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Existence and Stability Results for Caputo Fractional Stochastic Differential Equations with Lévy Noise

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Abstract. In this paper, the existence of solution of stochastic fractional differential equations with Lévy noise is established by the Picard-Lindelöf successive approximation scheme. The stability of nonlinear stochastic fractional dynamical system with Lévy noise is obtained using Mittag Leffler function. Examples are provided to illustrate the theory.

1. Introduction

The deterministic models often fluctuate due to noise and a natural extension of a deterministic model is stochastic, where the relevant parameters are modeled as suitable stochastic process. These equations plays an important role in characterizing many physical, biological and engineering problems. They are important from the viewpoint of applications since they incorporate randomness into the mathematical description of the phenomena and provide a more accurate description of it. Therefore, the theory of stochastic differential equations (SDEs) has developed quickly and the investigation for SDEs has attracted considerable attention [9, 12, 25, 26]. Environmental changes, rapid fluctuations in financial markets and many other real discontinuous systems are described by Lévy noise [1]. This process have application in mathematical finance, stochastic control and quantum field theory. So the study of stochastic differential equations with Lévy noise gained much interest.

Fractional calculus has its applications in several areas of mathematical physical and engineering sciences. Its generalized the ideas of integer order differentiation[11, 19]. It describes the dynamical behavior of the models more accurately than the integer order equations because of its hereditary properties. The increasing interest of fractional equations is motivated by the applications in various fields of science such as physics, fluid mechanics, viscoelasticity, heat conduction in materials with memory, chemistry and engineering.

Considering fractional differential equations with random elements comes from the fact that many phenomena in science and engineering have been modeled by fractional differential equations with some uncertainty. These equations have immense physical applications in many fields such as turbulence, heterogeneous flows and materials, viscoelasticity and electromagnetic theory [3, 6, 17, 27, 32, 33, 35].

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The problems of existence and uniqueness of solutions of differential equations provide the basis for the model validation and further undertaking a study of the corresponding dynamic processes. Many authors [12, 20, 26, 28] discussed the existence and uniqueness of the solution of stochastic differential equations. This problem was studied by Pedjeu and Ladde [27] using independent set of time scales. The concept of stability is extremely important because almost every workable control system is designed to be stable. It means that system remains in a constant state unless affected by an external action and returns to a constant state when the external action is removed. Luo [22], Khasminskii [18] and Balachandran et al. [7, 8] disscussed the stability of stochastic differential equations. Exponential stability for stochastic neutral partial functional differential equations was obtained by Govindan using semigroup theory [13–15]. Zhu et al [36] studied the stability of stochastic systems with Poisson jumps. Further the stability of fractional dynamical systems is studied by many authors [10, 16, 30]. Abouagwa and Li [4, 5] discussed the stochastic fractional systems with Levy noise under Caratheodory conditions.
The fractional Brownian motion introduced by Mandelbrot and Van Ness [24] which incorporates memory with randomness. These fractional Brownian motions are fractional integrals or fractional derivatives of Brownian motion. However these models only take into consideration the memory effects of the noises in the system and not the memory associated with the system state dynamics. Li et al. [21] gives the comparative study of the classical stochastic model for European option pricing, namely, the Black Scholes model with the stochastic equation with fractional Brownian motion and stochastic equation with fractional time derivative. It has been shown that the stochastic model with derivative in time replaced by fractional derivative performs better than the model with fractional Brownian noise. In this paper we prove the existence of solution of stochastic fractional differential equations with Lévy noise and stability of such equations.

2. Preliminaries

In this section we present a few well-known concepts of fractional and stochastic differential equations.

Definition 2.1. (Riemann-Liouville fractional integral). The Riemann-Liouville fractional integral operator of order $\alpha > 0$ of a function $f \in L^1(\mathbb{R}^+)$ is defined as

$$I^{\alpha}_{0+}f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}f(s)ds, \ t > 0, \tag{2.1}$$

where $\Gamma(\cdot)$ is the Euler gamma function.

Definition 2.2. (Riemann-Liouville fractional derivative). The Riemann-Liouville fractional derivative of order $\alpha > 0$, $n-1 < \alpha < n$, $n \in \mathbb{N}$, is defined as

$$D^{\alpha}_{0+}f(t) = \left(\frac{d}{dt}\right)^n I^{n-\alpha}_{0+}f(t) = \frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dt}\right)^n \int_0^t (t-s)^{n-\alpha-1}f(s)ds, \tag{2.2}$$

where the function $f(t)$ has absolutely continuous derivatives upto order (n - 1).

Definition 2.3. (Caputo fractional derivative). The Caputo fractional derivative of order $\alpha > 0$, $n-1 < \alpha < n$, $n \in \mathbb{N}$, is defined as

$$^{C}D^{\alpha}_{0+}f(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-s)^{n-\alpha-1}f^{(n)}(s)ds, \tag{2.3}$$

where the function $f(t)$ has absolutely continuous derivatives upto order (n - 1).

Definition 2.4. (Mittag-Leffler function). The one parameter Mittag-Leffler function is defined by

$$E_{\alpha}(z) = \sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+1)}, \ (z \in \mathbb{C}, \ Re(\alpha) > 0). \tag{2.4}$$

A two parameter Mittag-Leffler function is defined by

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}, \ (z, \beta \in \mathbb{C}, \ Re(\alpha) > 0). \tag{2.5}$$

In particular when $\beta = 1$ *then* $E_{\alpha,1}(z) = E_\alpha(z)$. *The Mittag Leffler function of a matrix A is defined by*

$$E_{\alpha,\beta}(At) = \sum_{k=0}^{\infty} \frac{(At)^k}{\Gamma(\alpha k + \beta)}, \ (\alpha, \beta > 0, \ A \in \mathbb{R}^{n\times n}).$$

Definition 2.5. (Stochastic Process). A collection $\{X(t)| \ t \geq 0\}$ of random variables is called a stochastic process.

