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An Application of Fuzzy Soft Multisets to Algebra

Canan Akın[a]

[a]*Department of Mathematics, Faculty of Arts and Science, Giresun University, 28200, Giresun, Turkey.*

Abstract. In this paper, notions of multi and soft multi LA-Γ-semigroup are defined. Some generalizations of certain operations on the families of the soft and fuzzy soft multisets are introduced. A concept of fuzzy multi LA-Γ-semigroup is presented. The concept of fuzzy soft multisets is applied to LA-Γ-semigroups and various characteristics of these all structures defined on LA-Γ-semigroups are investigated. Some properties of generalized operations on multi soft and fuzzy multi soft LA-Γ-semigroups are studied.

1. Introduction

The notion of LA-semigroup was first introduced by Kazım and Naseeruddin [16, 17] in 1972. A groupoid A is called an LA-semigroup if it satisfies the left invertive law $(ab)c = (cb)a$ for all $a, b, c \in A$. LA-semigroups which have wide applications in the theory of flocks and in automata theory are non-associative structures between a commutative semigroup and a groupoid. In 1981, the concept of Γ-semigroup was introduced by Sen [28] as a generalization of semigroups. In 2010, Shah and Rehman introduced the concept of LA-Γ-semigroup as a generalization of LA-semigroups [29].

Most of the concepts in real world are involving vagueness than certainty. So many researchers have become interested in modelling vagueness recently because of that their applicability to mathematical structures, real life problems, computer science, engineering, medicine and other many areas. In addition to the known theories such as probability theory and fuzzy set theory [39], rough set theory [25], vague set theory [14] and soft set theory [23] are often useful mathematical approaches to model vagueness.

The elements of a set are pair wise different since a set is a well-defined collection of distinct object. If we allow the elements to be occurrences more times than one, then we can obtain a multiset (or a bag mentioned as in some studies) [15, 22, 35]. In 1986, Yager introduced the fuzzy multiset as a fuzzy extension of the multisets [35]. Recently, Nazmul et al. [19] have introduced the notion of multigroups and studied its important properties. Shinoj et al. [30] have introduced the concept of fuzzy multigroups.

The soft set theory which was proposed by Molodtsov in 1999 [23] can be combined with other mathematical models or algebraic structures and even both of them. Maji et al. present the concept of fuzzy soft set [21] which is based on a combination of the fuzzy set and soft set models. Presentations of lots of methods which for solving some important problems of some fields such as medicine and decision making in the last decade [1, 5, 8, 18, 32, 34, 38] show that applied studies on soft set theory continue without slowing down. Moreover, theoretical studies on soft set theory have been continuing rapidly in recent years

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Email address: cananekiz28@gmail.com (Canan Akın)

[2, 4, 6, 7, 11, 12, 33]. Soft set theory was first applied to algebra by Aktaş and Çağman [3] in 2007. Fuzzy soft set theory was applied to algebra by Aygünoğlu and Aygün [7] in 2009. Recently, Babıtha and John [9] have introduced the concept of soft multisets, and then soft multisets have been applied to algebra by Nazmul and Samanta [20]. The notion of multi-fuzzy sets has been defined by Sebastian and Ramakrishnan [26, 27]. Shinoj and John [31] have presented intuitionistic fuzzy multisets as a new concept and they have discussed an application of this concept in medical diagnosis. Yang et al. [37] have introduced the concept of multi-fuzzy soft sets (in this paper, it is mentioned as fuzzy soft multisets) which is a combination of the multi-fuzzy sets and soft sets, and they have solved a decision making problem which is based on the concept of the multi-fuzzy soft set. Then their studies have been generalized by Dey and Pal [13].

The main purpose of this paper is to apply the fuzzy soft multisets to algebra and to form a new concept which is called fuzzy soft multi LA-Γ-semigroup as a beginning of the study of various algebraic structures of the fuzzy soft multisets and to discuss its various properties. In this paper, we also apply the soft multisets to LA-Γ-semigroup and we obtain a notion which is called soft multi LA-Γ-semigroup. Before defining these concepts, we introduce the notions of multi LA-Γ-semigroup and fuzzy multi LA-Γ-semigroup which we need and we mention some properties of them. To facilitate our discussion, we first review some background on LA-Γ-semigroup, multiset and fuzzy multiset, soft and fuzzy soft set, soft multiset and fuzzy soft multiset in Section 2. In Section 3, concepts of multi LA-Γ-semigroup, soft multi LA-Γ-semigroup, fuzzy multi LA-Γ-semigroup and fuzzy soft multi LA-Γ-semigroup are presented and their some properties are investigated in related subheadings.

2. Preliminaries

Let U be a nonempty set. The power set of U is denoted by $\mathcal{P}(U)$. A function f from U to the unit interval $[0, 1]$ is called a fuzzy subset of U. $F(U)$ denotes the set of all of the fuzzy subsets of U. Let f, g be fuzzy subsets of U, then $f \subseteq g$ means that $f(a) \leq g(a)$ for all $a \in U$. Let $A \subseteq U$. Then χ_A is denoted the characteristic function of A with the value 1 if $x \in A$ and 0 elsewhere.

Let S and Γ be nonempty sets. Then S is called an LA-Γ-semigroup if there exists a mapping $S \times \Gamma \times S \to S$, written as (a, γ, b) and denoted by $a\gamma b$ such that S satisfies the identity $(a\gamma b)\alpha c = (c\gamma b)\alpha a$ for all $a, b, c \in S$ and $\gamma, \alpha \in \Gamma$. If $A, B \subseteq S$, then $A\Gamma B := \{a\gamma b \mid a \in A, b \in B \, \text{and} \, \gamma \in \Gamma\}$. For a positive integer m, $B^m = (...((B\Gamma B)\Gamma B)...)\Gamma B$. A nonempty subset A of S is called an LA-Γ-subsemigroup of S if $A\Gamma A \subseteq A$. A nonempty subset A of S is called a left (right) LA-Γ-ideal of S if $S\Gamma A \subseteq A$ ($A\Gamma S \subseteq A$). A nonempty subset A of S is called an LA-Γ-ideal of S if it is both a left and a right LA-Γ-ideal of S. A nonempty subset A of S is called a generalized LA-Γ-bi-ideal of S if $(A\Gamma S)\Gamma A \subseteq A$. An LA-Γ-subsemigroup A of S is called an LA-Γ-bi-ideal of S if $(A\Gamma S)\Gamma A \subseteq A$. A nonempty subset A of S is called an LA-Γ-interior ideal of S if $(S\Gamma A)\Gamma S \subseteq A$.

