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Radical Screen Transversal Slant Lightlike Submanfolds of Indefinite Kaehler Manifolds

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Abstract. In this paper, we define Radical screen transversal slant lightlike submanifolds of an indefinite Kaehler manifold and give an example. We prove two characterization theorems for the existence of the Radical screen transversal slant lightlike submanifolds and obtain the necessary and sufficient conditions for Radical screen transversal slant lightlike submanifolds to be Radical screen slant lightlike product.

1. Introduction

In Riemannian geometry, concept of slant submanifolds were introduced by Chen[2, 3] as a generalization of invariant, anti-invariant and CR-submanifolds. In [4], Papaghuic defined semi-slant submanifolds of Kaehler manifolds which include holomorphic submanifolds, totally real submanifolds, CR-submanifolds and slant submanifolds as their particular cases.
In present time, we are dealing with the semi-Riemannian geometry which is almost similar to Riemannian geometry but it is most popular due to the fact that it can deal with semi-definite metric. Another interesting theory is that it has a common portion between the tangent bundle and normal bundle. The notion of transversal and screen transversal lightlike submanifolds of an indefinite Kaehler manifold were introduced by Sahin [10]. Further, they studied slant lightlike [11] and screen slant lightlike submanifolds [12] of indefinite Kaehler manifolds. Recently, Haider et al [14] introduced hemi-slant lightlike submanifolds.
In this paper, we introduce Radical screen transversal slant lightlike submanifolds of an indefinite Kaehler manifold which includes radical screen transversal , ST-anti-invariant and isotropic lightlike submanifolds at $\theta = 0, \theta = \frac{\pi}{2}$ and $D^{\theta} = 0$, respectively.
The paper is arranged as follows. In section 2, we summarize basic material on lightlike submanifolds and indefinite Kaehler manifold which will be used throughout this paper. In section 3, we define radical screen transversal slant lightlike submanifolds of an indefinite Kaehler manifold supported by an example. We investigate integrability conditions and totally geodesicness of leaves of the distribution involved in definition. We also prove two characterization theorems for the existence of radical screen transversal slant lightlike submanifolds and obtain necessary and sufficient condition for radical screen transversal lightlike

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submanifold to be radical transversal lightlike product. Further we obtain geometric condition for the induced connection to be a metric connection.

2. Preliminaries

We follow [5] for the notation and the formulae used in this paper. A submanifold (M^m, g) immersed in a semi-Riemannian manifold $(\overline{M}^{m+n}, \overline{g})$ is called a lightlike submanifold if the metric connection g induced from $\overline{g}$ is degenerate and the radical distribution $RadTM$ is of rank r, where $1 \leq r \leq m$. Let $S(TM)$ be a screen distribution which is a semi-Riemannian complementary distribution of $RadTM$ in TM, i.e,

$$TM = RadTM \perp S(TM).$$

Consider a screen transversal vector bundle $S(TM^{\perp})$,which is a semi-Riemannian complementary vector bundle of $RadTM$ in $TM^{\perp}$. Since for any local basis $\{\xi_i\}$ of $RadTM$, there exist a local null frame $\{N_i\}$ of sections with values in the orthogonal complement of $S(TM^{\perp})$ in $[S(TM)]^{\perp}$ such that $\overline{g}(\xi_i, N_i) = \delta_{ij}$, it follows that there exists a lightlike transversal vector bundle $ltr(TM)$ locally spanned by $[\{N_i\}\ \ [5], pg-144]$. Let $tr(TM)$ be complementary (but not orthogonal) vector bundle to TM in $T\overline{M}|_M$. Then

$$tr(TM) = ltr(TM) \perp S(TM^{\perp}),$$

$$T\overline{M}|_M = S(TM) \perp [RadTM \oplus ltr(TM)] \perp S(TM^{\perp}).$$

Following are four subcases of lightlike submanifold $(M, g, S(TM), S(TM^{\perp}))$.
Case 1 : r-lightlike if $r < min\{m, n\}$.
Case 2 : co-isotropic if $r = n < m; S(TM^{\perp}) = \{0\}$
Case 3 : Isotropic if $r = m < n; S(TM) = \{0\}$
Case 4 : Totally lightlike if $r = m = n; S(TM) = \{0\} = S(TM^{\perp})$.
The Gauss and Weingarten equations are