Definition 2.6. (Lévy Process). Let $\{X(t)| \ t \geq 0\}$ be a stochastic process defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$. Then X is a Lévy process if

1. $X(0) = 0$ (a.s).
2. X has an independent and stationary increments.
3. X is stochastically continuous. That is, for all $a > 0$ and for all $s > 0$ $\lim_{t\to s} P(|X(t) - X(s)| > a) = 0$.

Definition 2.7. (Chebyshev's Inequality). If X is a random variable and $1 \leq p < \infty$, then

$$\mathbb{P}(|X| \geq \lambda) \leq \frac{1}{\lambda^p}\mathbb{E}(|X|^p) \text{ for all } \lambda > 0.$$

Lemma 2.8. (Borel Cantelli Lemma). If $\{A_k\} \subset \mathcal{F}$ and $\sum_{k=1}^{\infty} \mathbb{P}(A_k) < \infty$, then

$$\mathbb{P}(\limsup_{k\to\infty} \ A_k) = 0.$$

Theorem 2.9. *(i) If* $\{X_n\}_{n=1}^{\infty}$ *is a submartingale, then*

$$\mathbb{P}\left(\max_{1\leq k\leq n} X_k \geq \lambda\right) \leq \mathbb{E}(X_n^+)$$

for all $n = 1, 2, \ldots$ *and* $\lambda > 0$.
(ii) If $\{X_n\}_{n=1}^{\infty}$ *is a martingale and* $1 < p < \infty$, *then*

$$\mathbb{E}\left(\max_{1\leq k\leq n} |X_k|^p\right) \leq \left(\frac{p}{p-1}\right)^p \mathbb{E}(|X_n|^p)$$

for all $n = 1, 2, \ldots$.

3. Existence and Uniqueness

In this section we prove the existence and uniqueness of solution of nonlinear stochastic fractional differential equations and stochastic fractional delay differential equations with Lévy noise. Here the results are obtained by using the classical Picard-Lindelöf method of successive approximation scheme [31, 34].

3.1. Nonlinear Equations

Let $W(t)$ be an m-dimensional Brownian motion and $\tilde{N}(dt, dz) = N(dt, dz) - \nu(dz)dt$ which is the l-dimensional compensated jump measure of $\eta(.)$ an independent compensated Poisson random measure on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$. Here $N(dt, dz)$ is the l-dimensional jump measure (or Poisson measure) and $\nu(dz)$ is the Lévy measure of l-dimensional Lévy process $\eta(.)$. For convenience $x(t, \omega), t \geq 0$ and $\omega \in \Omega$ can be written as $x(t)$ throughout this paper. Consider the stochastic fractional differential equation with Lévy noise of the form

$$\left.\begin{aligned} {}^{C}D^{\alpha}x(t) &= b(t, x(t)) + \sigma(t, x(t))\frac{dW(t)}{dt} + \int_{z} g(t, x(t), z)\frac{d\tilde{N}(t, z)}{dt}, \ t \in J = [0, T] \\ x(0) &= x_0, \end{aligned}\right\} \tag{3.1}$$

where $\alpha \in (1/2, 1)$ and $z \in \mathbb{R}_0^n = \mathbb{R}^n/\{0\}$. Here $b : J \times \mathbb{R}^n \to \mathbb{R}^n$, $\sigma : J \times \mathbb{R}^n \to \mathbb{R}^{nm}$, $g : J \times \mathbb{R}^n \times \mathbb{R}_0^n \to \mathbb{R}^{nl}$ are given functions such that for all t, $b(t, x(t))$, $\sigma(t, x(t))$ and $g(t, x(t), z)$ are $\mathcal{F}_t$ measurable for all $x \in \mathbb{R}^n$ and $z \in \mathbb{R}_0^n$. We can rewrite the equation (3.1) in its equivalent integral form as:

$$\begin{aligned} x(t) = x_0 &+ \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1} b(s, x(s))ds + \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\sigma(s, x(s))dW(s) \\ &+ \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\int_z g(s, x(s), z)d\tilde{N}(s, z). \end{aligned} \tag{3.2}$$

Theorem 3.1. (Existence and Uniqueness). *Assume that* $(t, x) \in J \times \mathbb{R}^n$, $\alpha \in (1/2, 1)$, $z \in \mathbb{R}_0^n$, $b \in C(J \times \mathbb{R}^n, \mathbb{R}^n)$, $\sigma \in C(J \times \mathbb{R}^n, \mathbb{R}^{nm})$, $g \in C(J \times \mathbb{R}^n \times \mathbb{R}_0^n, \mathbb{R}^{nl})$*and* $W = \{W(t), t \geq 0\}$ *is an m-dimensional Brownian motion on a complete probability space* $(\Omega, \mathcal{F}, \mathcal{P})$*. Suppose the following inequalities hold:*
(i) Linear growth condition :

$$|b(t, x)|^2 + |\sigma(t, x)|^2 + \int_z |g(t, x, z)|^2\nu(dz) \leq K^2(1 + |x|^2) \tag{3.3}$$

for some constant $K > 0$.
(ii)The Lipschitz condition :

$$\begin{aligned} |b(t, x) - b(t, y)|^2 + |\sigma(t, x) - \sigma(t, y)|^2 &+ \int_z |g(t, x, z) - g(t, y, z)|^2\nu(dz) \\ &\leq L^2(|x - y|^2) \end{aligned} \tag{3.4}$$

for some constant $L > 0$.