A multiset M drawn from U is represented by a function *Count M*, C_M in short, defined as $C_M : U \to \mathbb{N}$, where $\mathbb{N}$ is the set of natural numbers including zero. For each $x \in U$, $C_M(x)$ is the characteristic value of x in M and it denotes the number of occurrences of x in M. Let M_1 and M_2 be two multisets drawn from U. Then M_1 is a submultiset of M_2 if $C_{M_1}(x) \leq C_{M_2}(x)$ for all $x \in U$ and it denoted by $M_1 \subseteq M_2$. The union of two multisets M_1 and M_2 drawn from U is a multiset $M_1 \cup M_2$ such that $C_{M_1 \cup M_2}(x) = max\{C_{M_1}(x), C_{M_2}(x)\} = C_{M_1}(x) \vee C_{M_2}(x)$ for all $x \in U$. The intersection of two multisets M_1 and M_2 drawn from U is a multiset $M_1 \cap M_2$ such that $C_{M_1 \cap M_2}(x) = min\{C_{M_1}(x), C_{M_2}(x)\} = C_{M_1}(x) \wedge C_{M_2}(x)$ for all $x \in U$. In the present paper, $MS(U)$ denotes the set of all multisets drawn from U and, for any positive integer w, $[U]^w$ denotes the set of all multisets whose elements drawn from U such that no elements in the multiset occurs more than w times and the set of all the submultisets of a multiset M is denoted by $PW(M)$.

A fuzzy multiset A drawn from U is characterized by a function which is called "count membership of A" and denoted by CM_A such that $CM_A : U \to MS([0, 1])$. For $x \in U$, the membership sequence is defined to be a decreasingly ordered sequence of the elements in $CM_A(x)$. It is denoted by $\left(\mu_A^1(x), \ldots, \mu_A^w(x)\right)$, where $\mu_A^1(x) \geq \cdots \geq \mu_A^w(x)$ and the integer w is called the dimension of A. The set of all w dimensional fuzzy multisets drawn from U is denoted by $M^wFS(U)$.

Let $A, B \in M^w FS(U)$. Then, $A \subseteq B \Leftrightarrow \mu^j_A(x) \leq \mu^j_B(x)$; $A = B \Leftrightarrow \mu^j_A(x) = \mu^j_B(x)$; $\mu^j_{A\cup B}(x) = max\{\mu^j_A(x), \mu^j_B(x)\} = \mu^j_A(x) \vee \mu^j_B(x)$; $\mu^j_{A\cap B}(x) = min\{\mu^j_A(x), \mu^j_B(x)\} = \mu^j_A(x) \wedge \mu^j_B(x)$, $(1 \leq j \leq w, \forall x \in U)$. By $CM_A(x) \leq CM_A(y)$, it is demonstrated $\mu^j_A(x) \leq \mu^j_A(y)$; by $CM_A(x) \wedge CM_A(y)$, it is demonstrated $\mu^j_A(x) \wedge \mu^j_A(y)$ and by $CM_A(x) \vee CM_A(y)$, it is demonstrated $\mu^j_A(x) \vee \mu^j_A(y)$,$(1 \leq j \leq w, \forall x, y \in U)$. The fuzzy multiset $(u/(0, 0, ..., 0) : u \in U)$, denoted by $\tilde{\emptyset}_w$, is called null fuzzy multiset of dimension w and the fuzzy multiset $(u/(1, 1, ..., 1) : u \in U)$, denoted by $\tilde{1}_w$, is called the absolute fuzzy multiset of dimension w.

Let U be a nonempty set and P be a set of parameters. Let A be a subset of P. Then a pair (F, A) is called a soft set over U where F is a mapping given by $F : A \to \mathcal{P}(U)$. Let (F, A) and (G, B) be two soft sets over U, (F, A) is called a soft subset of (G, B), denoted by $(F, A) \subseteq (G, B)$, if $A \subseteq B$ and $F(x) \subseteq G(x)$ for each $x \in A$. A soft set (F, A) is said to be a soft LA-Γ-semigroup (LA-Γ-left ideal, LA-Γ-right ideal, generalized LA-Γ-bi-ideal, LA-Γ-bi-ideal, LA-Γ-interior ideal, respectively) over U if and only if $F(x)$ is an LA-Γ-semigroup (LA-Γ-left ideal, LA-Γ-right ideal, generalized LA-Γ-bi-ideal, LA-Γ-bi-ideal, LA-Γ-interior ideal, respectively) of U for all $x \in A$ (see [10, 36]).

Let $E \subseteq P$. A pair (f, E) is called a fuzzy soft set over U, where f is a mapping given by $f : E \to F(U)$. Let (f, E) and (g, H) be two fuzzy soft sets over U. Then (f, E) is called a fuzzy soft subset of (g, H), denoted by $(f, E)\tilde{\subseteq}(g, H)$, if $E \subseteq H$ and $f(a) \leq g(a)$ for each $a \in E$.

Let U be a nonempty set, P be a set of parameters, M be a multiset drawn from U and $A \subseteq P$. Then a pair (F, A) is called a soft multiset over M, where F is a mapping given by $F : A \to PW(M)$. Let (F, A) and (G, B) be two fuzzy soft multisets over a multiset M. Then (F, A) is said to be soft submultiset of (G, B) if, for all $x \in A$, $A \subseteq B$ and $F(x) \subseteq G(x)$. (F, A) and (G, B) are said to be equal if (F, A) is soft submultiset of (G, B) and (G, B) is soft submultiset of (F, A).

A pair (f, E) is called a fuzzy soft multiset of dimension w over U, where f is a mapping given by $f : E \to M^w FS(U)$. Let (f, A), (g, B) be two fuzzy soft multisets of dimension w over U. Then (f, A) is said to be a fuzzy soft submultiset of (g, B) if, for all $x \in A$, $A \subseteq B$ and $f(x) \subseteq g(x)$. Let (f, A), (g, B) be two fuzzy soft multisets of dimension w over U. (f, A) and (g, B) are said to be equal if (f, A) is fuzzy soft submultiset of (g, B) and (g, B) is fuzzy soft submultiset of (f, A).