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y), \quad \forall X, Y \in \Gamma(TM)$$

$$\overline{\nabla}_X U = -A_U X + \nabla^t_X U, \quad \forall X \in \Gamma(TM), U \in \Gamma(tr(TM))$$

where $\{\nabla_x Y, -A_U X\}$ and $\{h(X, Y), \nabla^t_X U\}$ belongs to $\Gamma(TM)$ and $\Gamma tr(TM)$ respectively, ∇ and ∇^t are linear connection on M and on the vector bundle $tr(TM)$, respectively.
Moreover, we have

$$\overline{\nabla}_X Y = \nabla_X Y + h^l(X, Y) + h^s(X, Y) \tag{1}$$

$$\overline{\nabla}_X N = -A_N X + \nabla^l_X N + D^s(X, N) \tag{2}$$

$$\overline{\nabla}_X W = -A_W X + \nabla^s_X W + D^l(X, W) \tag{3}$$

for each $X, Y \in \Gamma(TM)$, $N \in \Gamma(ltr(TM))$ and $W \in \Gamma(S(TM^{\perp}))$.
Denote the projection of TM on $S(TM)$ by P. Then, by using $(1)-(3)$ and the fact that $\overline{\nabla}$ is a metric connection, we get

$$\overline{g}(h^s(X, Y), W) + \overline{g}(Y, D^l(X, W)) = g(A_W X, Y) \tag{4}$$

$$\overline{g}(D^s(X, N), W) = \overline{g}(N, A_W X)$$

From the decomposition of the tangent bundle of a lightlike submanifolds, we have

$$\nabla_X PY = \nabla^*_X PY + h^*(X, PY) \tag{5}$$

$$\nabla_X \xi = -A^*_\xi X + \nabla^{*t}_X \xi$$

for $X, Y \in \Gamma(TM)$ and $\xi \in \Gamma(RadTM)$.
It is important to note that the induced connection ∇ on M is not a metric connection whereas ∇^* is a metric connection on $S(TM)$.
We denote curvature tensor of $\overline{M}$ and M by $\overline{R}$ and R respectively. The Gauss equation for M is given by

$$\begin{aligned}\overline{R}(X,Y)Z &= R(X,Y)Z + A_{h^l(X,Z)}Y + A_{h^l(Y,Z)}X + A_{h^s(X,Z)}Y - A_{h^s(Y,Z)} \\ &+ (\nabla_X h^l)(Y,Z) - (\nabla_Y h^l)(X,Z) + D^l(X, h^s(Y,Z)) - D^l(Y, h^s(X,Z)) \\ &+ (\nabla_X h^s)(Y,Z) - (\nabla_Y h^s)(X,Z) + D^s(X, h^l(Y,Z)) - D^s(Y, h^l(X,Z))\end{aligned} \tag{6}$$

for all $X, Y \in \Gamma(TM)$.
The curvature tensor $\overline{R}$ of an indefinite complex space form $\overline{M}(c)$ is given by [1].

$$\overline{R}(X,Y)Z = c/4[\overline{g}(Y,Z)X - \overline{g}(X,Z)Y + \overline{g}(JY,Z)JX - \overline{g}(JX,Z)JY + 2\overline{g}(X,JY)JZ] \tag{7}$$

for any $X, Y \in \Gamma(T\overline{M})$.
A $2n$-dimensional semi-Riemannian manifolds $(\overline{M}, \overline{g}, J)$ of constant index q, $0 < q < 2n$, is called an indefinite almost Hermitian manifold if there exists a tensor field J of type $(1,1)$ on $\overline{M}$ such that

$$J^2 = -I \quad and \quad \overline{g}(JX, JY) = \overline{g}(X,Y) \quad \forall X, Y \in \Gamma(T\overline{M}), \tag{8}$$

where I denotes the identity transformation of $T_P M$. An indefinite almost Hermitian manifold $\overline{M}$ is said to be indefinite Kaehler [1] if

$$(\overline{\nabla}_X J)Y = 0 \quad \forall X, Y \in \Gamma(TM) \tag{9}$$

where $\overline{\nabla}$ is Levi-Civita ecconnection on $\overline{M}$ with respect to $\overline{g}$.
From now on, we denote $(M, g, S(TM), S(TM^\perp))$ by M in this paper.