Let x_0 *be a random variable defined on* $(\Omega, \mathcal{F}, \mathcal{P})$ *and independent of the σ-algebra* $\mathcal{F}_s^t \subset \mathcal{F}$ *generated by* $\{W(s), t \geq s \geq 0\}$ *and such that* $\mathbb{E}|x_0|^2 < \infty$*. Then the initial value problem (3.1) has a unique solution which is t-continuous with the property that* $x(t, \omega)$ *is adapted to the filtration* $\mathcal{F}_t^{x_0}$ *generated by* x_0 *and* $\{W(s)(\cdot), s \leq t\}$ *and*

$$\sup_{0 \leq t \leq T} \mathbb{E}[|x(t)|^2] < \infty. \tag{3.5}$$

Proof. ***Existence***: First we establish the existence of solution of the initial value problem. Let us define $x^{(0)}(t) = x_0$ and $x^{(k)}(t) = x^{(k)}(t, \omega)$ inductively as follows:

$$\begin{aligned} x^{(k+1)}(t) = x_0 &+ \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1} b(s, x^{(k)}(s))ds + \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\sigma(s, x^{(k)}(s))dW(s) \\ &+ \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\int_z g(s, x^{(k)}(s), z)\tilde{N}(ds, dz), \end{aligned} \tag{3.6}$$

for $k = 0, 1, 2, \ldots$. If, for fixed $k \geq 0$, the approximation $x^{(k)}(t)$ is $\mathcal{F}_t$-measurable and continuous on J, then it follows from (3.3)-(3.4), that the integrals in (3.6) are meaningful and that the resulting process $x^{(k+1)}(t)$ is

$\mathcal{F}_t$-measurable and continuous on J. As $x^{(0)}(t)$ is obviously $\mathcal{F}_t$-measurable and continuous on J, it follows by induction that so too is each $x^{(k)}(t)$ for $k = 1, 2, \ldots$.

Since x_0 is $\mathcal{F}_t$-measurable with $\mathbb{E}(|x_0|^2) < \infty$, it is clear that

$$\sup_{0\le t\le T} \mathbb{E}(|x^{(0)}(t)|^2) < \infty.$$

Applying the algebraic inequality $(a+b+c+d)^2 \le 3(a^2+b^2+c^2+d^2)$, the Cauchy-Schwarz inequality, the Itô isometry and the linear growth condition (3.3) we obtain from (3.6) that

$$\begin{aligned}\mathbb{E}(|x^{(k+1)}(t)|^2) \le 4\mathbb{E}[|x_0|^2] &+ \frac{4}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\mathbb{E}\left[\int_0^t \left|b(s,x^{(k)}(s)\right|^2 ds\right]\\ &+ \frac{4}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\mathbb{E}\left[\int_0^t \left|\sigma(s,x^{(k)}(s))\right|^2 ds\right]\\ &+ \frac{4}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\mathbb{E}\left[\int_0^t \left|\int_z g(s,x^{(k)}(s),z)\nu(dz)\right|^2 ds\right]\end{aligned}$$

Therefore

$$\mathbb{E}(|x^{(k+1)}(t)|^2) \le 4\mathbb{E}[|x_0|^2] + 3K^2\frac{4}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\mathbb{E}\left(\int_0^t \left(1+|x^{(k)}(s)|^2\right)ds\right),$$

for $k = 0, 1, 2, \ldots$ and $m > 0$. By induction, we have

$$\sup_{0\le t\le T} \mathbb{E}(|x^{(k)}(t)|^2) \le C_0 < \infty,$$

for $k = 1, 2, 3, \ldots$. Let

$$d^{(k)}(t) = \mathbb{E}(|x^{(k+1)}(t) - x^{(k)}(t)|).$$

We claim that

$$d^{(k)}(t) \le \frac{(Mt)^{(k+1)}}{(k+1)!}, \text{ for all } k = 0, 1, 2, \ldots, \tag{3.7}$$

for some constants M, depending in K, L and x_0. From equation (3.6) by applying the Schwarz inequality and Itô isometry and the Lipchitz condition (3.4) we obtain

$$\begin{aligned}d^{(k)}(t) &= \mathbb{E}[|x^{(k+1)}(t) - x^{(k)}(t)|^2]\\ &\le \frac{3}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \mathbb{E}\left[\left|b(s,x^{(k)}(s) - b(s,x^{(k-1)}(s))\right|^2\right]ds\\ &\quad+ \frac{3}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \mathbb{E}\left[\left|\sigma(s,x^{(k)}(s) - \sigma(s,x^{(k-1)}(s))\right|^2\right]ds\\ &\quad+ \frac{3}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \mathbb{E}\left[\int_z \left|g(s,x^{(k)}(s),z) - g(s,x^{(k-1)}(s),z)\right|^2 \nu(dz)\right]ds\\ &\le 3\frac{L^2}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \mathbb{E}\left[\left|x^{(k)}(s) - x^{(k-1)}(s))\right|^2 ds\right]\\ &\quad+ 3\frac{L^2}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \mathbb{E}\left[\left|x^{(k)}(s) - x^{(k-1)}(s))\right|^2 ds\right]\\ &\quad+ 3\frac{L^2}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \mathbb{E}\left[\left|x^{(k)}(s) - x^{(k-1)}(s))\right|^2 ds\right].\end{aligned} \tag{3.8}$$