Example 2.1. Let $U = \{p_1, p_2, p_3, p_4\}$ be a set of some wristbands. $E = \{e_1, e_2\}$ is the set of parameters, where e_1 stands for the parameter "color" which consists of black, grey and transparent and e_2 stands for the parameter "ingredient" which is made from leather, metal and plastic. We define a fuzzy soft multiset of dimension 3 as follows
$F(e_1) = \{p_1/(0.6, 0.3, 0.1); p_2/(0.6, 0.2, 0.1); p_3/(0.6, 0.2, 0.2); p_4/(0.6, 0.1, 0.1)\}$
$F(e_2) = \{p_1/(0.5, 0.2, 0.1); p_2/(0.5, 0.4, 0.1); p_3/(0.5, 0.5, 0.0); p_4/(0.5, 0.1, 0.1)\}$.

3. Main Results

In the present section, multi LA-Γ-semigroups are introduced and also, their fuzzy and soft extensions are studied. Moreover, in this section, a notion of fuzzy multi soft LA-Γ-semigroup is defined as an algebraic extension of fuzzy soft multisets and some properties of them are examined.

3.1. *Multi LA-Γ-Semigroups*

Definition 3.1. Let U be an LA-Γ-semigroup. Let $S_1, S_2 \in [U]^w$. Then the composition of S_1 and S_2 is denoted by $S_1 \circ S_2$ and defined as follows:

$$C_{S_1 \circ S_2}(x) = \begin{cases} \bigvee_{x=y\gamma z} C_{S_1}(y) \wedge C_{S_2}(z), & \text{if } \exists\ y, z \in U \text{ and } \gamma \in \Gamma : x = y\gamma z; \\ 0 \quad , & \text{otherwise.} \end{cases}$$

Example 3.2. Let $U = \{a, b, c, e\}$ be Klein's 4 group. Then, for any nonempty set Γ, let $U \times \Gamma \times U \to U$ be defined by $x\gamma y = x \bullet y$, then U is an LA-Γ-semigroup. The composition of the multisets $A = (a, a, a, e, e, e, e, e)$ and $B = (b, b, b, b, e, e, e, e, e)$ is the multiset $A \circ B = (a, a, a, b, b, b, b, c, c, c, e, e, e, e, e)$.

Theorem 3.3. *Let U be an LA-Γ-semigroup. Then*

(i) $([U]^w, \circ)$ *is an LA-semigroup.*

(ii) If, for any $A, B, C, D \in [U]^w$, $C \subseteq A$ *and* $D \subseteq B$, *then* $C \circ D \subseteq A \circ B$.

Proof.

(i) $[U]^w \neq \emptyset$, since U is an LA-Γ-semigroup. Let $A, B, C \in [U]^w$. Let $x \in U$. If, for all $a, b \in U$ and $\gamma \in \Gamma$, $x \neq a\gamma b$, then $C_{(A\circ B)\circ C}(x) = 0 = C_{(C\circ B)\circ A}(x)$. Let there exist $a, b \in U$ and $\gamma \in \Gamma$ such that $x = a\gamma b$. If there exist $k, t \in U$ and $\alpha \in \Gamma$ such that $a = k\alpha t$, then $C_{(A\circ B)\circ C}(x) = \bigvee_{x=a\alpha b}\big(C_{A\circ B}(a) \wedge C_C(b)\big) = \bigvee_{x=a\alpha b}\Big(\big(\bigvee_{a=k\alpha t} C_A(k) \wedge C_B(t)\big) \wedge C_C(b)\Big) = \bigvee_{x=(k\alpha t)\gamma b}\big(C_A(k) \wedge C_B(t) \wedge C_C(b)\big) = \bigvee_{x=(b\alpha t)\gamma k}\big(C_C(b) \wedge C_B(t) \wedge C_A(k)\big) = \bigvee_{x=s\alpha k}\Big(\big(\bigvee_{s=b\alpha t} C_C(b) \wedge C_B(t)\big) \wedge C_A(k)\Big) = \bigvee_{x=s\alpha k}\big(C_{C\circ B}(s) \wedge C_A(k)\big) = C_{(C\circ B)\circ A}(x)$. Conversely, that is, if $a \neq k\alpha t$ for all $k, t \in U$ and $\alpha \in \Gamma$, then $C_{(A\circ B)\circ C}(x) = 0$ and also $C_{(C\circ B)\circ A}(x) = 0$ since $C_{(C\circ B)}(a) = 0$. Thus, $(A \circ B) \circ C = (C \circ B) \circ A$ for all $A, B, C \in [U]^w$.

(ii) Let $x \in U$. If, for all $a, b \in U$ and $\gamma \in \Gamma$, $x \neq a\gamma b$, then $C_{(A\circ B)}(x) = 0 = C_{(C\circ D)}(x)$. Let there exist $a, b \in U$ and $\gamma \in \Gamma$ such that $x = a\gamma b$. $C_{(C\circ D)}(x) = \bigvee_{x=a\gamma b} C_C(a) \wedge C_D(b) \leq \bigvee_{x=a\gamma b} C_A(a) \wedge C_B(b) = C_{(A\circ B)}(x)$. Hence, $C \circ D \subseteq A \circ B$.

Definition 3.4. Let U be an LA-Γ-semigroup.

(i) A multiset A over U is called a multi LA-Γ-semigroup over U if the count function C_A satisfies $C_A(x\gamma y) \geq C_A(x) \wedge C_A(y)$ for all $x, y \in U$ and $\gamma \in \Gamma$.

(ii) A multiset A over U is called a multi LA-Γ-right (-left) ideal over U if the count function C_A satisfies $C_A(x\gamma y) \geq C_A(x)$ $(C_A(x\gamma y) \geq C_A(y))$ for all $x, y \in U$ and $\gamma \in \Gamma$. A is called a multi LA-Γ-ideal if it is both a multi LA-Γ-right and multi LA-Γ-left ideal over U.

(iii) A multiset A over U is called a multi generalized LA-Γ-bi-ideal over U if the count function CM_A satisfies $C_A((x\alpha z)\gamma y) \geq C_A(x) \wedge C_A(y)$ for all $x, y, z \in U$ and $\alpha, \gamma \in \Gamma$. A multi LA-$\Gamma$-subsemigroup over U is called a multi LA-Γ-bi-ideal if it is a multi generalized LA-Γ-bi-ideal over U.