3. Radical Screen Transversal Slant Lightlike Submanifolds

In this section, we first recall the following lemma[9], with the help of which we shall define radical screen transversal slant lightlike submanifolds in indefinite Kaehler manifolds. Further, we introduce and study Radical screen transversal slant lightlike submanifolds of an indefinite Kaehler manifold.

Lemma 3.1. *Let M be a $2q$-lightlike submanifolds of an indefinite Kaehler manifold $\overline{M}$ with constant index $2q$ such that $2q < dim(M)$. Then the screen distribution $S(TM)$ of lightlike submanifold M is Riemannian.*

Using the above lemma, we define radical screen transversal slant lightlike submanifolds as follows.

Definition 3.2. . *A $2q$-lightlike submanifold of M of an indefinite Hermitian manifold $\overline{M}$ of index $2q$ is said to be radical screen transversal slant lightlike submanifold if the following conditions are satisfied:*
i) RadTM is a distribution on M such that

$$J(RadTM) \subset S(TM^\perp).$$

ii) For each non-zero vector field X tangent to $S(TM)$ at $X \in U \subset M$ the angle $\theta(X)$ between JX and $S(TM)$ is constant, that is, it is independent of the choice of $X \in U \subset M$ and $X \in \Gamma(S(TM))$.

The angle $\theta(X)$ is called the slant angle of the distribution $S(TM)$. A radical screen transversal slant lightlike submanifold is said to be proper if $S(TM) \neq 0$ and $\theta \neq 0, \pi/2$.

Moreover, Radical ST-lightlike submanifolds and ST-anti invariant lightlike submanifolds are the particular cases of Radical screen Transversal Slant lightlike submanifolds when $\theta = 0$ and $\theta = \pi/2$ respectively.

On the otherhand , our study differ from slant lightlike submanifolds and screen slant lightlike submanifolds, on the behaviour $RadTM$ after application of ϕ on it. In slant lightlike submanifolds $\phi RadTM$ is subset of $S(TM)$ and in screen slant lightlike submanifolds $\phi RadTM$ is invariant while in our case $\phi RadTM$ is a subset of $S(TM^{\perp})$

From the definition 3.2, we have the following decomposition :

$$TM = RadTM \perp S(TM)$$

$$tr(TM) = ltr(TM) \perp FS(TM) \perp \{Fltr(TM) \oplus FRadTM\} \perp \mu$$

$$T\overline{M} = (RadTM \oplus ltr(TM)) \perp S(TM) \perp \{Fltr(TM) \oplus FRadTM\} \perp FS(TM) \perp \mu$$

Now, we give an example of a proper radical screen transversal slant lightlike submanifold of indefinite Kaehler manifolds in support of the definition3.2.

Example 3.3. *Let $\overline{M}$ be a semi-Riemannian manifolds R^8_2 is a semi-Euclidean space of signature $(-,-,+,+,+,+,+,+)$ and let M be a submanifold of R^8_2 defined by*

$$x^1 = s, \quad x^2 = ucos\alpha, \quad x^3 = usin\alpha, \quad x^4 = s\ cos\alpha$$

$$y^1 = 0, \quad y^2 = vcos\alpha, \quad y^3 = -vsin\alpha, \quad y^4 = s\ sin\alpha$$

where $\alpha > 0$. Then the tangent bundle TM is spanned by

$$\xi = \frac{\partial}{\partial x_1} + cos\alpha\frac{\partial}{\partial x_4} + sin\alpha\frac{\partial}{\partial y_4}$$

$$X_1 = cos\alpha\frac{\partial}{\partial x_2} + sin\alpha\frac{\partial}{\partial x_3}$$

$$X_2 = cos\alpha\frac{\partial}{\partial y_2} - sin\alpha\frac{\partial}{\partial y_3}$$