By applying again the Schwarz inequality, the Itô isometry together with the growth conditions (3.3) for $k = 0$,

$$\begin{aligned}
d^{(0)}(t) &= \mathbb{E}[|x^{(1)}(t) - x^{(0)}(t)|^2] \\
&\leq \frac{3}{(\Gamma(\alpha))^2}\mathbb{E}\left(\left|\int_0^t (t-s)^{\alpha-1} b(s, x^{(0)}(s))ds\right|^2\right) \\
&\quad + \frac{3}{(\Gamma(\alpha))^2}\mathbb{E}\left(\left|\int_0^t (t-s)^{\alpha-1} \sigma(s, x^{(0)}(s))dW(s)\right|^2\right) \\
&\quad + \frac{3}{(\Gamma(\alpha))^2}\mathbb{E}\left(\left|\int_0^t (t-s)^{\alpha-1} \int_z g(s, x^{(0)}(s), z)\tilde{N}(ds, dz)\right|^2\right) \\
&\leq \frac{3}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \mathbb{E}\left[\left|b(s, x^{(0)}(s)\right|^2 ds\right] \\
&\quad + \frac{3}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \mathbb{E}\left[\left|\sigma(s, x^{(0)}(s)\right|^2 ds\right] \\
&\quad + \frac{3}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \mathbb{E}\left[\left|\int_z g(s, x^{(0)}(s), z)\nu(dz)\right|^2 ds\right] \\
&\leq K^2\frac{3^2}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\mathbb{E}\left(\int_0^t (1+|x_0|^2)ds\right) \\
&\leq K^2\frac{3^2}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}(t)(1+\mathbb{E}(|x_0|^2)). \qquad (3.9)
\end{aligned}$$

Now, for $k = 1$, replacing $\mathbb{E}[|x^{(1)}(t) - x^{(0)}(t)|^2]$ in the inequality (3.8) with the value on the right hand side of inequality (3.9) and integrating, we obtain

$$\begin{aligned}
\mathbb{E}[|x^{(2)}(t) - x^{(1)}(t)|^2] &\leq L^2\frac{3^2}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \mathbb{E}[|x^{(1)}(s) - x^{(0)}(s)|^2]ds \\
&\leq K^2(1+\mathbb{E}(|x_0|^2))\left(L^2\frac{3^2}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\right)^2 \int_0^t sds \\
&\leq K^2(1+\mathbb{E}(|x_0|^2))\left(L^2\frac{3^2}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\right)^2 \times \frac{t^2}{2!}. \qquad (3.10)
\end{aligned}$$

For $k = 2$, proceeding as before, we have

$$\mathbb{E}[|x^{(3)}(t) - x^{(2)}(t)|^2] \leq K^2(1+\mathbb{E}(|x_0|^2))\left(L^2\frac{3^2}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\right)^3 \times \frac{t^3}{3!}. \qquad (3.11)$$

Thus, by the principle of mathematical induction, we have

$$d^{(k)}(t) = \mathbb{E}[|x^{(k+1)}(t) - x^{(k)}(t)|^2] \leq \frac{BM^{k+1}t^{(k+1)}}{(k+1)!}, \ k = 0, 1, 2, \ldots, \ 0 \leq t \leq T, \qquad (3.12)$$

where $B = K^2(1 + \mathbb{E}|x_0|^2)$ and $M = \left(L^2\frac{3^2}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\right)$ is a constant depending only on α, T, L^2 and $\mathbb{E}|x_0|^2$.

Note that

$$\begin{aligned}\max_{0\le t\le T}|x^{(k+1)}(t)-x^{(k)}(t)|^2 &\le 3\max_{0\le t\le T}\int_0^t (t-s)^{\alpha-1}|b(s,x^{(k)}(s))-b(s,x^{(k-1)}(s))|^2 ds\\ &+3\max_{0\le t\le T}\int_0^t (t-s)^{\alpha-1}|\sigma(s,x^{(k)}(s))-\sigma(s,x^{(k-1)}(s))|^2 dW(s)\\ &+3\max_{0\le t\le T}\int_0^t (t-s)^{\alpha-1}\int_z |g(s,x^{(k)}(s),z)-g(s,x^{(k-1)}(s),z)|^2\tilde{N}(ds,dz).\end{aligned}$$

Taking expectation on both sides we have

$$\begin{aligned}\mathbb{E}\left(\max_{0\le t\le T}|x^{(k+1)}(t)-x^{(k)}(t)|^2\right) &\le 3L^2\frac{T^{2\alpha-1}}{2\alpha-1}\mathbb{E}\left(\max_{0\le t\le T}\int_0^t |x^{(k)}(s)-x^{(k-1)}(s)|^2 ds\right)\\ &+3\mathbb{E}\left(\max_{0\le t\le T}\int_0^t (t-s)^{\alpha-1}|\sigma(s,x^{(k)}(s))-\sigma(s,x^{(k-1)}(s))|^2 dW(s)\right)\\ &+3L^2\frac{T^{2\alpha-1}}{2\alpha-1}\mathbb{E}\left(\max_{0\le t\le T}\int_0^t |x^{(k)}(s)-x^{(k-1)}(s)|^2 ds\right).\end{aligned}$$

Using second part of the Theorem 2.9 gives

$$\begin{aligned}\mathbb{E}\left(\max_{0\le t\le T}|x^{(k+1)}(t)-x^{(k)}(t)|^2\right) &\le 3L^2\frac{T^{2\alpha-1}}{2\alpha-1}\mathbb{E}\left(\int_0^T |x^{(k)}(s)-x^{(k-1)}(s)|^2 ds\right)\\ &+12L^2\frac{T^{2\alpha-1}}{2\alpha-1}\mathbb{E}\left(\int_0^T |x^{(k)}(s)-x^{(k-1)}(s)|^2 ds)\right)\\ &+3L^2\frac{T^{2\alpha-1}}{2\alpha-1}\mathbb{E}\left(\int_0^T |x^{(k)}(s)-x^{(k-1)}(s)|^2 ds\right).\\ &\le B\frac{M^{k+1}}{(k+1)!}T^{(k+1)}, \end{aligned} \tag{3.13}$$

where B is a constant depending on L and T. By using Chebyshev's inequality gives

$$\mathbb{P}\left(\max_{0\le t\le T}|x^{(k+1)}(t)-x^{(k)}(t)|^2>\frac{1}{k^2}\right)\le\frac{1}{(1/k^2)^2}\mathbb{E}\left(\max_{0\le t\le T}|x^{(k+1)}(t)-x^{(k)}(t)|^2\right).$$