(iv) A multiset A over U is called a multi LA-Γ-interior ideal over U if the count function C_A satisfies $C_A((x\alpha z)\gamma y) \geq C_A(z)$ for all $x, y, z \in U$ and $\alpha, \gamma \in \Gamma$.

Example 3.5. $U = \{a, b, c\}$ is an LA-semigroup with the following binary operation:

$\bullet$	a	b	c
a	c	c	b
b	b	b	b
c	b	b	b

For any nonempty set Γ, let $U \times \Gamma \times U \to U$ be defined by $x\gamma y = x \bullet y$, then U is an LA-Γ-semigroup [24, 29]. The multiset $S = (b, b, b, c, c, a)$ is a multi LA-Γ-semigroup over U but $H = (b, b, b, c, c, a, a, a)$ is not a multi LA-Γ-semigroup over U, since $C_H(a\gamma a) = C_H(c) = 2 \ngeq 3 = C_H(a) = C_H(a) \wedge C_H(a)$.

Definition 3.6. Let S be a multi LA-Γ-semigroup over U. A multi LA-Γ-semigroup (LA-Γ-right (-left) ideal, (generalized) LA-Γ-bi-ideal, LA-Γ-interior ideal, respectively) A over U is called a multi LA-Γ-subsemigroup (LA-Γ-right (-left) ideal, (generalized) LA-Γ-bi-ideal, LA-Γ-interior ideal, respectively) of S if $A \subseteq S$.

Example 3.7. Let $U = \mathbb{Z}$ and $\Gamma = \{-1, 1\}$. Define a mapping $U \times \Gamma \times U \to U$ by $a\gamma b = a.\gamma.b$ for all $a, b \in U$ and $\gamma \in \Gamma$. Then U is an LA-Γ-semigroup. The multiset $S = (-1, -1, -1, 0, 0, 0, 0, 1, 1, 1)$ is a multi LA-Γ-semigroup over U and $A = (0, 0, 0)$ is a multi LA-Γ-ideal of S. The multiset $B = (-1, -1, 0, 1, 1)$ is an LA-Γ-subsemigroup of S but neither a multi LA-Γ-right nor a multi LA-Γ-left ideal of S since $C_B((-1)(1)0) \ngeq C_B(-1)$ and $C_B(0(1)(-1)) \ngeq C_B(-1)$.

Proposition 3.8. *Let U be an LA-Γ-semigroup. A multi LA-Γ-right or multi LA-Γ-left ideal A over U is a multi LA-Γ-semigroup and multi LA-Γ-bi-ideal over U.*

Proof. Straightforward.

Theorem 3.9. *Let U be an LA-Γ-semigroup.*

(i) $A \in [U]^w$ is a multi LA-Γ-semigroup over U if and only if $A \circ A \subseteq A$.

(ii) If $A, B \in [U]^w$ are multi LA-Γ-semigroups over U, then $A \circ B$ is a multi LA-Γ-semigroup over U.

Proof.

(i) Let $x \in U$. If $C_{A\circ A}(x) = 0$, then $A \circ A \subseteq A$. If $C_{A\circ A}(x) = \bigvee_{x=a\gamma b} C_A(a) \wedge C_A(b)$, then $C_{A\circ A}(x) \leq C_A(x)$ since A is a multi LA-Γ-semigroup over U. Thus, $A \circ A \subseteq A$. On the contrary, let $A \circ A \subseteq A$. Then $C_A(a) \wedge C_A(b) \leq \bigvee_{a\gamma b=k\alpha t} C_A(k) \wedge C_A(t) = C_{A\circ A}(a\gamma b) \leq C_A(a\gamma b)$ for all $a, b \in U$ and $\gamma \in \Gamma$. Hence, A is a multi LA-Γ-semigroup over U.

(ii) $(A \circ B) \circ (A \circ B) = ((A \circ B) \circ B) \circ A = ((B \circ B) \circ A) \circ A = (A \circ A) \circ (B \circ B) \subseteq A \circ B$ since $([U]^w, \circ)$ is an LA-semigroup with the Theorem 3.3. Thus, $A \circ B$ is a multi LA-Γ-semigroup over U with i).

Theorem 3.10. *Let U be an LA-Γ-semigroup. If $\{A_i \mid i \in I\}$ is a family of multi LA-Γ-semigroups (LA-Γ-right (left) ideals, (generalized) LA-Γ-bi-ideals, LA-Γ-interior ideals, respectively) over U, then $\bigcap_{i\in I} A_i$ is a multi LA-Γ-semigroup (LA-Γ-right (left) ideal, (generalized) LA-Γ-bi-ideal, LA-Γ-interior ideal, respectively) over U.*

Proof. $\bigcap_{i\in I} A_i$ is a multi LA-Γ-semigroup over U since $C_{\bigcap_{i\in I} A_i}(x) \wedge C_{\bigcap_{i\in I} A_i}(y) = (\bigwedge_{i\in I} C_{A_i}(x)) \wedge (\bigwedge_{i\in I} C_{A_i}(y)) = \bigwedge_{i\in I}(C_{A_i}(x) \wedge C_{A_i}(y)) \leq \bigwedge_{i\in I} C_{A_i}(x\gamma y) = C_{\bigcap_{i\in I} A_i}(x\gamma y)$ for all $x, y \in U$ and $\gamma \in \Gamma$. Similarly, it is easy to see the other situations by using Definition 3.4.

The following example shows $\bigcup_{i\in I} A_i$ is not a multi LA-Γ-semigroup over U in general.

Example 3.11. Let U be the LA-Γ-semigroup in Example 3.2. The union of the multi LA-Γ-semigroups $A = (a, a, a, e, e, e, e, e)$ and $B = (b, b, b, b, e, e, e, e, e)$ is the multiset $A \cup B = (a, a, a, b, b, b, b, e, e, e, e, e)$ and it is not a multi LA-Γ-semigroup since $C_{A\cup B}(a\gamma b) = 0 \ngeq 3 = C_{A\cup B}(a) \wedge C_{A\cup B}(b)$.

3.2. Soft Multi LA-Γ-Semigroups

Some generalizations of certain operations on soft multisets which is given by Nazmul et al. [19] are presented in the following definition.