It follows that M is a 1-lightlike submanifolds of R^8_2 with $RadTM = span\ \{\xi\}$. Let $S(TM)= span\{X_1, X_2\}$. Then, $S(TM)$ is Riemannian vector subbundle and it can be easily prove that $S(TM)$ is a slant distribution with slant angle 2α. Moreover, the screen transversal bundle $S(TM^{\perp})$ is spanned by

$$W_1 = sin\alpha\frac{\partial}{\partial y_2} - cos\alpha\frac{\partial}{\partial y_3}$$

$$W_2 = sin\alpha\frac{\partial}{\partial x_2} - cos\alpha\frac{\partial}{\partial x_3}$$

and $ltr(TM)$ is spanned by

$$N = \frac{1}{2}(-\frac{\partial}{\partial x_1} + cos\alpha\frac{\partial}{\partial x_4} + sin\alpha\frac{\partial}{\partial y_4})$$

By direct calculation, we get
$J(RadTM) \subset S(TM^{\perp})$ and $J(ltr(TM)) \subset S(TM^{\perp})$.

For any $X \in \Gamma(TM)$, we write

$$JX = TX + FX, \tag{10}$$

where TX is the tangential component of JX and FX is the transversal component of JX. Similarly, for $V \in \Gamma(tr(TM))$, we write

$$JV = BV + CV \tag{11}$$

$$JV = B_1V + B_2V + C_1V + C_2V + \mu. \tag{12}$$

where BV is the tangential component of JV and CV is the transversal component of JV. We note that B_1V, B_2V, C_1V, C_2V are the components in $S(TM), RadTM, ltr(TM), F(ltr(TM))$ respectively.

Let Q_1 and Q_2 be the projection on the distribution $S(TM)$ and $RadTM$ respectively. Then for any X tangent to M, we can write

$$X = Q_1X + Q_2X$$

Applying J to the above equation, we obtain

$$JX = TQ_1X + FQ_1X + FQ_2X \tag{13}$$

Using (10) and (13), we see that

$$TX = TQ_1X,$$

$$JQ_2X = FQ_2X \subset S(TM^{\perp}), \quad TQ_2X = 0 \qquad TQ_1X \in \Gamma(S(TM) \quad and \quad FQ_1X \subset S(TM^{\perp}).$$

Now, we prove two characterization theorems for radical screen transversal slant lightlike submanifolds, similar to the characterization of slant submanifolds in indefinite Hermitian manifold given by Sahin[11] .

Theorem 3.4. *A $2q$- lightlike submanifold M of an indefinite Hermitian manifold $\overline{M}$ of index $2q$ is a radical screen transversal slant lightlike submanifolds if and only if*
i) $J(ltr(TM))$ is a distribution in $S(TM^{\perp})$,
ii) There exists a constant $\lambda \in [-1, 0]$ such that

$$(Q_1T)^2Q_1X = \lambda Q_1X \quad \forall\ X \in \Gamma(S(TM)) \tag{14}$$

where $\lambda = -cos^2\theta$.

Proof. Suppose M is a radical screen transversal slant lightlike submanifold $\overline{M}$ of index $2q$. Then $J(RadTM) \subset S(TM^{\perp})$. Since $\overline{g}(JRadTM, J(ltr(TM))) \neq 0$, so $\overline{g}(JX, JX) = \overline{g}(X, X)$ is possible only when $Jltr(TM) \subset S(TM^{\perp})$, which proves (i). Further, as the angle between JQ_1X and $X \in \Gamma(S(TM))$ is constant, we have

$$cos\theta(Q_1X) = -\frac{g(Q_1X, TQ_1TQ_1X)}{|Q_1X||TQ_1X|} \tag{15}$$

On the other hand

$$cos\theta(Q_1X) = \frac{|TQ_1X|}{|JQ_1X|} \tag{16}$$

Combining (15) and (16), we obtain

$$cos^2\theta(Q_1X) = -\frac{g(Q_1X, (Q_1T)^2Q_1X)}{|Q_1X|^2} \tag{17}$$

Using (17) and the fact that $\theta(Q_1X)$ is constant on $S(TM)$, we get

$$(Q_1T)^2Q_1X = \lambda Q_1X, \quad \lambda \in [-1, 0],$$

where $\lambda = -cos^2\theta$. Converse part directly follows from (i) and (ii). □

Theorem 3.5. *A $2q$- lightlike submanifold M of an indefinite Hermitian manifold $\overline{M}$ of index $2q$ is a radical screen transversal slant lightlike submanifold if and only if*
i) $J(ltr(TM))$ is a distribution in $S(TM^{\perp})$
ii) There exists a constant $\mu \in [-1, 0]$, such that

$$B_1FQ_1X = \mu Q_1X \quad \forall\ X \in \Gamma(TM),$$

where $\mu = -sin^2\theta, \theta$ is the slant angle of M and Q_1 is the projection on $S(TM)$ which is complementary to $RadTM$.

