finally, for* $\alpha = k = 1$, *we can get some new special Hermite–Hadamard–Fejér type inequalities via classical integrals. We omit their proofs and the details are left to the interested readers.*

**Remark 2.21.** *Also, applying our Theorems 2.3 and 2.12 for* $0 < f'(x) \leq K$, $\forall\, x \in O$, *we can get some new k–fractional integral inequalities.*

## 3. Conclusion

In this paper, we defined a new interesting class of functions involving Raina's function and some Hermite–Hadamard–Fejér type integral inequalities are provided as well. This results can be applied in convex analysis, optimization and different areas of pure and applied sciences. The authors hope that these results will serve as a motivation for future work in this fascinating area.

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