Using the equation (3.13) and summing up the resultant inequalities gives,

$$\sum_{k=0}^{\infty}\mathbb{P}\left(\max_{0\le t\le T}|x^{(k+1)}(t)-x^{(k)}(t)|^2>\frac{1}{k^2}\right)\le\sum_{k=0}^{\infty}\frac{BM^{k+1}k^4T^{(k+1)}}{(k+1)!}.$$

where the series on the right side converges by ratio test. Hence the series on the left side also converges, so by the Borel-Cantelli lemma, we conclude that $\max_{0\le t\le T}\left(|x^{(k+1)}(t)-x^{(k)}(t)|^2\right)$ converges to 0, almost surely, that is, the successive approximations $x^{(k)}(t)$ converge, almost surely, uniformly on J to a limit $x(t)$ defined by

$$\lim_{n\to\infty}\left(x^{(0)}(t)+\sum_{k=1}^{n}(x^{(k)}(t)-x^{(k-1)}(t))\right)=\lim_{n\to\infty}x^{(n)}(t)=x(t). \tag{3.14}$$

From (3.6), we have

$$\begin{aligned}x(t)&=x_0+\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}b(s,x(s))ds+\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\sigma(s,x(s))dW(s)\\ &+\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\int_z g(s,x(s),z)d\tilde{N}(ds,dz).\end{aligned} \tag{3.15}$$

for all $t \in J$. This completes the proof of the existence of solution of (3.1).
Uniqueness: The uniqueness follows from the Itô isometry, the Lipschitz conditions (3.4).
Let $x(t,\omega)$ and $y(t,\omega)$ be solution processes through the initial data $(0,x_0)$ and $(0,y_0)$ respectively, that is, $x(0,\omega)=x_0(\omega)$ and $y(0,\omega)=y_0(\omega)$, $\omega\in\Omega$. Let

$$\gamma_1(s,\omega) = b\,(s,x(s)) - b\,(s,y(s)),$$
$$\gamma_2(s,\omega) = \sigma\,(s,x(s)) - \sigma\,(s,y(s))$$
$$\gamma_3(s,\omega) = \int_z g(s,x(s),z)\nu(dz) - \int_z g(s,y(s),z)\nu(dz).$$

Then by virtue of the Schwarz inequality and the Itô isometry, we have

$$\begin{aligned}\mathbb{E}[|x(t)-y(t)|^2] \;\le\;& \frac{4}{(\Gamma(\alpha))^2}\mathbb{E}[|x_0-y_0|^2] + \frac{4}{(\Gamma(\alpha))^2}\frac{t^{2\alpha-1}}{2\alpha-1}\mathbb{E}\Big[\int_0^t |\gamma_1(s,\omega)|^2 ds\Big]\\ &+\frac{4}{(\Gamma(\alpha))^2}\frac{t^{2\alpha-1}}{2\alpha-1}\mathbb{E}\Big[\int_0^t |\gamma_2(s,\omega)|^2 ds\Big]\\ &+\frac{4}{(\Gamma(\alpha))^2}\frac{t^{2\alpha-1}}{2\alpha-1}\mathbb{E}\Big[\int_0^t |\gamma_3(s,\omega)|^2 ds\Big]\\ \le\;& \frac{3}{(\Gamma(\alpha))^2}\mathbb{E}[|x_0-y_0|^2] + 2^2L^2\frac{3}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \mathbb{E}[|x(s)-y(s)|^2]ds.\end{aligned}$$

We define $v(t)=\mathbb{E}[|x(t)-y(t)|^2]$. Then the function v satisfies $v(t)\le F + A\int_0^t v(s)ds$, where $F=\frac{3}{(\Gamma(\alpha))^2}\mathbb{E}[|x_0-y_0|^2]$ and $A = 2^2L^2\frac{3}{(\Gamma(\alpha))^2}\frac{T^{2\alpha-1}}{2\alpha-1}$. By the application of the Gronwall inequality, we conclude that

$$v(t)\le F\exp(At).$$

Now assume that $x_0=y_0$. Then $F=0$ and so $v(t)=0$ for all $t\ge 0$. That is,

$$\mathbb{E}[|x(t)-y(t)|^2]=0.$$

Which gives

$$\int_0^t |x(t)-y(t)|^2 d\mathbb{P} = 0.$$

This implies that $x(t)=y(t)$ a.s for all $t\in J$. That is

$$P\Big\{|x(t,\omega)-y(t,\omega)|=0 \quad \text{for all } t\in J\Big\}=1,$$

that is, the solution is unique. This completes the proof of existence and uniqueness of solution of the given stochastic fractional differential equation (3.1). □