Proof. Let M be a radical screen transversal slant lightlike submanifolds of an indefinite Hermitian manifold $\overline{M}$ of a index $2q$. Then obviously $J(ltr(TM))$ is a distribution in $S(TM^{\perp})$. On the otherhand by applying J to (13) and using (10) and (11), we get

$$-X = (Q_1T)^2Q_1X + FQ_1TQ_1X + B_1FQ_1X + B_2FQ_2X$$

Comparing screen component on both sides of above equation, we obtain

$$-Q_1X = (Q_1T)^2Q_1X + B_1FQ_1X \tag{18}$$

Since M is a radical screen transversal slant lightlike submanifold, from Theorem 3.4, we have

$$(Q_1T)^2Q_1X = -cos^2\theta(Q_1X) \tag{19}$$

Thus (*ii*) follows from (18) and (19).
Conversely, assume that (*i*) and (*ii*) hold good. From (*ii*) and (18), we conclude that

$$(Q_1T)^2Q_1X = -(1 + \mu)Q_1X.$$

Let $-(1 + \mu) = \lambda$. Then $\lambda \in [-1, 0]$. Thus the proof follows from Theorem 3.4. □

As a consequence of the above theorem, we have

Corollary 3.6. *Let M be a radical screen transversal slant lightlike submanifold of an indefinite Hermitian manifold $\overline{M}$. Then we have*
$g(TQ_1X, TQ_1Y) = cos^2\theta g(Q_1X, Q_1Y)$
and
$g(FQ_1X, FQ_1Y) = sin^2\theta g(Q_1X, Q_1Y) \ \ \forall X, Y \in \Gamma(TM)$

Differentiating , (13), using Kaehler Character of $\overline{M}$ and then comparing tangential, lightlike transversal and screen transversal parts of the resulting equation , we get

$$(\nabla_XT)Q_1Y = A_{FQ_1Y}X + A_{FQ_2Y}X + Bh^s(X, Y) \tag{20}$$

$$h^l(X, TQ_1Y) + D^l(X, FQ_1Y) + D^l(X, FQ_2Y) = 0 \tag{21}$$

$$(\nabla_XF)Q_1Y + (\nabla_XF)Q_2Y + h^s(X, TQ_1Y) = Ch^l(X, Y) + Ch^s(X, Y) \tag{22}$$

The integrability of the distributions involved in the definition of radical screen transversal slant lightlike submanifolds immersed in Kaehler manifold, given by the following theorems.

Theorem 3.7. *Let M be a radical screen transversal slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$ of index $2q$. Then the distribution $RadTM$ is integrable if and only if*
(i)$A_{FQ_2Y}X = A_{FQ_2X}Y$
(ii)$\nabla^s_XFQ_2Y - \nabla^s_YFQ_2X \in F(RadTM) \subset S(TM^{\perp})$.
for all $X, Y \in \Gamma(RadTM)$.

Proof. From (20), we have

$$-TQ_1\nabla_X Y = A_{FQ_2Y}X + Bh^s(X, Y) \tag{23}$$

for each $X, Y \in \Gamma(RadTM.)$. Interchanging the role of X and Y and substracting (23) from the resulting equation, we get

$$TQ_1[X, Y] = A_{FQ_2X}Y - A_{FQ_2Y}X. \tag{24}$$

On the otherhand, from (22), we have

$$-FQ_1\nabla_X Y + \nabla^s_X FQ_2Y - FQ_2\nabla_X Y = Ch^l(X, Y) + Ch^s(X, Y) \tag{25}$$

for all $X, Y \in \Gamma(RadTM)$. Interchanging the roles of X and Y and subtracting (25) from the resulting equation, we obtain

$$FQ_1[X, Y] + FQ_2[X, Y] = \nabla^s_X FQ_2Y - \nabla^s_Y FQ_2X. \tag{26}$$

Thus, our assertion follows from (24) and (26). □

Theorem 3.8. *Let M be a radical screen transversal slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$ of index $2q$. Then the screen distribution $S(TM)$ is integrable if and only if*

$$\nabla^s_X FQ_1Y - \nabla^s_Y FQ_1X + h^s(X, TQ_1Y) - h^s(Y, TQ_1X) \in F(S(TM)) \subset S(TM^\perp).$$

for all $X, Y \in \Gamma(S(TM))$.