Definition 3.12. Let S be a multiset over U and $\{(F_i, E_i) \mid i \in I\}$ be a family of soft multisets over S. Then,

(i) their restricted intersection is denoted by $\overline{(\bigcap_r)}_{i\in I}(F_i, E_i) = \left(\overline{\bigcap}_{i\in I} F_i, \bigcap_{i\in I} E_i\right)$, where $(\overline{\bigcap}_{i\in I} F_i)(x) = \bigcap_{i\in I} F_i(x)$ for all $x \in \bigcap_{i\in I} E_i$.

(ii) their extended intersection is denoted by $\overline{(\bigcap_e)}_{i\in I}(F_i, E_i) = \left(\overline{\bigcap}_{i\in I} F_i, \bigcup_{i\in I} E_i\right)$, where $(\overline{\bigcap}_{i\in I} F_i)(x) = \bigcap_{i\in\Lambda(x)} F_i(x)$ for all $x \in \bigcup_{i\in I} E_i$ and $\Lambda(x) = \{i \in I \mid x \in E_i\}$.

(iii) their restricted union is denoted by $\overline{(\bigcup_r)}_{i\in I}(F_i, E_i) = \left(\overline{\bigcup}_{i\in I} F_i, \bigcap_{i\in I} E_i\right)$, where $(\overline{\bigcup}_{i\in I} F_i)(x) = \bigcup_{i\in I} F_i(x)$ for all $x \in \bigcap_{i\in I} E_i$.

(iv) their extended union is denoted by $\overline{(\bigcup_e)}_{i\in I}(F_i, E_i) = \left(\overline{\bigcup}_{i\in I} F_i, \bigcup_{i\in I} E_i\right)$, where $(\overline{\bigcup}_{i\in I} F_i)(x) = \bigcup_{i\in\Lambda(x)} F_i(x)$ for all $x \in \bigcup_{i\in I} E_i$.

(v) their $\wedge$-intersection is denoted by $\overline{\wedge}_{i\in I}(F_i, E_i) = (\overline{F}, \prod_{i\in I} E_i)$, where $\overline{F}((x_i)_{i\in I}) = \bigcap_{i\in I} F_i(x_i)$ for all $(x_i)_{i\in I} \in \prod_{i\in I} E_i$.

(vi) their $\vee$-union is denoted by $\overline{\vee}_{i\in I}(F_i, E_i) = (\overline{F}, \prod_{i\in I} E_i)$, where $\overline{F}((x_i)_{i\in I}) = \bigcup_{i\in I} F_i(x_i)$ for all $(x_i)_{i\in I} \in \prod_{i\in I} E_i$.

Definition 3.13. Let S be a multi LA-Γ-semigroup over U.

(i) A soft multiset (F, E) over S is called a soft multi LA-Γ-semigroup drawn from S if and only if $F(x)$ is a multi LA-Γ-subsemigroup of S for all $x \in E$.

(ii) A soft multiset (F, E) over S is called a soft multi LA-Γ-right (left) ideal drawn from S if and only if $F(x)$ is a multi LA-Γ-right (left) ideal of S for all $x \in E$. (F, E) is called soft multi LA-Γ-ideal drawn from S if and only if it is both soft multi LA-Γ-right and left ideal drawn from S.

(iii) A soft multiset (F, E) over S is called a soft multi (generalized) LA-Γ-bi-ideal drawn from S if and only if $F(x)$ is a multi (generalized) LA-Γ-bi-ideal of S for all $x \in E$.

(iv) A soft multiset (F, E) over S is called a soft multi LA-Γ-interior ideal drawn from S if and only if $F(x)$ is a multi LA-Γ-interior ideal of S for all $x \in E$.

Example 3.14. Let S be the multi LA-Γ-semigroup in Example 3.5 and $E = \{e_1, e_2\}$. (F, E), defined by $F(e_1) = (b, b, c, c, a)$ and $F(e_2) = (b, b, c)$, is a soft multi LA-Γ-semigroup drawn from S. However, (F, E) is not a soft multi LA-Γ-ideal drawn from S since $C_{F(e_2)}(a\gamma b) = 1 \ngeq 2 = C_{F(e_2)}(b)$.

Theorem 3.15. *Let S be a multi LA-Γ-semigroup over U and A be a nonempty family $\{(F_i, E_i) \mid i \in I\}$ of soft multi sets over S and let $\bigcap_{i\in I} E_i \neq \emptyset$. If A is a family of soft multi LA-Γ-subsemigroups (LA-Γ-right (left) ideals, (generalized) LA-Γ-bi-ideals, LA-Γ-interior ideals, respectively) of S, then $\overline{(\bigcap_r)}_{i\in I} (F_i, E_i)$, $\overline{(\bigcap_e)}_{i\in I} (F_i, E_i)$ and $\overline{\wedge}_{i\in I} (F_i, E_i)$ are soft multi LA-Γ-subsemigroups (LA-Γ-right (left) ideals, (generalized) LA-Γ-bi-ideals, LA-Γ-interior ideals, respectively) of S.*