3.2. Delay Differential Equations

Delay and Poisson jumps always coexist in real dynamic systems. Thus it is reasonable to consider them together, leading us to investigate the existence of solution of stochastic fractional delay differential equations with Lévy noise [36]. Let $\xi(\cdot)\in C[-\delta,0]$ be the initial path of x, where $\delta>0$ is a given finite time delay. Moreover, denote by $L^p_{\mathcal{F}_0}([-\delta,0];\mathbb{R}^n)$ the family of $\mathbb{R}^n$ valued adapted stochastic processes $\xi(s)$, $-\delta\le s\le 0$ such that $\xi(s)$ is $\mathcal{F}_0$-measurable and $\mathbb{E}(\sup_{-\delta\le t\le 0}|\xi(t)|^2)<\infty$. Consider the nonlinear stochastic fractional delay differential equations of the form

$$\begin{aligned}{}^CD^\alpha x(t) &= b(t,x(t),x(t-\delta)) + \sigma(t,x(t),x(t-\delta))\frac{dW(t)}{dt} + \int_z g(t,x(t),x(t-\delta),z)\frac{d\tilde{N}(t,z)}{dt},\\ &\qquad t\in J=[0,T]\\ x(t) &= \xi(t),\quad t\in[-\delta,0],\end{aligned} \tag{3.16}$$

where $\alpha \in (1/2, 1)$ and $z \in \mathbb{R}_0^n = \mathbb{R}^n/\{0\}$. Here $b : J \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$, $\sigma : J \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{nm}$, $g : J \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_0^n \to \mathbb{R}^{nl}$ are given functions such that for all t, $b(t, x(t), x(t-\delta))$, $\sigma(t, x(t), x(t-\delta))$ and $g(t, x(t), x(t-\delta), z)$ are $\mathcal{F}_t$ measurable for all $x \in \mathbb{R}^n$, $y \in \mathbb{R}^n$ and $z \in \mathbb{R}_0^n$. We can rewrite the equation (3.1) in its equivalent integral form as:

$$\begin{aligned} x(t) = \xi(0) &+ \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} b(s, x(s), x(s-\delta)) ds \\ &+ \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \sigma(s, x(s), x(s-\delta)) dW(s) \\ &+ \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \int_z g(s, x(s), x(s-\delta), z) \tilde{N}(ds, dz). \end{aligned} \tag{3.17}$$

Assume the following conditions

(H1) There exists a constant $K_i > 0$, $i = 1, 2$ such that

$$|b(t, x, y)|^2 + |\sigma(t, x, y)|^2 \le K_1(1 + |x|^2 + |y|^2),$$
$$\int_z |g(t, x, y)|^2 \nu(dz) \le K_2(1 + |x|^2 + |y|^2).$$

(H2) There exists a constant $L_i > 0$, $i = 1, 2, 3$ such that

$$|b(t, x_1, y_1) - b(t, x_2, y_2)|^2 \le L_1^2(|x_1 - x_2|^2 + |y_1 - y_2|^2),$$
$$|\sigma(t, x_1, y_1) - \sigma(t, x_2, y_2)|^2 \le L_2^2(|x_1 - x_2|^2 + |y_1 - y_2|^2),$$
$$\int_z |g(t, x_1, y_1) - g(t, x_2, y_2)|^2 \nu(dz) \le L_3^2(|x_1 - x_2|^2 + |y_1 - y_2|^2).$$

Theorem 3.2. *Assume that (H1) and (H2) holds. Let $\xi(t) \in L^2_{\mathcal{F}_0}([-\delta, 0]; \mathbb{R}^n)$ be a random variable defined on $(\Omega, \mathcal{F}, \mathcal{P})$ and independent of the σ-algebra $\mathcal{F}_s^t \subset \mathcal{F}$ generated by $\{W(s), t \ge s \ge 0\}$ and such that $\mathbb{E}(\sup_{-\delta \le t \le 0} |\xi(t)|^2) < \infty$. Then the initial value problem (3.16) has a unique solution which is t-continuous with the property that $x(t, \omega)$ is adapted to the filtration $\mathcal{F}_t^{x_0}$ generated by x_0 and $\{W(s)(\cdot), s \le t\}$ and $\sup_{0 \le t \le T} \mathbb{E}[|x(t)|^2] < \infty$.*

By using successive approximation technique one can prove the existence and uniqueness of solutions.

4. Stability Analysis

In this section we study the exponentially asymptotic stability in the quadratic mean of a trivial solution. Consider the following stochastic fractional nonlinear system with Lévy noise of the form

$$\left. \begin{aligned} {}^C D^\alpha x(t) &= Ax(t) + f(t, x(t)) + \sigma(t, x(t)) \frac{dW(t)}{dt} + \int_z g(t, x(t), z) \frac{d\tilde{N}(t, z)}{dt}, \ t \in J \\ x(0) &= x_0, \end{aligned} \right\} \tag{4.1}$$

where $\alpha \in (1/2, 1)$, $z \in \mathbb{R}_0^n = \mathbb{R}^n/\{0\}$, $f \in C(J \times \mathbb{R}^n, \mathbb{R}^n)$, $\sigma \in C(J \times \mathbb{R}^n, \mathbb{R}^{nm})$, $g \in C(J \times \mathbb{R}^n \times \mathbb{R}_0^n, \mathbb{R}^{nl})$, $\tilde{N}(dt, dz) = N(dt, dz) - \nu(dz)dt$ which is the l-dimensional compensated jump measure of $\eta(.)$ an independent compensated Poisson random measure and $W = \{W(t), t \ge 0\}$ is an m-dimensional Brownian motion on a complete probability space $\Omega \equiv (\Omega, \mathcal{F}, P)$, $A \in \mathbb{R}^{n \times n}$ is a diagonal stability matrix. Assume from now on that $f(t, 0) = \sigma(t, 0) \equiv 0$ a.e t so that equation (4.1) admits a trivial solution.

Definition 4.1. *The trivial solution of equation (4.1) is said to be exponentially stable in the quadratic mean if there exist positive constants C, ν such that*

$$\mathbb{E}(|x(t)|^2) \le C\mathbb{E}(|x_0|^2) \exp(-\nu t), \quad t \ge 0.$$

The following lemmas are necessary to obtain the main results. For that we assume the following hypothesis [29]:

(H3) There exists a constant $M > 0$ such that for $t \geq 0$,

$$|E_{\alpha,\beta}(At^{\alpha})| \leq Me^{-at},$$

where $0 < \alpha < 1$ and $\beta = 1, 2$ and α.