Proof. Using (22), we obtain

$$\nabla^s_X FQ_1Y - FQ_1\nabla_X Y - FQ_2\nabla_X Y + h^s(X, TQ_1Y) = Ch^l(X, Y) + Ch^s(X, Y)$$

for all $X, Y \in \Gamma(S(TM))$.
Interchanging the role of X and Y and subtracting the above equation from the resulting equation, we get

$$FQ_1[X, Y] + FQ_2[X, Y] = \nabla^s_X FQ_1Y - \nabla^s_Y FQ_1X + h^s(X, TQ_1Y) - h^s(Y, TQ_1X)$$

from which our assumption follows. □

In the following two results, we discuss the condition under which the distribution $S(TM)$ and $RadTM$ defines a totally geodesic foliations.

Theorem 3.9. *Let M be a radical screen transversal slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$ of index $2q$. Then the distribution $S(TM)$ defines a totally geodesic foliation if and only if $A_{FQ_1TQ_1Y}X - B\nabla^s_X FQ_1Y$ has no component in $RadTM$ $\forall X, Y \in \Gamma(S(TM))$ and $N \in \Gamma(ltr(TM))$.*

Proof. For any $X, Y \in \Gamma(S(TM))$ and $N \in \Gamma(ltr(TM))$, from (1), (3), (13) and (14), we have

$$\overline{g}(\overline{\nabla}_X Y, N) = cos^2\theta g(\overline{\nabla}_X Y, N) + g(A_{FQ_1TQ_1Y}X - B\nabla^s_X FQ_1Y, N)$$

which can be rewritten as

$$sin^2\theta g(\nabla_X Y, N) = g(A_{FQ_1TQ_1Y}X - B\nabla^s_X FQ_1Y, N). \tag{27}$$

Thus, our result follows from (27). □

Theorem 3.10. *Let M be a radical screen transversal slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$ of index $2q$. Then the distribution $RadTM$ defines a totally geodesic foliation if and only if $h^s(\xi_1, TQ_1Z) + \nabla^s_{\xi_1} FQ_1Z$ has no component in $J(ltr(TM)) \subset S(TM^\perp)$ $\forall \xi_1, \xi_2 \in \Gamma(RadTM), Z \in \Gamma(S(TM))$.*

Proof. Using (1), (3) and (13), Obtain

$$\overline{g}(\overline{\nabla}_{\xi_1}\xi_2, Z) = -\overline{g}(J\xi_2, h^s(\xi_1, TQ_1Z) + \nabla^s_{\xi_1} FQ_1Z) \tag{28}$$

for all $\xi_1, \xi_2 \in \Gamma(RadTM)$ and $Z \in \Gamma(S(TM))$. Thus our assertion follows from (28) □

In general the induced connection ∇ on a lightlike submanifold M in a semi-Riemannian manifold is not a metric connection . Here we are giving the condition under which induced connection ∇ on a radical screen transversal slant lightlike submanifold of an indefinite Kaehler manifold to be a metric connection.

Theorem 3.11. *Let M be a radical screen transversal slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$ of index $2q$. Then the induced connection ∇ is a metric connection on M if and only if $TQ_1A_{FQ_2Y}X = B\nabla^s_X FQ_2Y$ for all $X \in \Gamma(TM)$ and $Y \in \Gamma(RadTM)$.*

Proof. From (8), (9) and (13), we get

$$\overline{\nabla}_X Y = -J\overline{\nabla}_X FQ_2Y \tag{29}$$

Using (1), (3), (11), (13) and (29), we get

$$\begin{aligned} \nabla_x Y + h^l(X, Y) + h^s(X, Y) &= TQ_1A_{FQ_2Y}X + FQ_1A_{FQ_2Y}X + FQ_2A_{FQ_2Y}X - B\nabla^s_X FQ_2Y \\ &- C\nabla^s_X FQ_2Y - CD^l(X, FQ_2Y). \end{aligned} \tag{30}$$