Proof. Let $\{(F_i, E_i) \mid i \in I\}$ be a family of soft multi LA-Γ-subsemigroups of S.
1) Let $x \in \bigcap_{i\in I} E_i$. Then $\overline{(\bigcap_r)}_{i\in I} (F_i, E_i)$ is a soft multi LA-Γ-subsemigroup of S since $C_{(\overline{\bigcap}_{i\in I} F_i)(x)}(a\gamma b) = C_{\bigcap_{i\in I}(F_i(x))}(a\gamma b) = \bigwedge_{i\in I} C_{F_i(x)}(a\gamma b) \geq \bigwedge_{i\in I}(C_{F_i(x)}(a) \wedge C_{F_i(x)}(b)) = (\bigwedge_{i\in I} C_{F_i(x)}(a)) \wedge (\bigwedge_{i\in I} C_{F_i(x)}(b)) = C_{\bigcap_{i\in I}(F_i(x))}(a) \wedge C_{\bigcap_{i\in I}(F_i(x))}(b) = C_{(\overline{\bigcap}_{i\in I} F_i)(x)}(a) \wedge C_{(\overline{\bigcap}_{i\in I} F_i)(x)}(b)$ for all $a, b \in U$ and $\gamma \in \Gamma$.
2) Let $x \in \bigcup_{i\in I} E_i$. Then, $(\overline{\bigcap}_{i\in I} F_i)(x) = \bigcap_{i\in\Lambda(x)} F_i(x)$. Hence, $\overline{(\bigcap_e)}_{i\in I} (F_i, E_i)$ is a soft multi LA-Γ-subsemigroup of S since $\bigcap_{i\in\Lambda(x)} F_i(x)$ is a multi LA-Γ-subsemigroup of S in a similar way by a).
3) Let $(x_i)_{i\in I} \in \prod_{i\in I} E_i$. Then, $\overline{\wedge}_{i\in I} (F_i, E_i)$ is a soft multi LA-Γ-subsemigroup of S since $C_{\overline{F}((x_i)_{i\in I})}(a\gamma b) = C_{\bigcap_{i\in I} F_i(x_i)}(a\gamma b) = \bigwedge_{i\in I} C_{F_i(x_i)}(a\gamma b) \geq \bigwedge_{i\in I}(C_{F_i(x_i)}(a) \wedge C_{F_i(x_i)}(b)) = (\bigwedge_{i\in I} C_{F_i(x_i)}(a)) \wedge (\bigwedge_{i\in I} C_{F_i(x_i)}(b)) = C_{\bigcap_{i\in I}(F_i(x_i))}(a) \wedge C_{\bigcap_{i\in I}(F_i(x_i))}(b) = C_{\overline{F}((x_i)_{i\in I})}(a) \wedge C_{\overline{F}((x_i)_{i\in I})}(b)$ for all $a, b \in U$ and $\gamma \in \Gamma$. The proofs of the other situations have analogues processes via Definition 3.12.

The following example shows the restricted union of the soft multi LA-Γ-semigroups is not a soft multi LA-Γ-semigroup in general.

Example 3.16. Let U be the LA-Γ-semigroup and $S = A \circ B$ in Example 3.2. Let $E_1 = \{e_1, e_2\}$ and $E_2 = \{e_1, e_3\}$. Then (F_1, E_1) defined by $F_1(e_1) = A$ and $F_1(e_2) = (a, a, b, b, c, c)$ and (F_2, E_2) defined by $F_2(e_1) = B$ and $F_2(e_3) = (a, b, c, e)$ are soft multi LA-Γ-semigroups drawn from S. However, the restricted union of them, (F, E), is not a soft multi LA-Γ-semigroup drawn from S since $C_{F(e_1)}(a\gamma b) = 0 \ngeq 3 = C_{F(e_1)}(a) \wedge C_{F(e_1)}(b)$. So, their extended union is also not a soft multi LA-Γ-semigroup. The $\vee$-union of them, (G, B), is not a soft multi LA-Γ-semigroup drawn from S since $C_{G(e_1,e_1)}(a\gamma b) = 0 \ngeq 3 = C_{G(e_1,e_1)}(a) \wedge C_{G(e_1,e_1)}(b)$.

3.3. *Fuzzy Multi LA-Γ-Semigroups*

Recall $M^wFS(U)$ denotes the set of all w dimensional fuzzy multisets drawn from U.

Definition 3.17. Let U be an LA-Γ-semigroup and $S_1, S_2 \in M^wFS(U)$. Then the composition of S_1 and S_2 is denoted by $S_1 \circ S_2$ and defined as follows:

$$CM_{S_1\circ S_2}(x) = \begin{cases} \bigvee_{x=y\gamma z} CM_{S_1}(y) \wedge CM_{S_2}(z), \text{if } \exists\, y, z \in U \text{ and } \gamma \in \Gamma : x = y\gamma z; \\ \quad 0 \quad , \quad \text{otherwise.} \end{cases}$$

Example 3.18. Let U be the LA-Γ-semigroup in Example 3.2. Then, the composition of the fuzzy multisets $A = \{a/(0.4, 0.3, 0.2), e/(0.9, 0.8, 0.6)$ and $B = \{b/(0.6, 0.6, 0.3), e/(0.9, 0.8, 0.6)$ is
$A \circ B = \{a/(0.4, 0.3, 0.2), b/(0.6, 0.6, 0.3), c/(0.4, 0.3, 0.2), e/(0.9, 0.8, 0.6)\}$.

Theorem 3.19. *Let U be an LA-Γ-semigroup. Then*

(i) $(M^w FS(U), \circ)$ *is an LA-semigroup.*

(ii) If, for any $A, B, C, D \in M^w FS(U)$, $C \subseteq A$ and $D \subseteq B$, then $C \circ D \subseteq A \circ B$.

Proof. By following similar steps in the proof of Theorem 3.3, it is straightforward from the definition of the composition of fuzzy multi sets.

Definition 3.20. Let U be an LA-Γ-semigroup.

(i) A fuzzy multiset A over U is called a fuzzy multi LA-Γ-semigroup over U if the count function CM_A satisfies $CM_A(x\gamma y) \geq CM_A(x) \wedge CM_A(y)$ for all $x, y \in U$ and $\gamma \in \Gamma$.

(ii) A fuzzy multiset A over U is called a fuzzy multi LA-Γ-right (-left) ideal over U if the count function CM_A satisfies $CM_A(x\gamma y) \geq CM_A(x)$ $(CM_A(x\gamma y) \geq CM_A(y))$ for all $x, y \in U$ and $\gamma \in \Gamma$. A is called a fuzzy multi LA-Γ-ideal if it is both a fuzzy multi LA-Γ-right and fuzzy multi LA-Γ-left ideal over U.

(iii) A fuzzy multiset A over U is called a fuzzy multi generalized LA-Γ-bi-ideal over U if the count function CM_A satisfies $CM_A((x\alpha z)\gamma y) \geq CM_A(x) \wedge CM_A(y)$ for all $x, y, z \in U$ and $\alpha, \gamma \in \Gamma$. A fuzzy multi LA-Γ-subsemigroup over U is called a fuzzy multi LA-Γ-bi-ideal if it is a fuzzy multi generalized LA-Γ-bi-ideal over U.

(iv) A fuzzy multiset A over U is called a fuzzy multi LA-Γ-interior ideal over U if the count function CM_A satisfies $CM_A((x\alpha z)\gamma y) \geq CM_A(z)$ for all $x, y, z \in U$ and $\alpha, \gamma \in \Gamma$.