Lemma 4.2. *Assume that the hypothesis (H3) holds. Then for any stochastic process* $F : [0, \infty) \to \mathbb{R}^n$ *which is strongly measurable with* $\int_0^T \mathbb{E}|F(t)|^2 ds < \infty$, $0 < T \leq \infty$, *the following inequality holds for* $0 < t \leq T$,

$$\mathbb{E}\left|\int_0^t E_{\alpha,\beta}(A(t-s)^{\alpha})F(s)ds\right|^2 \leq (M^2/a)\int_0^t \exp(-a(t-s))\mathbb{E}|F(s)|^2 ds,$$

where $\alpha \in (1/2, 1)$ *and* $\beta = 1, 2$ *and* α.

Lemma 4.3. *Assume that the hypothesis (H3) holds. Then for any* B_t- *adapted predictable process* $\Phi : [0, \infty) \to \mathbb{R}^n$ *with* $\int_0^T \mathbb{E}|\Phi(s)|^2 ds < \infty,\ t \geq 0$, *the following inequality holds for* $0 < t \leq T$,

$$\mathbb{E}\left|\int_0^t E_{\alpha,\beta}(A(t-s)^{\alpha})\Phi(s)dW(s)\right|^2 \leq M^2\int_0^t \exp(-a(t-s))\mathbb{E}|\Phi(s)|^2 ds,$$

where $\alpha \in (1/2, 1)$ *and* $\beta = 1, 2$ *and* α.

Theorem 4.4. *Let the assumptions of Theorem 3.1 holds. Then the solution of equation (4.1) is exponentially stable in the quadratic mean provided*

$$a > \beta = \beta(a, K, M) = \frac{4M^2(2K^2/a + K^2)T^{2\alpha-1}}{2\alpha - 1}.$$

Proof. The integral form of the equation (4.1) can be given by [7, 23]

$$\begin{aligned} x(t) = E_{\alpha}(At^{\alpha})x_0 &+ \int_0^t (t-s)^{\alpha-1}E_{\alpha,\alpha}(A(t-s)^{\alpha})b(s, x(s))ds \\ &+ \int_0^t (t-s)^{\alpha-1}E_{\alpha,\alpha}(A(t-s)^{\alpha})\sigma(s, x(s))dW(s) \\ &+ \int_0^t (t-s)^{\alpha-1}E_{\alpha,\alpha}(A(t-s)^{\alpha})\int_z g(s, x(s), z)\tilde{N}(ds, dz). \end{aligned} \tag{4.2}$$

Applying the algebraic inequality $(a + b + c + d)^2 \leq 4(a^2 + b^2 + c^2 + d^2)$ we have

$$\begin{aligned} |x(t)|^2 \leq 4\,(|E_{\alpha}(At^{\alpha})x_0|)^2 &+ 4\left(\left|\int_0^t (t-s)^{\alpha-1}E_{\alpha,\alpha}(A(t-s)^{\alpha})b(s, x(s))ds\right|\right)^2 \\ &+ 4\left(\left|\int_0^t (t-s)^{\alpha-1}E_{\alpha,\alpha}(A(t-s)^{\alpha})\sigma(s, x(s))dW(s)\right|\right)^2 \\ &+ 4\left(\left|\int_0^t (t-s)^{\alpha-1}E_{\alpha,\alpha}(A(t-s)^{\alpha})\int_z g(s, x(s), z)\tilde{N}(ds, dz)\right|\right)^2. \end{aligned}$$

By using Hölder inequality and Lemmas 4.2 and 4.3 we get

$$\begin{aligned}\mathbb{E}|x(t)|^2 \leq 4M^2\exp(-at)\mathbb{E}|x_0|^2 &+ 4(M^2/a)\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \exp(-a(t-s))\mathbb{E}|b(s,x(s))|^2 ds \\ &+ 4M^2\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \exp(-a(t-s))\mathbb{E}|\sigma(s,x(s))|^2 ds \\ &+ 4(M^2/a)\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \exp(-a(t-s))\mathbb{E}|\int_z g(s,x(s),z)\nu(dz)|^2 ds.\end{aligned}$$

Linear growth assumption (3.3) when $b(t,0) = \sigma(t,0) \equiv 0$ a.e t yields

$$\begin{aligned}\exp(at)\mathbb{E}|x(t)|^2 &\leq 4M^2\mathbb{E}|x_0|^2 + 4(M^2/a)K^2\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \exp(as)\mathbb{E}|x(s))|^2 ds \\ &\quad + 4M^2K^2\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \exp(as)\mathbb{E}|x(s))|^2 ds \\ &\quad + 4(M^2/a)K^2\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \exp(as)\mathbb{E}|x(s))|^2 ds \\ &\leq 4M^2\mathbb{E}|x_0|^2 + 4M^2(2K^2/a + K^2)\frac{T^{2\alpha-1}}{2\alpha-1}\int_0^t \exp(as)\mathbb{E}|x(s))|^2 ds\end{aligned}$$

Applying Gronwall's inequality, we obtain

$$\exp(at)\mathbb{E}|x(t)|^2 \leq 4M^2\mathbb{E}|x_0|^2\exp\left(4M^2(2K^2/a + K^2)\frac{T^{2\alpha-1}}{2\alpha-1}t\right)$$