Comparing tangential component component of (30), we obtain

$$\nabla_X Y = TQ_1A_{FQ_2Y}X - B\nabla^s_X FQ_2Y \tag{31}$$

Suppose the induced connection ∇ is a metric connection. Then $\nabla_X Y \in \Gamma(RadTM)$. Thus from (31), we have

$$TQ_1A_{FQ_2Y}X - B\nabla^s_X FQ_2Y = 0$$

which proves our assertion. Converse part directly follofws from (31). □

Definition 3.12. *A radical screen transversal slant lightlike submanifold of an indefinite Hermitian manifold $\overline{M}$ of index $2q$ is said to be radical screen transversal slant lightlike product if both the distribution RadTM and $S(TM)$ are integrable and their leaves are totally geodesic in M.*

A characterization of radical screen transversal slant lightlike product is given by the following theorem.

Theorem 3.13. *Let M be a radical screen transversal slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$ of index $2q$. Then M is a radical screen transversal slant lightlike product if and only if $\nabla T = 0$.*

Proof. Suppose M is a radical screen transversal slant lightlike product,i.e, leaves of $RadTM$ and $S(TM)$ are totally geodesic in M. Using (20), for $X \in \Gamma(TM)$ and $Y \in \Gamma(RadTM)$, we get

$$(\nabla_X T)Q_1Y = A_{FQ_2Y}X + Bh^s(X, Y). \tag{32}$$

Taking inner product of (32) with $Z \in \Gamma(S(TM))$ and using (1), (11), we obtain

$$g(\nabla_X T)Q_1Y, Z) = g(A_{FQ_2Y}X + Bh^s(X, Y), Z) \tag{33}$$

from which we have

$$g((\nabla_X T)Q_1Y, Z) = 0 \tag{34}$$

where we have used (3), (13) and (33) with the fact that the leaves of $RadTM$ are totally geodesic in M. Similarly, for $X \in \Gamma(TM)$ and $Y \in \Gamma(S(TM))$, from (20), we have

$$(\nabla_X T)Q_1Y = A_{FQ_1Y}X + Bh^s(X, Y).$$

Taking inner product of the above equation with $N \in \Gamma(ltr(TM))$ and using (1), (11), we arrive at

$$\overline{g}((\nabla_X T)Q_1 Y, N) = g(A_{FQ_1 Y} X, N) + \overline{g}(\overline{\nabla}_X JY, N). \tag{35}$$

Making use of (3), (13) and (35) with totally geodesicness of leaves of $S(TM)$, we get

$$\overline{g}((\nabla_X T)Q_1 Y, N) = 0 \tag{36}$$

Thus from (34) and (36), we have

$$(\nabla_X T)Q_1 Y = 0 \quad i.e \quad \nabla T = 0.$$

Conversely, suppose $\nabla T = 0$. Then from (1), (3), (11) and (13), we have

$$h^l(X, TQ_1 Y) + h^s(X, TQ_1 Y) - A_{FQ_1 Y} X + \nabla^s_X FQ_1 Y + D^l(X, FQ_1 Y) - FQ_1 \nabla_X Y - FQ_2 \nabla_X Y - Ch^l(X, Y) - Bh^s(X, Y) - Ch^s(X, Y) = 0$$

for all $X, Y \in \Gamma(S(TM))$.
Interchanging the role of X and Y in the above equation and subtracting resulting equation from the above equation we arrive at

$$-h^l(X, TQ_1 Y) + h^l(Y, TQ_1 X) - h^s(X, TQ_1 Y) + h^s(Y, TQ_1 X) + A_{FQ_1 Y} X - A_{FQ_1 X} Y + \nabla^s_Y FQ_1 X - \nabla^s_X FQ_1 Y$$

$$+D^l(Y, FQ_1 X) - D^l(X, FQ_1 Y) + FQ_1[X, Y] + FQ_2[X, Y] = 0 \tag{37}$$

Comparing the components of $S(TM^\perp)$ on both sides of (3.27), we get

$$FQ_1[X, Y] + FQ_2[X, Y] = -h^s(X, TQ_1 Y) + h^s(Y, TQ_1 X) + \nabla^s_Y FQ_1 X - \nabla^s_X FQ_1 Y \tag{38}$$