Example 3.21. Let $U = \mathbb{Z}$ and $\Gamma = \{-1, 1\}$. Define the mapping $U \times \Gamma \times U \to U$ by $a\gamma b = a.\gamma.b$ for all $a, b \in U$ and $\gamma \in \Gamma$. Then U is an LA-Γ-semigroup.
Thus, $S \in M^5 FS(U)$ which is given by $\{0/(1, 1, 1, 1, 1), 1/(0.9, 0.9, 0.9), (-1)/(0.9, 0.9, 0.9)\}$ is a fuzzy multi LA-Γ-semigroup over U and $A = \{0/(1, 1, 1), 1/(0.9, 0.8, 0.7), (-1)/(0.9, 0.8, 0.7)\}$ is a fuzzy multi LA-Γ-ideal over U which is included in S. $B = \{1/(0.8, 0.8, 0.7), (-1)/(0.9)\}$ is not a fuzzy multi LA-Γ-semigroup over U since $CM_B((-1)(1)(-1)) \ngeq CM_B(-1)$.

Proposition 3.22. *Let U be an LA-Γ-semigroup. A fuzzy multi LA-Γ-right or fuzzy multi LA-Γ-left ideal A over U is a fuzzy multi LA-Γ-semigroup and fuzzy multi LA-Γ-bi-ideal over U.*

Proof. Straightforward.

Theorem 3.23. *Let U be an LA-Γ-semigroup.*

(i) $A \in M^w FS(U)$ is a fuzzy multi LA-Γ-semigroup over U if and only if $A \circ A \subseteq A$.
(ii) If $A, B \in M^w FS(U)$ are fuzzy multi LA-Γ-semigroups over U and $A \circ B = B \circ A$, then $A \circ B$ is a fuzzy multi LA-Γ-semigroup over U.

Proof. It is similar to the proof of Theorem 3.9 by the analogous steps.

Theorem 3.24. *Let U be an LA-Γ-semigroup. If $\{A_i \mid i \in I\}$ is a family of fuzzy multi LA-Γ-semigroups (LA-Γ-right (left) ideals, (generalized) LA-Γ-bi-ideals, LA-Γ-interior ideals, respectively) over U, then $\bigcap_{i \in I} A_i$ is a fuzzy multi LA-Γ-semigroups (LA-Γ-right (left) ideals, (generalized) LA-Γ-bi-ideals, LA-Γ-interior ideals, respectively) over U.*

Proof. The proof is similar to the proof of Theorem 3.10 by the analogous steps.

Example 3.25. Let U be the LA-Γ-semigroup in Example 3.2. Then, the fuzzy multisets $A = \{a/(0.4, 0.3, 0.2), e/(0.9, 0.8, 0.6)\}$ and $B = \{b/(0.6, 0.6, 0.3), e/(0.9, 0.8, 0.6)\}$ are fuzzy multi LA-Γ-semigroups. But $A \cup B = \{a/(0.4, 0.3, 0.2), b/(0.6, 0.6, 0.3), e/(0.9, 0.8, 0.6)\}$ is not a fuzzy multi LA-Γ-semigroup, since $CM_{A\cup B}(a\gamma b) \ngeq CM_{A\cup B}(a) \wedge CM_{A\cup B}(b)$.

Definition 3.26. Let S be a fuzzy multi LA-Γ-semigroup over U. A fuzzy multi LA-Γ-semigroup (LA-Γ-right (-left) ideal, (generalized) LA-Γ-bi-ideal, LA-Γ-interior ideal, respectively) A over U is called a fuzzy multi LA-Γ-subsemigroup (LA-Γ-right (-left) ideal, (generalized) LA-Γ- bi-ideal, LA-Γ-interior ideal, respectively) of S if $A \subseteq S$.

3.4. Fuzzy Soft Multi LA-Γ-Semigroups

Yang et al. [37] introduce the definition of the fuzzy soft multiset with the name "multi-fuzzy soft sets". Some generalizations of certain operations which are presented by Yang et al. on fuzzy soft multisets are given in the following definition.

Definition 3.27. Let $\{(f_i, E_i)|i \in I\}$ be a family of fuzzy soft multisets.
The restricted intersection of the family $\{(f_i, E_i)|i\in I\}$, denoted by $\overline{(\bigcap_r)}_{i\in I}(f_i, E_i)$, is a fuzzy soft multiset (f, E), $E=\bigcap_{i\in I} E_i$ and $f(x) = \cap_{i\in I} f_i(x)$ for all $x\in E$.
The extended intersection of the family $\{(f_i, E_i)|i\in I\}$, denoted by $\overline{(\bigcap_e)}_{i\in I}(f_i, E_i)$, is a fuzzy soft multiset (f, E), $E=\bigcup_{i\in I} E_i$ and $f(x) = \cap_{i\in\Lambda(x)} f_i(x)$ for all $x\in E$,, where $\Lambda(x) = \{i \in I \mid x\in E_i\}$.
The restricted union of the family $\{(f_i, E_i)|i\in I\}$, denoted by $\overline{(\bigcup_r)}_{i\in I}(f_i, E_i)$, is a fuzzy soft multiset (f, E), $E=\bigcap_{i\in I} E_i$ and $f(x) = \cup_{i\in I} f_i(x)$ for all $x\in E$.
The extended union of the family $\{(f_i, E_i)|i\in I\}$, denoted by $\overline{(\bigcup_e)}_{i\in I}(f_i, E_i)$, is a fuzzy soft multiset (f, E), $E=\bigcup_{i\in I} E_i$ and $f(x) = \cup_{i\in\Lambda(x)} f_i(x)$ for all $x\in E$.
The fuzzy $\wedge$-intersection of the family $\{(f_i, E_i)|i\in I\}$, denoted by $\overline{\wedge}_{i\in I}(f_i, E_i)$, is the soft multiset (f, E) defined as $E=\prod_{i\in I} E_i$ and $f((x_i)_{i\in I}) = \cap_{i\in I} f_i(x_i)$ $(\forall (x_i)_{i\in I}\in E)$.
The fuzzy $\vee$-union of the family $\{(f_i, E_i)|i\in I\}$, denoted by $\overline{\vee}_{i\in I}(f_i, E_i)$, is the soft multiset (f, E) defined as $E=\prod_{i\in I} E_i$ and $f((x_i)_{i\in I}) = \cup_{i\in I} f_i(x_i)$ $(\forall (x_i)_{i\in I}\in E)$.