Consequently,

$$\mathbb{E}|x(t)|^2 \leq C\mathbb{E}|x_0|^2\exp(-\nu t), \ t \geq 0, \tag{4.3}$$

where $\nu = a - \beta$ *and* $C = 4M^2$. □

Next consider the nonlinear stochastic fractional delay differential equation of the form

$$\begin{aligned}{}^{C}D^{\alpha}x(t) &= Ax(t) + f(t,x(t),x(t-\delta)) + \sigma(t,x(t),x(t-\delta))\frac{dW(t)}{dt} \\ &\quad + \int_z g(t,x(t),x(t-\delta),z)\frac{d\tilde{N}(t,z)}{dt}, \quad t \in J = [0,T] \\ x(t) &= \xi(t), \ t \in [-\delta, 0],\end{aligned} \tag{4.4}$$

where $\alpha \in (1/2, 1)$ and $z \in \mathbb{R}_0^n = \mathbb{R}^n/\{0\}$. Here $f \in C(J \times \mathbb{R}^n \times \mathbb{R}^n, \mathbb{R}^n)$, $\sigma \in C(J \times \mathbb{R}^n \times \mathbb{R}^n, \mathbb{R}^{nm})$, $g \in C(J \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_0^n, \mathbb{R}^{nl})$, $\tilde{N}(dt,dz) = N(dt,dz) - \nu(dz)dt$ which is the l-dimensional compensated jump measure of $\eta(.)$ an independent compensated Poisson random measure and $W = \{W(t), t \geq 0\}$ is an m-dimensional Brownian motion on a complete probability space $\Omega \equiv (\Omega, \mathcal{F}, P)$, $A \in \mathbb{R}^{n\times n}$ is a diagonal stability matrix. Assume from now on that $f(t,0) = \sigma(t,0) \equiv 0$ a.e t so that equation (4.1) admits a trivial solution. The integral form of the equation (4.4) in terms of the Mittag Leffler function is given by

$$\begin{aligned}x(t) = E_\alpha(At^\alpha)\xi(0) &+ \int_0^t (t-s)^{\alpha-1}E_{\alpha,\alpha}(A(t-s)^\alpha)b(s,x(s),x(s-\delta))ds \\ &+ \int_0^t (t-s)^{\alpha-1}E_{\alpha,\alpha}(A(t-s)^\alpha)\sigma(s,x(s),x(s-\delta))dW(s) \\ &+ \int_0^t (t-s)^{\alpha-1}E_{\alpha,\alpha}(A(t-s)^\alpha)\int_z g(s,x(s),x(s-\delta),z)\tilde{N}(ds,dz).\end{aligned} \tag{4.5}$$

Theorem 4.5. *Let the assumptions of the Theorem 3.2 holds. Then the solution of the delay differential equation (4.4) is exponentially stable in the quadratic mean provided*

$$a > \beta = \beta(a, K_1, K_2, M) = \frac{4M^2(K_1^2(1/a+1) + K_2^2)T^{2\alpha-1}}{2\alpha - 1}.$$

Proof. Using the hypothesis (H3), Lemmas 4.2 and 4.3 one can prove the theorem. The proof is similar to the previous theorem and hence omitted. □

5. Examples

Example 5.1. *Consider the following stochastic fractional differential equation with Lévy noise of the form*

$$\left.\begin{aligned} {}^{C}D^{\alpha}x(t) + 0.6x(t) &= \frac{t^{2-\alpha}}{\Gamma(1-\alpha)} + \frac{1}{1+t}\frac{dW(t)}{dt} + \int_{\mathbb{R}/\{0\}} tz\frac{d\tilde{N}(t,z)}{dt},\ t \in J \\ x(0) &= 1. \end{aligned}\right\} \tag{5.1}$$

Here $b(t, x(t)) = -0.6x(t) + \frac{t^{2-\alpha}}{\Gamma(1-\alpha)}$, $\sigma(t, x(t)) = \frac{1}{1+t}$ *and* $g(t, x(t), z) = tz$. *It can be easily seen that* $b(t, x(t))$, $\sigma(t, x(t))$ *and* $g(t, x(t), z)$ *satisfies the condition of (3.3) and (3.4) of Theorem 3.1. Hence by the Theorem 3.1 the stochastic fractional differential equation (5.1) has a unique solution. Also the equation (5.1) satisfy the condition of Theorem 4.4. So from Theorem 4.4 the stochastic fractional differential equation with* $A = 0.6$ *is exponentially stable.*

Example 5.2. *Consider the following stochastic fractional delay differential equation of the form*

$$\left.\begin{aligned} {}^{C}D^{\alpha}x(t) + 0.4x(t) &= -\frac{t^3 y}{\Gamma(2-\alpha)} + t^2\frac{dW(t)}{dt} + \int_{\mathbb{R}/\{0\}} zy\frac{d\tilde{N}(t,z)}{dt},\ t \in J \\ x(t) &= 0.\ \ t \in [-1, 0] \end{aligned}\right\} \tag{5.2}$$

Here $b(t, x(t), y(t)) = -0.4x(t) - \frac{t^3 y}{\Gamma(2-\alpha)}$, $\sigma(t, x(t), y(t)) = t^2$ *and* $g(s, x(t), y(t), z) = zy$. *It can be easily seen that* $b(t, x(t), y(t))$, $\sigma(t, x(t), y(t))$ *and* $g(t, x(t), y(t), z)$ *satisfies the assumptions (H1) and (H2) of Theorem 3.2. Hence by the Theorem 3.2 the stochastic fractional differential equation (5.2) has a unique solution. Also the equation (5.2) satisfy the condition of Theorem 4.5. So from Theorem 4.5 the stochastic fractional differential equation with* $A = 0.4$ *is exponentially stable.*

Conclusion

This paper discusses the existence and stability of results of stochastic fractional differential equations with Lévy noise. Existence and uniqueness of solutions are established by the Picard-Lindelöf successive approximation scheme. The stability of nonlinear stochastic fractional dynamical system with Lévy noise is obtained using Mittag Leffler function. Further we can extend this results for stochastic impulsive systems with Lévy noise [2], stochastic neutral systems and stochastic delay integrodifferential systems.

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