Thus, the integrability of $S(TM)$ follows from Theorem 3.8 and (38). In respect of the integrability of $RadTM$, we have

$$\overline{\nabla}_X JZ = J\overline{\nabla}_X Z$$

for each $Z \in \Gamma(RadTM)$. Using(1), (3), (11), (13) and $\nabla T = 0$, we arrive at

$$-A_{FQ_2 Z} X + \nabla^s_X FQ_2 Z + D^l(X, FQ_2 Z) = FQ_1 \nabla_X Z + FQ_2 \nabla_X Z + Ch^l(X, Z) + Bh^s(X, Z) + Ch^s(X, Z).$$

Interchanging the role of X and Z and subtracting resulting equation from the above equation, we get

$$A_{FQ_2 X} Z - A_{FQ_2 Z} X + \nabla^s_X FQ_2 Z - \nabla^s_Z FQ_2 X + D^l(X, FQ_2 Z) - D^l(Z, FQ_2 X) = FQ_1[Z, X] - FQ_2[Z, X] \tag{39}$$

Taking the inner product of (39) with JY, $Y \in \Gamma(S(TM))$, we get

$$\overline{g}(FQ_1[Z, X], JY) = g(A_{FQ_2 X} Z - A_{FQ_2 Z} X, TQ_1 Y) + \overline{g}(\nabla^s_X FQ_2 Z - \nabla^s_Z FQ_2 X, FQ_1 Y)$$

from which we have

$$sin^2\theta g([Z, X], Y) = g(A_{FQ_2 X} Z - A_{FQ_2 Z} X, TQ_1 Y) + \overline{g}(\nabla^s_X FQ_2 Z - \nabla^s_Z FQ_2 X, FQ_1 Y) \tag{40}$$

Thus the integrability of $RadTM$ follows from Theorem 3.7 and (40).
Now, we prove that the leaves of $S(TM)$ and $RadTM$ are totally geodesic in M. Using $\nabla T = 0$ and (20) for any $Z \in \Gamma(TM)$ and $X, Y \in \Gamma(S(TM))$, we obtain

$$A_{FQ_1 Y} X + Bh^s(X, Y) = 0.$$

Taking inner product of above equation with $N \in \Gamma(ltr(TM))$ and using (1), (3) and (13), we get

$$\overline{g}(\nabla_X TQ_1 Y, N) = 0.$$

From which we conclude that the leaves of $S(TM)$ are totally geodesic in M.
In a similar way, from $\nabla T = 0$ and (20),we have

$$A_{FQ_2 Y} X + Bh^s(X, Y) = 0 \tag{41}$$

for any $X, Y \in \Gamma(RadTM)$. Taking inner product of (41) with $Z \in \Gamma(S(TM))$ and using (1), (3), (13), we obtain

$$g(\nabla_X Y, TQ_1 Z) = 0$$

which shows that the leaves of $RadTM$ are totally geodesic in M. This completes the proof. □

In view of the above Theorem, one can easily have :

Theorem 3.14. *Let M be a radical screen transversal slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$ of index $2q$. Then M is a radical screen transversal slant lightlike product if and only if $A_{FQ_1Z}Y = A_{FQ_1}Z$ and $\nabla_X TQ_1 Y \in \Gamma(S(TM))$ $\forall X \in \Gamma(TM)$, $Y, Z \in \Gamma(S(TM))$.*

Proof. From (4), (13) and (20), we have

$$g((\nabla_X T)Q_1 Y, Z) = g(A_{FQ_1Y}Z - A_{FQ_1Z}Y, X) \tag{42}$$

for any $X \in \Gamma(TM)$ and $Y, Z \in \Gamma(S(TM))$.
Also using (1), (3), (13) and (20) for $X \in \Gamma(TM)$, $Y \in \Gamma(S(TM))$ and $N \in \Gamma(ltr(TM))$, we get

$$g((\nabla_X T)Q_1 Y, N) = g(\nabla_X TQ_1 Y, N). \tag{43}$$

Thus our proof follows from (42), (43) Theorem 3.13 □

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