Definition 3.28. Let (F, A) be a soft set over U and let $t : A \to M^{\omega}FS(U)$ be defined by $t(e) = \{u/(\chi_{F(e)}(u), 0, ..., 0) \mid u \in U\}$ for all $e \in A$. Then (t, A) is a fuzzy soft multiset and is called a fuzzy soft multiset derived by the soft set (F, A) over U.

Definition 3.29. Let U be an LA-Γ-semigroup and (f, A) be a fuzzy soft multiset over U.

(i) (f, A) is called a fuzzy soft multi LA-Γ-semigroup over U if and only if $f(x)$ is a fuzzy multi LA-Γ-semigroup over U for all $x \in A$.
(ii) (f, A) is called a fuzzy soft multi LA-Γ-right (-left) ideal over U if and only if $f(x)$ is a fuzzy multi LA-Γ-right (-left) ideal over U for all $x \in A$. (f, A) is said to be fuzzy soft multi LA-Γ-ideal over U if and only if it is both fuzzy soft multi LA-Γ-right and LA-Γ-left ideal over U.
(iii) (f, A) is called a fuzzy soft multi generalized LA-Γ-bi-ideal over U if and only if $f(x)$ is a fuzzy multi generalized LA-Γ-bi-ideal over U for all $x \in A$. A fuzzy soft multi LA-Γ-semigroup (f, A) is called a fuzzy soft multi LA-Γ-bi-ideal over U if and only if $f(x)$ is a fuzzy multi generalized LA-Γ-bi-ideal over U for all $x \in A$.
(iv) (f, A) is called a fuzzy soft multi LA-Γ-interior ideal over U if and only if $f(x)$ is a fuzzy multi LA-Γ-interior ideal over U for all $x \in A$.

Example 3.30. Let $U = \{a, b, c\}$ is an LA-semigroup with the following multiplication table:

$\bullet$	a	b	c
a	a	a	a
b	c	c	c
c	a	a	a

For any nonempty set Γ, let $U \times \Gamma \times U \to U$ be defined by $x\gamma y = x \bullet y$, then U is an LA-Γ-semigroup [24, 29]. Let $A = \{e_1, e_2, e_3\}$ be the set of parameters and let a fuzzy soft multi set be defined as follows $f(e_1) = \{a/(0.9, 0.8, 0.7), b/(0.1, 0.1), c/(0.2, 0.1)\}$ $f(e_2) = \{a/(0.2, 0.1), b/(0.1), c/(0.1)\}$ $f(e_3) = \{a/(0.4, 0.1), b/(0.2), c/(0.4)\}$. Therefore (f, A) is a fuzzy soft multi LA-Γ-semigroup over U.

Example 3.31. Let $A \subseteq P$ and let $t_A : P \to M^w FS(U)$ be a mapping with the value $\tilde{1_w}$ if $p \in A$ and $\tilde{\emptyset_w}$ elsewhere. Then the fuzzy soft multiset (t_A, P) is a fuzzy soft multi LA-Γ-semigroup.

Theorem 3.32. *Let U be an LA-Γ-semigroup. (F, A) is a soft LA-Γ-semigroup (LA-Γ-left (right) ideal, (generalized) LA-Γ-bi-ideal, LA-Γ-interior ideal, respectively) over U if and only if (t, A) is a fuzzy soft multi LA-Γ-semigroup (LA-Γ-left (right) ideal, (generalized) LA-Γ-bi-ideal, LA-Γ-interior ideal, respectively) over U.*

Proof. Let (F, A) is a soft LA-Γ-semigroup over U and $a \in A$. Suppose that $CM_{t(a)}(x) \wedge CM_{t(a)}(y) = (1, 0, ..., 0)$ for any $x, y \in U$ and $\gamma \in \Gamma$. Thus, $CM_{t(a)}(x) = (1, 0, ..., 0)$ and $CM_{t(a)}(y) = (1, 0, ..., 0)$. Hence, $x \in F(a)$ and $y \in F(a)$. So, $x\gamma y \in F(a)$ since (F, A) is a soft LA-Γ-semigroup over U. Therefore, (t, A) is a fuzzy soft multi LA-Γ-semigroup since $CM_{t(a)}(x\gamma y) = (1, 0, ..., 0)$. Conversely, let $x, y \in F(a)$ and $\gamma \in \Gamma$. Hence, $CM_{t(a)}(x) = (1, 0, ..., 0)$ and $CM_{t(a)}(y) = (1, 0, ..., 0)$. Thus, $CM_{t(a)}(x) \wedge CM_{t(a)}(y) = (1, 0, ..., 0)$. So, $CM_{t(a)}(x\gamma y) = (1, 0, ..., 0)$. Therefore, (F, A) is a soft LA-Γ-semigroup over U since $x\gamma y \in F(a)$. The proofs of the other situations have analogues processes via related definitions.

Theorem 3.33. *Let U be an LA-Γ-semigroup and A be a family $\left\{(f_i, E_i) \,\middle|\, i \in I\right\}$ of fuzzy soft multi sets over U and let $\bigcap_{i \in I} E_i \neq \emptyset$. If A is a family of fuzzy soft multi LA-Γ-semigroups (LA-Γ-right (left) ideals, (generalized) LA-Γ-bi-ideals, LA-Γ-interior ideals, respectively) over U, then restricted and extended intersection and fuzzy $\wedge$-intersection of the family are fuzzy soft multi LA-Γ-semigroups (LA-Γ-right (left) ideals, (generalized) LA-Γ-bi-ideals, LA-Γ-interior ideals, respectively) over U.*

Proof. The proof has analogues processes with the proof of Theorem 3.15 in which include the definitions in Definition 3.29.

4. Conclusions

In this paper we apply the concept of fuzzy soft multisets to the LA-Γ-semigroups as an algebraic structure by introducing a concept of the fuzzy soft multi LA-Γ-semigroups. We also generalize some operations on the soft and fuzzy soft multisets and, study their properties. It would be an interesting topic to study fuzzy soft multisets on algebraic sets. Our further work is to study on groups by the extension of the unit interval to the complete lattices. In future one can think of the algebraic nature of some extensions of fuzzy soft multisets such as intuitionistic fuzzy soft multisets and thus still extend it. We hope that our work would help advanced on researchers whose interested in this topics.

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